New Concept of Statistical Ensembles

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1. Statistical Ensembles. Textbook Examples.
2. Generalization.
3. New Example: MCE/sVF.

1. ***Statistical Ensembles with Fluctuating Extensive Quantities***
   Gorenstein and Hauer, Phys. Rev. C 78, 041902 (2008)

2. ***Statistical Ensembles with Volume fluctuations***
   Gorenstein, J. Phys. G 25, 125102 (2008)

3. ***Power Law in Micro-Canonical Ensemble with scaling Volume Fluctuations***
   Begun, Gazdzicki, Gorenstein, Phys. Rev. C 78, 024904 (2008)

4. ***Semi-Inclusive Observables in Statistical Models***
   Begun, Gazdzicki, Gorenstein, Phys. Rev. C 80, 064903 (2009)

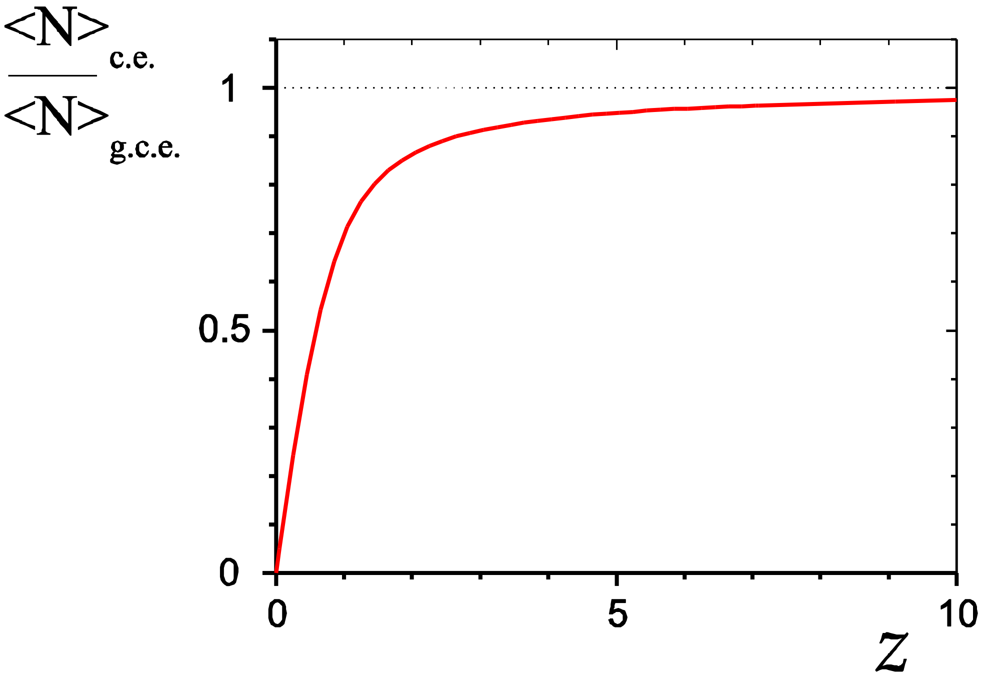
$$Z_{gce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

$$z = \frac{V}{2\pi^2} T m^2 K_2(m/T)$$

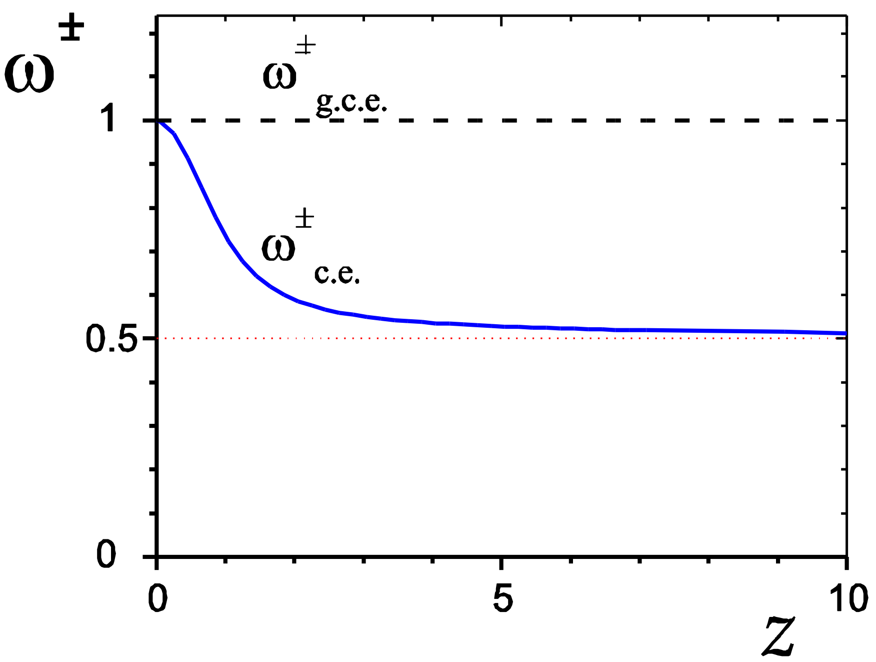
$$Z_{ce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2 \rangle - \langle N_- \rangle^2}{\langle N_- \rangle}, \qquad \langle N_- \rangle_{gce} = z , \qquad \omega^-_{gce} = 1$$

$$\langle N_- \rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \qquad \omega^-_{ce} = 1 - z \left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)} \right]$$

