Next-to-leading power threshold effects

Melissa van Beekveld (PhD student Radboud University / NIKHEF)

1905.08741 [MvB, Wim Beenakker, Eric Laenen, Chris White]
1905.11771 [MvB, Wim Beenakker, Rahul Basu, Eric Laenen, Anuradha Misra, Patrick Motylinski]

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Warning!
Talk is going to be filled with acronyms

Sorry…

What is NLP?

Consider a general production process, plus additional radiation near threshold.
For a generic threshold variable $(1-z) \to 0$ we can write:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

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Leading-power contributions

LP LP

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

- Universal process-independent form
- Localized at threshold
- Resummation well understood

Next-to-leading-power contributions

NLP

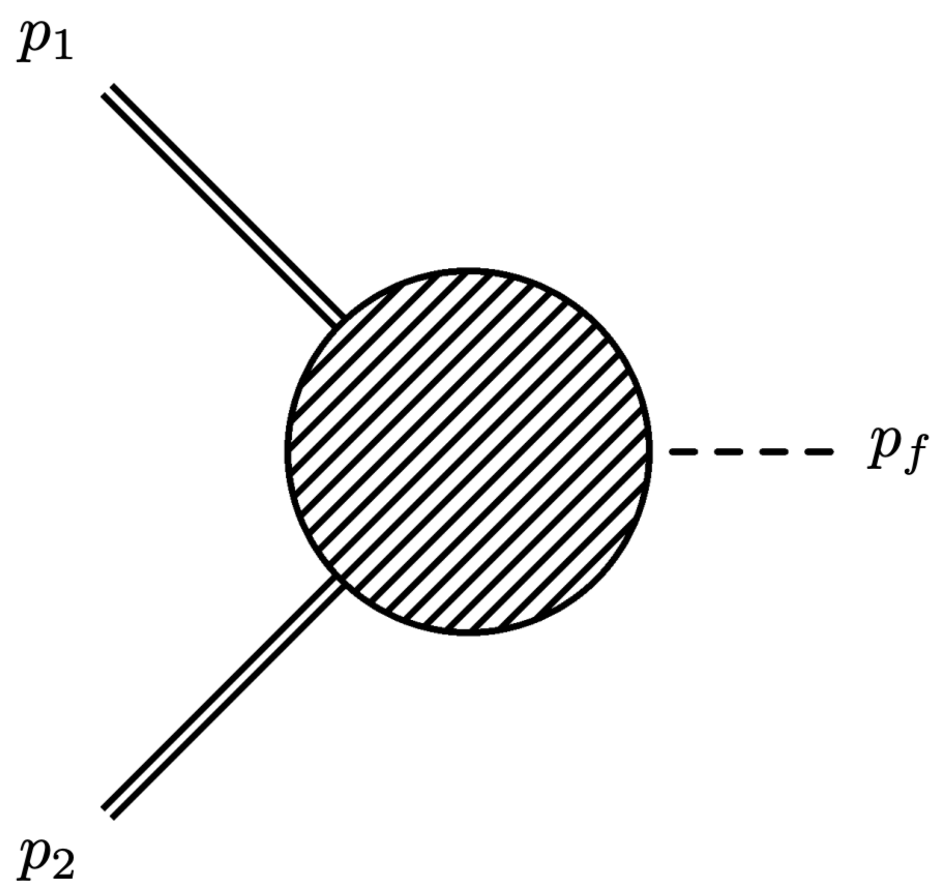
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

- Suppressed to leading power, but still singular
- No *general* resummation framework for these!
- Might be relevant experimentally
- Important check of higher order corrections

