Multi-differential resummation in SCET+

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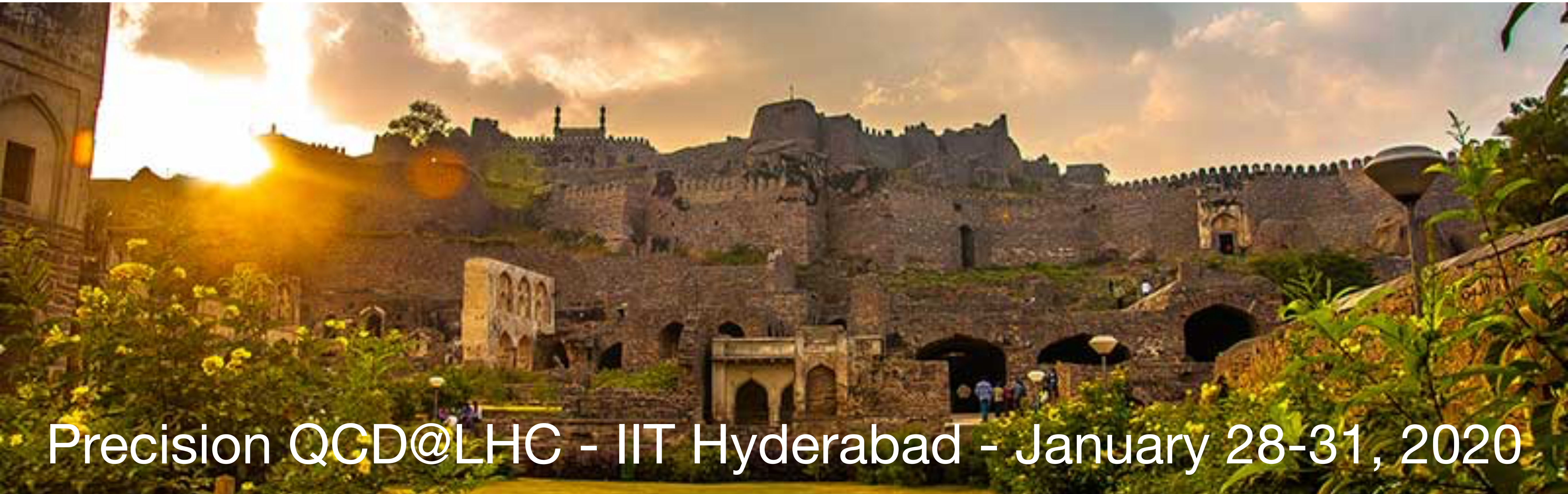
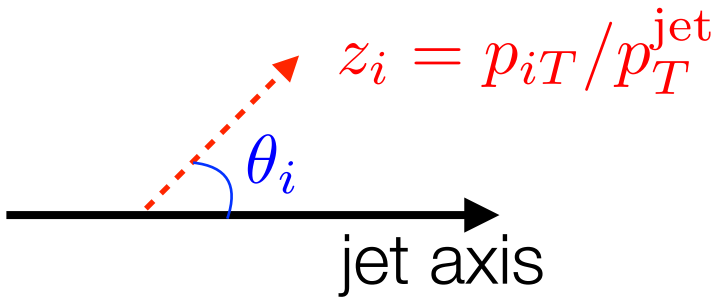


Precision QCD@LHC - IIT Hyderabad - January 28-31, 2020

Motivation

- Resummation captures dominant effect of higher-orders in α_s, enhanced in collinear/soft limit
- Resummation has mostly been considered to single variables
[threshold+transverse momentum: Laenen, Sterman, Vogelsang; …]
- LHC analyses involve multiple cuts
→ describe correlations beyond accuracy of parton shower
- Ratio observables require double-differential resummation
E.g. N-subjettiness, energy correlation functions, planar flow, …
- Toy example: ratio of two angularities e_α/e_β

$$e_\alpha = \sum_{i \in \text{jet}} z_i \left(\frac{\theta_i}{R}\right)^\alpha$$

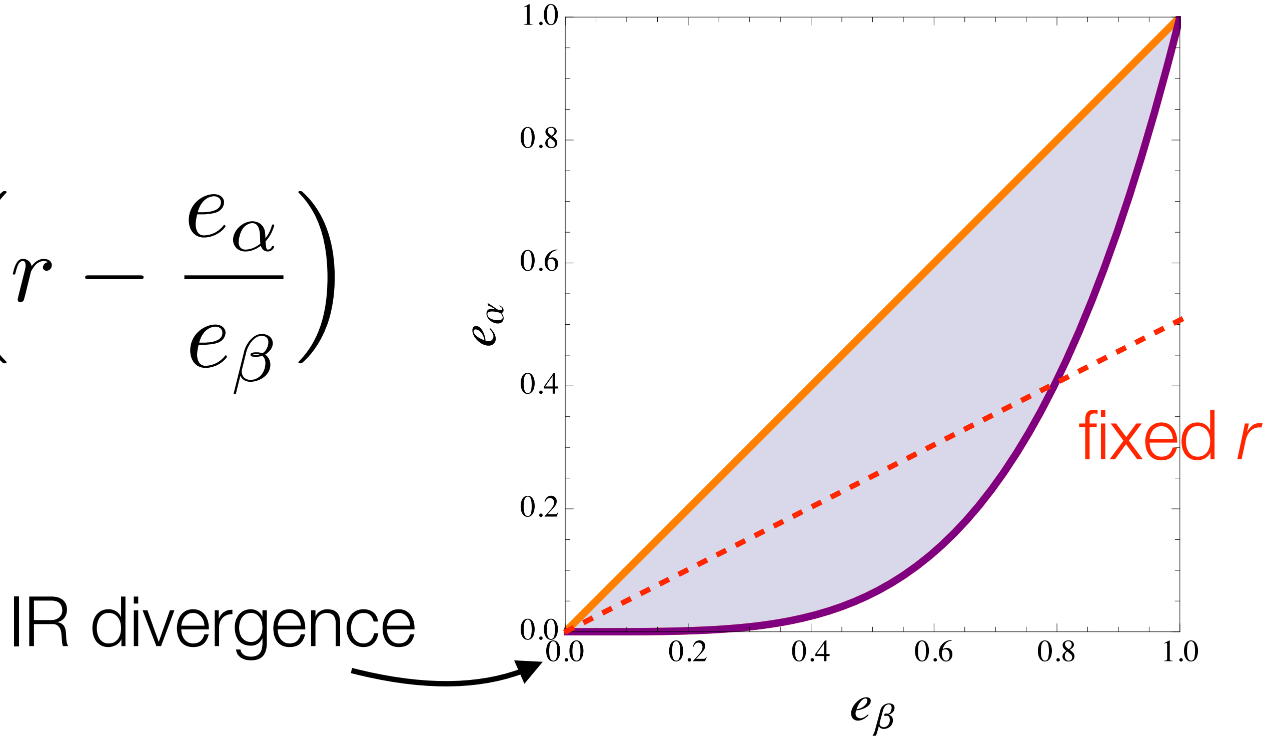


[Berger, Kucs, Sterman; Almeida et al.]

Ratios require double-differential resummation

- Ratio $r = e_\alpha / e_\beta$ is not IR safe
  [Soyez, Salam, Kim, Dutta, Cacciari]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \int \mathrm{d}e_\alpha \, \mathrm{d}e_\beta \, \frac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha \, \mathrm{d}e_\beta} \, \delta\!\left(r - \frac{e_\alpha}{e_\beta}\right)$$



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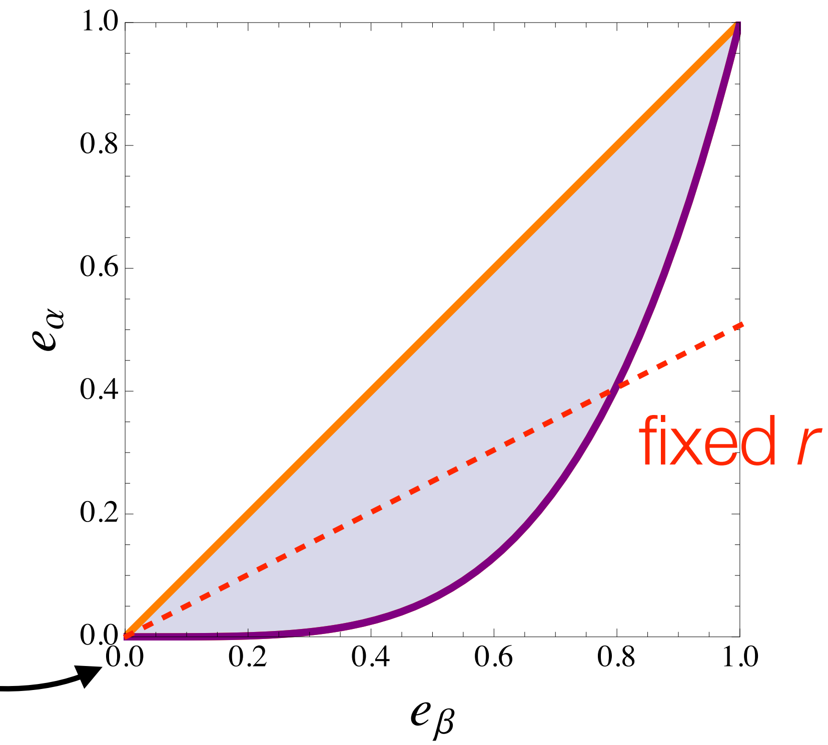


IR divergence

- IR region is Sudakov suppressed [Larkoski, Thaler]
  Requires simultaneous resummation of $\ln e_\alpha, \ln e_\beta$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \sqrt{\alpha_s} \, \frac{\sqrt{C_F \beta}}{\alpha - \beta} \, \frac{1}{r} + \ldots$$