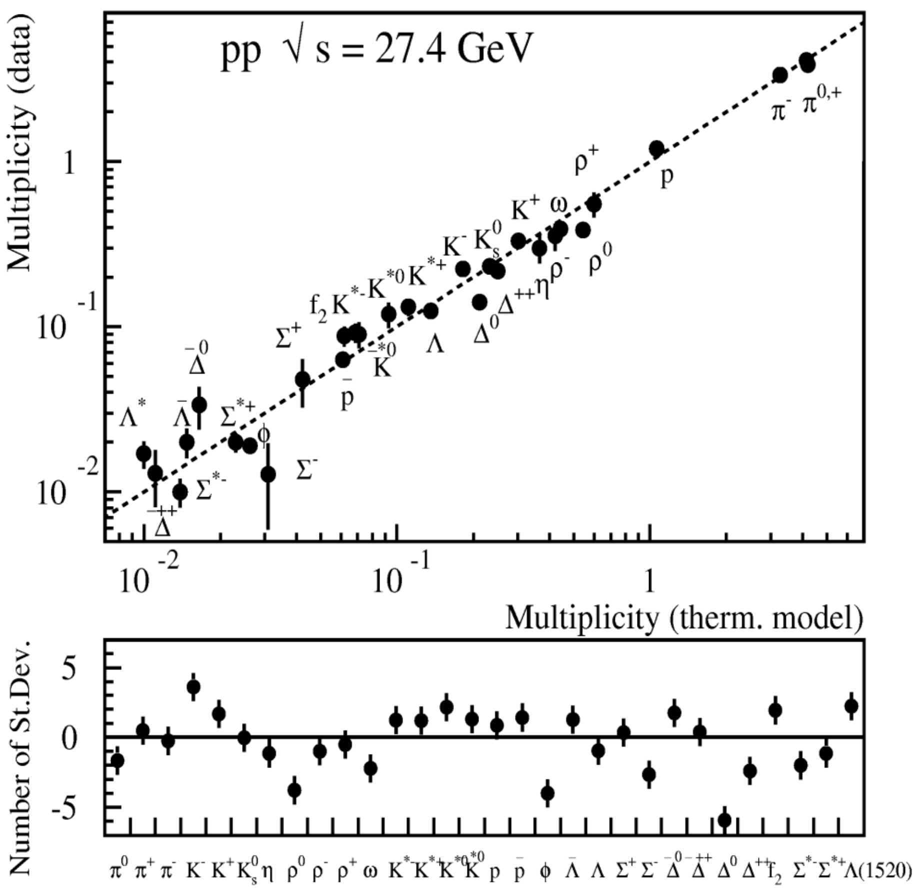


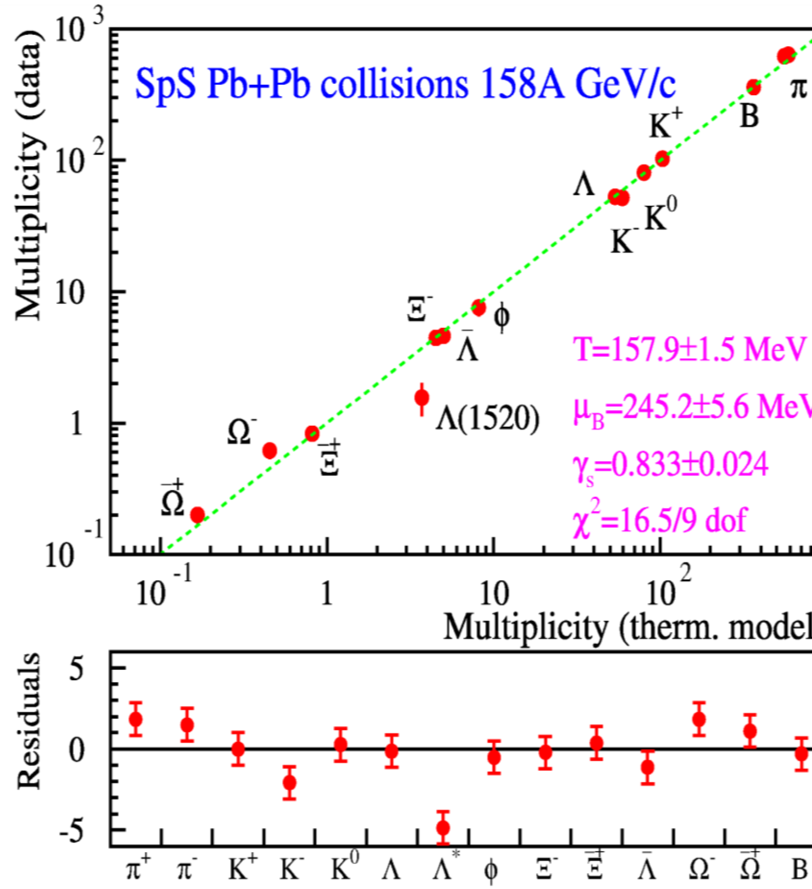
<N>c.e.
<N>g.c.e.
1
0.5
0
0
5
10
z



ω±
ω±g.c.e.
ω±c.e.
1
0.5
0
0
5
10
z

Mean Multiplicities





Becattini, Heinz, Z. Phys. (1997) CE in p+p

Becattini, Manninen, Gazdzicki,
Phys. Rev. C (2006) GCE in A+A

Gorenstein., Gazdzicki, Greiner,
Phys. Lett. B (2000) **CE** for antibaryons in p+A

Gorenstein, Kostyuk, Stoecker, Greiner,
Phys. Lett. B (2001) **CE** for charmed hadrons

Momentum Spectra

Becattini, Passaleva,

Eur. Phys. J. (2002),

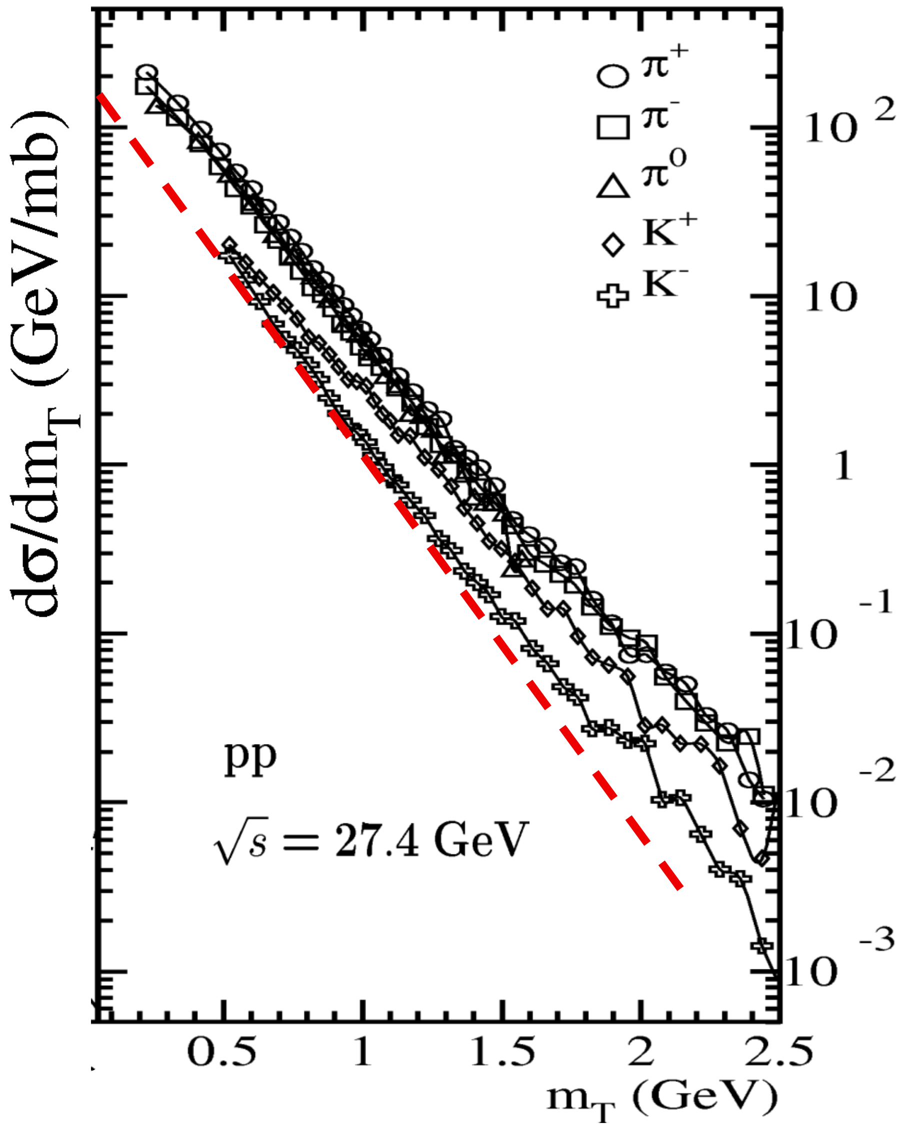
data from

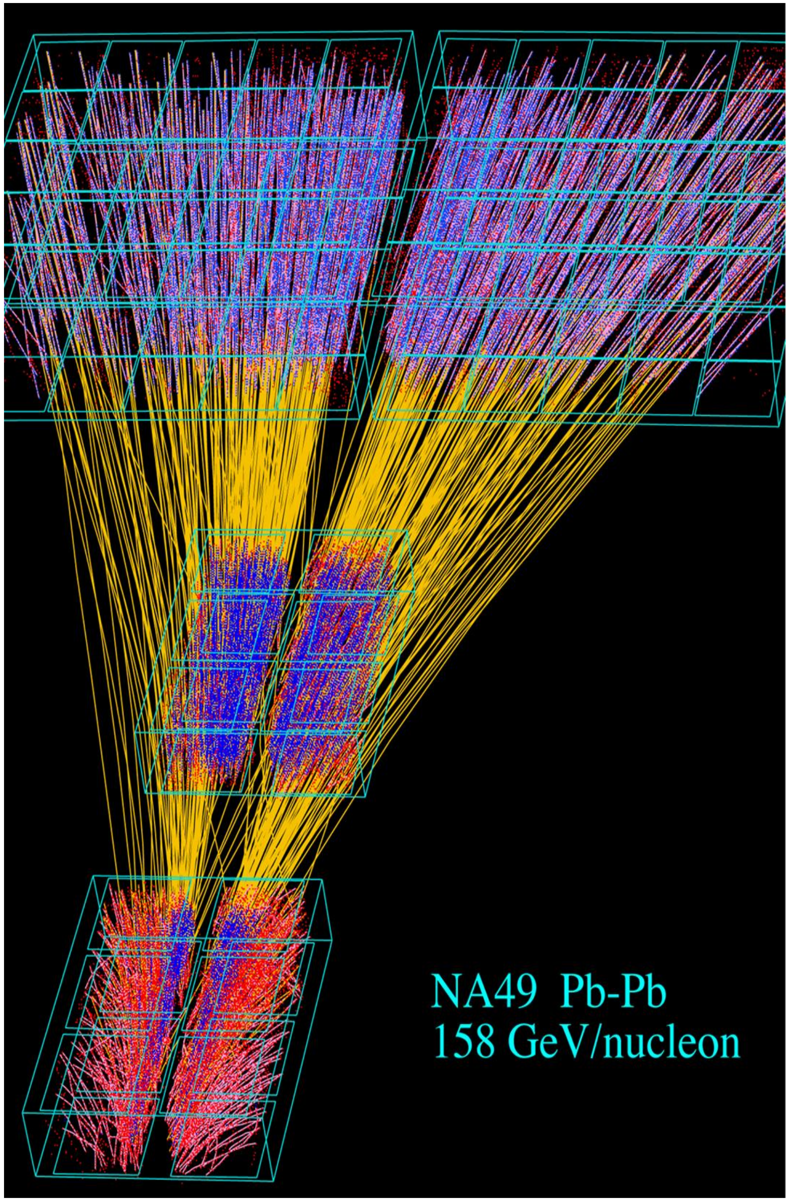
Aguilar-Benitez et al.,

Z. Phys. C (1990)

$$\boxed{\exp\left(-\frac{\mathrm{m_T}}{\mathrm{T}}\right)}$$

$$\mathrm{m_T} = \sqrt{p_T^2 + m^2}$$





$$N = 10^2 \div 10^4$$

$$P(N)\ , \qquad \langle N^k \rangle = \sum_N N^k \, P(N)$$

$$Var(N) = \langle N^2 \rangle - \langle N \rangle^2$$
$$= \langle (N - \langle N \rangle)^2 \rangle = \langle (\Delta N)^2 \rangle$$

