One-Loop Angularity Distributions with Recoil using SCET

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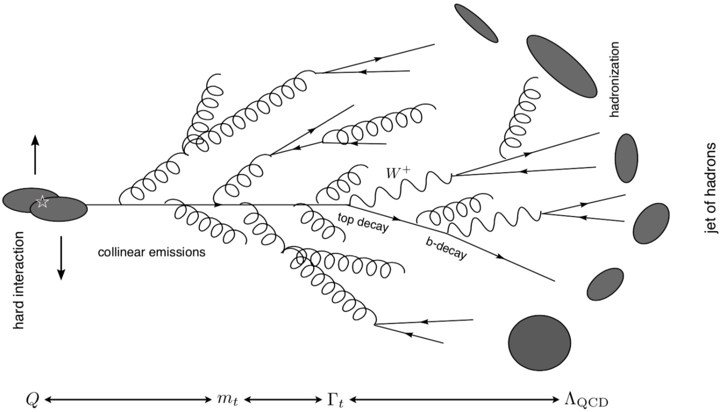
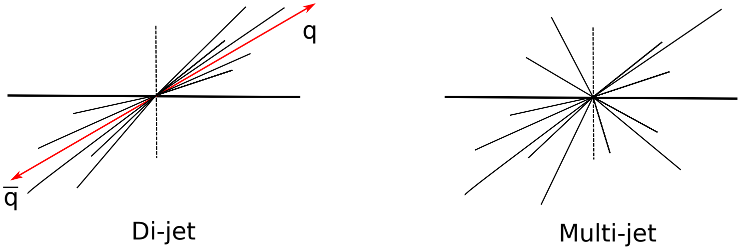


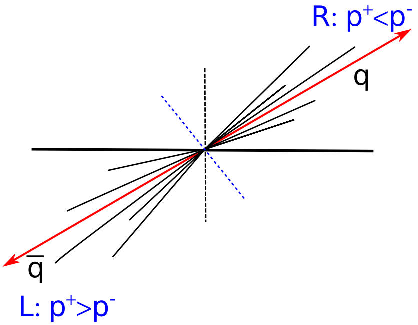
Figure: An energetic top quark evolving into a jet of hadrons. The various scales represent the intermediate stages of the jet evolution.

Image courtesy: Dr. Ambar Jain



- **Thrust–** $\tau \equiv 1 - \mathrm{T} = 1 - \max_{\hat{n}} \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_i \cdot \hat{n}|$
  - Provides a measure of jet-likeness of the final state.
  - $\tau \to 0 \Rightarrow$ back-to-back collimated jets, $\tau \sim 0.5$ for spherically distributed final state particles.
  - Thrust axis, $\hat{n}$, provides a natural way of dividing the event into hemispheres.
- **Jet Broadening–** $\mathrm{B} = \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_\perp^{\,i}|$
  - Studies the transverse spread in a jet.
  - $p_\perp$ measured relative to $\hat{n}$.

$$p^{\pm} = E \mp p_z$$



Thrust: $\tau = \frac{1}{Q}\left[\sum_{i\epsilon L} |p_i^-| + \sum_{i\epsilon R} |p_i^+|\right]$

Broadening: $\mathrm{B} = \frac{1}{Q}\left[\sum_{i\epsilon L}\sqrt{p_i^+ p_i^-} + \sum_{i\epsilon R}\sqrt{p_i^- p_i^+}\right]$

$$\textbf{Thrust: } \tau = \frac{1}{Q}\left[\sum_{i\epsilon L} |p_i^-| + \sum_{i\epsilon R} |p_i^+|\right]$$

$$\textbf{Broadening: } \mathrm{B} = \frac{1}{Q}\left[\sum_{i\epsilon L}\sqrt{p_i^+ p_i^-} + \sum_{i\epsilon R}\sqrt{p_i^- p_i^+}\right]$$