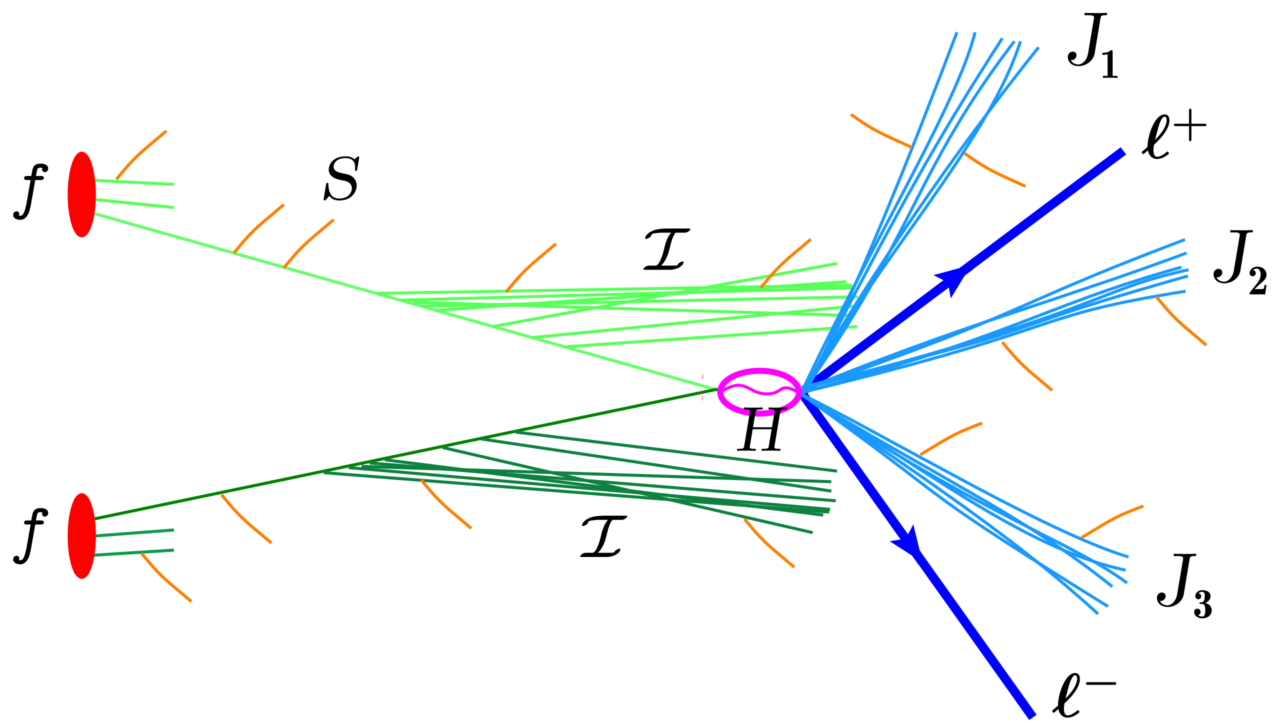
can't get this from fixed-order

Outline

1. Resummation in SCET
2. Factorization for q_T and 0-jettiness
3. Numerical results
4. Summary and outlook

1. Resummation in SCET

Large logarithms and resummation



- LHC collisions involve multiple scales: $\Lambda_{\rm QCD}, m, p_T^{\rm jet}, p_T^{\rm cut}, R, \ldots$
- Scale hierarchies can induce large logarithms in cross section

E.g. $\sigma(H + 0 \text{ jets}) = \sum_{m \le 2n} c_{n,m}\, \alpha_s^n \ln^m \frac{m_H}{p_T^{\rm cut}} \left[1 + \mathcal{O}\Big(\frac{p_T^{\rm cut}}{m_H}\Big)\right]$

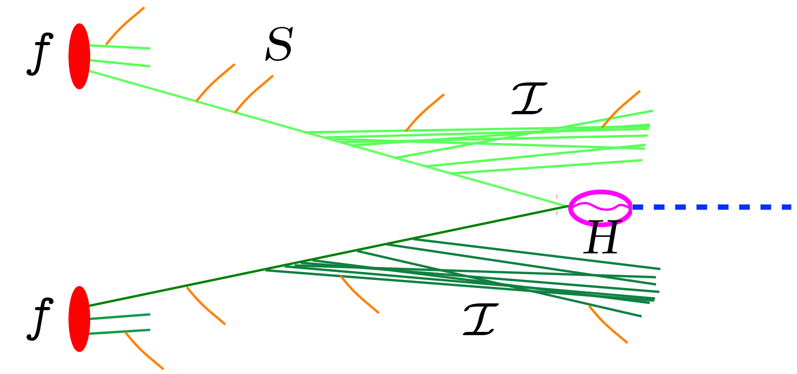
- In collinear and soft limit, gluons emissions are $\propto \alpha_s \int \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}z}{z}$ which is responsible for double logarithms

Soft-Collinear Effective Theory

- Effective theory of QCD for collinear and soft radiation

  [Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart]

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{coll}} + \mathcal{L}_{\text{coll}} + \mathcal{L}_{\text{soft}} + \sum_i C_i O_i$$



- Hard virtual corrections are integrated out

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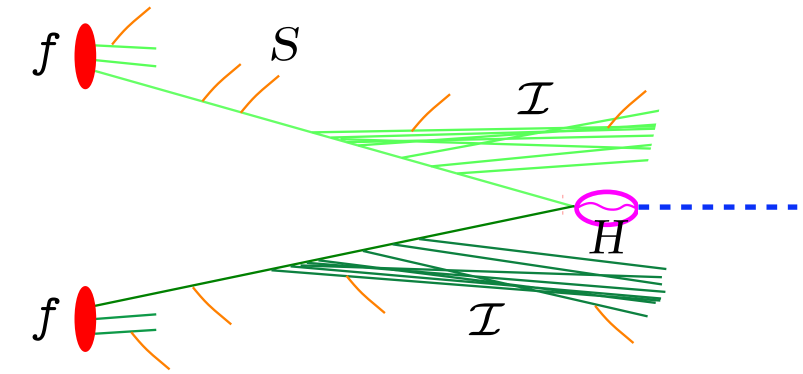
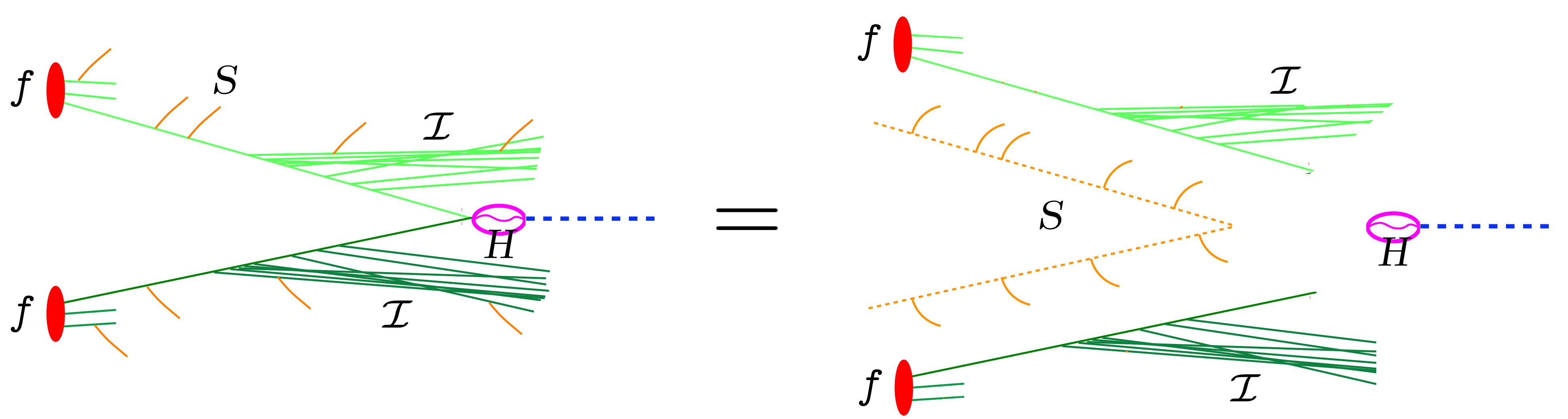


- Hard virtual corrections are integrated out

- Factorize collinear and soft modes in Lagrangian



- Factorize obs. $\mathcal{T} = \sum_i (E_i - |p_i^z|) = \mathcal{T}^{\rm coll} + \mathcal{T}^{\rm coll} + \mathcal{T}^{\rm soft}$

Factorization for 0-jettiness

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} = H(Q,\mu) \int \mathrm{d}\mathcal{T}^{\mathrm{coll}}\, B_a(Q\mathcal{T}^{\mathrm{coll}}, x_a, \mu) \int \mathrm{d}\mathcal{T}^{\mathrm{coll}}\, B_b(Q\mathcal{T}^{\mathrm{coll}}, x_b, \mu)$$
$$\times \int \mathrm{d}\mathcal{T}^{\mathrm{soft}}\, S(\mathcal{T}^{\mathrm{soft}}, \mu)\, \delta(\mathcal{T} - \mathcal{T}^{\mathrm{coll}} - \mathcal{T}^{\mathrm{coll}} - \mathcal{T}^{\mathrm{soft}})$$

- Hard function contains hard virtual corrections at the scale Q
- Beam functions describe the contribution from collinear radiation at scale $\sqrt{Q\mathcal{T}}$ and parton distribution functions

$$B_i(t = Q\mathcal{T}^{\mathrm{coll}}, x, \mu) = \sum_{i'} \int \frac{\mathrm{d}x'}{x'}\, \mathcal{I}_{ii'}\Big(t, \frac{x}{x'}, \mu\Big) f_{i'}(x', \mu)$$

[Stewart, Tackmann, WW]

- Soft function captures the soft radiation effects at scale $\mathcal{T}$

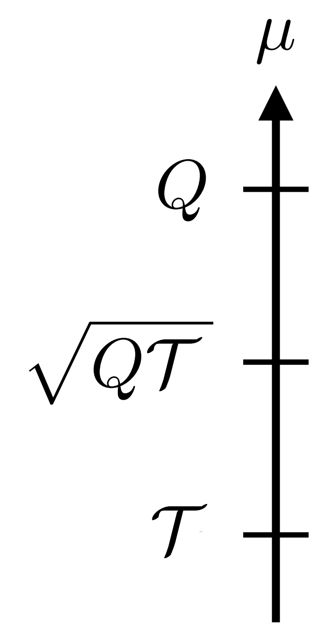
Resummation in SCET

- Achieve resummation by evaluating each ingredient at its natural scale and RG evolving to a common scale