1. What is the origin of these next-to-leading power logarithms?

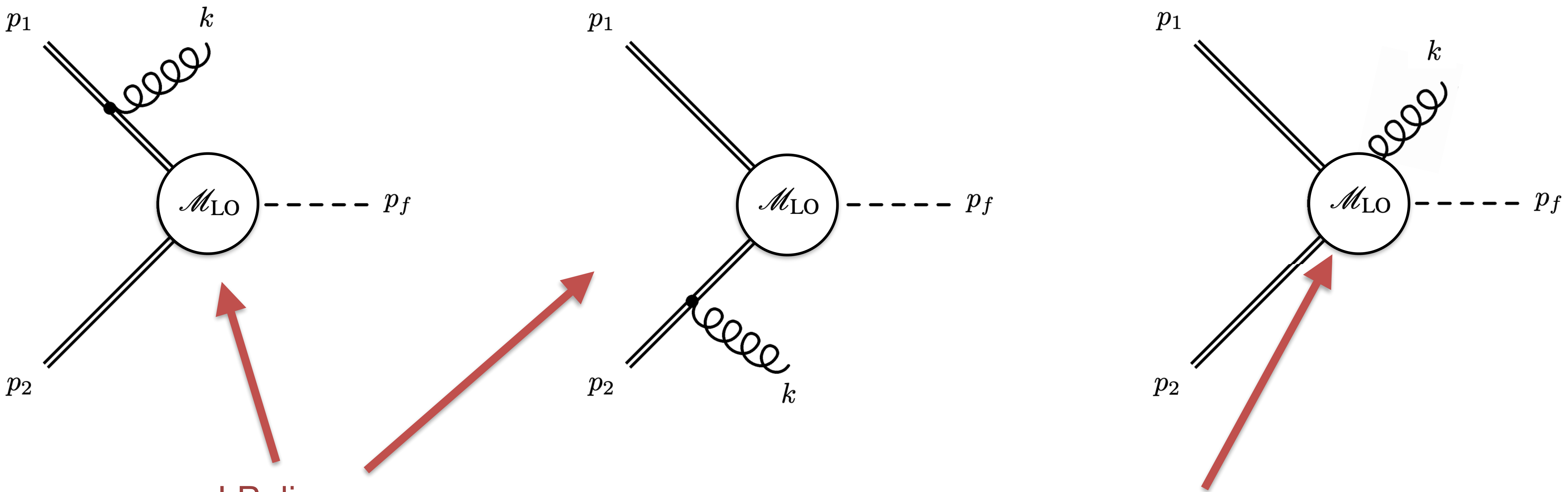
Universality of NLP logs

Let us first examine what happens when a *colorless* final state is produced



[1706.04018]