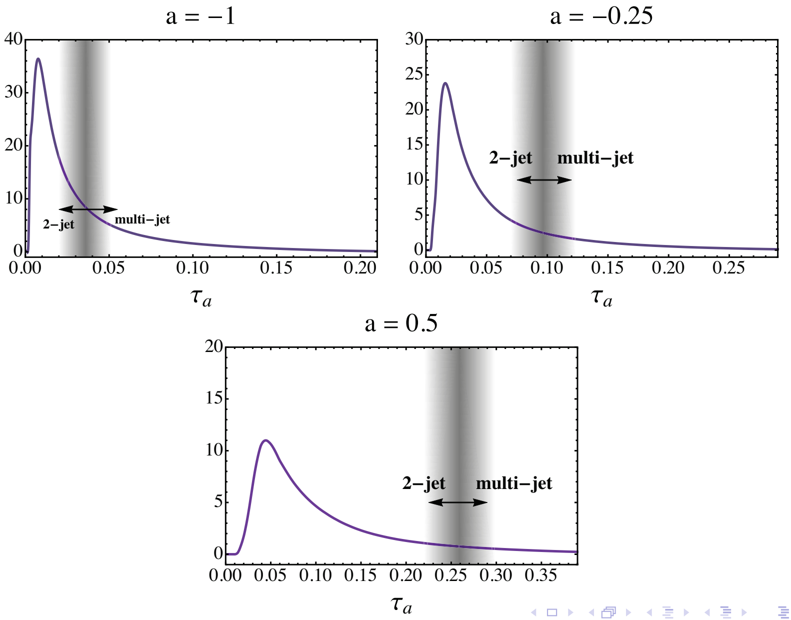
- **Berger, Kucs, Sterman, 03**

$$\tau_b = \frac{1}{Q}\left[\sum_{i\epsilon L}(p_i^+)^{\frac{1-b}{2}}(p_i^-)^{\frac{1+b}{2}} + \sum_{i\epsilon R}(p_i^+)^{\frac{1+b}{2}}(p_i^-)^{\frac{1-b}{2}}\right] \tag{1}$$

- For Infrared safety : $-1 < b < \infty$.
- Generalization to *'thrust'* ($b = 1$) and jet *'broadening'* ($b = 0$).
- Varying *'b'* changes the sensitivity of the observable to the substructure of the jet.

a=1-b

arXiv:1808.07867v3



EFT modes

$$p^\mu \equiv (p^+, p^-, p_\perp)$$

Thrust:

- $\tau = \frac{1}{Q}\left[\sum_{i\epsilon L}|p_i^-| + \sum_{i\epsilon R}|p_i^+|\right]$
- On-shell $\Rightarrow p^+ p^- = p_\perp^2$
- $p_i^+ \sim \tau \quad \forall\ i\,\epsilon\,R$

  & $p_i^- \sim \tau \quad \forall\ i\,\epsilon\,L$
- $\lambda \sim \sqrt{\tau}$

$$\Rightarrow p_{\bar{n}}(p^- > p_\perp > p^+) \sim Q(\lambda^2, 1, \lambda),$$
$$p_n(p^- < p_\perp < p^+) \sim Q(1, \lambda^2, \lambda),$$
$$p_s(p^- \sim p_\perp \sim p^+) \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

SCET$_{\rm I}$

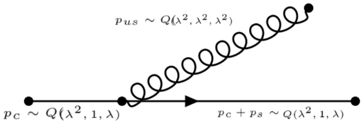
Broadening:

- $\mathrm{B} = \frac{1}{Q}\left[\sum_{i\epsilon L}\sqrt{p_i^+ p_i^-} + \sum_{i\epsilon R}\sqrt{p_i^- p_i^+}\right]$
- On-shell $\Rightarrow p^+ p^- = p_\perp^2$
- $p_i^\perp \sim \mathrm{B} \quad \forall\ i\,\epsilon\,L, R$
- $\lambda \sim \mathrm{B}$

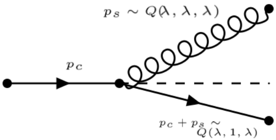
$$\Rightarrow p_{\bar{n}}(p^- > p_\perp > p^+) \sim Q(\lambda^2, 1, \lambda),$$
$$p_n(p^- < p_\perp < p^+) \sim Q(1, \lambda^2, \lambda),$$
$$p_s(p^- \sim p_\perp \sim p^+) \sim Q(\lambda, \lambda, \lambda)$$