$$\omega = \frac{Var(N)}{\langle N \rangle}$$

Scaled Variances are not equal to each other in different SE

Statistical Ensembles E, V, Q

$E \Longleftrightarrow T$

$V \Longleftrightarrow p$

$Q \Longleftrightarrow \mu_Q$

$2^3 = 8$

E, V, Q **MCE**

T, V, Q **CE**

T, V, μ_Q **GCE**

E, V, μ_Q MGCE

Gorenstein,
J. Phys. G (2008)

$$\left.\begin{array}{ll} E, p, Q & T, p, Q \\ T, p, \mu_Q & E, p, \mu_Q \end{array}\right\}$$

Pressure Ensembles

$$\vec{A} = (E, V, Q_1, \ldots, Q_k)$$

Alpha-Enesmble

$$P_\alpha(X) = \int d\vec{A} \; P_\alpha(\vec{A}) \; P_{mce}(X; \vec{A})$$

Gorenstein and Hauer, Phys. Rev. C (2008)

Problems of the statistical approach

I. Multiplicity distribution in $e^+e^-,\ pp,\ p\bar{p}$

II. Power law at high p_t and high m

$$\frac{d^3N_i}{dp^3} \sim C\ p_T^{-K_p} \qquad p_T \gg m_i$$

$$\langle N_i \rangle \sim C\ m_i^{-K_m} \qquad K_p \approx 8$$

$$K_m \approx K_p - 3$$

I. Multiplicity distribution

KNO scaling & Large fluctuations

Data:

$$\mathrm{P(N)} = \frac{1}{\langle \mathrm{N} \rangle} \, \Psi_\alpha\left(\frac{\mathrm{N}}{\langle \mathrm{N} \rangle}\right) ,$$

Koba, Nielsen, Olesen, Nucl. Phys. B (1972)

$$\omega \equiv \frac{\langle \mathrm{N}^2 \rangle - \langle \mathrm{N} \rangle^2}{\langle \mathrm{N} \rangle} \propto \langle \mathrm{N} \rangle$$

Statistical Models:

$$\mathrm{P(N)} \cong \frac{1}{\sqrt{2\pi\,\omega\,\langle N \rangle}} \exp\left[-\frac{(\mathrm{N} - \langle \mathrm{N} \rangle)^2}{2\,\omega\,\langle \mathrm{N} \rangle}\right] ,$$

$$\omega \approx \mathrm{const} \approx 1$$

Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$\boxed{\mathbf{P}_{\alpha}(\mathbf{X};\mathbf{E}) \;=\; \int_{0}^{\infty} \mathbf{dV}\;\mathbf{P}_{\alpha}(\mathbf{V})\;\mathbf{P}_{\mathbf{mce}}(\mathbf{X};\mathbf{E},\mathbf{V})}$$

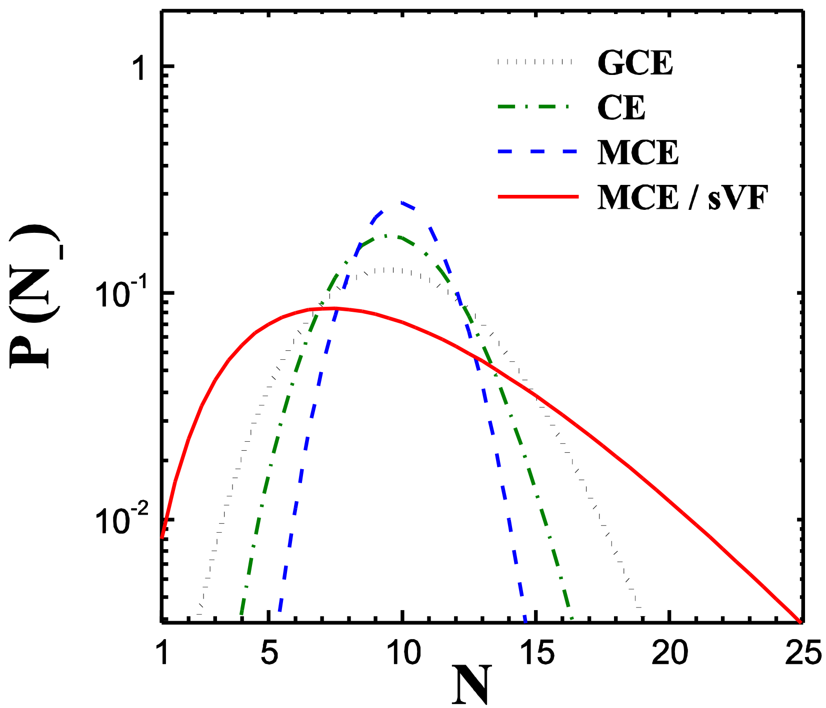
$$\mathbf{X} = \mathbf{N},\;\mathbf{p}$$

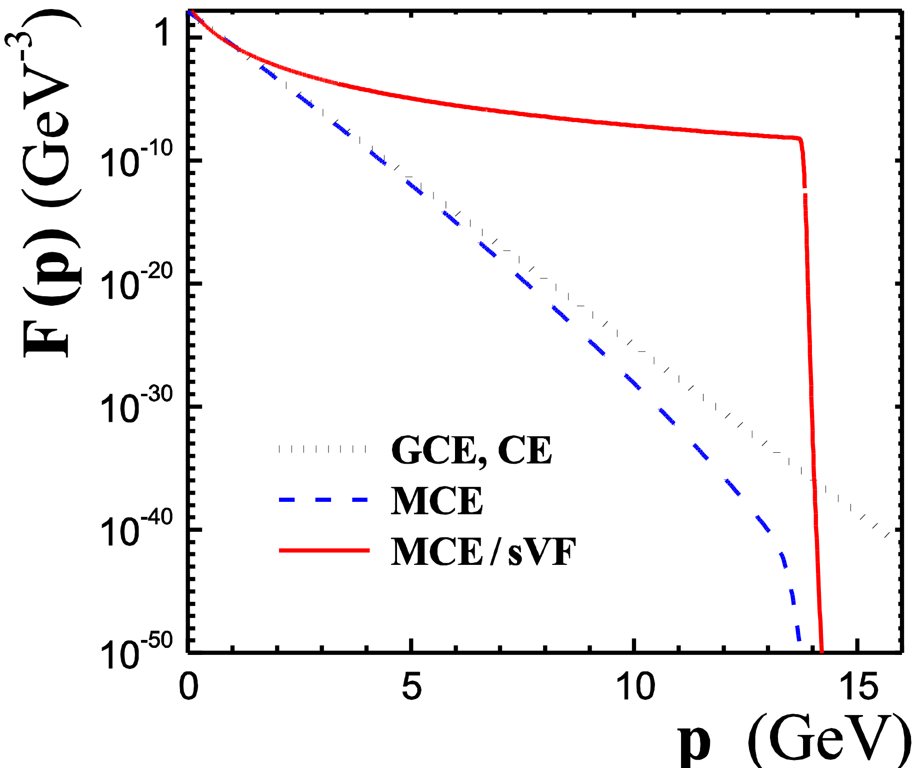
Begun, Gazdzicki, Gorenstein, Phys. Rev. C (2008)

$$\mathbf{P}_{\alpha}(\mathbf{V}) \;=\; \frac{1}{\overline{\mathbf{V}}}\,\Phi_{\alpha}(\mathbf{V}/\overline{\mathbf{V}})$$

