Double-differential Higgs and jet p_t resummation in momentum space

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Mainly based on

`Monni, Re, PT, 1604.02191`
`Bizon, Monni, Re, Rottoli, PT, 1705.09127`
`Monni, Rottoli, PT, 1909.04704`

Multi-differential Higgs distributions

- Kinematic distributions of Higgs and QCD radiation in gluon fusion sensitive to potential BSM effects.
- Higgs transverse momentum may be used to constrain models with heavy states such as top partners [Banfi, Martin, Sanz, 1308.4771], modifications to light Yukawa couplings [Bishara, et al., 1606.09253], ...
- Experimental prospects for precise measurements of Higgs distributions in different jet bins. 0-jet bin defined through a veto on the p_t of QCD radiation.
- Theoretical control required at the multi-differential level (Higgs and QCD jets). Focus on Higgs transverse momentum with a veto on p_t of accompanying jets.

Fixed-order vs resummation

- Fixed-order prediction for cumulative cross section Σ for observable V ($V = 0$ at Born).

$$\Sigma(V < \nu) = \int_0^{\nu} dV \frac{d\sigma}{dV} \sim \alpha_S^b [\underbrace{\mathcal{O}(1)}_{LO} + \underbrace{\mathcal{O}(\alpha_S)}_{NLO} + \underbrace{\mathcal{O}(\alpha_S^2)}_{NNLO} + \ldots].$$

- In regions dominated by soft/collinear radiation, fixed order spoiled by large logarithms

$$\frac{d\sigma}{d\nu} \sim \frac{1}{\nu}\alpha_S^n L^m, \qquad m \leq 2n - 1, \qquad L = \ln(1/\nu).$$

- Enhanced logarithmic contributions to be resummed at all orders.

- Logarithmic accuracy defined on the logarithm of Σ:

$$\ln \Sigma(V < \nu) \sim \underbrace{\mathcal{O}(\alpha_S^n L^{n+1})}_{LL} + \underbrace{\mathcal{O}(\alpha_S^n L^n)}_{NLL} + \underbrace{\mathcal{O}(\alpha_S^n L^{n-1})}_{NNLL} + \underbrace{\mathcal{O}(\alpha_S^n L^{n-2})}_{N^3LL} + \ldots$$

Conjugate- vs direct-space resummation

$$\Sigma(V < \nu) \sim \sum_n \int d\Phi_{\mathrm{rad},n} \, |M(k_1, ..., k_n)|^2 \, \Theta(\nu - V(k_1, ..., k_n)) \, .$$

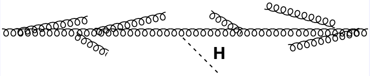
- Traditional approach to resummation of V: find a conjugate space where observable $V(k_1, ..., k_n)$ factorises, and resum there.

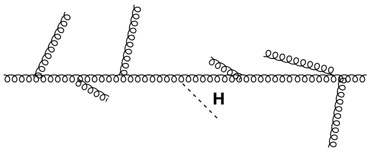
- Often complicated/impossible, but not necessary. V resummable if recursive IRC (rIRC) safe [Banfi, Salam, Zanderighi, 0112156, 0304148, 0407286], allowing exponentiation of leading logarithms.
  - * Same soft/collinear scaling properties for any number of emissions.
  - * The more soft/collinear the emission, the less it contributes to the value of V.

- 'CAESAR/ARES' approach follows [Banfi et al., 1412.2126, 1607.03111, 1807.11487]: resummation of many rIRC observables in direct space.

- CAESAR/ARES cannot predict rIRC-safe observables with azimuthal cancellations, as p_t^{H} in gluon fusion $\rightarrow$ included in the RadISH direct-space resummation [Monni, Re, PT, 1604.02191], [Bizon, Monni, Re, Rottoli, PT, 1705.09127].

Higgs in gluon fusion at small p_t





- Left. Commensurate emissions' transverse momenta k_{ti}: $m_H \gg \max(k_{ti}) \equiv k_{t1} \sim p_t^{\mathrm{H}}$. Exponential Sudakov suppression of $\Sigma(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}})$ at small p_t^{HV}.
- Right. Large azimuthal cancellations: $m_H \gg k_{t1} \gg p_t^{\mathrm{H}}$. Power law $\Sigma(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}) \sim (p_t^{\mathrm{HV}})^2$ at small p_t^{HV} [Parisi, Petronzio, 1979] dominates over Sudakov.

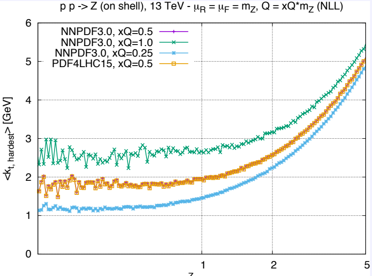
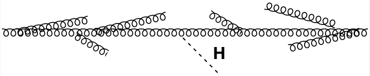
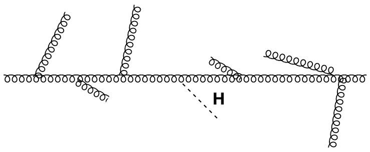


Figure credit: P.F. Monni

Higgs in gluon fusion at small p_t





- Left. Commensurate emissions' transverse momenta k_{ti}: $m_H \gg \max(k_{ti}) \equiv k_{t1} \sim p_t^{\mathrm{H}}$.
  Exponential Sudakov suppression of $\Sigma(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}})$ at small p_t^{HV}.

- Right. Large azimuthal cancellations: $m_H \gg k_{t1} \gg p_t^{\mathrm{H}}$.
  Power law $\Sigma(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}) \sim (p_t^{\mathrm{HV}})^2$ at small p_t^{HV} [Parisi, Petronzio, 1979] dominates over Sudakov.

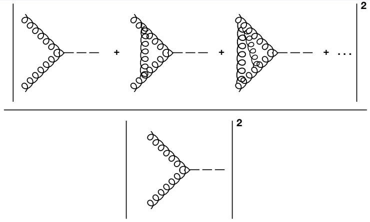
- All configurations accounted for in direct space in the RadISH approach: p_t^{H} up to N^{3}LL.

- Method applicable to generic colour singlet, not only Higgs, [Bizon et al., 1805.05916, 1905.05171] and all transverse observables (p_t^{J}, ϕ_η^*, E_t, ...), and extendible to more complicated ones.