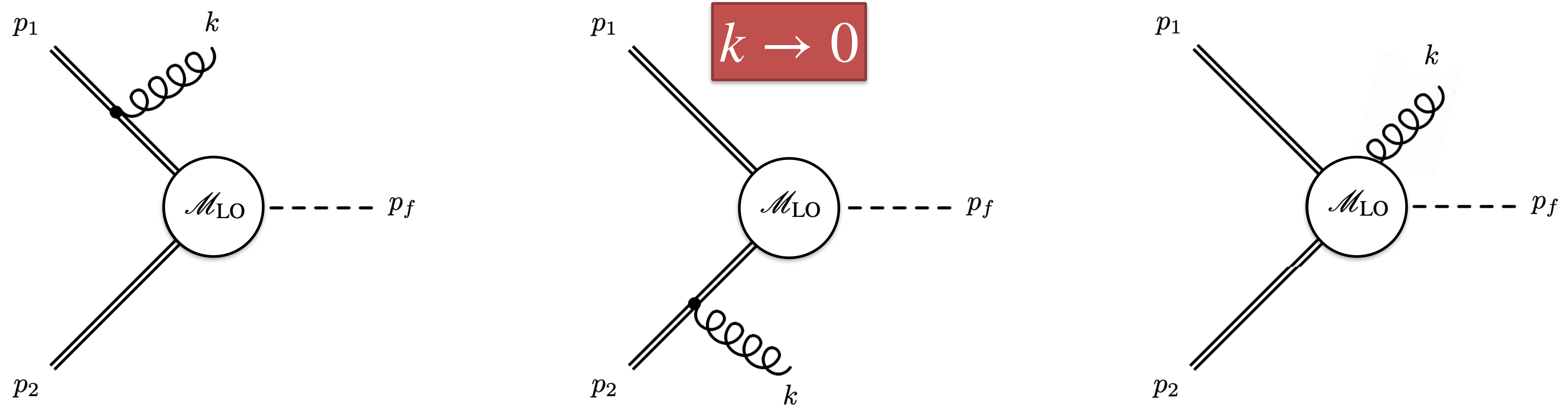
NLO Amplitude at NLP



LP diagrams

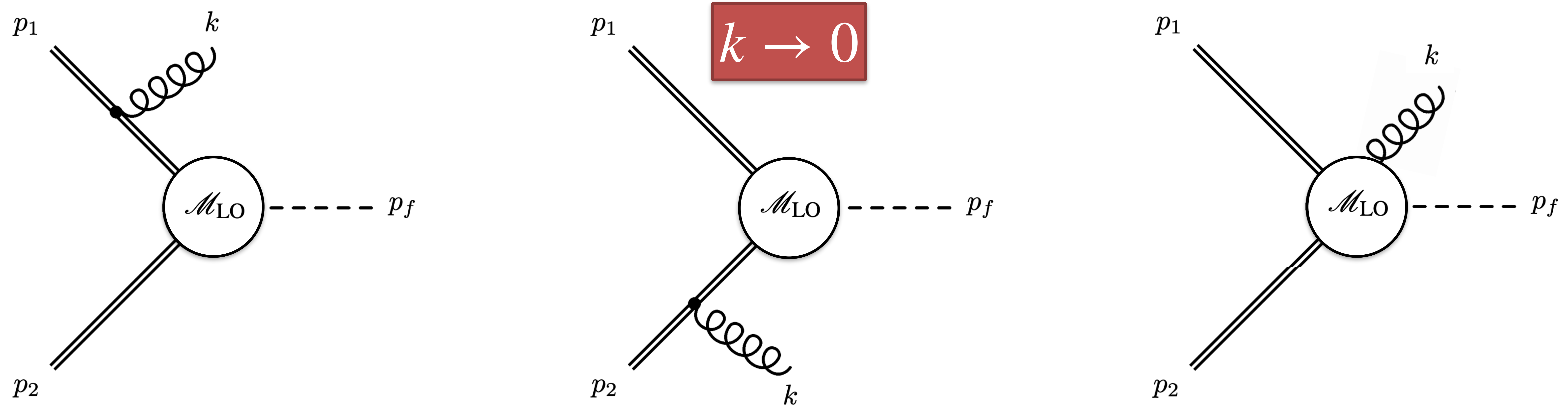
At NLP, also one ‘internal’ emission contributes (Low’s theorem)

NLO Amplitude at NLP



$$\mathcal{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

NLO Amplitude at NLP



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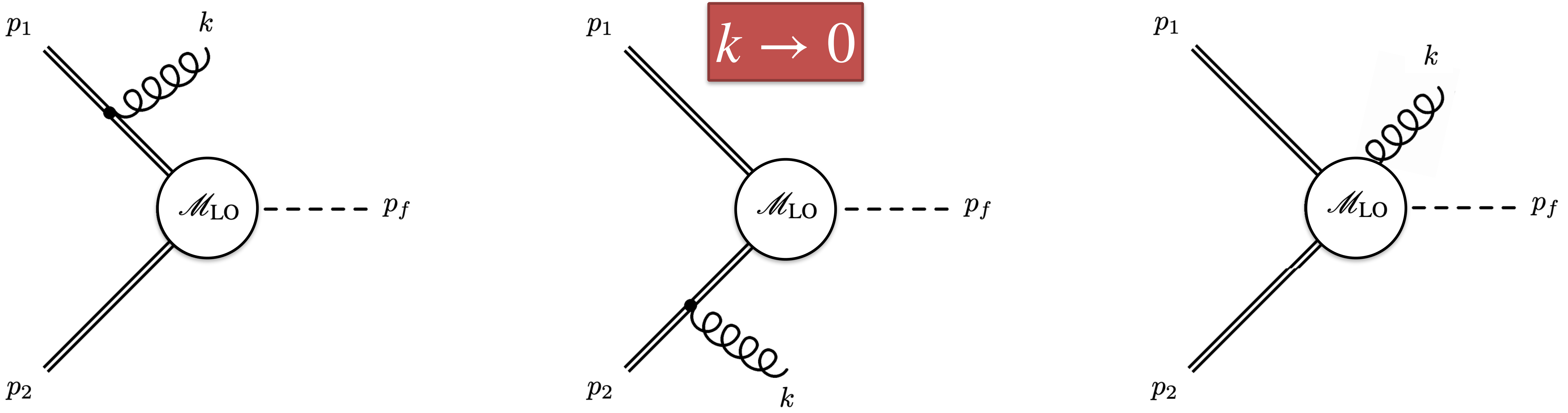
NLO Amplitude at NLP