Scaling volume fluctuations selected to fit experimental multiplicity distribution

Particle Number Distributions and Spectra



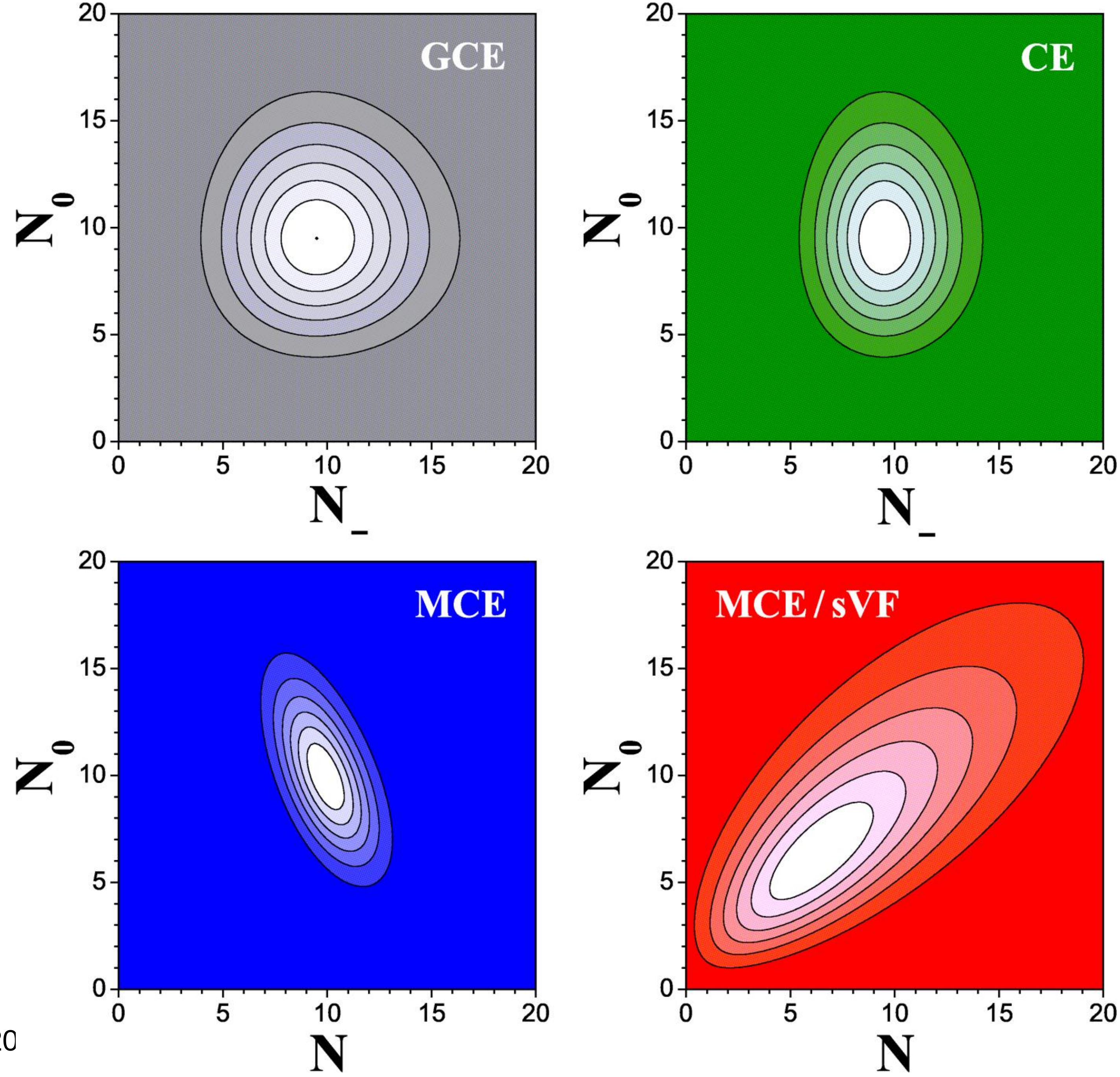


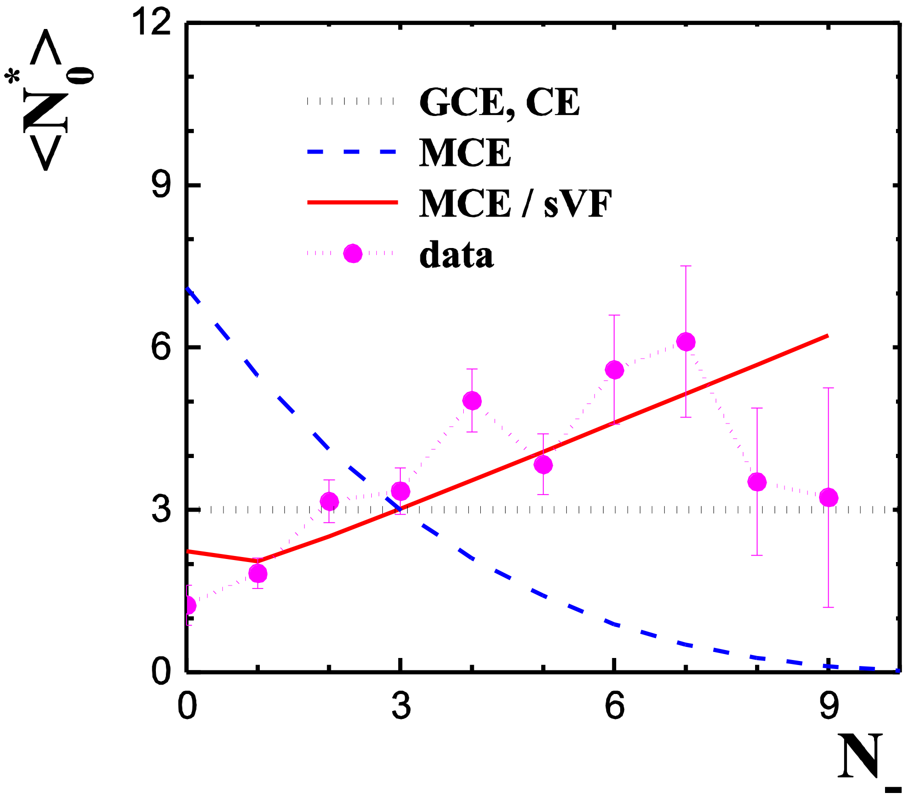
$$P_\alpha(N) = \frac{1}{\overline{\overline{N}}}\,\Psi_\alpha(N/\overline{N})$$

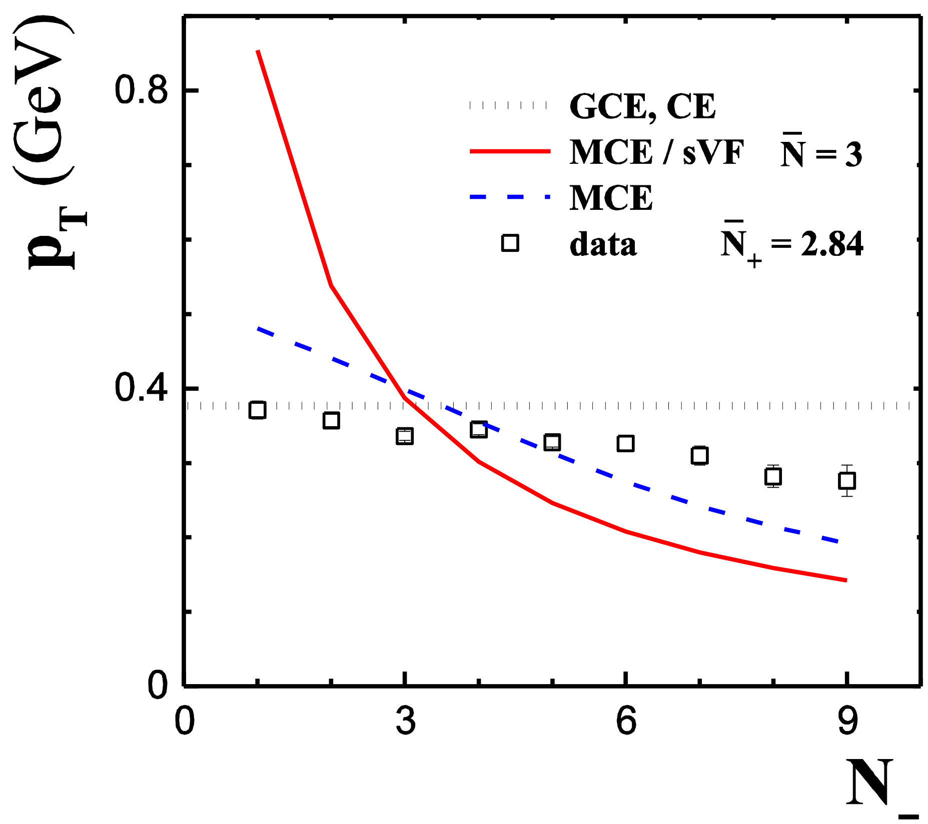
$$\Psi_\alpha(y) = \frac{k^k}{3!}\, y^{k-1}\, \exp(-k\,y)$$