Resummation in direct space: virtual and real radiation

$$\Sigma(V < \nu) = \int d\Phi_B \, \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(k_1, ..., k_n)|^2 \, \Theta(\nu - V(k_1, ..., k_n))$$

- $\mathcal{V}(\Phi_B)$ = all-order virtual form factor (see [Dixon, Magnea, Sterman, 0805.3515]).



- $|M(k_1, ..., k_n)|^2$ = all-order real matrix element.
  Can be systematically organised into contributions entering at given logarithmic accuracy

- Virtuals and reals are separately divergent.

Resummation in direct space: regularisation of virtuals and reals

$$\Sigma(V < \nu) = \int d\Phi_B \, \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(k_1, \ldots, k_n)|^2 \, \Theta(\nu - V(k_1, \ldots, k_n))$$

- Introduce a slicing parameter ϵk_{t1}.
  Real emissions categorised as unresolved (or resolved) if $k_{ti} < \epsilon k_{t1}$ (or $k_{ti} > \epsilon k_{t1}$).

- Unresolved contribute negligibly to V (rIRC safety), exponentiate and regularise the virtual form factor $\mathcal{V}(\Phi_B) \implies$ Sudakov radiator $R(\epsilon k_{t1})$

$$\mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=2}^{n} [dk_i] |M(k_1, \ldots, k_n)|^2 \Theta(\epsilon k_{t1} - k_{ti}) \propto e^{-R(\epsilon k_{t1})},$$

$$-R(k_t) = \underbrace{L_t \, g_1(\alpha_s L_t)}_{LL} + \underbrace{g_2(\alpha_s L_t)}_{NLL} + \underbrace{\frac{\alpha_s}{\pi} g_3(\alpha_s L_t)}_{NNLL} + \ldots, \qquad L_t = \ln(m_H/k_t).$$

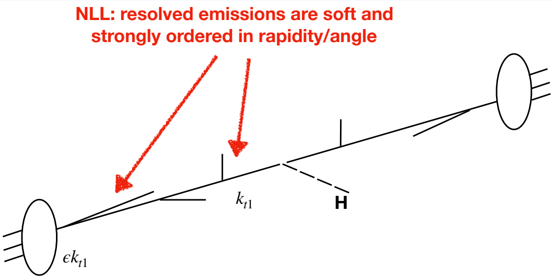
- Resolved emissions parametrised in terms of $R'(k_{ti}) = dR/d\ln(m_H/k_{ti})$.
  Expand ϵk_{t1} and k_{ti} around k_{t1} and truncate to eliminate subleading effects.
  Retain subleading terms only for 0, 1, 2, ... emissions at NLL, NNLL, N^{3}LL, ...

Higgs p_t at NLL

- Resummed cross section for Higgs transverse momentum $p_t^{\rm H}$ below a certain value $p_t^{\rm HV}$.
- $L_{t1} = \ln(m_H/k_{t1})$, $\mathcal{L}_{NLL}$ = NLL luminosity, $p_t^{\rm H}(\{k_j\}) = |\sum_j \vec{k}_{tj}|$.

$$\Sigma_{NLL}(p_t^{\rm H} < p_t^{\rm HV}) = \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \frac{d}{dL_{t1}} \left[- e^{-R_{NLL}(k_{t1})} \mathcal{L}_{NLL}(k_{t1}) \right] \times$$
$$\times \underbrace{\epsilon^{R'_{LL}(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \frac{d\phi_i}{2\pi} R'_{LL}(k_{t1})}_{\equiv \int d\mathcal{Z}} \Theta(p_t^{\rm HV} - |\sum_{j=1}^{n+1} \vec{k}_{tj}|)$$

- Generated as a k_t-ordered (semi-inclusive) parton shower.



Higgs p_t at NNLL

$$\Sigma_{NNLL}(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}) = \int_0^{\infty} \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(k_{t1})} \mathcal{L}_{NNLL}(k_{t1}) \right] \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}| \right) \right\}$$

- Luminosity $\mathcal{L}_{NNLL}$ with $O(\alpha_S)$ coefficient functions, radiator R_{NNLL} with $O(\alpha_S^n L_{t1}^{n-1})$ terms.

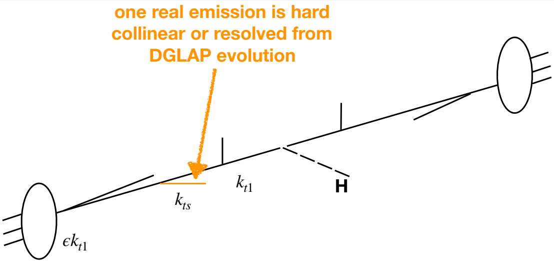
Higgs p_t at NNLL

$$\Sigma_{NNLL}(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}) = \int_0^{\infty} \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(k_{t1})} \mathcal{L}_{NNLL}(k_{t1}) \right] \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}| \right) \right.$$

$$+ e^{-R_{NLL}(k_{t1})} R'_{LL}(k_{t1}) \int_0^{k_{t1}} \frac{dk_{ts}}{k_{ts}} \frac{d\phi_s}{2\pi} \left[\left(\delta R'(k_{t1}) + R''_{LL}(k_{t1}) \ln \frac{k_{t1}}{k_{ts}} - \frac{d}{dL_{t1}} \right) \mathcal{L}_{NLL}(k_{t1}) \right]$$

$$\left. \times \left[\Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts}| \right) - \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}| \right) \right] \right\}$$

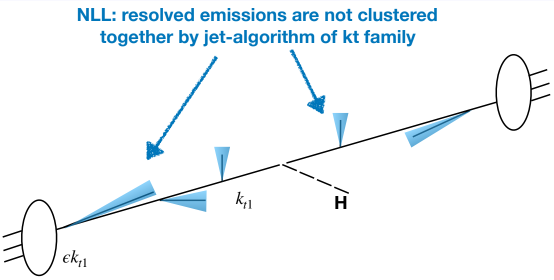
- Correction of one emission k_s (only one at NNLL) in the resolved ensemble (finite in $d = 4$)