SCET$_{\rm II}$

- **Thrust:**

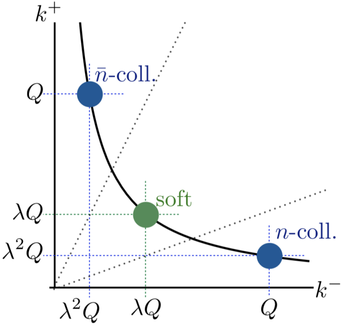


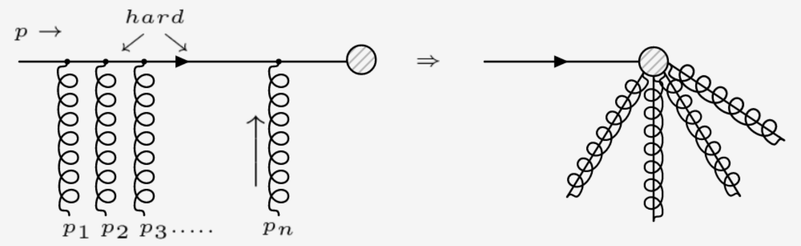
- **Broadening:**



Jet Angularities are novel observables that allow us to transform between recoil-insensitive to recoil-sensitive observables in a continuous manner making a unified framework particularly challenging.

- Appears when $k^+ k^-$ is fixed but the ratio k^+/k^- diverges.
- Dimensional regularization does not suffice.
- Need a regulator that essentially breaks boost-invariance.
- No rapidity divergence in the full theory.





$$W_n = \Big[\sum_{perm} \exp\Big(-g\frac{\bar{n}\cdot A_n}{\bar{n}\cdot p}\Big)\Big]$$

⇒ For regulating the divergence,

$$W_n = \Big[\sum_{perm} \exp\Big(-\frac{g}{\bar{n}\cdot p} w \frac{|\bar{n}\cdot p|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\Big)\Big] \quad (2)$$

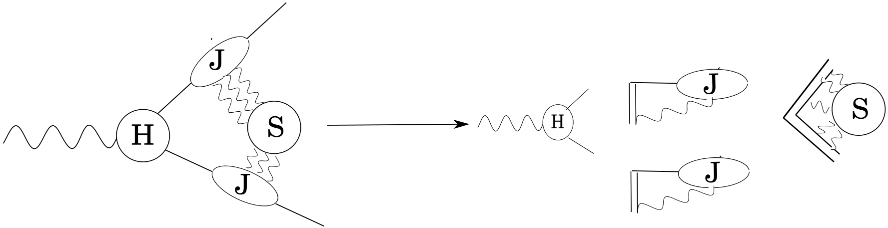


Figure: Factorization of the hard scattering process into individual hard, jet and soft functions.

$$\mathcal{L}^{(0)}_{SCET} = [\mathcal{L}_{\text{n-coll}}]_{\mathcal{J}} + [\mathcal{L}_{\bar{n}\text{-coll}}]_{\mathcal{J}} + [\mathcal{L}_{\text{soft}}]_{\mathcal{S}} + \text{power-correc}^{s}$$
$$d\sigma = Hard \cdot \mathcal{J}_n \otimes \mathcal{J}_{\bar{n}} \otimes \mathcal{S} \tag{3}$$

- Factorization properties of QCD in the soft/collinear limit allows for the separation of the process into hard, jet and soft sectors.
- All factorized sectors depend only on a single dynamical scale and the scale of factorization.