$$F_\alpha(p) \cong \frac{k^k\Gamma(k+4)}{2\Gamma(k)}\, T^{k+1}\,(p+kT)^{-k-4}$$

$$\cong 11.27\ \mathrm{GeV}^5\,(p + 4T)^{-8}$$


GCE
CE
MCE
MCE / sVF
N_0
N_
20
15
10
5
0
0
5
10
15
20





Begun, Gazdzicki, Gorenstein, Phys. Rev. C (2009)

Summary

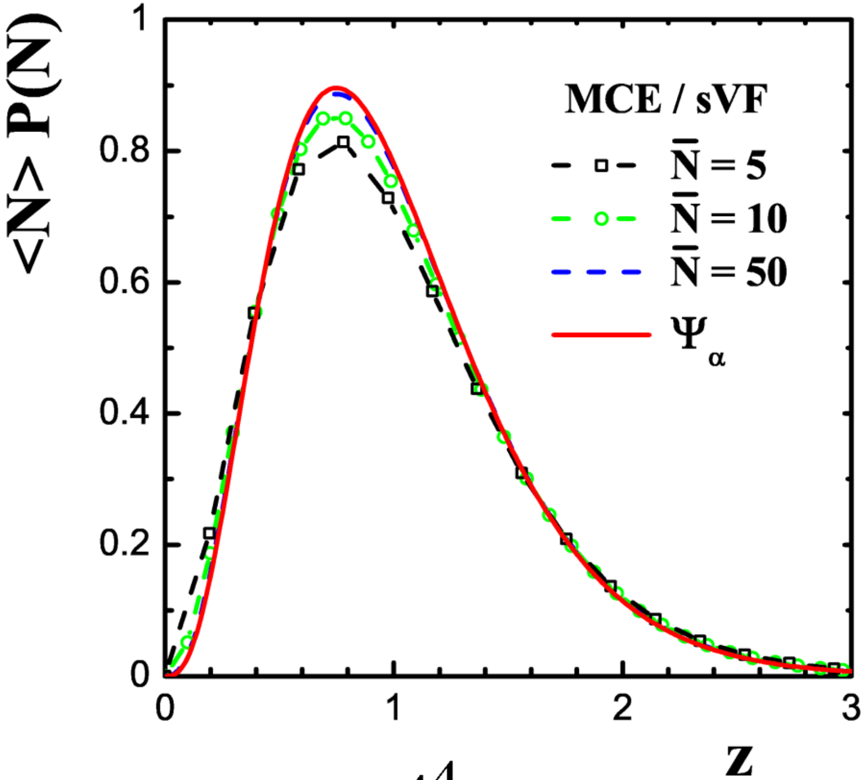
1. Statistical Ensembles with Fluctuating Extensive Quantities

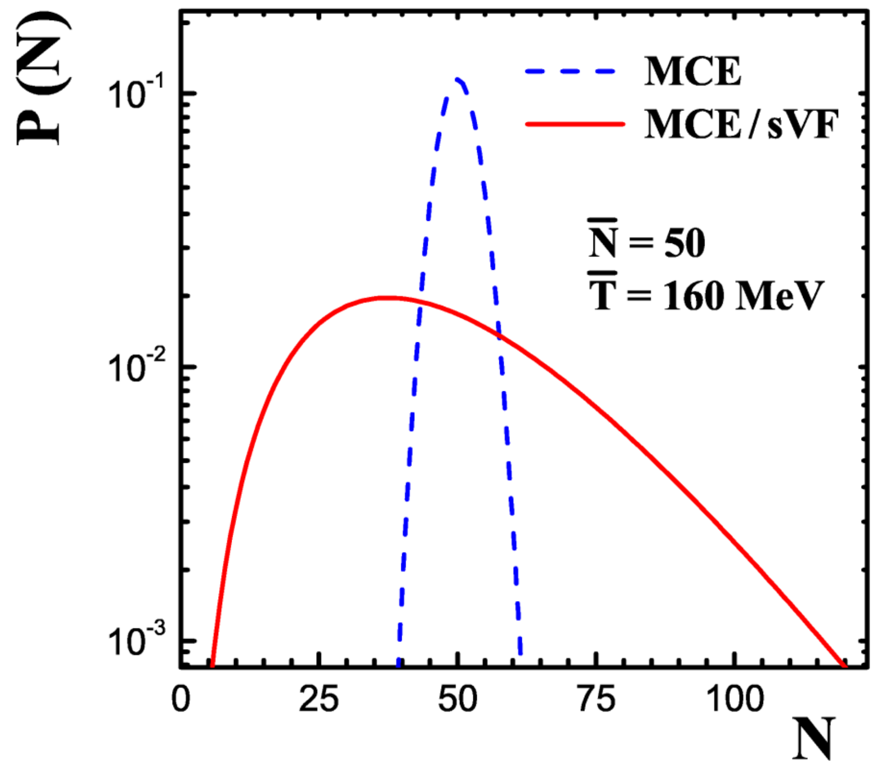
2. MCE/sVF
   a). Large Particle Number Fluctuations
   b). Power Law at Large Momenta

3. Semi-Inclusive Observables in Statistical Mechanics

Thank you!

Multiplicity distribution in the MCE/sVF





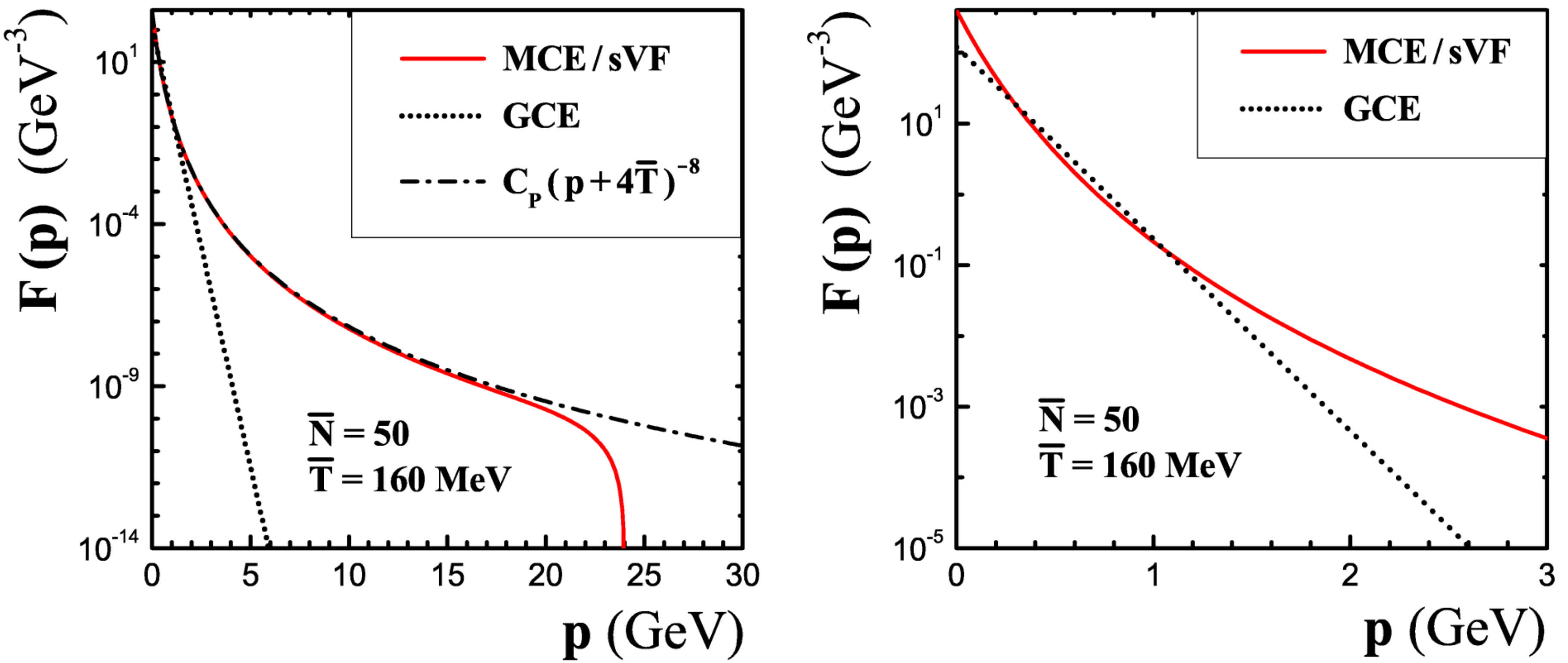
$$\Psi_\alpha(y) = \frac{4^4}{3!}\, y^3 \, \exp(-4\,y)$$