Hardest jet p_t at NLL

- Observables $p_t^{\mathrm{J}}(\{k_j\}) = \max(k_{tj})$ and $p_t^{\mathrm{H}}(\{k_j\}) = |\sum_j \vec{k}_{tj}|$ have the same radiator R.
- At NLL the (anti)-k_t jet algorithm does not cluster emissions together.
  Same NLL resummation as p_t^{H} but for the measurement function.

$$\Sigma_{NLL}(p_t^{\mathrm{J}} < p_t^{\mathrm{JV}}) = \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \frac{d}{dL_{t1}} \left[-e^{-R_{NLL}(k_{t1})} \mathcal{L}_{NLL}(k_{t1}) \right] \times$$
$$\times \epsilon^{R'_{LL}(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \frac{d\phi_i}{2\pi} R'_{LL}(k_{t1}) \, \Theta(p_t^{\mathrm{JV}} - k_{t1})$$



Hardest jet p_t at NNLL

$$\Sigma_{NNLL}(p_t^{J} < p_t^{JV}) = \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(k_{t1})} \mathcal{L}_{NNLL}(k_{t1}) \right] \Theta\left(p_t^{JV} - k_{t1}\right) \right\}$$

- Luminosity $\mathcal{L}_{NNLL}$ with $O(\alpha_S)$ coefficient functions, radiator R_{NNLL} with $O(\alpha_S^n L_{t1}^{n-1})$ terms.

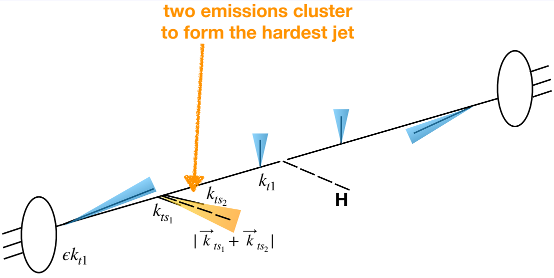
Hardest jet p_t at NNLL

$$\Sigma_{NNLL}(p_t^{J} < p_t^{JV}) = \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(k_{t1})} \mathcal{L}_{NNLL}(k_{t1}) \right] \Theta\left(p_t^{JV} - k_{t1}\right) \right.$$

$$+ \frac{1}{2!} \mathcal{L}_{NLL}(k_{t1})\, e^{-R_{NLL}(k_{t1})} \left(R'_{LL}(k_{t1})\right)^2 \int_0^{k_{t1}} \frac{dk_{ts_1}}{k_{ts_1}} \frac{dk_{ts_2}}{k_{ts_2}} \int d\Delta\eta_{s_1 s_2} \frac{d\phi_{s_1}}{2\pi} \frac{d\Delta\phi_{s_1 s_2}}{2\pi} \left(2 C_A \frac{\alpha_S(k_{t1})}{\pi}\right)$$

$$\left. \times\; J_R(s_1, s_2) \left[\Theta\left(p_t^{JV} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\right) - \Theta\left(p_t^{JV} - \max(k_{ts_1}, k_{ts_2})\right) \right] \Theta\left(p_t^{JV} - k_{t1}\right) + \ldots \right\}$$

- Clustering correction, [Banfi, et al., 1206.4998], with $J_R(s_1, s_2) \equiv \Theta\left(R^2 - \Delta\eta^2_{s_1 s_2} - \Delta\phi^2_{s_1 s_2}\right)$: at NNLL the jet algorithm may cluster two emissions to form the hardest jet (ellipses: $s_{1,2} \to 1$).



Hardest jet p_t at NNLL

$$\Sigma_{NNLL}(p_t^{\rm J} < p_t^{\rm JV}) = \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(k_{t1})} \mathcal{L}_{NNLL}(k_{t1}) \right] \Theta\left(p_t^{\rm JV} - k_{t1}\right) \right.$$

$$+ \frac{1}{2!} \mathcal{L}_{NLL}(k_{t1})\, e^{-R_{NLL}(k_{t1})} \left(R'_{LL}(k_{t1})\right)^2 \int_0^{k_{t1}} \frac{dk_{ts_1}}{k_{ts_1}} \frac{dk_{ts_2}}{k_{ts_2}} \int d\Delta\eta_{s_1 s_2} \frac{d\phi_{s_1}}{2\pi} \frac{d\Delta\phi_{s_1 s_2}}{2\pi} \left(2C_A \frac{\alpha_S(k_{t1})}{\pi}\right)$$

$$\times \left[J_R(s_1, s_2) \left[\Theta\left(p_t^{\rm JV} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\right) - \Theta\left(p_t^{\rm JV} - \max(k_{ts_1}, k_{ts_2})\right) \right] \Theta\left(p_t^{\rm JV} - k_{t1}\right) \right.$$

$$\left. \left. + \mathcal{C}(s_1, s_2) \left[1 - J_R(s_1, s_2) \right] \left[\Theta\left(p_t^{\rm JV} - \max(k_{ts_1}, k_{ts_2})\right) - \Theta\left(p_t^{\rm JV} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\right) \right] \right] + \ldots \right\}$$

- Correlated correction, [Banfi, et al., 1206.4998], with $\mathcal{C}(a,b) = \frac{|\tilde{M}(k_a,k_b)|^2}{|M(k_a)|^2 |M(k_b)|^2}$, and $|\tilde{M}(k_a,k_b)|^2$ is the non-factorisable part of the double-soft matrix element.

- $|\tilde{M}(k_a,k_b)|^2$ appears in $R_{NLL}(k_{t1})$, resulting in the CMW coupling [Catani, Marchesini, Webber, 1991]. Integrated inclusively there, and veto applied on $|\vec{k}_{ta} + \vec{k}_{tb}|$.
  Correct for configurations where the two correlated emissions are not clustered together.

Double-differential Higgs and jet p_t resummation at NNLL

- From contributions detailed above just need to combine measurement functions!