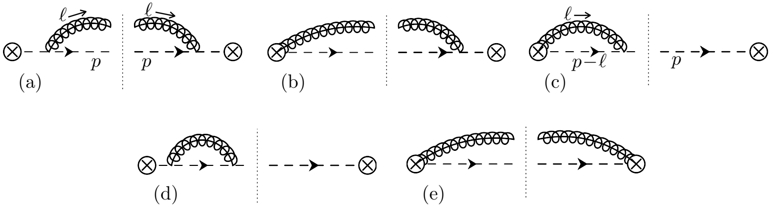
Factorization of angularities

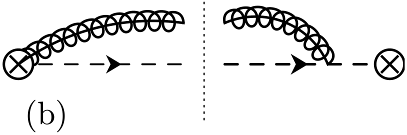
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_L \mathrm{d}\tau_R} = H(Q;\mu)\int \mathrm{d}\tau_n\,\mathrm{d}\tau_{\bar{n}}\,\mathrm{d}\tau_n^s\,\mathrm{d}\tau_{\bar{n}}^s\,\delta(\tau_R-\tau_n-\tau_n^s)\,\delta(\tau_L-\tau_{\bar{n}}-\tau_{\bar{n}}^s)$$
$$\int \mathrm{d}\vec{p}_t^{\,2}\,\mathrm{d}\vec{k}_t^{\,2}\,\mathcal{J}(\tau_n,\vec{p}_t^{\,2};\mu,\nu)\,\mathcal{J}(\tau_{\bar{n}},\vec{k}_t^{\,2};\mu,\nu)\,\mathcal{S}(\tau_n^s,\tau_{\bar{n}}^s,\vec{p}_t^{\,2},\vec{k}_t^{\,2};\mu,\nu)$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_L\mathrm{d}\tau_R} = H(Q;\mu)\int \mathrm{d}\tau_n\,\mathrm{d}\tau_{\bar{n}}\,\mathrm{d}\tau_n^s\,\mathrm{d}\tau_{\bar{n}}^s\,\delta(\tau_R-\tau_n-\tau_n^s)\,\delta(\tau_L-\tau_{\bar{n}}-\tau_{\bar{n}}^s)$$
$$\int \mathrm{d}\vec{p}_t^{\,2}\,\mathrm{d}\vec{k}_t^{\,2}\,\mathcal{J}(\tau_n,\vec{p}_t^{\,2};\mu,\nu)\,\mathcal{J}(\tau_{\bar{n}},\vec{k}_t^{\,2};\mu,\nu)\,\mathcal{S}(\tau_n^s,\tau_{\bar{n}}^s,\vec{p}_t^{\,2},\vec{k}_t^{\,2};\mu,\nu)$$

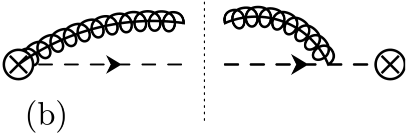
- The definition of the observable changes from broadening to more generic angularities.
- The scaling of the gluon fields in soft Wilson line changes to $A_s \sim Q(\lambda^{1+b},\lambda^{1+b},\lambda^{1+b})$ instead of $A_s \sim Q(\lambda,\lambda,\lambda)$ for jet broadening.

Angularity Jet function at one-loop





$$\mathcal{J}^{(1)}_{\mathrm{b,unsub}}(\tau_R,0) = \frac{2}{1+b}\,\frac{\alpha_s(\mu)C_F}{\pi}\,\frac{e^{\epsilon\gamma_E}w^2}{\Gamma(1-\epsilon)}\left(\frac{\mu}{Q}\right)^{2\epsilon}\left(\frac{\nu}{Q}\right)^{\eta}\frac{1}{\tau_R^{1+\frac{2\epsilon}{1+b}}}$$
$$\times\int_0^1 \mathrm{d}x\,\left[(1-x)^{-b}+x^{-b}\right]^{\frac{2\epsilon}{1+b}}\,\frac{(1-x)}{x^{1+\eta}}$$



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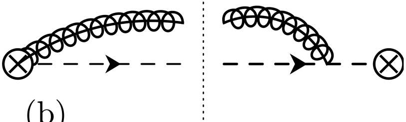
$$\times\int_0^1 \mathrm{d}x\,[(1-x)^{-b}+x^{-b}]^{\frac{2\epsilon}{1+b}}\,\frac{(1-x)}{x^{1+\eta}}$$

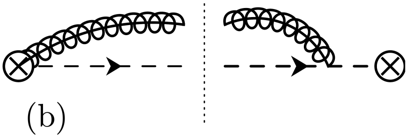
- For positive-b, $x \to 0$ already regulated by ϵ.

$$\mathcal{J}^{(1)}_{\mathrm{b,unsub}}(\tau_R,0) = \frac{2}{1+b}\,\frac{\alpha_s(\mu)C_F}{\pi}\,\frac{e^{\epsilon\gamma_E}w^2}{\Gamma(1-\epsilon)}\left(\frac{\mu}{Q}\right)^{2\epsilon}\frac{1}{\tau_R^{1+\frac{2\epsilon}{1+b}}}$$
$$\times\int_0^1 \mathrm{d}x\left[1+\left(\frac{x}{1-x}\right)^{b}\right]^{\frac{2\epsilon}{1+b}}\frac{(1-x)}{x^{1+\frac{2\epsilon\, b}{1+b}}}$$



(b)