$$z = N/\langle N\rangle_\alpha\,, \quad y = (V/\overline{V})^{1/4}$$

$$\boxed{\omega_\alpha = \frac{\langle N^2\rangle_\alpha - \langle N\rangle_\alpha^2}{\langle N\rangle_\alpha} \cong \frac{1}{4}\,\langle N\rangle_\alpha}$$

Mrowczynski, Z. Phys. C (1985);
Gorenstein, Sov. J. Nucl. Phys. (1980); Yad. Fiz. (1980)

Power law in momentum spectrum



$$F_\alpha(p) = \frac{1}{\langle N \rangle_\alpha} \left\langle \frac{dN}{p^2 dp} \right\rangle_\alpha$$

Advantages of the statistical approach

$$\mathbf{A+A;} \quad \mathbf{e^+ + e^-,\ p+p,\ p+\bar{p}} \;\rightarrow\; \mathbf{hadrons}$$

I. Spectra

$$\boxed{\frac{\mathbf{dN_i}}{\mathbf{p^2dp}} \sim \mathbf{V}\ \exp\left(-\frac{\sqrt{p_T^2+m_i^2}}{\mathbf{T}}\right), \quad \mathbf{T \simeq 160\,MeV}}$$

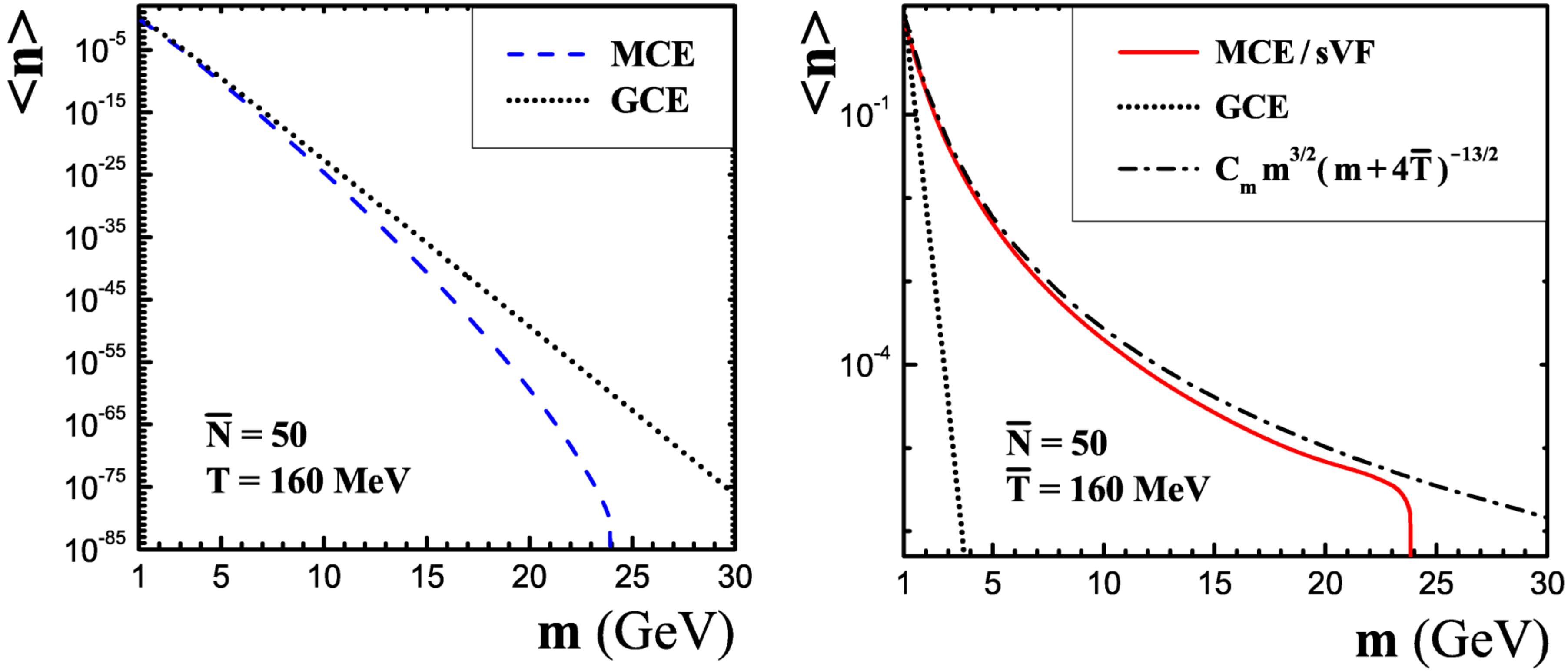
$$\mathbf{p_T \gg m} \qquad \frac{\mathbf{dN_i}}{\mathbf{p^2dp}} \sim \mathbf{V}\ \exp\left(-\frac{\mathbf{p_T}}{\mathbf{T}}\right)$$

II. Mean multiplicity

$$\mathbf{m_i \gg T} \qquad \langle \mathbf{N_i} \rangle \sim \mathbf{V}\ (\mathbf{m_iT})^{3/2}\ \exp\left(-\frac{\mathbf{m_i}}{\mathbf{T}}\right)$$

$$\pi^{\pm},\ \mathbf{K}^{\pm},\ \phi,\ \mathbf{\Lambda},\ \overline{\mathbf{\Lambda}},\ \mathbf{\Xi}^{\pm},\ \mathbf{\Omega},\ \overline{\mathbf{\Omega}},\ \mathbf{p},\ \bar{\mathbf{p}}$$