'inclusive' contribution

with ptH and ptj measurement functions

$$
\begin{aligned}
&\Sigma_{NNLL}(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}, p_t^{\mathrm{J}} < p_t^{\mathrm{JV}}) = \\
&\int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \Bigg\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(L_{t1})} \mathcal{L}_{NNLL}(L_{t1}) \right] \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}| \right) \Theta\left(p_t^{\mathrm{JV}} - k_{t1} \right) \\
&+ e^{-R_{NLL}(L_{t1})} R'_{LL}(k_{t1}) \int_0^{k_{t1}} \frac{dk_{ts}}{k_{ts}} \frac{d\phi_s}{2\pi} \left[\left(\delta R'(k_{t1}) + R''_{LL}(k_{t1}) \ln \frac{k_{t1}}{k_{ts}} - \frac{d}{dL_{t1}} \right) \mathcal{L}_{NLL}(L_{t1}) \right] \\
&\quad \times \left[\Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts}| \right) - \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}| \right) \right] \Theta\left(p_t^{\mathrm{JV}} - k_{t1} \right) \\
&+ \frac{1}{2!} \mathcal{L}_{NLL}(L_{t1}) e^{-R_{NLL}(L_{t1})} \left(R'_{LL}(k_{t1}) \right)^2 \int_0^{k_{t1}} \frac{dk_{ts_1}}{k_{ts_1}} \frac{dk_{ts_2}}{k_{ts_2}} \int d\Delta\eta_{s_1 s_2} \frac{d\phi_{s_1}}{2\pi} \frac{d\Delta\phi_{s_1 s_2}}{2\pi} \left(2C_A \frac{\alpha_S(k_{t1})}{\pi} \right) \\
&\quad \times \Theta\left(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts_1} + \vec{k}_{ts_2}| \right) \Big[J_R(s_1, s_2) \left(\Theta\left(p_t^{\mathrm{JV}} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}| \right) - \Theta\left(p_t^{\mathrm{JV}} - \max(k_{ts_1}, k_{ts_2}) \right) \right) \\
&\quad + \mathcal{C}(s_1, s_2) \left[1 - J_R(s_1, s_2) \right] \left(\Theta\left(p_t^{\mathrm{JV}} - \max(k_{ts_1}, k_{ts_2}) \right) - \Theta\left(p_t^{\mathrm{JV}} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}| \right) \right) \Big] \Theta\left(p_t^{\mathrm{JV}} - k_{t1} \right) + \ldots \Bigg\}
\end{aligned}
$$

Joint Higgs and jet p_t resummation at NNLL

- From contributions detailed above just need to combine measurement functions!

NNLL correction to ptH

with ptj measurement function

$$\Sigma_{NNLL}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV}) =$$

$$\int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \Bigg\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(L_{t1})} \mathcal{L}_{NNLL}(L_{t1}) \right] \Theta\Big(p_t^{\rm HV} - |\sum_{j=1}^{n+1} \vec{k}_{tj}|\Big) \Theta\Big(p_t^{\rm JV} - k_{t1}\Big)$$

$$+ e^{-R_{NLL}(L_{t1})} R'_{LL}(k_{t1}) \int_0^{k_{t1}} \frac{dk_{ts}}{k_{ts}} \frac{d\phi_s}{2\pi} \left[\left(\delta R'(k_{t1}) + R''_{LL}(k_{t1}) \ln \frac{k_{t1}}{k_{ts}} - \frac{d}{dL_{t1}} \right) \mathcal{L}_{NLL}(L_{t1}) \right]$$

$$\times \left[\Theta\Big(p_t^{\rm HV} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts}|\Big) - \Theta\Big(p_t^{\rm HV} - |\sum_{j=1}^{n+1} \vec{k}_{tj}|\Big) \right] \Theta\Big(p_t^{\rm JV} - k_{t1}\Big)$$

$$+ \frac{1}{2!} \mathcal{L}_{NLL}(L_{t1}) e^{-R_{NLL}(L_{t1})} \Big(R'_{LL}(k_{t1})\Big)^2 \int_0^{k_{t1}} \frac{dk_{ts_1}}{k_{ts_1}} \frac{dk_{ts_2}}{k_{ts_2}} \int d\Delta\eta_{s_1 s_2} \frac{d\phi_{s_1}}{2\pi} \frac{d\Delta\phi_{s_1 s_2}}{2\pi} \left(2 C_A \frac{\alpha_S(k_{t1})}{\pi} \right)$$

$$\times \Theta\Big(p_t^{\rm HV} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) \Bigg[J_R(s_1, s_2) \Big(\Theta\Big(p_t^{\rm JV} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) - \Theta\Big(p_t^{\rm JV} - \max(k_{ts_1}, k_{ts_2})\Big) \Big)$$

$$+ \mathcal{C}(s_1, s_2) \Big[1 - J_R(s_1, s_2) \Big] \Big(\Theta\Big(p_t^{\rm JV} - \max(k_{ts_1}, k_{ts_2})\Big) - \Theta\Big(p_t^{\rm JV} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) \Big) \Bigg] \Theta\Big(p_t^{\rm JV} - k_{t1}\Big) + \ldots \Bigg\}$$

Joint Higgs and jet p_t resummation at NNLL

- From contributions detailed above just need to combine measurement functions!

clustering and correlated NNLL ptj corrections
with ptH measurement function

$$\Sigma_{NNLL}(p_t^{\mathrm{H}} < p_t^{\mathrm{HV}}, p_t^{\mathrm{J}} < p_t^{\mathrm{JV}}) =$$

$$\int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \Bigg\{ \frac{d}{dL_{t1}} \left[-e^{-R_{NNLL}(L_{t1})} \mathcal{L}_{NNLL}(L_{t1}) \right] \Theta\Big(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}|\Big) \Theta\Big(p_t^{\mathrm{JV}} - k_{t1}\Big)$$

$$+ e^{-R_{NLL}(L_{t1})} R'_{LL}(k_{t1}) \int_0^{k_{t1}} \frac{dk_{ts}}{k_{ts}} \frac{d\phi_s}{2\pi} \left[\left(\delta R'(k_{t1}) + R''_{LL}(k_{t1}) \ln \frac{k_{t1}}{k_{ts}} - \frac{d}{dL_{t1}} \right) \mathcal{L}_{NLL}(L_{t1}) \right]$$