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} H(Q,\mu) = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} B_i(t,x,\mu) = \int \mathrm{d}t'\, \gamma_B^i(t-t',\mu)\, B(t',x,\mu)$$

$$\ldots$$



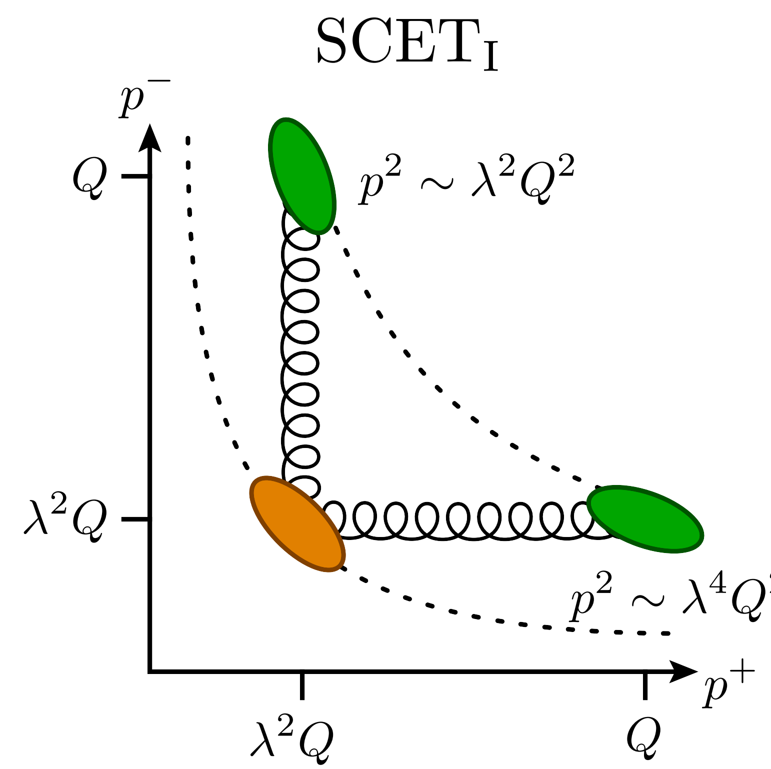
- E.g. hard function for Higgs production with $Q = m_H$

$$H(m_H,\mu) \propto 1 - \frac{\alpha_s C_A}{2\pi} \ln^2 \frac{m_H^2}{\mu^2} + \ldots$$

$$H(m_H,\mu) = \exp\Big[-\frac{\alpha_s C_A}{2\pi} \ln^2 \frac{m_H^2}{\mu^2} + \ldots \Big] H(m_H,m_H)$$

Power counting

	$\mathrm{SCET_I}$
n-collinear	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$
soft	$Q(\lambda^2, \lambda^2, \lambda^2)$



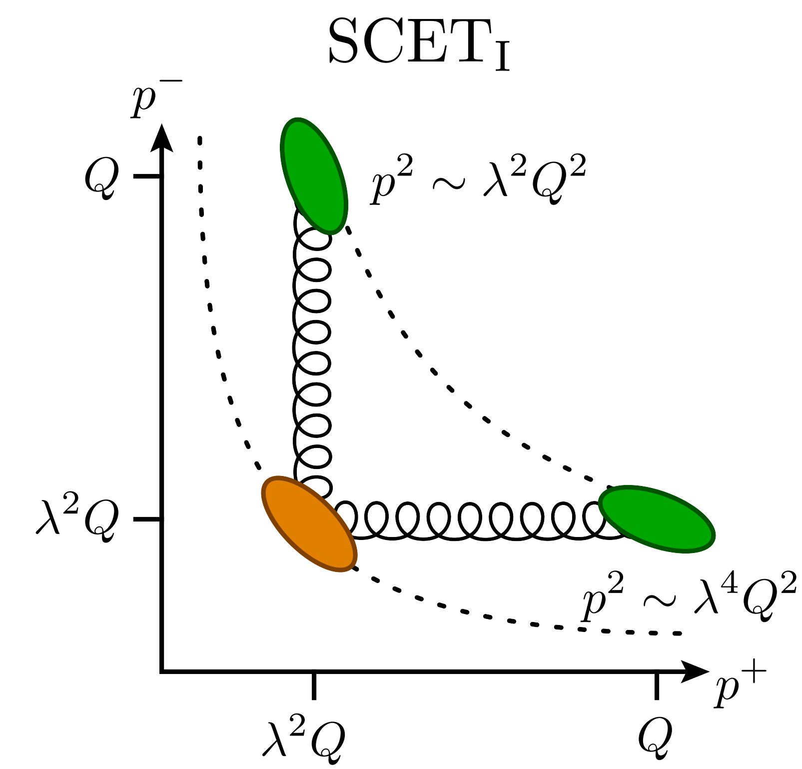
- Light cone coordinates

$$p^\mu = (p^+, p^-, p_\perp^\mu) = (p^0 - p^3, p^0 + p^3, p_\perp^\mu)$$

- Measurement determines modes:
  - $\mathcal{T} = \sum_i (E_i - |p_i^z|) = \sum_i \min\{p_i^+, p_i^-\}$
    described by $\mathrm{SCET_I}$ with $\lambda^2 = \mathcal{T}/Q$

Power counting

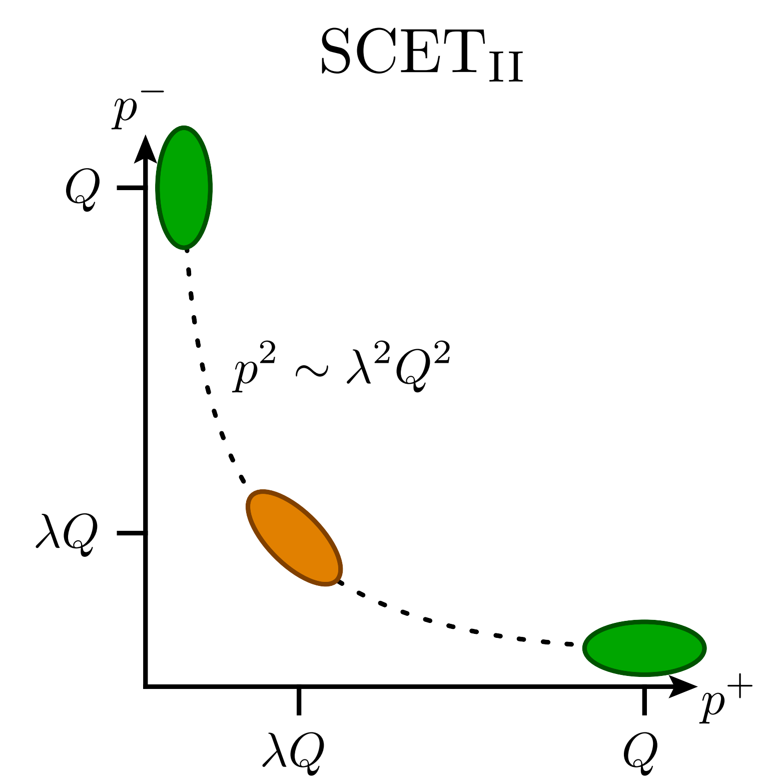
	$\mathrm{SCET_I}$	$\mathrm{SCET_{II}}$
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q(1, \lambda^2, \lambda)$
soft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q(\lambda, \lambda, \lambda)$



- Light cone coordinates

  $$p^\mu = (p^+, p^-, p_\perp^\mu) = (p^0 - p^3, p^0 + p^3, p_\perp^\mu)$$

- Measurement determines modes:
  - $\mathcal{T} = \sum_i (E_i - |p_i^z|) = \sum_i \min\{p_i^+, p_i^-\}$ described by $\mathrm{SCET_I}$ with $\lambda^2 = \mathcal{T}/Q$
  - $\vec{q}_T = \sum_i \vec{p}_{i\perp}$ is $\mathrm{SCET_{II}}$ with $\lambda = q_T/Q$

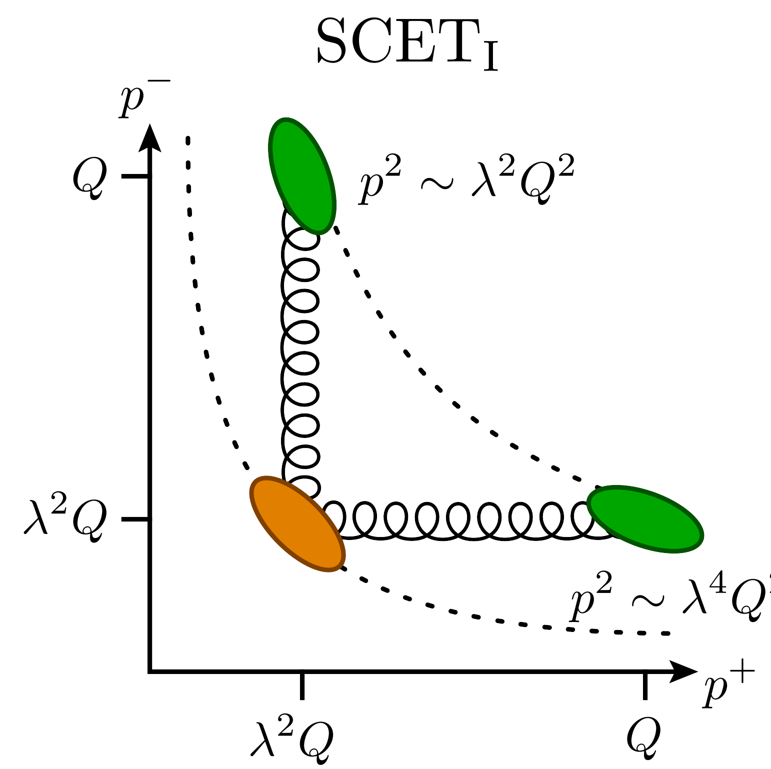


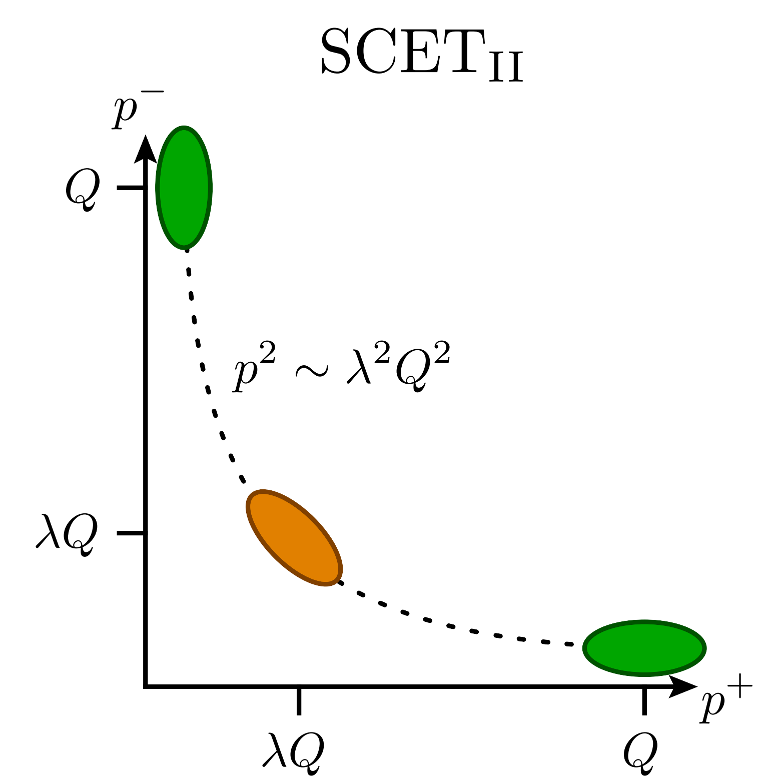
Rapidity divergences

- For SCET$_{\text{II}}$ the soft radiation decouples at the matching step

- SCET$_{\text{II}}$ involves rapidity divergences

$$S^{(1)}(q_T) \propto \alpha_s \frac{\mu^{2\epsilon}}{q_T^{1+2\epsilon}} \int \mathrm{d}y$$