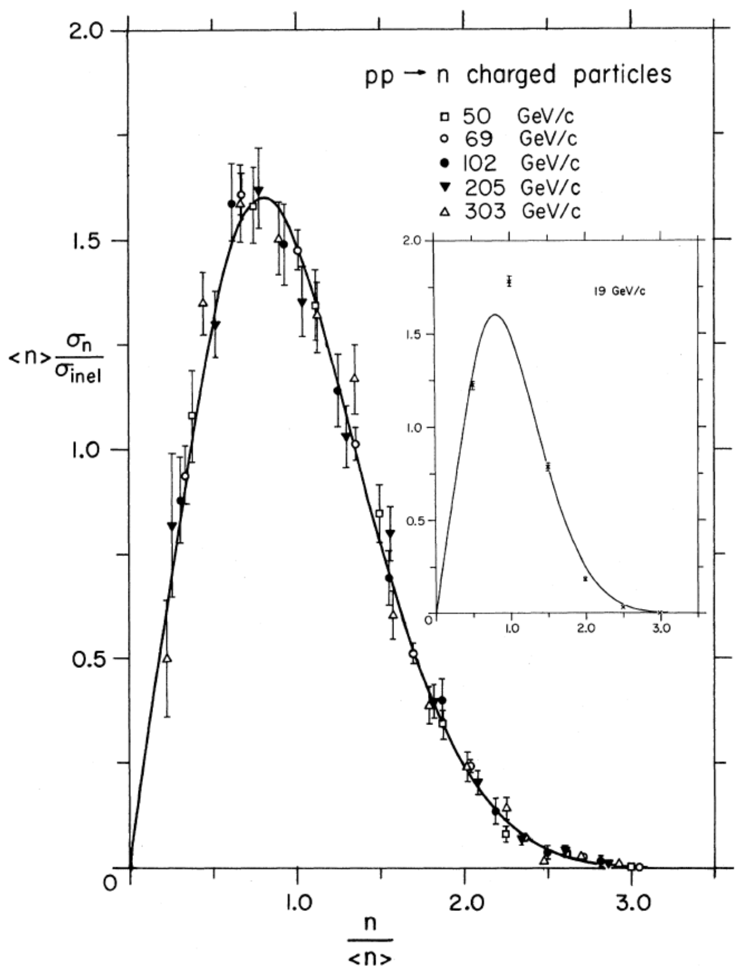
Power law in mass dependence

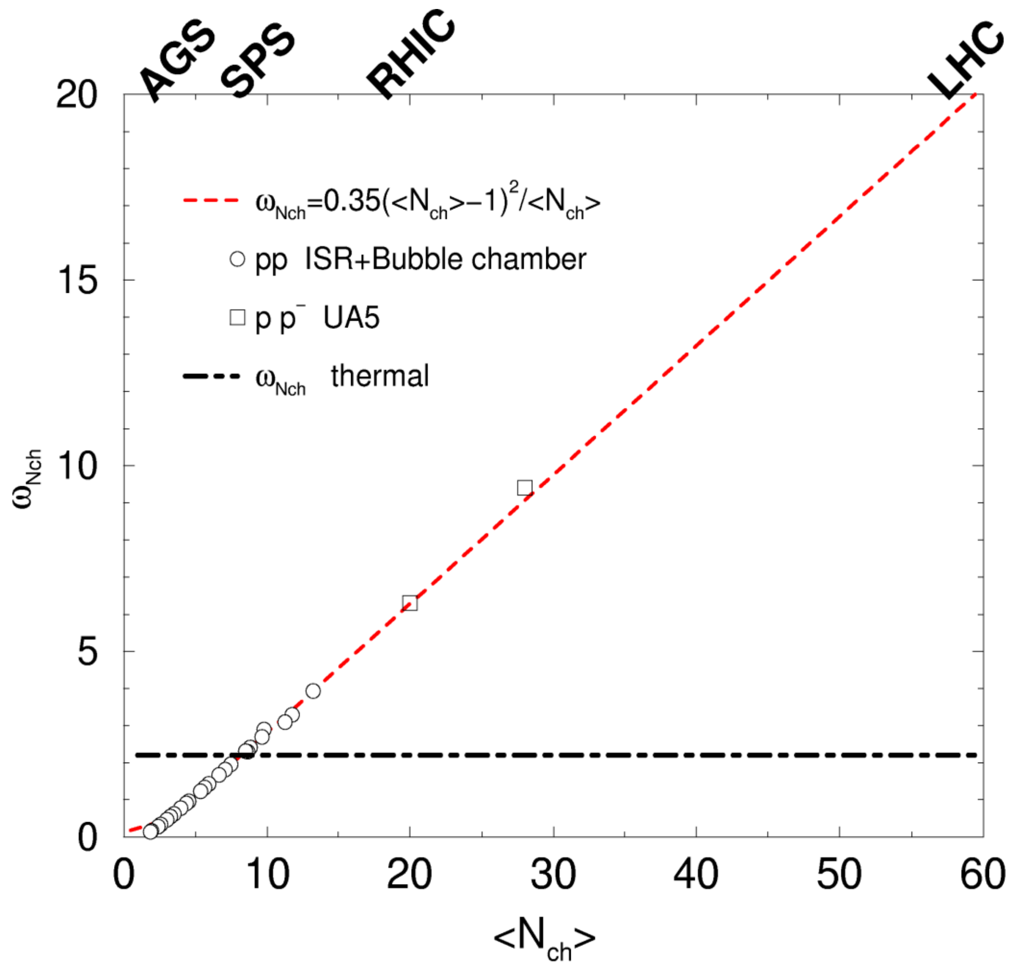


$$\bar{n} \;=\; V\;(m\,T/2\pi)^{3/2}\;\exp\left(-\,\frac{m}{T}\right)$$

$$\langle n\rangle_\alpha \;\propto\; m^{-5}$$

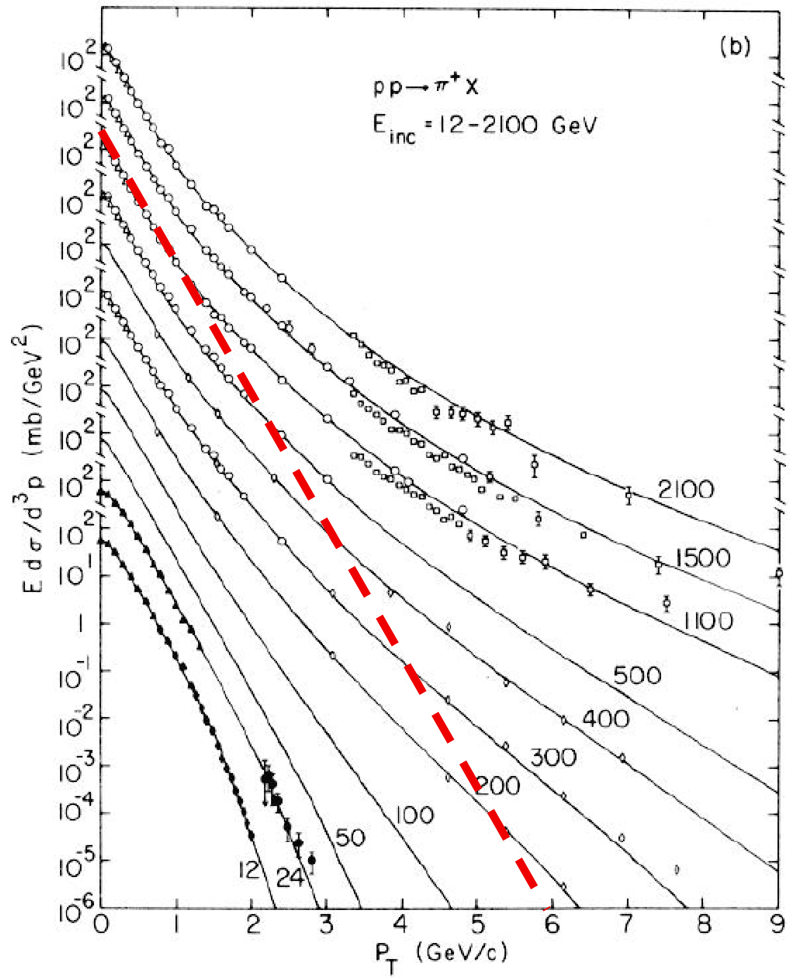
KNO scaling & Large fluctuations



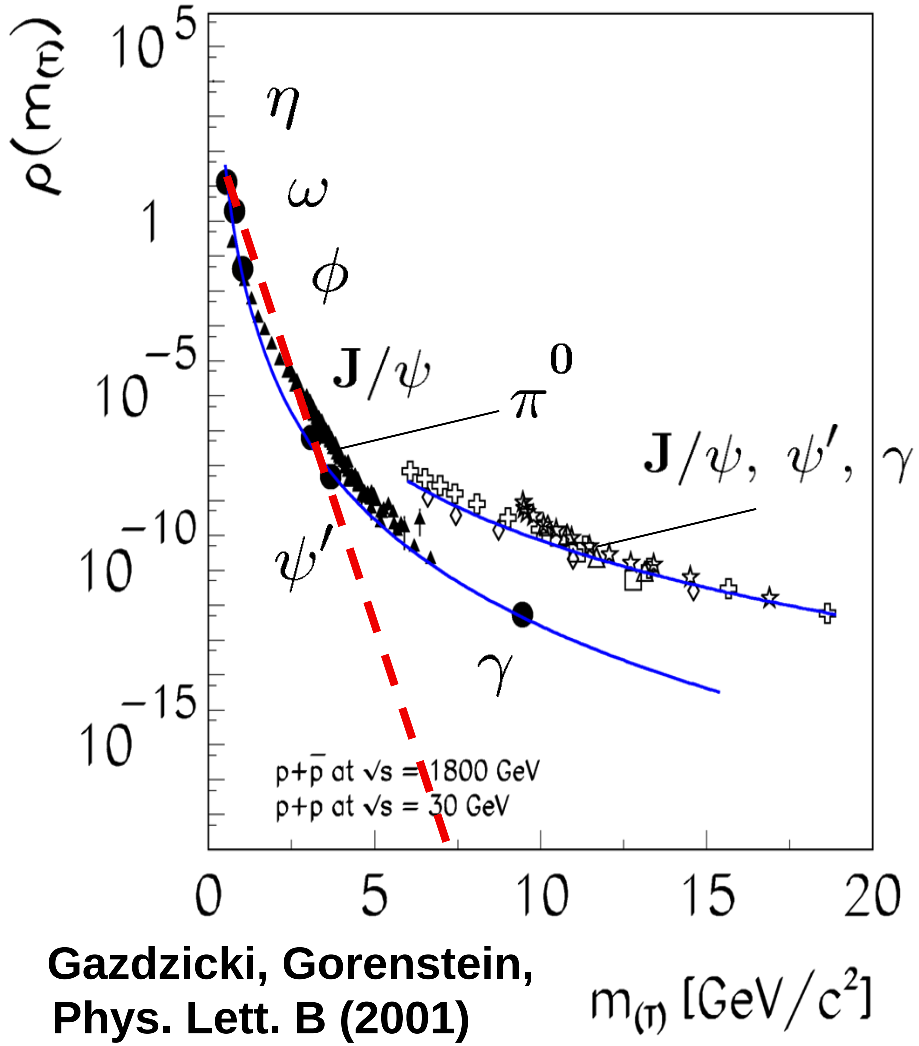


Slattery, Phys. Rev. Lett. (1972); Heiselberg, Phys. Rept. (2001)

Power law at high p_t and high m



Beier *et al.*, Phys. Rev. D (1978)



Gazdzicki, Gorenstein, Phys. Lett. B (2001)

II. Power law at high p_t and high m

$\mathrm{p} \geq 3\ \mathrm{GeV}$ $$\frac{d^3N_i}{dp^3} \sim C \left(\sqrt{p_T^2 + m_i^2}\right)^{-K_p}$$

$$K_p \approx 8$$

$$\boxed{\begin{array}{l} \dfrac{d^3N_i}{dp^3} \sim C\ p_T^{-K_p} \\[2em] \langle N_i \rangle \sim C\ m_i^{-K_m} \end{array}}$$

$p_T \gg m_i$

$m_i \geq 3\ \mathrm{GeV}$

$K_m = K_p - 3$ **Gazdzicki, Gorenstein, Phys. Lett. B (2001)**

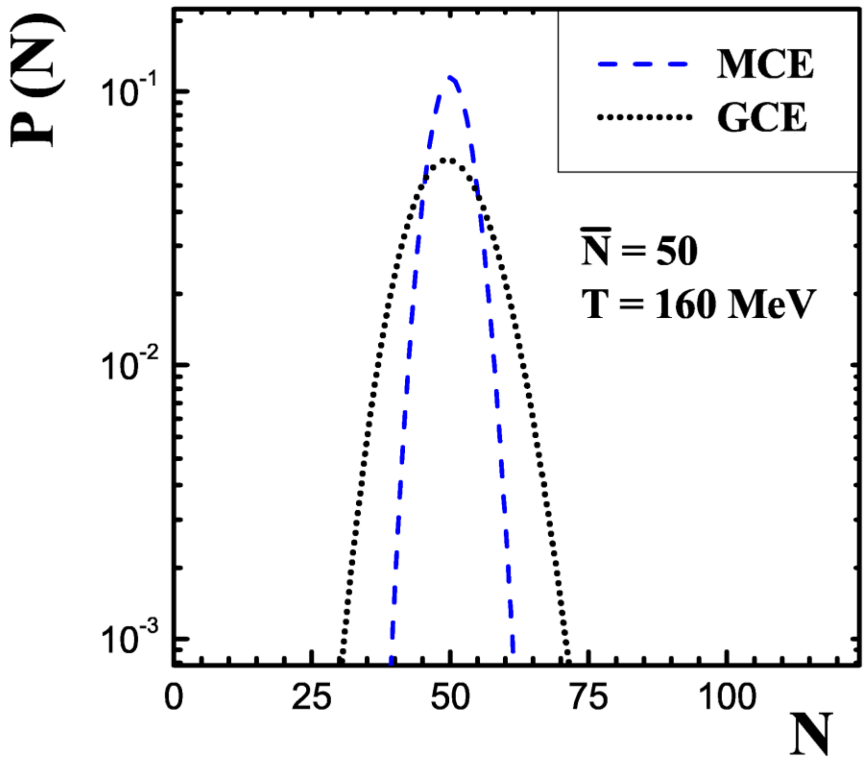
Micro-Canonical Ensemble of massless neutral particles

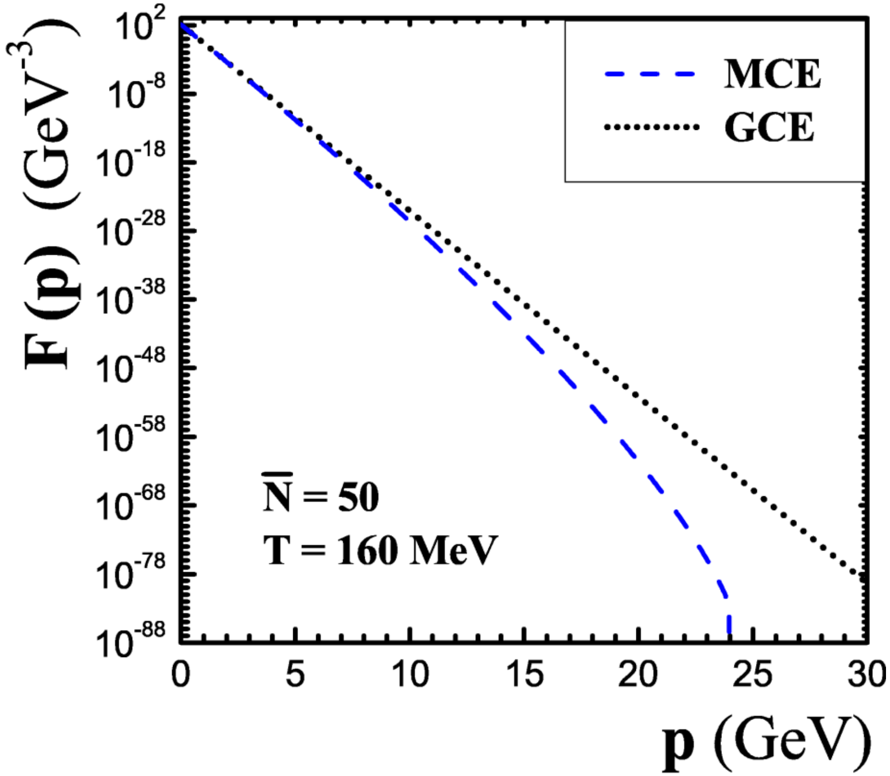