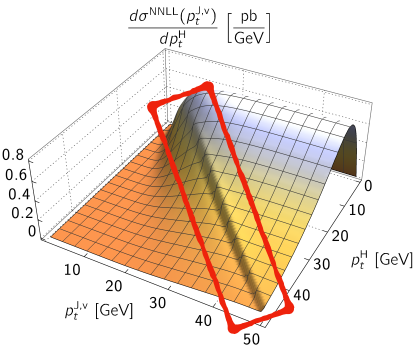
$$\times \left[\Theta\Big(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts}|\Big) - \Theta\Big(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj}|\Big) \right] \Theta\Big(p_t^{\mathrm{JV}} - k_{t1}\Big)$$

$$+ \frac{1}{2!} \mathcal{L}_{NLL}(L_{t1}) e^{-R_{NLL}(L_{t1})} \Big(R'_{LL}(k_{t1})\Big)^2 \int_0^{k_{t1}} \frac{dk_{ts_1}}{k_{ts_1}} \frac{dk_{ts_2}}{k_{ts_2}} \int d\Delta\eta_{s_1 s_2} \frac{d\phi_{s_1}}{2\pi} \frac{d\Delta\phi_{s_1 s_2}}{2\pi} \left(2 C_A \frac{\alpha_S(k_{t1})}{\pi} \right)$$

$$\times \Theta\Big(p_t^{\mathrm{HV}} - |\sum_{j=1}^{n+1} \vec{k}_{tj} + \vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) \Big[J_R(s_1, s_2) \Big(\Theta\Big(p_t^{\mathrm{JV}} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) - \Theta\Big(p_t^{\mathrm{JV}} - \max(k_{ts_1}, k_{ts_2})\Big) \Big)$$

$$+ \mathcal{C}(s_1, s_2) \Big[1 - J_R(s_1, s_2) \Big] \Big(\Theta\Big(p_t^{\mathrm{JV}} - \max(k_{ts_1}, k_{ts_2})\Big) - \Theta\Big(p_t^{\mathrm{JV}} - |\vec{k}_{ts_1} + \vec{k}_{ts_2}|\Big) \Big) \Big] \Theta\Big(p_t^{\mathrm{JV}} - k_{t1}\Big) + \dots \Bigg\}$$

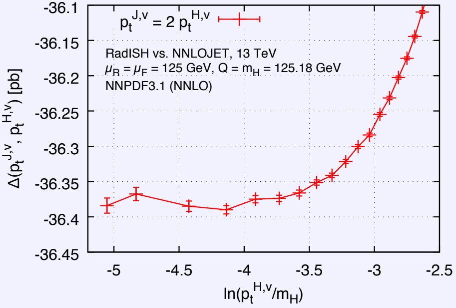
NNLL cross section differential in p_t^{H}, cumulative in $p_t^{\mathrm{J}} < p_t^{\mathrm{JV}}$



- At given p_t^{JV} this is the resummed p_t^{H} spectrum in the 0-jet bin.
- Peaked structure (power like) at small p_t^{H}; Sudakov suppression at small p_t^{J}.
- Sudakov shoulder [Catani, Webber, 9710333]: integrable singularity beyond LO around $p_t^{\mathrm{H}} \sim p_t^{\mathrm{JV}}$.
- Logarithms associated with the shoulder are resummed in the region $p_t^{\mathrm{H}} \sim p_t^{\mathrm{JV}} \ll m_H$ (absence of the integrable singularity there).

Accuracy check

Difference between $\Sigma_{NNLL}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})$ expanded at $\mathcal{O}(\alpha_s^2)$ and fixed order at $\mathcal{O}(\alpha_s^2)$



- Difference tends to an $\mathcal{O}(\alpha_s^2)$ constant (i.e. N^{3}LL) at very small $p_t^{\rm HV}$ in the region of the shoulder, $p_t^{\rm JV} = 2\, p_t^{\rm HV}$.
- Very strong check: NNLL control of logarithms of the shoulder when $p_t^{\rm J} \sim p_t^{\rm H} \ll m_H$.
- Analogously for $p_t^{\rm H} \ll p_t^{\rm J} \ll m_H$, and $p_t^{\rm J} \ll p_t^{\rm H} \ll m_H$.
  Logarithms correctly accounted for regardless of hierarchy between $p_t^{\rm H}$ and $p_t^{\rm J}$ (if $\ll m_H$).

Multiplicative matching to fixed order

$$\Sigma_{MATCH}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})$$
$$= \frac{\Sigma_{NNLL}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})}{\Sigma_{NNLL}(p_t^{\rm J} < p_t^{\rm JV})} \left[\Sigma_{NNLL}(p_t^{\rm J} < p_t^{\rm JV}) \frac{\Sigma_{NNLO}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})}{\Sigma_{EXP}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})}\right]_{\mathcal{O}(\alpha_{\rm S}^2)}$$

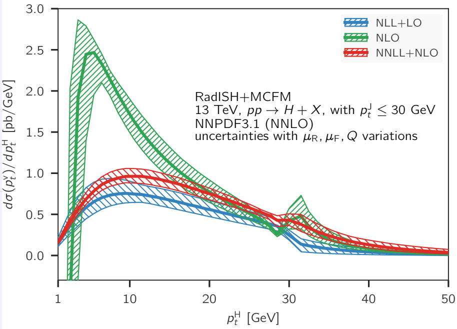
where

$$\Sigma_{NNLO}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV}) \quad = \quad \sigma_{NNLO}^{pp\to H} - \int \left(\Theta(p_t^{\rm J} > p_t^{\rm JV}) \ \vee \ \Theta(p_t^{\rm H} > p_t^{\rm HV})\right) d\sigma_{NLO}^{pp\to Hj}.$$

- $\Sigma_{EXP}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})$ = expansion of $\Sigma_{NNLL}(p_t^{\rm H} < p_t^{\rm HV}, p_t^{\rm J} < p_t^{\rm JV})$ up to $\mathcal{O}(\alpha_{\rm S}^2)$ relative to Born.
- $\Sigma_{NNLL}(p_t^{\rm J} < p_t^{\rm JV}) \equiv \Sigma_{NNLL}(p_t^{\rm H} < \infty, p_t^{\rm J} < p_t^{\rm JV})$ avoids (N^{3}LL) K factors at large $p_t^{\rm HV}$. NNLL+NLO $p_t^{\rm J}$ ($p_t^{\rm H}$) cross section recovered for $p_t^{\rm HV} \to \infty$ ($p_t^{\rm JV} \to \infty$) (NLO refers to the spectrum).
- At NLO, the multiplicative scheme includes constant terms of $\mathcal{O}(\alpha_{\rm S}^2)$ from the fixed order, absent in an additive scheme (NNLL').