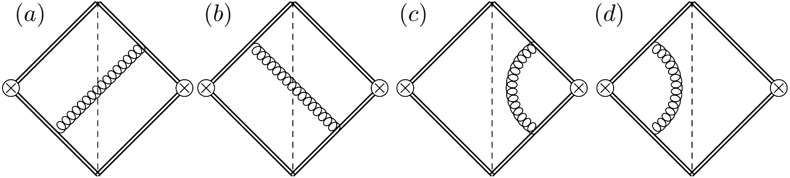
$$\mathcal{J}^{(1)}_{\rm b,unsub}(\tau_R, 0) = \frac{2}{1+b}\,\frac{\alpha_s(\mu) C_F}{\pi}\,\frac{e^{\epsilon\gamma_E} w^2}{\Gamma(1-\epsilon)}\left(\frac{\mu}{Q}\right)^{2\epsilon}\left(\frac{\nu}{Q}\right)^{\eta}\frac{1}{\tau_R^{1+\frac{2\epsilon}{1+b}}}$$
$$\times \int_0^1 \mathrm{d}x\,\left[(1-x)^{-b} + x^{-b}\right]^{\frac{2\epsilon}{1+b}}\,\frac{(1-x)}{x^{1+\eta}}$$



$$\mathcal{J}^{(1)}_{\rm b,unsub}(\tau_R,0) = \frac{2}{1+b}\,\frac{\alpha_s(\mu)C_F}{\pi}\,\frac{e^{\epsilon\gamma_E}w^2}{\Gamma(1-\epsilon)}\left(\frac{\mu}{Q}\right)^{2\epsilon}\left(\frac{\nu}{Q}\right)^{\eta}\frac{1}{\tau_R^{1+\frac{2\epsilon}{1+b}}}$$
$$\times\int_0^1 {\rm d}x\,[(1-x)^{-b}+x^{-b}]^{\frac{2\epsilon}{1+b}}\,\frac{(1-x)}{x^{1+\eta}}$$

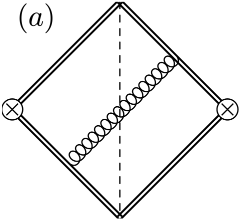
$$\mathcal{J}^{(1+)}_{\rm b,0}(\tau_R,0) = \frac{2}{1+b}\,\frac{\alpha_s(\mu)C_F}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\,w^2\left(\frac{\nu}{Q}\right)^{\eta}\left(\frac{\mu}{Q}\right)^{2\epsilon}\frac{1}{\tau_R^{1+\frac{2\epsilon}{1+b}}}\,\frac{\Gamma(\frac{\eta}{b})\,\Gamma(-\frac{2\epsilon}{1+b}-\frac{\eta}{b})}{b\,\Gamma(-\frac{2\epsilon}{1+b})}$$

Angularity soft function at one-loop



Angularity soft function at one-loop

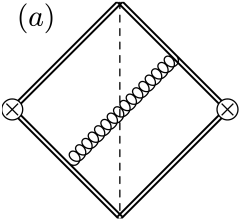
(a)



$$\mathcal{S}^{(1)}(\tau_L, \tau_R, \vec{p}_\perp^{\,2}, \vec{k}_\perp^{\,2}) = \frac{\alpha_s(\mu)\, C_F}{\pi} \frac{\mu^{2\epsilon} e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \nu^{\eta}\, w^2\, Q\, \delta(\tau_L)\, \delta(\vec{k}_\perp^{\,2})\, \theta\left(\left(\frac{|\vec{p}_\perp^{\,2}|}{Q\, \tau_R} \right)^{1/b} - 1 \right)$$

$$\times\, (\vec{p}_\perp^{\,2})^{-1-\epsilon-\frac{\eta+1}{2}} \frac{\left| 1 - \left(\frac{Q\,\tau_R}{|\vec{p}_\perp^{\,2}|} \right)^{2/b} \right|^{-\eta}}{|b| \left(\frac{Q\,\tau_R}{|\vec{p}_\perp^{\,2}|} \right)^{1-\eta/b}} + \left\{ \begin{array}{c} \tau_L \leftrightarrow \tau_R \\ \vec{k}_\perp^{\,2} \leftrightarrow \vec{p}_\perp^{\,2} \end{array} \right\}$$