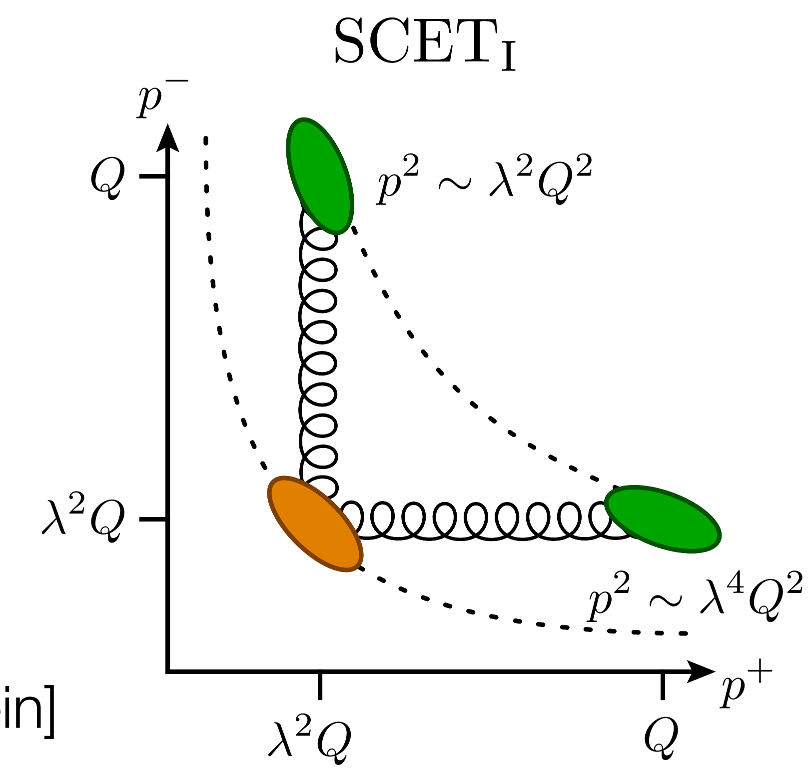
Rapidity divergences

- For SCET$_{II}$ the soft radiation decouples at the matching step

- SCET$_{II}$ involves rapidity divergences

$$S^{(1)}(q_T) \propto \alpha_s \frac{\mu^{2\epsilon}\,\nu^\eta}{q_T^{1+2\epsilon+\eta}} \int \mathrm{d}y\, |2\sinh y|^{-\eta}$$

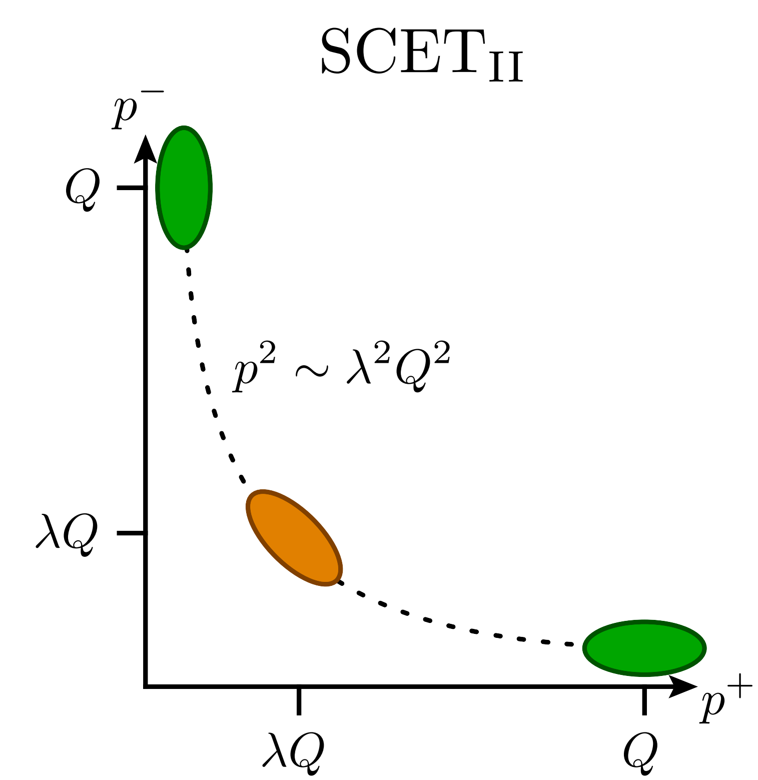
[Chiu, Jain, Neill, Rothstein]



- Many other choices for rapidity regulator
[Collins; Chiu, Fuhrer, Hoang, Kelley, Manohar; Becher, Bell; …]

- Must take $\eta \to 0$ before $\epsilon \to 0$

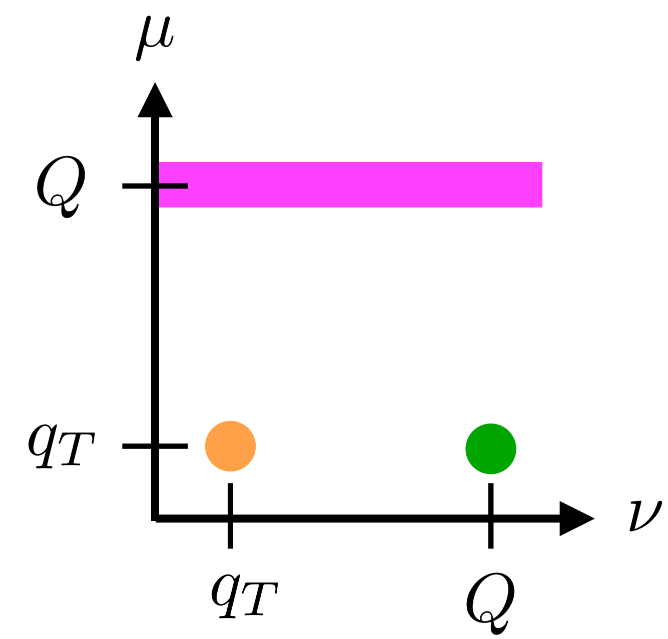
$$S^{(1)}(q_T) \propto \alpha_s \Big[\frac{2}{\eta}\Big(\frac{1}{\epsilon}\delta(q_T) - \frac{2}{\mu}\frac{1}{(q_T/\mu)_+}\Big) - \Big(\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{\mu}{\nu}\Big)\delta(q_T) + \dots\Big]$$



Rapidity resummation

- Rapidity resummation encoded in rapidity RG evolution
  [Chiu, Jain, Neill, Rothstein]

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} B_i(\vec{q}_T, x, \mu, \nu)$$
$$= \int \mathrm{d}\vec{q}_T^{\,\prime}\, \gamma_B^\nu(\vec{q}_T - \vec{q}_T^{\,\prime}, \mu)\, B_i(\vec{q}_T^{\,\prime}, x, \mu, \nu)$$



- μ and ν evolution commute

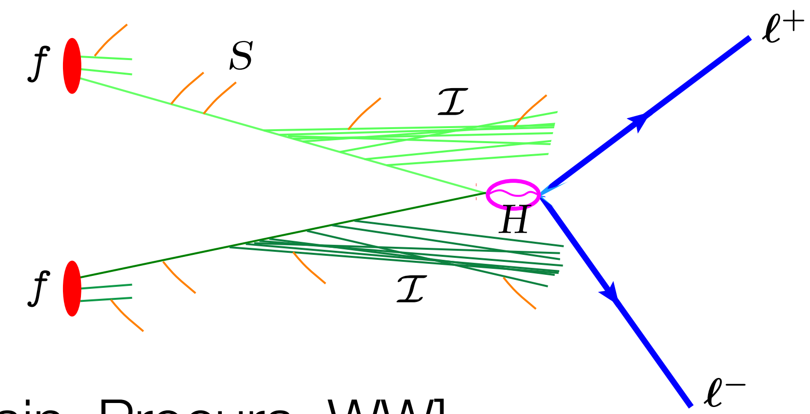
- Factorization and resummation of q_T spectrum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\vec{q}_T} = H(Q,\mu) \int \mathrm{d}\vec{q}_T^{\,\mathrm{coll}}\, B_a(\vec{q}_T^{\,\mathrm{coll}}, x_a, \mu, \nu) \int \mathrm{d}\vec{q}_T^{\,\mathrm{coll}}\, B_b(\vec{q}_T^{\,\mathrm{coll}}, x_b, \mu, \nu)$$
$$\times \int \mathrm{d}\vec{q}_T^{\,\mathrm{soft}}\, S(\vec{q}_T^{\,\mathrm{soft}}, \mu, \nu)\, \delta(\vec{q}_T - \vec{q}_T^{\,\mathrm{coll}} - \vec{q}_T^{\,\mathrm{coll}} - \vec{q}_T^{\,\mathrm{soft}})$$

2. Factorization for q_T and 0-jettiness

JHEP 1502 (2015) 117 - Procura, WW, Zeune

Beam thrust and transverse momentum resummation



- Make $\mathcal{T}$ or q_T factorization formulas more differential:

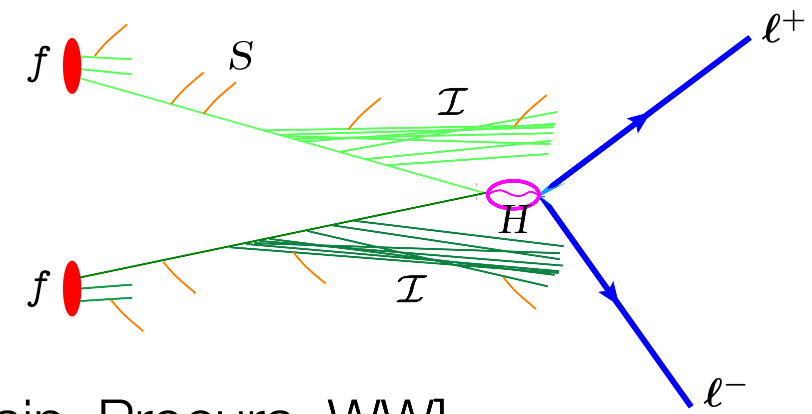
$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q)B(Q\mathcal{T}, \vec{q}_T) \otimes B(Q\mathcal{T}, \vec{q}_T) \otimes S(\mathcal{T})$$ [Jain, Procura, WW]

- Structure dictated by power counting

$$\mathrm{SCET_I}: \ \vec{q}_T = \vec{q}_T^{\,\mathrm{coll}} + \vec{q}_T^{\,\mathrm{coll}} + \mathcal{O}(\lambda^2) \qquad \rightarrow \qquad Q\mathcal{T} \sim \vec{q}_T^{\,2}$$

Beam thrust and transverse momentum resummation

- Make $\mathcal{T}$ or q_T factorization formulas more differential



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$$\mathrm{SCET_{II}}: \ \mathcal{T} = \mathcal{T}^{\mathrm{soft}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad \mathcal{T}^2 \sim \vec{q}_T^{\,2}$$

Beam thrust and transverse momentum resummation

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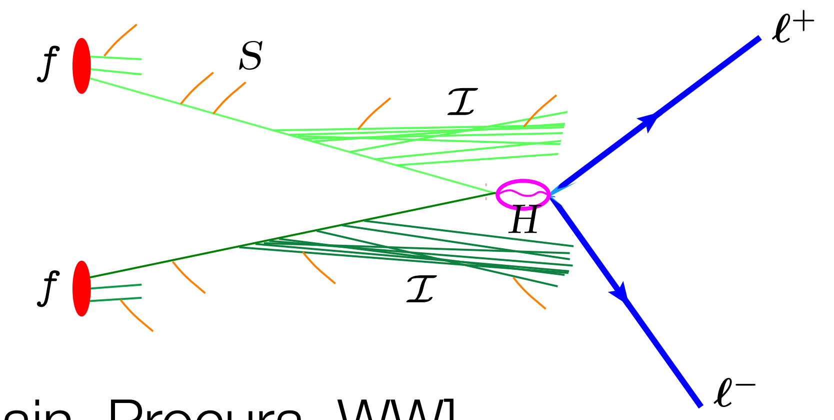

$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q)B(Q\mathcal{T},\vec{q}_T) \otimes B(Q\mathcal{T},\vec{q}_T) \otimes S(\mathcal{T})$$ [Jain, Procura, WW]

$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q)B(\vec{q}_T) \otimes B(\vec{q}_T) \otimes S_+(\mathcal{T},\vec{q}_T) \otimes S_+(\mathcal{T},\vec{q}_T) \otimes S(\mathcal{T})$$ [Procura, WW, Zeune]

$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q)B(\vec{q}_T) \otimes B(\vec{q}_T) \otimes S(\mathcal{T},\vec{q}_T)$$ [Larkoski, Moult, Neill]

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$$\mathrm{SCET_{II}}: \ \mathcal{T} = \mathcal{T}^{\mathrm{soft}} + \mathcal{O}(\lambda^2) \quad \to \quad \mathcal{T}^2 \sim \vec{q}_T^{\,2}$$

- Intermediate regime requires extra collinear-soft functions S_+

Collinear-soft function

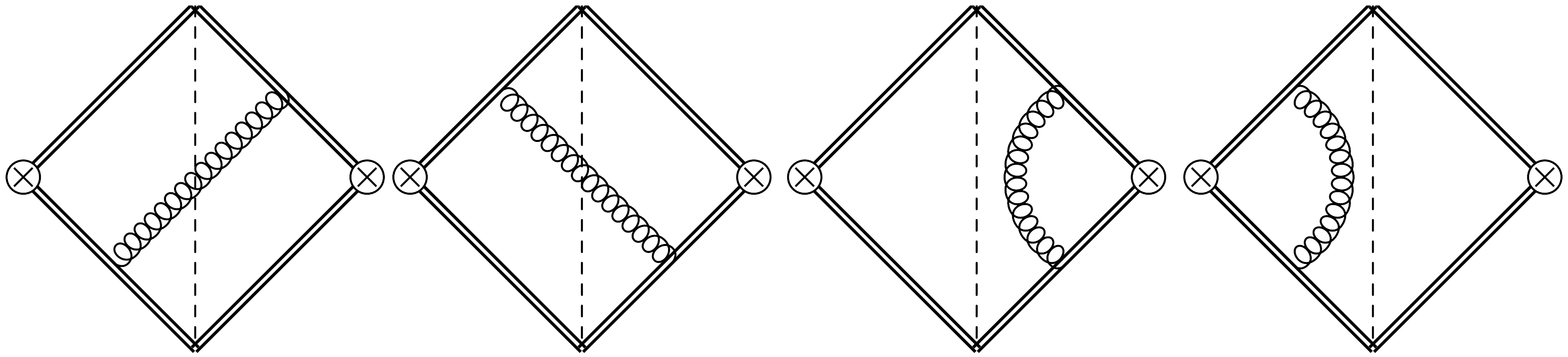
- Matrix-element of eikonal collinear-soft Wilson lines

$$S_+(\mathcal{T}^{\rm csoft}, \vec{q}_T^{\,\rm csoft}) = \frac{1}{N_c} {\rm tr}\, \langle 0 | \bar{T}\big[X_n^\dagger V_{\bar{n}}\big]\, \delta(\mathcal{T}^{\rm soft} - \hat{p}^+)\, \delta(\vec{q}_T^{\,\rm csoft} - \hat{\vec{p}}_T) T\big[V_n^\dagger X_n\big] |0\rangle$$

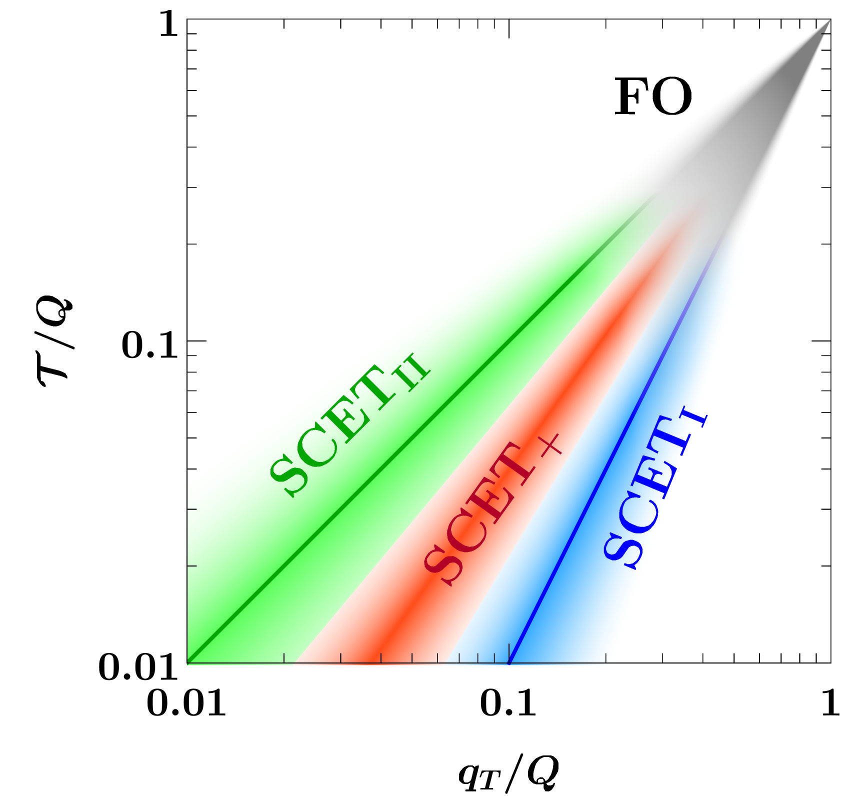
- Differs from double differential soft function because the collinear-soft radiation goes into one hemisphere, $\hat{\mathcal{T}} \to \hat{p}^+$

- At one-loop

$$S_+^{(1)}(k^+, \vec{k}_T, \mu, \nu) = \frac{\alpha_s C_F}{\pi^2} \Bigg\{ \frac{1}{\mu^3} \frac{1}{(k^+/\mu)_+} \frac{1}{(k_T^2/\mu^2)_+} + \delta(k^+) \Bigg[-\frac{1}{\mu^2} \bigg[\frac{\ln(k_T^2/\mu^2)}{k_T^2/\mu^2} \bigg]_+ + \frac{1}{\mu^2} \frac{1}{(k_T^2/\mu^2)_+} \ln\frac{\nu}{\mu} - \frac{\pi^2}{12} \delta(k_T^2) \Bigg] \Bigg\}$$



Consistency relations



- Between SCET$_I$ and SCET+

$$B(Q\mathcal{T}, \vec{q}_T) = B(\vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q\mathcal{T}}\right)\right]$$

- Between SCET$_{II}$ and SCET+

$$S(\mathcal{T}, \vec{q}_T) = S_+(\mathcal{T}, \vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \otimes S(\mathcal{T}) \left[1 + \mathcal{O}\left(\frac{\mathcal{T}^2}{q_T^2}\right)\right]$$