LHC results: Higgs p_t with a jet veto

Multiplicative matching to fixed order (NLO $H + j$ from MCFM, [Campbell, Ellis, Giele, 1503.06182])



- Resummed results display good perturbative convergence below 10 GeV. Above, effects from the large Higgs K factor.
- NNLL+NLO has less than 10% uncertainty for $p_t^H < p_t^{JV}$.
- Much reduced sensitivity to the shoulder at 30 GeV with respect to fixed order.

NLL joint resummation in b space

- Differential control in momentum space provides guidance to an analytic formula for double-differential resummation in impact-parameter space
- NLL $p_t^{\rm H}$ differential cross section (toy model with scale-independent PDFs for the sake of the argument): $p_t^{\rm H}$ measurement function completely factorises (by construction)

$$\frac{d\sigma}{d^2 \vec{p}_t^{\rm H}} = \sigma_B \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t^{\rm H}} \sum_{n=0}^{\infty} \frac{1}{n!} \int [dk_i]|M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{ti}} - 1 \right)$$

- Factorisation implies a factor $e^{i\vec{b}\cdot\vec{k}_{ti}}$ per emission: jet-veto constraints on k_{ti} can be applied at the level of b-space integrand!
- Jet veto on real radiation at NLL: $\Theta(p_t^{\rm JV} - \max(k_{t1},...,k_{tn})) = \prod_{i=1}^{n} \Theta(p_t^{\rm JV} - k_{ti})$
- Double-differential resummation at NLL in b space

$$\begin{aligned} \frac{d\sigma(p_t^{\rm J} < p_t^{\rm JV})}{d^2\vec{p}_t^{\rm H}} &= \sigma_B \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t^{\rm H}} \sum_{n=0}^{\infty} \frac{1}{n!} \int [dk_i]|M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{ti}} \Theta(p_t^{\rm JV} - k_{ti}) - 1 \right) \\ &= \sigma_B \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t^{\rm H}} e^{-\int [dk]|M(k)|^2 \left(1 - e^{i\vec{b}\cdot\vec{k}_t} \Theta(p_t^{\rm JV} - k_t)\right)} \end{aligned}$$

NNLL joint resummation in b space

- NNLL clustering/correlated corrections in the b-space integrand.

$$\mathcal{F}_{\rm clust} = \frac{1}{2!}\int [dk_a][dk_b]|M(k_a)|^2|M(k_b)|^2\, J_R(a,b)\, e^{i\vec{b}\cdot(\vec{k_{ta}}+\vec{k_{tb}})} \times \left[\Theta(p_t^{\rm JV} - |\vec{k}_{ta}+\vec{k}_{tb}|) - \Theta(p_t^{\rm JV} - \max\{k_{ta},k_{tb}\})\right]$$

$$\mathcal{F}_{\rm correl} = \frac{1}{2!}\int [dk_a][dk_b]|\tilde{M}(k_a,k_b)|^2\, \left[1 - J_R(a,b)\right]\, e^{i\vec{b}\cdot(\vec{k_{ta}}+\vec{k_{tb}})} \times \left[\Theta(p_t^{\rm JV} - \max\{k_{ta},k_{tb}\}) - \Theta(p_t^{\rm JV} - |\vec{k}_{ta}+\vec{k}_{tb}|)\right]$$

- Joint resummation at NNLL in b space

$$\frac{d\sigma(p_t^{\rm J,v})}{dy^{\rm H}d^2\vec{p}_t^{\rm H}} = M^2_{\rm gg\to H}\,\mathcal{H}(\alpha_s(m_{\rm H}))\int_{\mathcal{C}_1}\frac{d\nu_1}{2\pi i}\int_{\mathcal{C}_2}\frac{d\nu_2}{2\pi i}x_1^{-\nu_1}\,x_2^{-\nu_2}\int\frac{d^2\vec{b}}{4\pi^2}e^{-i\vec{b}\cdot\vec{p}_t^{\rm H}}\,e^{-S_{\rm NNLL}}\,(1+\mathcal{F}_{\rm clust}+\mathcal{F}_{\rm correl})$$
$$\times f_{\nu_1,a_1}(b_0/b)\,f_{\nu_2,a_2}(b_0/b)\left[\mathcal{P}\,e^{-\int_{p_t^{\rm J,v}}^{m_{\rm H}}\frac{d\mu}{\mu}\mathbf{\Gamma}_{\nu_1}(\alpha_s(\mu))J_0(b\mu)}\right]_{c_1a_1}\left[\mathcal{P}\,e^{-\int_{p_t^{\rm J,v}}^{m_{\rm H}}\frac{d\mu}{\mu}\mathbf{\Gamma}_{\nu_2}(\alpha_s(\mu))J_0(b\mu)}\right]_{c_2a_2}$$
$$\times C_{\nu_1,gb_1}(\alpha_s(b_0/b))\,C_{\nu_2,gb_2}(\alpha_s(b_0/b))\left[\mathcal{P}\,e^{-\int_{p_t^{\rm J,v}}^{m_{\rm H}}\frac{d\mu}{\mu}\mathbf{\Gamma}^{(C)}_{\nu_1}(\alpha_s(\mu))J_0(b\mu)}\right]_{c_1b_1}\left[\mathcal{P}\,e^{-\int_{p_t^{\rm J,v}}^{m_{\rm H}}\frac{d\mu}{\mu}\mathbf{\Gamma}^{(C)}_{\nu_2}(\alpha_s(\mu))J_0(b\mu)}\right]_{c_2b_2},$$

Outlook

- Theoretical control of multi-differential information to exploit LHC potential in the Higgs sector (and much more!).
- First simultaneous resummation for a double-differential kinematic observable defined through a jet algorithm in hadronic collisions.
- Formulation in direct space (RadISH) provides guidance to compact analytic formulation in b space.
- Logarithms of p_t^{H}/m_H and p_t^{J}/m_H controlled at NNLL when transverse momenta $\ll m_H$.
- This does not rely on hierarchy between p_t^{H} and p_t^{J}. Sudakov shoulder $p_t^{\mathrm{H}} \sim p_t^{\mathrm{J}}$ resummed in the small-p_t region.