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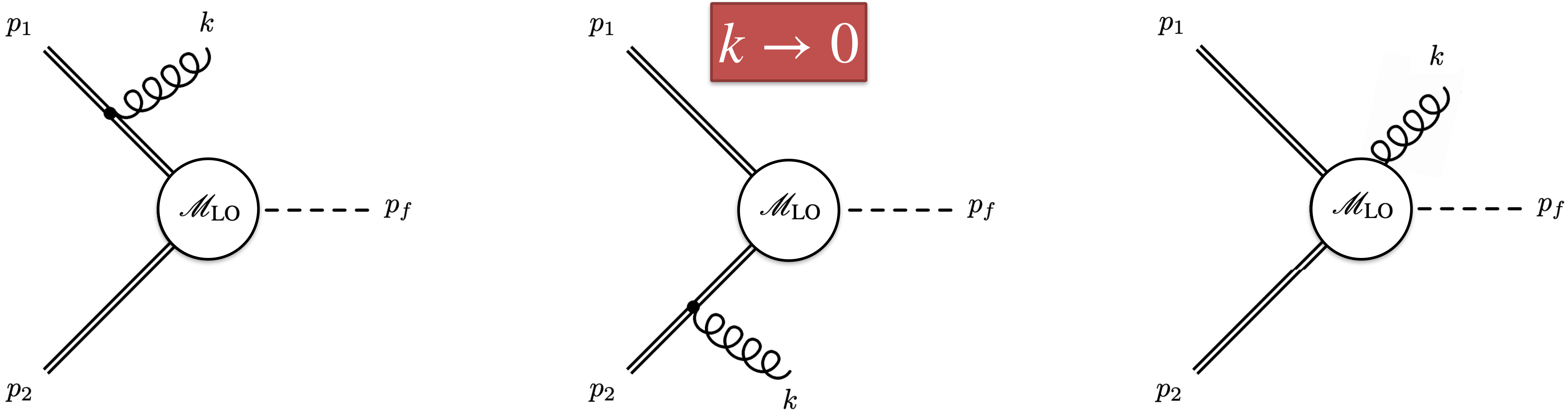
NLO Amplitude at NLP



$$\mathcal{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

NLO Amplitude at NLP

Eikonal

$$\mathcal{A}_{\text{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\text{LO}}\, \varepsilon_\sigma^*(k)$$

$$\mathcal{O}\left(\frac{1}{k}\right)$$

NLO Amplitude at NLP

Scalar

$$\mathcal{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

$$\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}(1)$$

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NLO Amplitude at NLP

$$\Sigma^{\sigma\alpha} \begin{cases} \frac{i}{4}[\gamma^\sigma, \gamma^\alpha] \equiv S^{\sigma\alpha} \\ i(g^{\rho\sigma}g^{\alpha\nu} - g^{\sigma\nu}g^{\alpha\rho}) \equiv M^{\sigma\alpha,\rho\nu} \end{cases}$$

Spin

$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k}L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

$\mathcal{O}(1)$

Needs to be inserted at the right place in the matrix element!