- Verified for anomalous dimensions and NLO ingredients

3. Numerical results

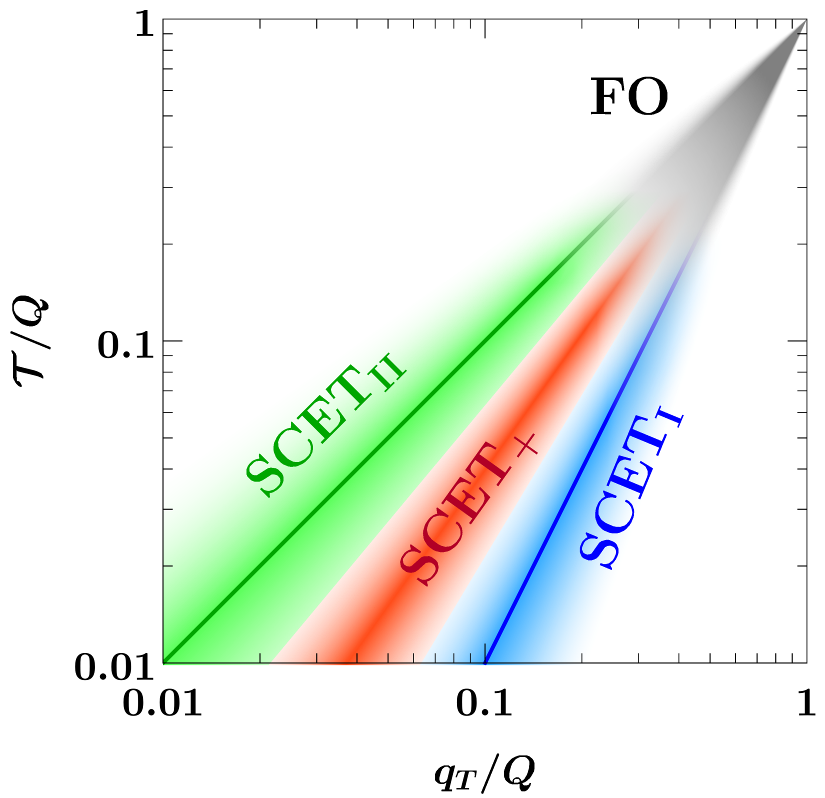
JHEP 1903 (2019) 124 - Lustermans, Michel, Tackmann, WW

Scales choices

- Natural scales connect regimes:

Scale	$\mathrm{SCET}_{\mathrm{I}}$	SCET_{+}	$\mathrm{SCET}_{\mathrm{II}}$
μ_H	Q	Q	Q
μ_B	$\sqrt{\mathcal{T}Q}$	q_T	q_T
ν_B		Q	Q
μ_{S_+}		q_T	
ν_{S_+}		$q_T^2/\mathcal{T}$	
μ_S	$\mathcal{T}$	$\mathcal{T}$	q_T
ν_S			q_T

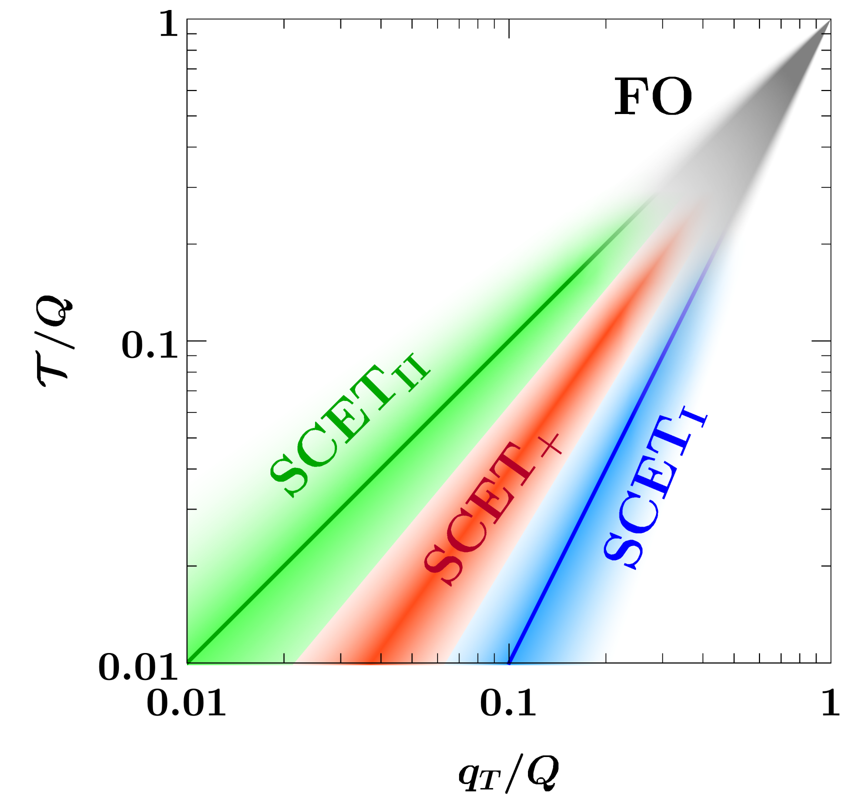
$q_T^2 \sim \mathcal{T}Q$ $\qquad$ $q_T \sim \mathcal{T}$



Scales choices

- Natural scales connect regimes:

Scale	$\mathrm{SCET_I}$	$\mathrm{SCET_+}$	$\mathrm{SCET_{II}}$
μ_H	Q	Q	Q
μ_B	$\sqrt{\mathcal{T}Q}$	b_0/b_T	b_0/b_T
ν_B		Q	Q
μ_{S_+}		b_0/b_T	
ν_{S_+}		$(b_0/b_T)^2/\mathcal{T}$	
μ_S	$\mathcal{T}$	$\mathcal{T}$	b_0/b_T
ν_S			b_0/b_T

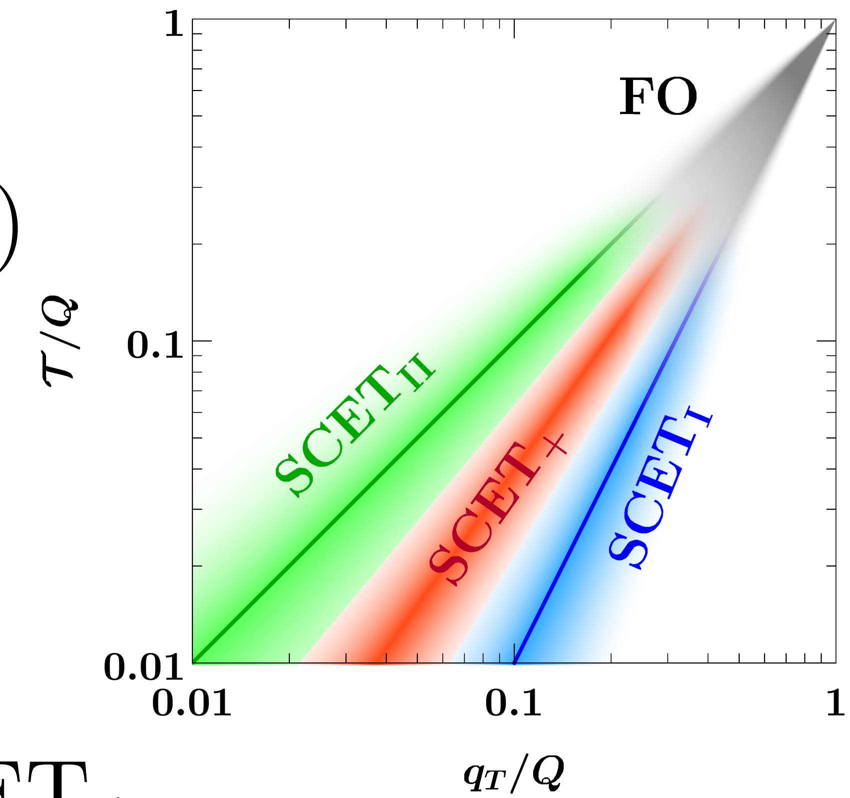


- Turn resummation off using profiles in q_T, $\mathcal{T}$ [Ligeti, Stewart, Tackmann]