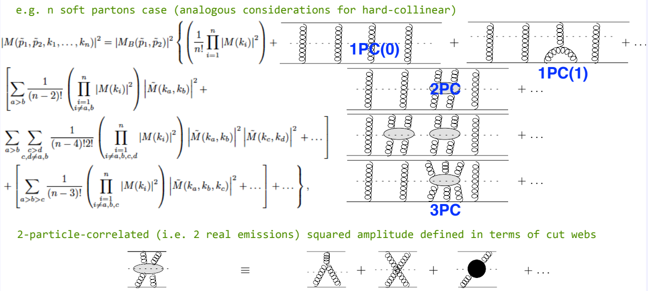
Thank you for your attention

Backup

RadISH resummation: organisation into correlated matrix elements

$$\Sigma(V<\nu) = \int d\Phi_B\, \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(k_1,\ldots,k_n)|^2\, \Theta(\nu - V(k_1,\ldots,k_n))$$

- $\mathcal{V}(\Phi_B)$ = all-order virtual form factor (see [Dixon, Magnea, Sterman, 0805.3515]).
- Multiple emission matrix element $|M(k_1,\ldots,k_n)|^2$ organised into *n*-particle-correlated (*n*PC) blocks $|\tilde{M}(k_1,\ldots,k_n)|^2$.



- Higher-orders in α_S at fixed *n*, or larger *n* $\Longrightarrow$ logarithmically suppressed

$p_t^{\rm H}$ resummation: finiteness in four dimensions, NLL case

$$\frac{d\Sigma_{\rm NLL}(p_t)}{d\Phi_B} = \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} k_{t1} \frac{\partial}{\partial k_{t1}} \left(-e^{-R(k_{t1})} \mathcal{L}_{\rm NLL}(k_{t1})\right) \times$$
$$\times \underbrace{\epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1})\right)}_{\equiv\ \int d\mathcal{Z}[\{R', k_i\}]} \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|).$$

- Luminosity $\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{a,b} \frac{d|M_B|^2_{ab}}{d\Phi_B} f_a(x_1, k_{t1}) f_b(x_2, k_{t1})$.
- $\int d\mathcal{Z}[\{R', k_i\}]\Theta$ finite as $\epsilon \to 0$:

$$\epsilon^{R'(k_{t1})} = 1 - R'(k_{t1}) \ln(1/\epsilon) + \ldots = 1 - \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_t}{k_t} R'(k_{t1}) + \ldots,$$

$$\int d\mathcal{Z}[\{R', k_i\}]\Theta = \left[1 - \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_t}{k_t} R'(k_{t1}) + \ldots\right] \left[\Theta(p_t - |\vec{k}_{t1}|) + \int_0^{2\pi} \frac{d\phi_2}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{t2}}{k_{t2}} R'(k_{t1}) \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) + \ldots\right]$$
$$= \Theta(p_t - |\vec{k}_{t1}|) + \int_0^{2\pi} \frac{d\phi_2}{2\pi} \underbrace{\int_0^{k_{t1}}}_{\epsilon \to 0} \frac{dk_{t2}}{k_{t2}} R'(k_{t1}) \underbrace{\left[\Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) - \Theta(p_t - |\vec{k}_{t1}|)\right]}_{\text{finite: real-virtual cancellation}} + \ldots$$

Small-$p_t^{\rm H}$ behaviour at NLL

$$\frac{d^2\Sigma(p_t)}{d^2\vec{p}_t d\Phi_B} \propto \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} R'(k_{t1}) \int d\mathcal{Z}[\{R', k_i\}] \delta^{(2)} \left(\vec{p}_t - \left(\vec{k}_{t1} + \cdots + \vec{k}_{t(n+1)}\right)\right).$$

- Fourier transform of the delta: $\delta^{(2)}\left(\vec{p}_t - |\sum_i \vec{k}_{ti}|\right) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n+1} e^{i\vec{b}\cdot\vec{k}_{ti}}.$
- Integrate over azimuthal direction of all $\vec{k}_{ti}$ and of $\vec{p}_t$:

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} = \sigma^{(0)}(\Phi_B)\, p_t \int b\, db J_0(p_t b) \int \frac{dk_{t1}}{k_{t1}} e^{-R(k_{t1})} R'(k_{t1}) J_0(bk_{t1}) \times \exp\left\{-R'(k_{t1}) \int_0^{k_{t1}} \frac{dk_t}{k_t}(1 - J_0(bk_t))\right\}.$$

- In the limit where $M \gg k_{t1} \gg p_t$ this gives

$$\int b\, db J_0(p_t b) J_0(bk_{t1}) \exp\left\{-R'(k_{t1}) \int_0^{k_{t1}} \frac{dk_t}{k_t}(1 - J_0(bk_t))\right\} \simeq 4 \frac{k_{t1}^{-2}}{R'(k_{t1})}$$

$$\Longrightarrow \quad \frac{d^2\Sigma(v)}{dp_t d\Phi_B} = 4\, \sigma^{(0)}(\Phi_B)\, p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})}.$$

Treatment of Landau pole

- Landau singularity in the radiator and in the coupling:

$$\alpha_S(\mu_R)\beta_0 \ln(Q/k_{t1}) = \frac{1}{2} \quad \Longrightarrow \quad k_{t1} \sim 0.1\ GeV \quad for \quad \mu_R \sim Q \sim m_H$$

- Perturbative prediction is cut off below this scale, by setting probabilities to 0.