NLO Amplitude at NLP

$$L_i^{\sigma\alpha} = -i\left(p_i^\sigma \frac{\partial}{\partial p_{i\alpha}} - p_i^\alpha \frac{\partial}{\partial p_{i\sigma}}\right)$$

Orbital

$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

$\mathcal{O}(1)$

NLO Amplitude at NLP

$$\mathscr{A}_{\text{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\text{LO}}\, \varepsilon_\sigma^*(k)$$

$$= \mathscr{A}_{\text{scal}} + \mathscr{A}_{\text{spin}} + \mathscr{A}_{\text{orb}}$$

Result is derived by using the soft approximation of the matrix element & the Ward identity
Can also be derived from an all-order factorization theorem (see e.g. 1503.05156, 1610.06842)

Towards the NLP cross section

$$|\mathscr{A}_{\text{NLP}}|^2 = \sum_{\text{colors}} |\mathscr{A}_{\text{scal}}|^2 + 2\text{Re}\left[\left(\mathscr{A}_{\text{spin}} + \mathscr{A}_{\text{orb}}\right)^\dagger \mathscr{A}_{\text{scal}}\right]$$

Towards the NLP cross section

$$
\begin{aligned}
|\mathscr{A}_{\rm NLP}|^2 &= \sum_{\rm colors} |\mathscr{A}_{\rm scal}|^2 + 2{\rm Re}\left[(\mathscr{A}_{\rm spin} + \mathscr{A}_{\rm orb})^\dagger \mathscr{A}_{\rm scal}\right] \\
&= K \frac{2p_1\cdot p_2}{p_1\cdot k p_2\cdot k} |\mathscr{M}_{\rm LO}(p_1+\delta p_1, p_2+\delta p_2)|^2
\end{aligned}
$$

Towards the NLP cross section

$$|\mathscr{A}_{\rm NLP}|^2 = \sum_{\rm colors} |\mathscr{A}_{\rm scal}|^2 + 2{\rm Re}\left[(\mathscr{A}_{\rm spin} + \mathscr{A}_{\rm orb})^\dagger \mathscr{A}_{\rm scal}\right]$$

$$= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\rm LO}(p_1 + \delta p_1, p_2 + \delta p_2)|^2$$

Eikonal factor

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Towards the NLP cross section

$$
\begin{aligned}
|\mathscr{A}_{\rm NLP}|^2 &= \sum_{\rm colors} |\mathscr{A}_{\rm scal}|^2 + 2{\rm Re}\left[\left(\mathscr{A}_{\rm spin} + \mathscr{A}_{\rm orb}\right)^\dagger \mathscr{A}_{\rm scal}\right] \\
&= K \frac{2p_1 \cdot p_2}{p_1 \cdot k\, p_2 \cdot k} \left|\mathscr{M}_{\rm LO}\left(p_1 + \delta p_1, p_2 + \delta p_2\right)\right|^2
\end{aligned}
$$

Shift in Born matrix element

Towards the NLP cross section

$$|\mathscr{A}_{\rm NLP}|^2 = \sum_{\rm colors} |\mathscr{A}_{\rm scal}|^2 + 2{\rm Re}\left[(\mathscr{A}_{\rm spin} + \mathscr{A}_{\rm orb})^\dagger \mathscr{A}_{\rm scal}\right]$$

$$= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\rm LO}(p_1 + \delta p_1, p_2 + \delta p_2)|^2$$

Integrate over phase space → one obtains all NLP terms!

Demonstrated for DY, (di-)Higgs, VV, in [1706.04018]