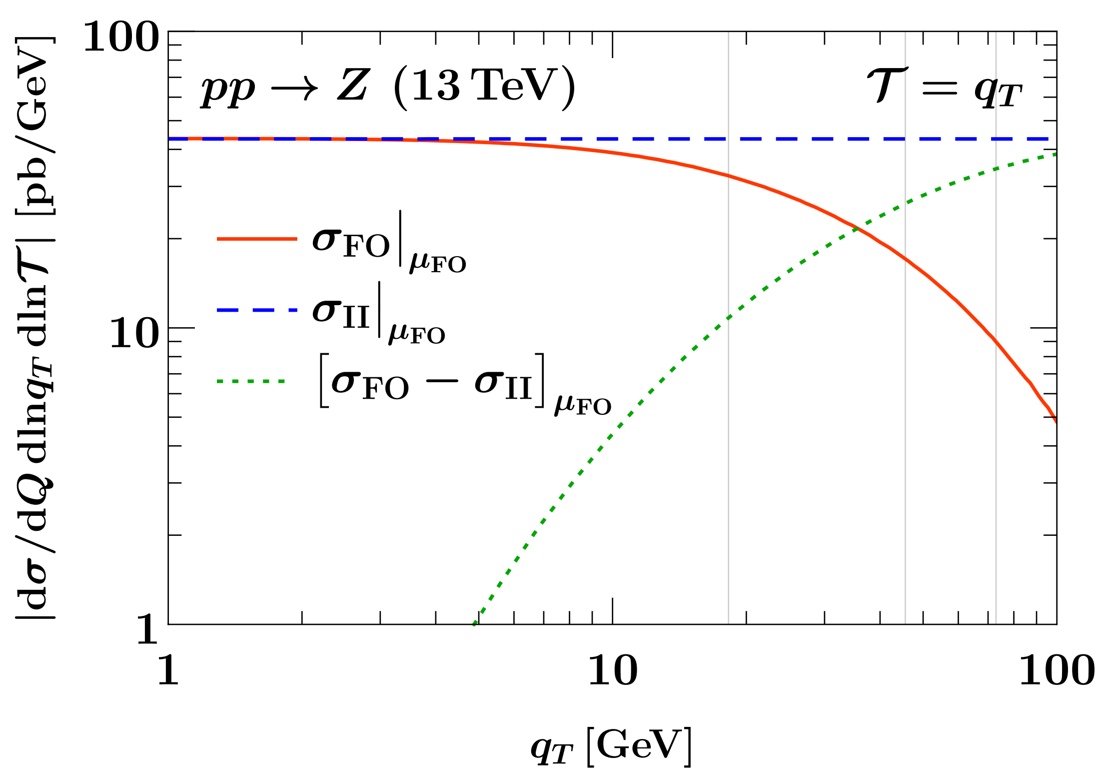
Matching

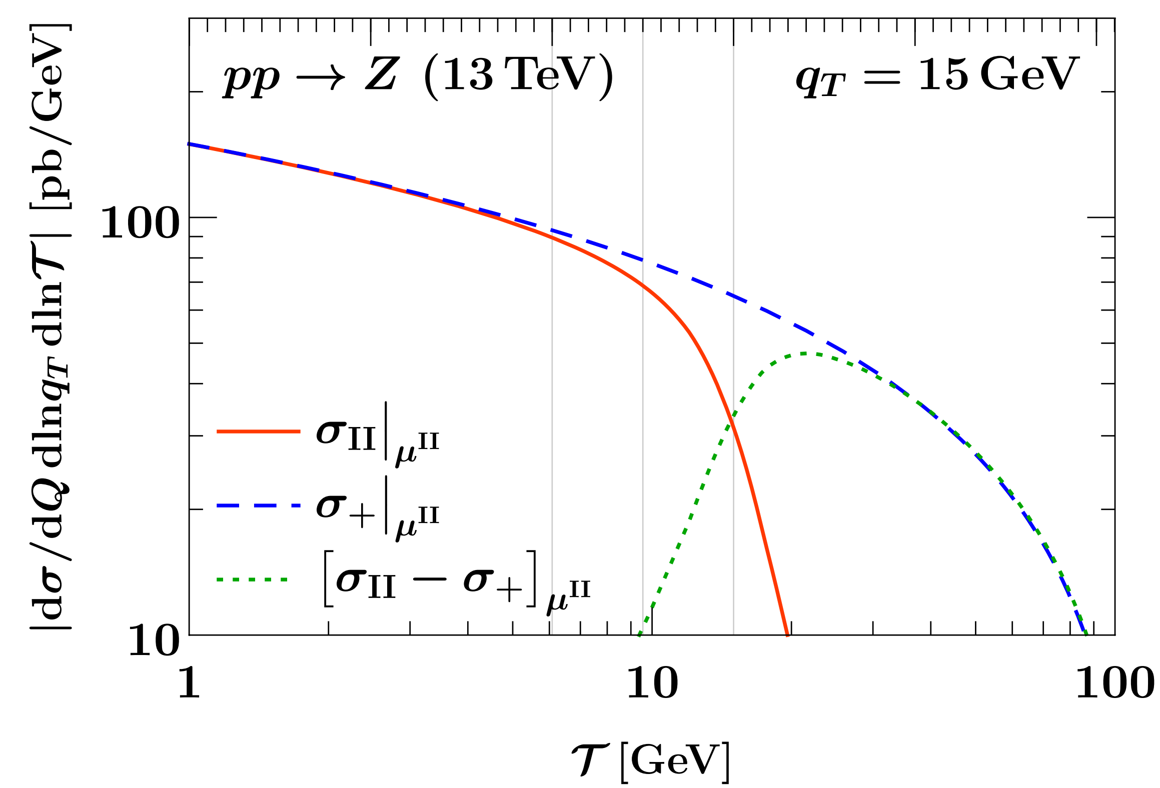
- Combining different regimes:

$$\sigma = \sigma_+(\mu) + [\sigma_{\mathrm{I}} - \sigma_+](\mu_{\mathrm{I}}) + [\sigma_{\mathrm{II}} - \sigma_+](\mu_{\mathrm{II}}) + [\sigma_{\mathrm{FO}} - \sigma_{\mathrm{I}} - \sigma_{\mathrm{II}} + \sigma_+](\mu_{\mathrm{FO}})$$

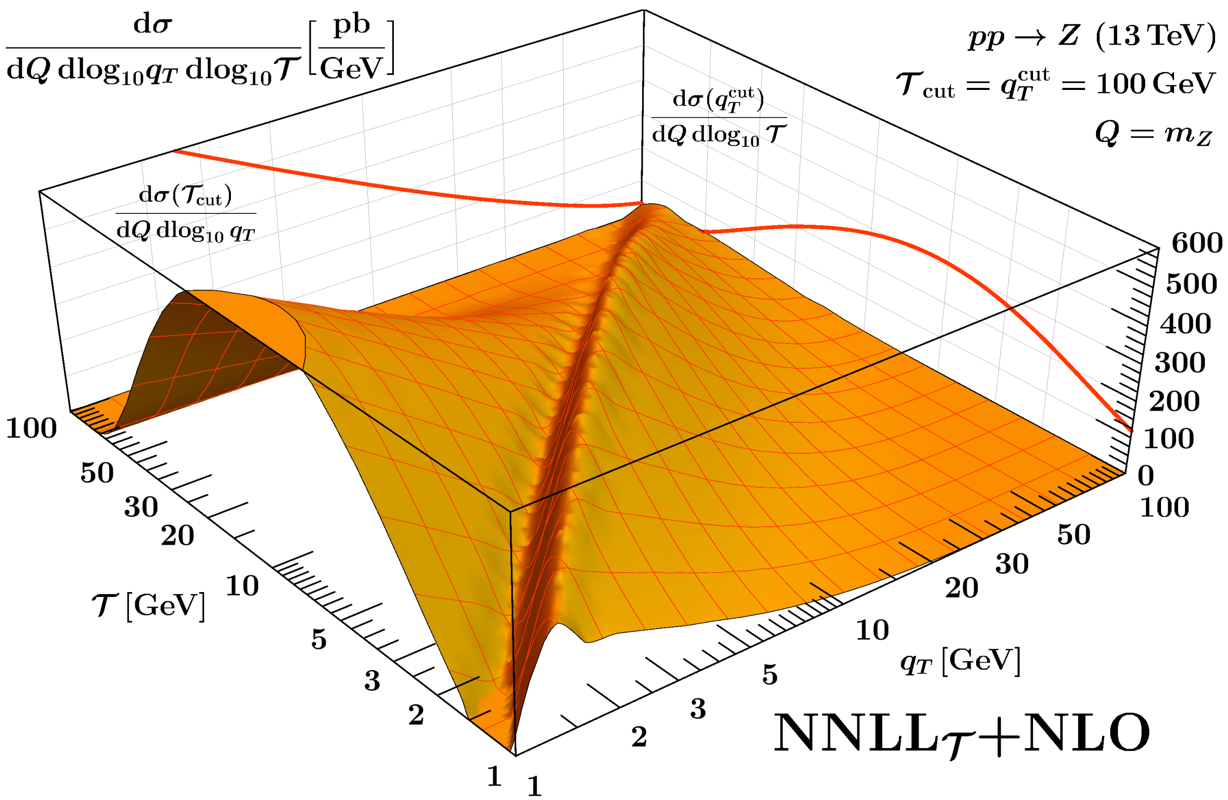


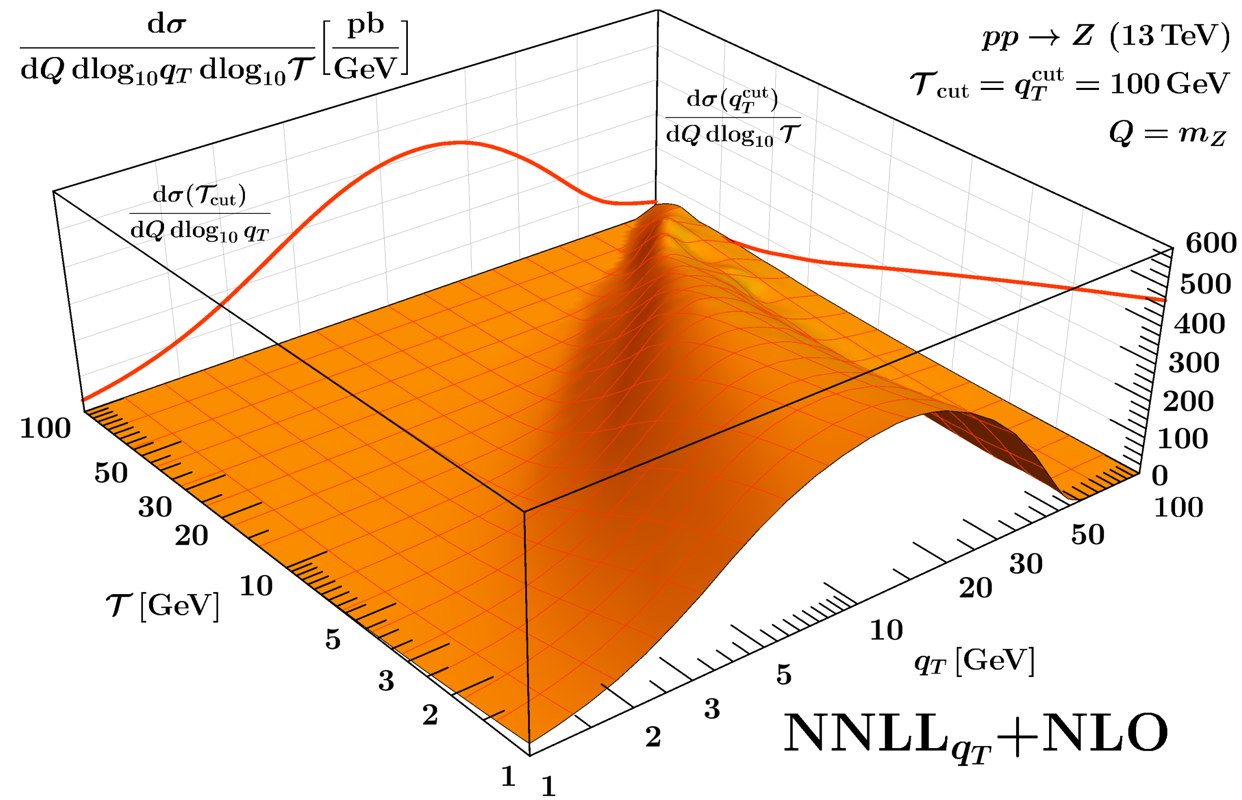
- E.g. transition from $\mathrm{FO} \to \mathrm{SCET_{II}} \to \mathrm{SCET_+}$