Angularity soft function at one-loop



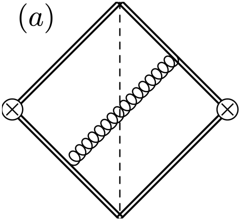
$$\mathcal{S}^{(1)}(\tau_L, \tau_R, \vec{p}_\perp^{\,2}, \vec{k}_\perp^{\,2}) = \frac{\alpha_s(\mu)\, C_F}{\pi} \frac{\mu^{2\epsilon} e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \nu^\eta\, w^2\, Q\, \delta(\tau_L)\, \delta(\vec{k}_\perp^{\,2})\, \theta\Big(\Big(\frac{|\vec{p}_\perp^{\,2}|}{Q\,\tau_R}\Big)^{1/b} - 1\Big)$$

$$\times\, (\vec{p}_\perp^{\,2})^{-1-\epsilon-\frac{\eta+1}{2}} \frac{\left|1 - \left(\frac{Q\,\tau_R}{|\vec{p}_\perp^{\,2}|}\right)^{2/b}\right|^{-\eta}}{|b| \left(\frac{Q\,\tau_R}{|\vec{p}_\perp^{\,2}|}\right)^{1-\eta/b}} + \left\{ \begin{array}{c} \tau_L \leftrightarrow \tau_R \\ \vec{k}_\perp^{\,2} \leftrightarrow \vec{p}_\perp^{\,2} \end{array} \right\}$$

For $b > 0$, we introduce the variables

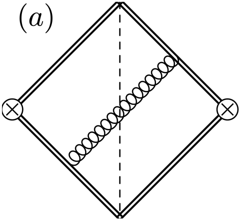
$$v_L = \frac{Q\tau_L}{k_\perp}, \qquad v_R = \frac{Q\tau_R}{p_\perp}$$

Angularity soft function at one-loop



$$\mathcal{S}^{(1+)}(\tau_L, \tau_R, \vec{p}_\perp^{\,2}, \vec{k}_\perp^{\,2}) = \frac{\alpha_s(\mu)\, C_F}{\pi} \frac{e^{\epsilon\gamma_E}\, w^2}{\Gamma(1-\epsilon)} \left(\frac{\nu}{\mu}\right)^{\eta} \delta(v_L)\, \delta(\vec{k}_\perp^{\,2})\, \frac{\theta(1-v_R)\,(1-v_R^{2/b})^{-\eta}}{b\, v_R^{1-\eta/b}}$$

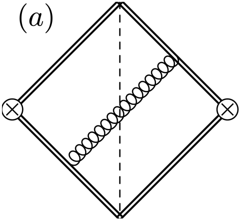
$$\times \frac{1}{\mu^2} \frac{1}{(\vec{p}_\perp^{\,2}/\mu^2)^{1+\epsilon+\frac{\eta}{2}}} \left[\left| \frac{dv_L}{d\tau_L} \right| \left| \frac{dv_R}{d\tau_R} \right| \right] + \left\{ \begin{array}{c} v_L \leftrightarrow v_R \\ \vec{k}_\perp^{\,2} \leftrightarrow \vec{p}_\perp^{\,2} \end{array} \right\}$$



$$\mathcal{S}^{(1+)}(\tau_L, \tau_R, \vec{p}_\perp^{\,2}, \vec{k}_\perp^{\,2}) = \frac{\alpha_s(\mu)\, C_F}{\pi} \frac{e^{\epsilon\gamma_E}\, w^2}{\Gamma(1-\epsilon)} \left(\frac{\nu}{\mu}\right)^{\eta} \delta(v_L)\, \delta(\vec{k}_\perp^{\,2})\, \frac{\theta(1-v_R)\,(1-v_R^{2/b})^{-\eta}}{b\, v_R^{1-\eta/b}}$$
$$\times \frac{1}{\mu^2} \frac{1}{(\vec{p}_\perp^{\,2}/\mu^2)^{1+\epsilon+\frac{\eta}{2}}} \left[\left|\frac{dv_L}{d\tau_L}\right| \left|\frac{dv_R}{d\tau_R}\right|\right] + \left\{ \begin{array}{c} v_L \leftrightarrow v_R \\ \vec{k}_\perp^{\,2} \leftrightarrow \vec{p}_\perp^{\,2} \end{array} \right\}$$

$$\left[\frac{\theta(1-v_R)\,(1-v_R^{2/b})^{-\eta}}{v_R^{1-\eta/b}}\right]_+^{\infty} = \theta(1-v_R)\left(\left[\frac{1}{v_R}\right]_+ + \frac{\eta}{b}\left[\frac{\ln v_R}{v_R}\right]_+ - \eta\left[\frac{\ln(1-v_R^{2/b})}{v_R}\right]_+\right)$$
$$+ a\,\delta(v_R) + \mathcal{O}(\eta^2)$$