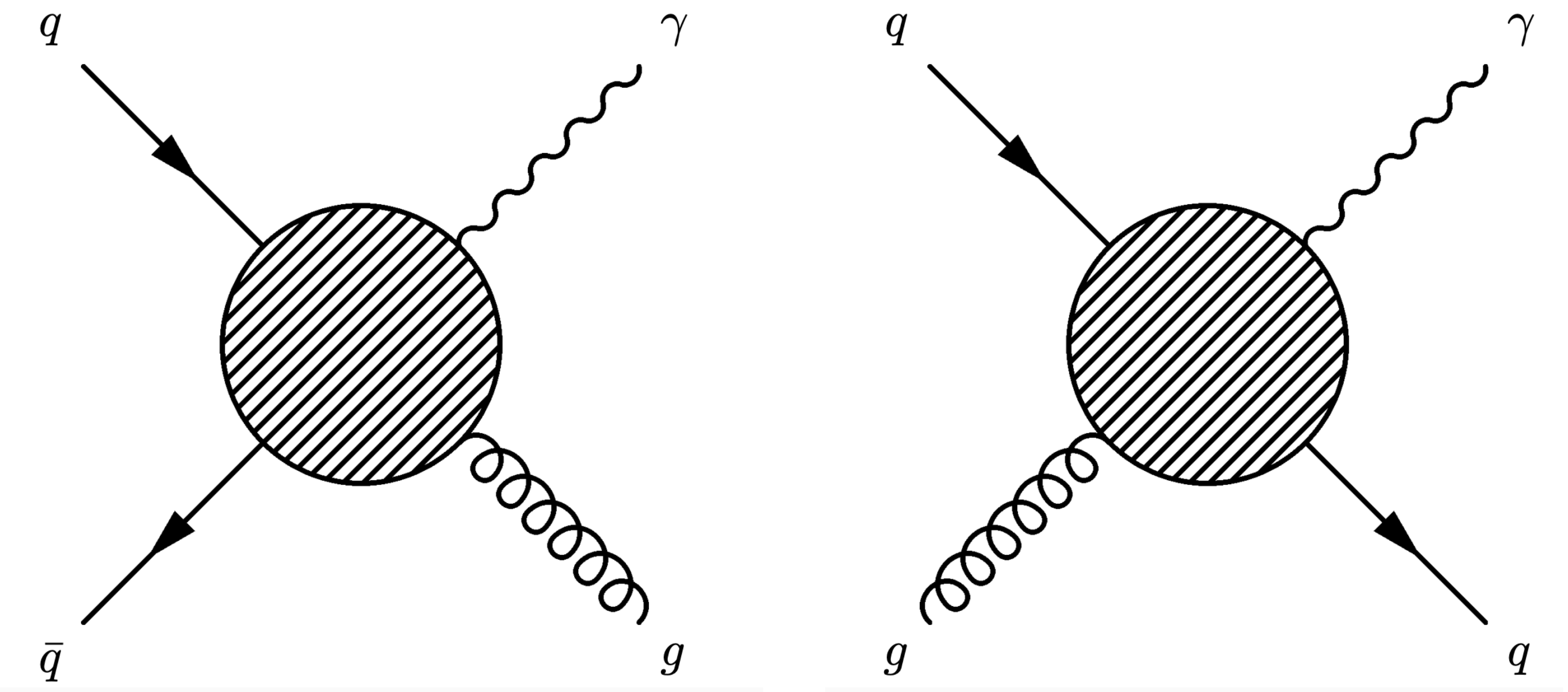
Let's extend these results

- What happens with colored particles in the final state?
- What role do soft quarks play?

[1905.08741]