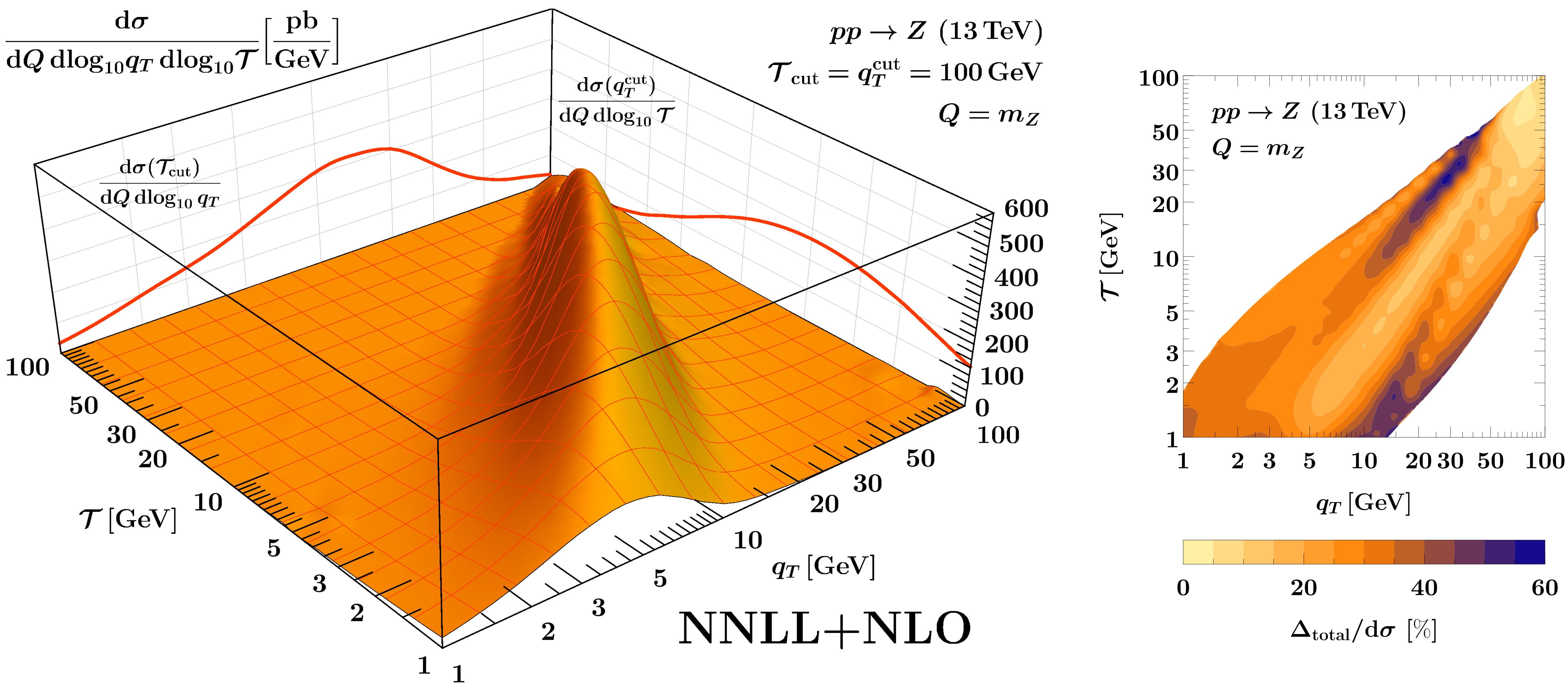
Result with only $\mathcal{T}$ or q_T resummation





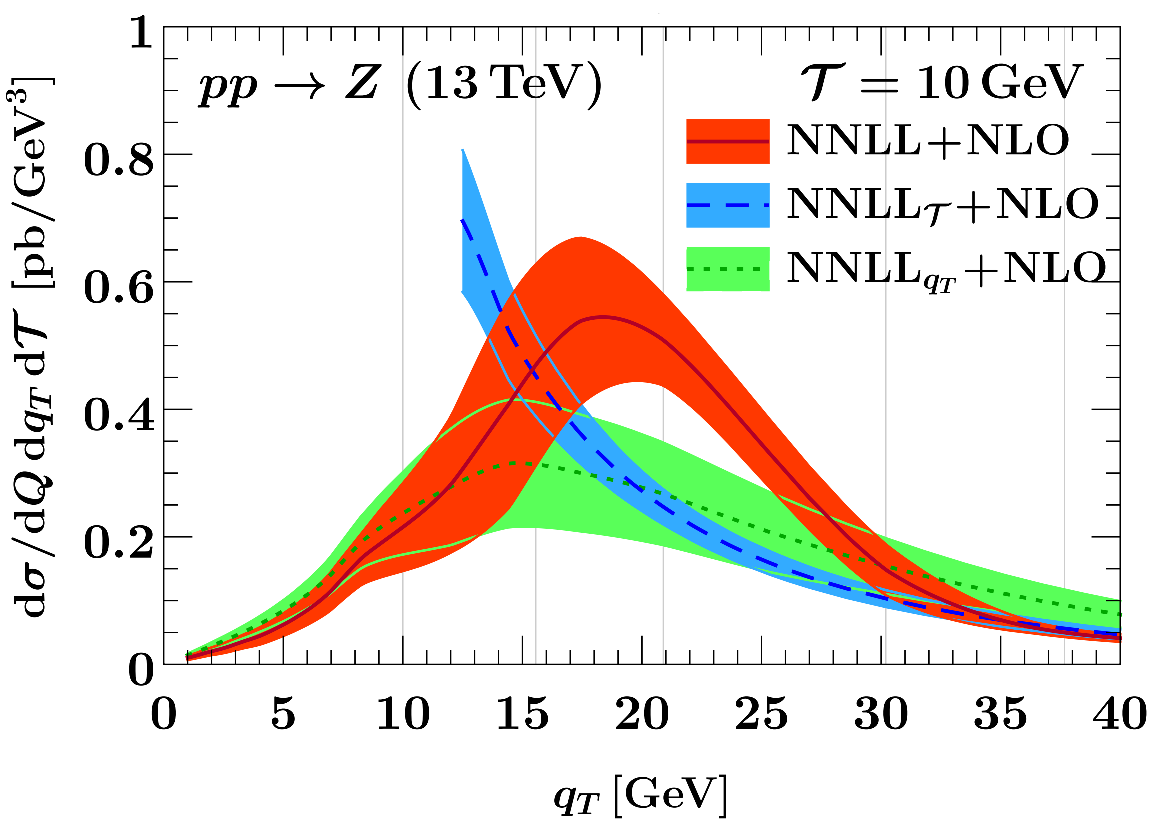
- Single differential distributions (on back wall) clearly show that only one variable is resummed while the other is not
- Sharp edge in left plot due to $\mathcal{T} \leq q_T$ for one emission

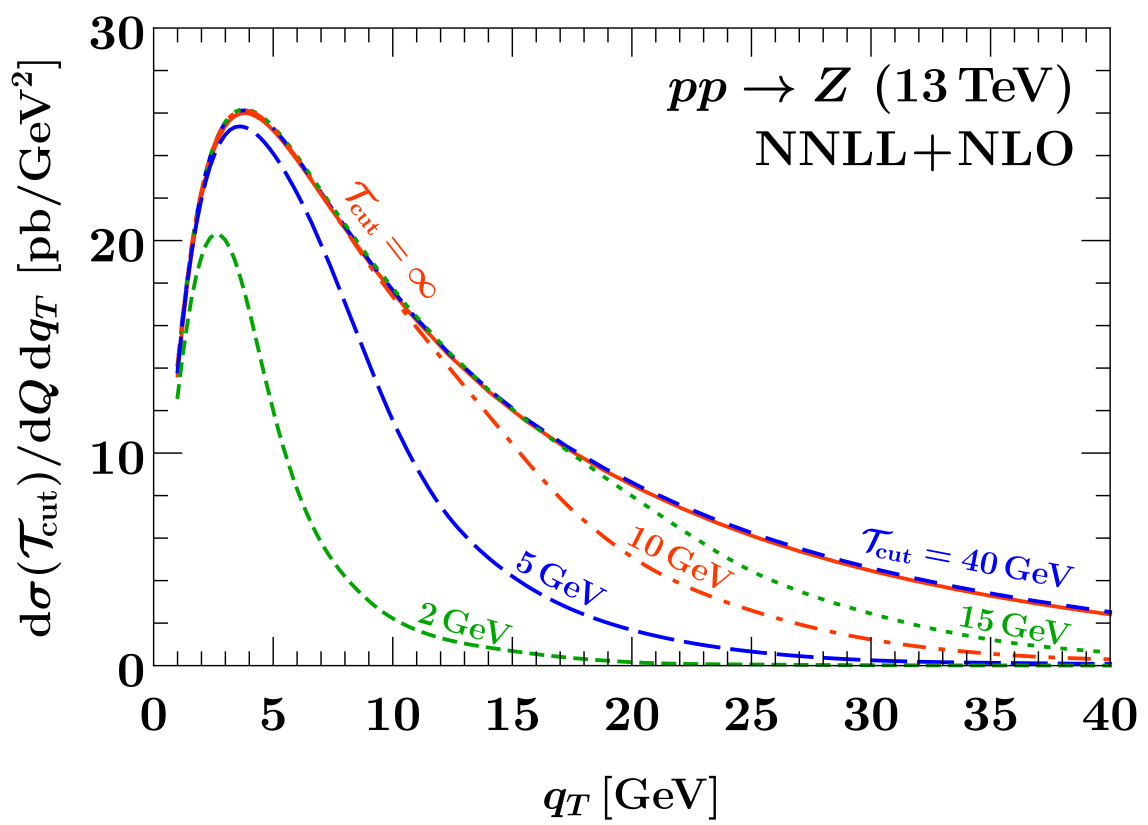
Result with joint $\mathcal{T}$ and q_T resummation



- Resummation yields a two-dimensional Sudakov peak
- Uncertainties in heat map (vary scales and transition points)

Two-dimensional slices





- SCET+ interpolates between SCET$_\mathrm{I}$ and SCET$_\mathrm{II}$
- Cut on beam thrust $\mathcal{T}$ changes shape of q_T spectrum

Summary and outlook

- SCET is powerful tool for factorization and resummation
- Multi-differential resum. involves collinear-soft modes = SCET_{+}
- SCET+ enables the simultaneous resummation of
  - Jet resolution parameters. E.g. q_T and 0-jettiness
  - Jet kinematic logarithms. E.g. hierarchies in jet energies
  - Jet radius logarithms
  - Non-global logarithms Larkoski, Moult, Neill; Becher, Neubert, Rothen, Shao]
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ध्यान देने के लिए आपका धन्यवाद