(a)



$$\mathcal{S}^{(1+)}(\tau_L,\tau_R,\vec{p}_\perp^{\,2},\vec{k}_\perp^{\,2}) = \frac{\alpha_s(\mu)\,C_F}{\pi}\,\frac{e^{\epsilon\gamma_E}\,w^2}{\Gamma(1-\epsilon)}\left(\frac{\nu}{\mu}\right)^{\eta}\delta(v_L)\,\delta(\vec{k}_\perp^{\,2})\,\frac{\theta(1-v_R)\,(1-v_R^{2/b})^{-\eta}}{b\,v_R^{1-\eta/b}}$$
$$\times\,\frac{1}{\mu^2}\,\frac{1}{(\vec{p}_\perp^{\,2}/\mu^2)^{1+\epsilon+\frac{\eta}{2}}}\left[\left|\frac{dv_L}{d\tau_L}\right|\left|\frac{dv_R}{d\tau_R}\right|\right] + \left\{\begin{matrix} v_L \leftrightarrow v_R \\ \vec{k}_\perp^{\,2} \leftrightarrow \vec{p}_\perp^{\,2}\end{matrix}\right\}$$

$$\left[\frac{\theta(1-v_R)\,(1-v_R^{2/b})^{-\eta}}{v_R^{1-\eta/b}}\right]_+^{\infty} = \theta(1-v_R)\left(\left[\frac{1}{v_R}\right]_+ + \frac{\eta}{b}\left[\frac{\ln v_R}{v_R}\right]_+ - \eta\left[\frac{\ln(1-v_R^{2/b})}{v_R}\right]_+\right)$$
$$+\,a\,\delta(v_R) + \mathcal{O}(\eta^2)$$

$$a = \tfrac{b}{\eta} + \tfrac{\pi^2}{12}\,b\,\eta$$

- $b > 0$

$$\left[\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{d\tau_b}\right]^{\mathrm{NLO}} = \frac{\alpha_s C_F}{\pi}\left\{\underbrace{-\frac{3}{(1+b)}\frac{1}{\tau_b} - \frac{4}{1+b}\frac{\ln\tau_b}{\tau_b}}_{\text{thrust}} \underbrace{-\frac{4}{(1+b)}\frac{\ln(1-r)}{\tau_b}}_{\text{recoil}}\right\}$$

thrust ⇓ leading-singular

recoil ⇓ **?**

where, r is given by the solution of

$$\frac{r}{(1-r)^{1+b}} = (\tau_b)^b \tag{4}$$

$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

- **Small-τ limit:**

$$r = a_1\tau^b + a_2\tau^{2b} + a_3\tau^{3b} + a_4\tau^{4b} + \ldots \tag{5}$$

$$\begin{aligned} \Rightarrow \frac{\ln(1-r)}{\tau} &= \sum_{n=1}^{\infty} \frac{c_n}{\tau^{1-n\,b}} \\ &= \sum_{m=1}^{\lceil 1/b \rceil - 1} \frac{c_m}{\tau^{1-m\,b}} + \text{power} - \text{corrections} \end{aligned} \tag{6}$$

$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

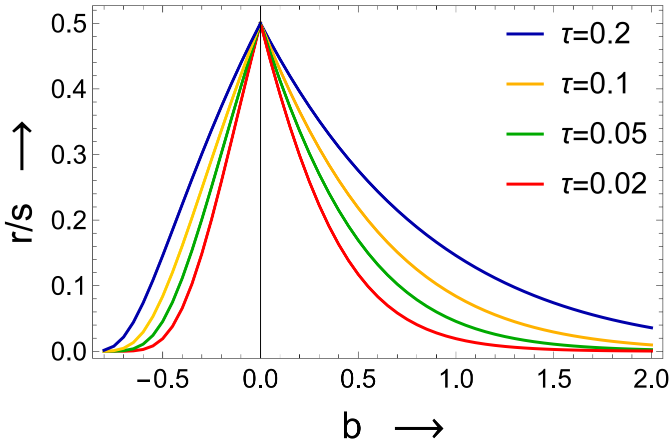
- **Small-b limit:**

$$r = r_0 + b\, r_1 + b^2\, r_2 + \ldots \qquad \text{where,} \frac{r_0}{1-r_0} = \tau^b$$