Prompt photon production

$$pp \to \gamma + X$$



$$u_1 = (p_1 - p_\gamma)^2 = -svw$$

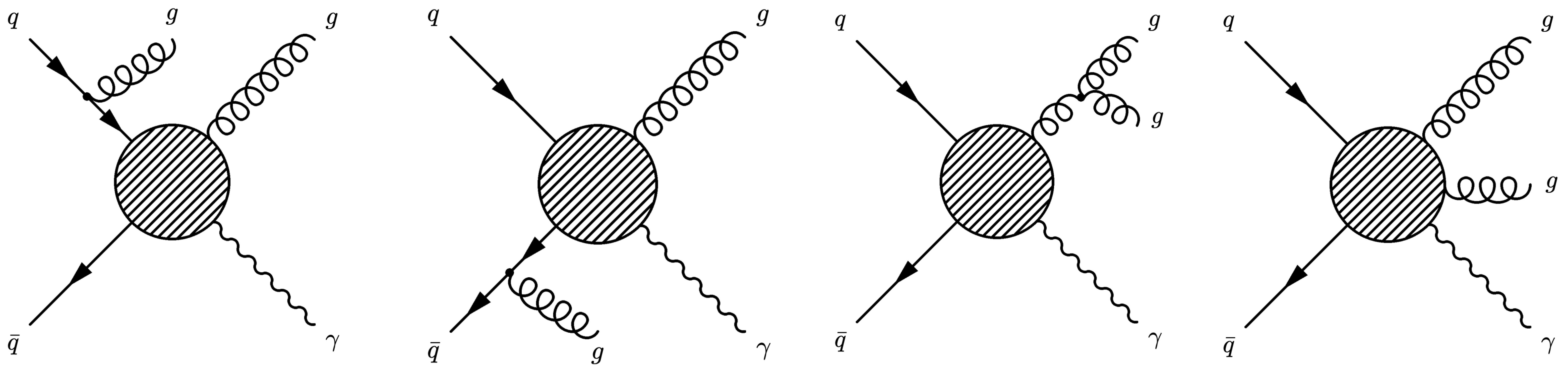
$$t_1 = (p_2 - p_\gamma)^2 = -s(1-v)$$

$$s_4 = (p_1 + p_2 - p_\gamma)^2 = sv(1-w)$$

Photon recoils against hard radiation, singular behavior for $w \to 1$

Simplest channel: $q\bar{q}$

$$q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$$



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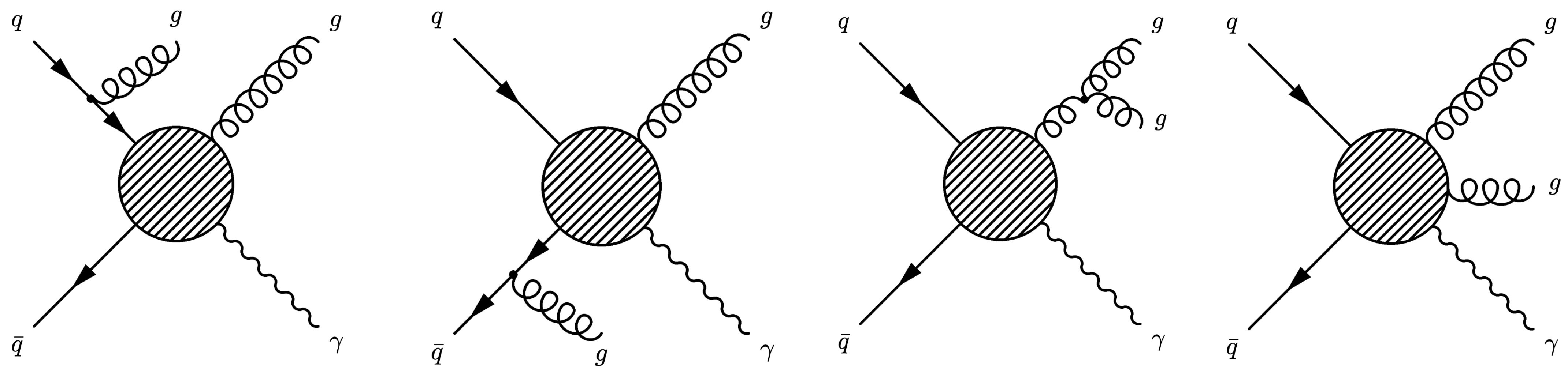
Similar NLP amplitude emerges!

$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=3} \mathbf{T}_i \left(\frac{2p_i^\sigma \pm k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$



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$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=3} \mathbf{T}_i \left(\frac{2p_i^\sigma \pm k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

Difference:
sign change for final state radiation

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Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 \quad = \quad & \frac{C_F}{C_A}\Bigg[C_F\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2},p_2+\delta p_{2;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R},p_R-\delta p_{R;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R},p_R-\delta p_{R;2})\right|^2 \\
& -\frac{1}{2}C_A\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2},p_2+\delta p_{2;1})\right|^2\Bigg]
\end{aligned}
$$

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Eikonal factors

$$|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 = \frac{C_F}{C_A}\Bigg[C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2$$
$$+\frac{1}{2}C_A \frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2$$
$$+\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2$$
$$-\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\Bigg]$$

Interferences are created!