$$\mathrm{W_N(E,V)}$$
$$= \frac{1}{\mathrm{N!}}\left(\frac{\mathrm{V}}{2\pi^2}\right)^{\mathrm{N}} \int_0^\infty \mathrm{p_1^2 dp_1} \ldots \int_0^\infty \mathrm{p_N^2 dp_N}\ \delta(\mathrm{E} - \sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{p_i})\Big)$$

$$\omega_{\mathrm{mce}} \equiv \frac{\langle \mathrm{N}^2\rangle - \langle \mathrm{N}\rangle^2}{\langle \mathrm{N}\rangle} = \mathbf{1/4}$$

Begun, Gorenstein, Kostyuk, Zozulya, Phys. Rev. C (2005)

$$\langle \mathrm{N^k}\rangle = \sum_{\mathrm{N}} \mathrm{N^k\, P_{mce}(N;E,V)} \qquad \mathrm{P_{mce}(N;E,V)} = \frac{\mathrm{W_N(E,V)}}{\sum_{\mathrm{N}} \mathrm{W_N(E,V)}}$$





$$\mathbf{P(N)} \;\cong\; \frac{1}{\sqrt{2\pi\cdot\omega\cdot\langle N\rangle}}\exp\left[-\frac{(\mathbf{N}-\langle\mathbf{N}\rangle)^2}{2\,\omega\cdot\langle\mathbf{N}\rangle}\right], \qquad \omega_{\mathbf{gce}} = 1$$

$$\boxed{\omega_{\mathbf{mce}} = 1/4}$$

$$\mathbf{F(p)} \;\propto\; \exp\left(-\frac{\mathbf{p}}{\mathbf{T}}\right)$$

$$\boxed{\mathrm{E} = 3\,\overline{\mathrm{N}}\,\mathrm{T}}$$