$$\Rightarrow \frac{ln(1-r)}{\tau} = -\ln 2 \left[\frac{1}{\tau}\right]_+ - \frac{b}{2}\left[\frac{\ln\tau}{\tau}\right]_+ + \frac{b\,\ln 2}{2}\left[\frac{1}{\tau}\right]_+ + \mathcal{O}(b^2) \tag{5}$$

⇒Recoil term provides a leading singular contribution for jet broadening (b=0) and reduces to only power corrections for thrust (b=1) while other angularities $0 < b < 1$ contain new additional singularities.

Variation of r/s-term with b



Numerical Size of the recoil term

b	% correction for $\tau_b = 0.05$	% correction for $\tau_b = 0.1$
1	2	6
0.5	8	16
0.25	16	26
0	31	45
-0.2	15	24
-0.5	2	5

Table: Relative size of the extra singular contribution compared to the leading singular contribution in the peak region for the τ_b distribution, for various values of b. A $2-6\%$ correction for $b=1$ or -0.5 shows the typical size of the power corrections due to the additional term.

Comparison against EVENT2

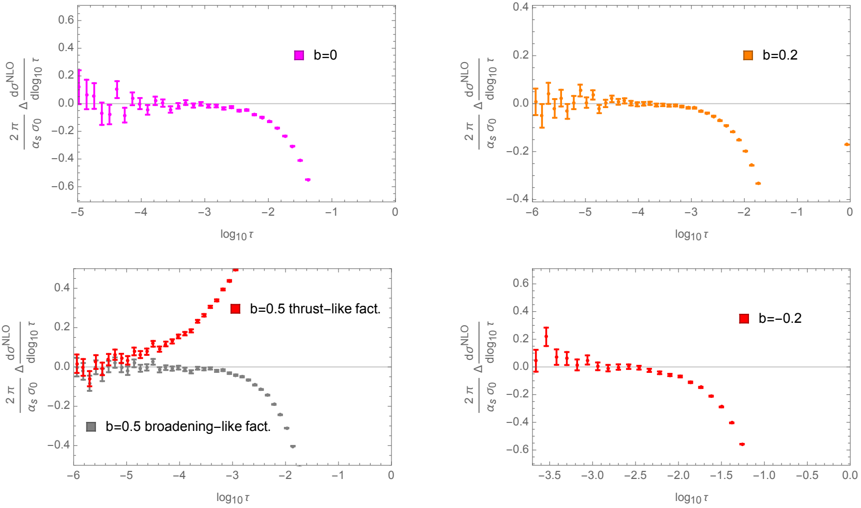


Figure: Differences between EVENT2 and our results from broadening-like factorization at NLO for $\mathrm{d}\sigma/\mathrm{d}\log_{10}\tau$ for different b values.

Summary

- Jet angularities provide a novel way of looking into the substructure which remains unexposed while looking at a single event shape observable.
- A broadening-like factorization for angularities provides the correct distribution for all $b > -1$ angularities while a thrust-like factorization works only in a certain range.
- Our analysis allows to smoothly interpolate between the thrust and jet broadening limits which have been defined within different effective theories so far.
- The fixed order angularity distributions with a broadening-like factorization suggest that the recoil effects are always important for $b < 1$ angularities.
- The recoil contributions, in the form of sub-leading singular terms, for $0 < b < 1$ provide a non-negligible contribution in the peak region. This is expected to effect the resummation of these observables.

- The fixed order results provided by SCET framework contain large logarithms of the angularity exponent which become dominant in the $\tau \to 0$ region and an all-order resummation is essential for obtaining predictive theoretical results, which is my next immediate goal.
- Resummation of these recoil-sensitive angularities will shed further light on the yet unexplored recoil effects.
- This analysis will open up the $b < 0.5$ range of angularities exponents which can then be utilized for novel one-loop constraints on α_s extractions from angularities, using the present and future LEP data.

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Thank you