- This cutoff has no visible consequence: low $p_t^{\rm H}$ dominated by k_{ti}'s > 1 GeV

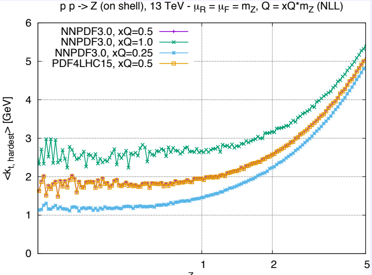


Figure credit: P.F. Monni

- This does not imply absence of non-perturbative corrections (not studied) at scales of a GeV

Equivalence of direct-space p_t^{H} resummation with b space

- Take direct-space formula for $d\Sigma/d\vec{p}_t$, Fourier-transform the $\delta^{(2)}(p_t - |\sum_i \vec{k}_{ti}|)$, and get

$$\frac{d}{dp_t}\hat{\Sigma}^{c_1 c_2}_{N_1,N_2}(p_t) = \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(M))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(M))\, p_t \int b\, db J_0(p_t b) \int_0^M \frac{dk_{t1}}{k_{t1}}$$
$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\Gamma_{N_{\ell_1}}(\alpha_s(k_{t1})) + \Gamma^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right) J_0(bk_{t1})$$
$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_{k_{t1}}^{M}\frac{dk_t}{k_t}\left(\mathbf{R}'_{\ell}(k_t) + \frac{\alpha_s(k_t)}{\pi}\Gamma_{N_\ell}(\alpha_s(k_t)) + \Gamma^{(C)}_{N_\ell}(\alpha_s(k_t))\right) J_0(bk_t)\right\}$$
$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_{\epsilon k_{t1}}^{M}\frac{dk_t}{k_t}\left(\mathbf{R}'_{\ell}(k_t) + \frac{\alpha_s(k_t)}{\pi}\Gamma_{N_\ell}(\alpha_s(k_t)) + \Gamma^{(C)}_{N_\ell}(\alpha_s(k_t))\right)(1 - J_0(bk_t))\right\}.$$

- Take limit $\epsilon \to 0$. Integrand in k_{t1} is a total derivative and integrates to 1, leaving

$$\frac{d}{dp_t}\hat{\Sigma}^{c_1 c_2}_{N_1,N_2}(p_t) = \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(M))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(M))\, p_t \int b\, db J_0(p_t b)$$
$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_{0}^{M}\frac{dk_t}{k_t}\left(\mathbf{R}'_{\ell}(k_t) + \frac{\alpha_s(k_t)}{\pi}\Gamma_{N_\ell}(\alpha_s(k_t)) + \Gamma^{(C)}_{N_\ell}(\alpha_s(k_t))\right)(1 - J_0(bk_t))\right\}.$$

- Transform $1 - J_0$ in a Θ up to subleading logarithms, and plug this into the hadronic cross section, to get the traditional b-space formulation.

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial \ln(Mb/b_0)^3}\Theta(k_t - \frac{b_0}{b}) + \ldots.$$

- ζ_3 term starts at N^{3}LL, is resummation-scheme change w.r.t. b space.

Generating secondary radiation as a simplified parton shower

- Secondary radiation:

$$
\begin{aligned}
d\mathcal{Z}[\{R', k_i\}] &= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})} \\
&= \sum_{n=0}^{\infty} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t(i-1)}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})}, \\
\epsilon^{R'(k_{t1})} &= e^{-R'(k_{t1}) \ln 1/\epsilon} = \prod_{i=2}^{n+2} e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}},
\end{aligned}
$$

with $k_{t(n+2)} = \epsilon\, k_{t1}$.

- Each secondary emissions has differential probability

$$
dw_i = \frac{d\phi_i}{2\pi} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = \frac{d\phi_i}{2\pi} d\left(e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} \right).
$$

- $k_{t(i-1)} \geq k_{ti}$. Scale k_{ti} extracted by solving $e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = r$, with r uniform random number in $[0, 1]$.

- Extract ϕ_i randomly in $[0, 2\pi]$.

Modified logarithms

- Ensure resummation does not affect the hard region of the spectrum.
- Supplement logarithms with power-suppressed terms, irrelevant at small k_{t1}, that enforce resummation to vanish at $k_{t1} \gg Q$.
- Modified logarithms

$$\ln\left(\frac{Q}{k_{t1}}\right) \quad \rightarrow \quad \tilde{L} = \frac{1}{p}\ln\left(\left(\frac{Q}{k_{t1}}\right)^{p} + 1\right).$$

- Q = resummation scale of $\mathcal{O}(M)$, varied to assess systematics due to higher logarithms.
- p = chosen so that resummation vanishes faster than fixed order in the hard region.
- Checked that variation of p does not induce visible effects.
- Modified logarithms map $k_{t1} = Q$ into $k_{t1} \to \infty$.

Luminosity to NNLL

$$\mathcal{L}_{NNLL}(k_{t1}) = \sum_{c,c'} \frac{d|\mathcal{M}_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} f_i\left(\mu_F e^{-L_{t1}}, \frac{x_1}{z_1}\right) f_j\left(\mu_F e^{-L_{t1}}, \frac{x_2}{z_2}\right)$$
$$\times \left\{ \delta_{ci}\delta_{c'j}\delta(1-z_1)\delta(1-z_2) \left(1 + \frac{\alpha_S(\mu_R)}{2\pi} H^{(1)}(\mu_R, x_Q)\right) \right.$$
$$\left. + \frac{\alpha_S(\mu_R)}{2\pi} \frac{1}{1 - 2\alpha_S(\mu_R)\beta_0 L_{t1}} \left(C^{(1)}_{ci}(z_1, \mu_F, x_Q)\delta(1-z_2)\delta_{c'j} + \{z_1 \leftrightarrow z_2; c, i \leftrightarrow c', j\}\right) \right\}$$

with $L_{t1} = \ln(Q/k_{t1})$.

Sudakov radiator for joint resummation in b space

$$
\begin{aligned}
S_{NNLL} &= \int [dk]|M(k)|^2 \left(1 - e^{i\vec{b}\cdot\vec{k}_t}\Theta(p_t^{\mathrm{JV}} - k_t)\right) \\
&= \int \frac{dk_t}{k_t} R'(k_t)\left(1 - J_0(bk_t)\Theta(p_t^{\mathrm{JV}} - k_t)\right) \\
&= \int \frac{dk_t}{k_t} R'(k_t)\left(1 - J_0(bk_t)\right) + \int \frac{dk_t}{k_t} R'(k_t) J_0(bk_t)\Theta(k_t - p_t^{\mathrm{JV}}) \\
&= -L_b g_1(\alpha_S L_b) - g_2(\alpha_S L_b) - \frac{\alpha_S}{\pi} g_3(\alpha_S L_b) + \int_{p_t^{\mathrm{JV}}}^{m_H} \frac{dk_t}{k_t} R'(k_t) J_0(bk_t)
\end{aligned}
$$

with $L_b = \ln(m_H b/b_0)$, $b_0 = 2e^{-\gamma_E}$.

Analogously for PDFs and coefficient functions.