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Shifts in Born amplitude

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 = \frac{C_F}{C_A}\Bigg[& C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \\
& +\frac{1}{2}C_A \frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2 \\
& +\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2 \\
& -\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \Bigg]
\end{aligned}
$$

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \rightarrow \gamma(p_\gamma)g(k)g(p_R)$

$$|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 = \frac{C_F}{C_A}\Bigg[C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2$$

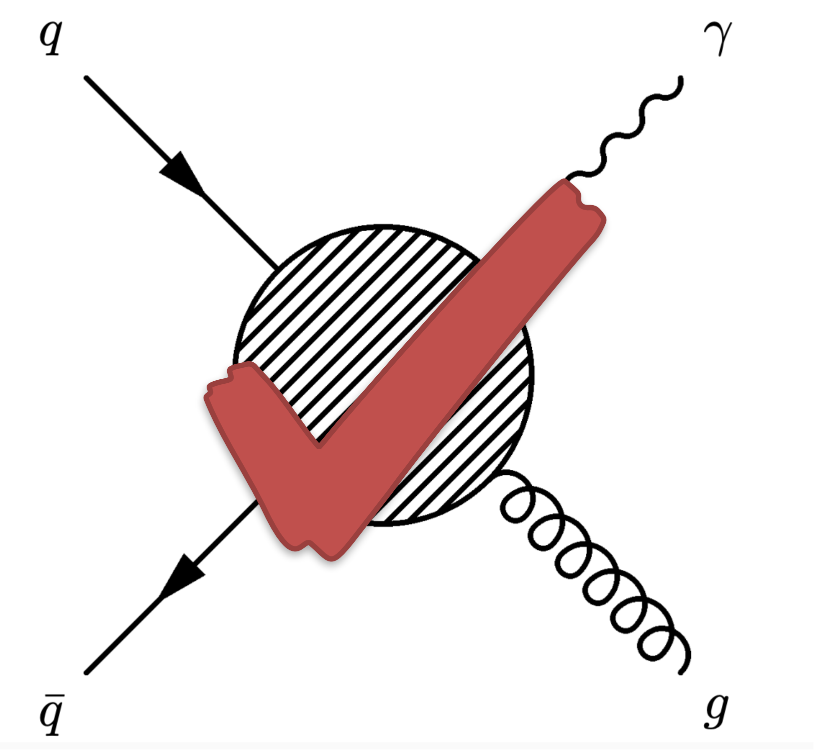
After integration over phase space all LL terms up to NLP are obtained. Missing LP NLL terms are recovered by adding the $g \rightarrow gg(q\bar{q})$ splittings.

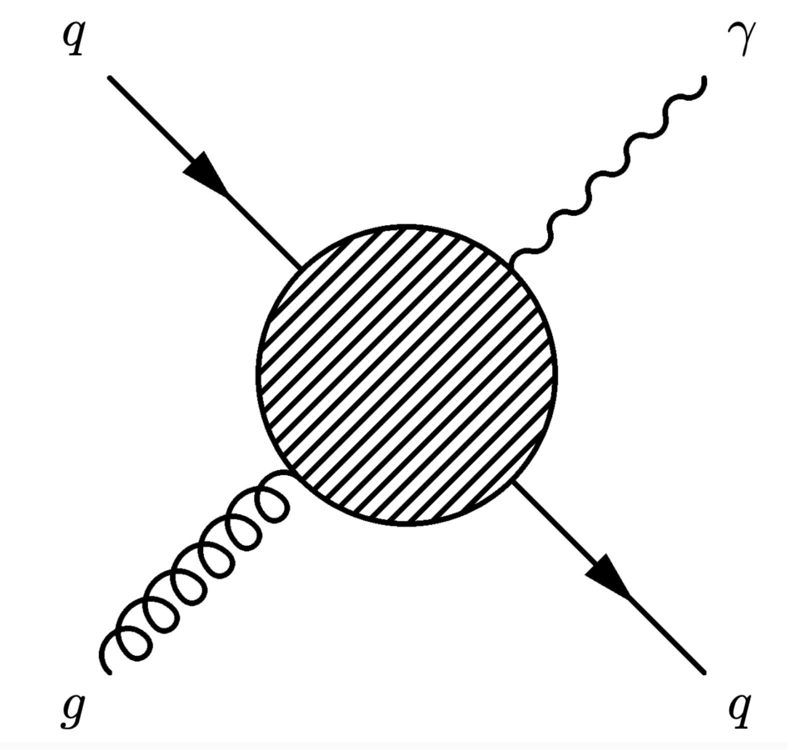
$$+\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2$$

$$-\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\Bigg]$$

Process that addresses all questions

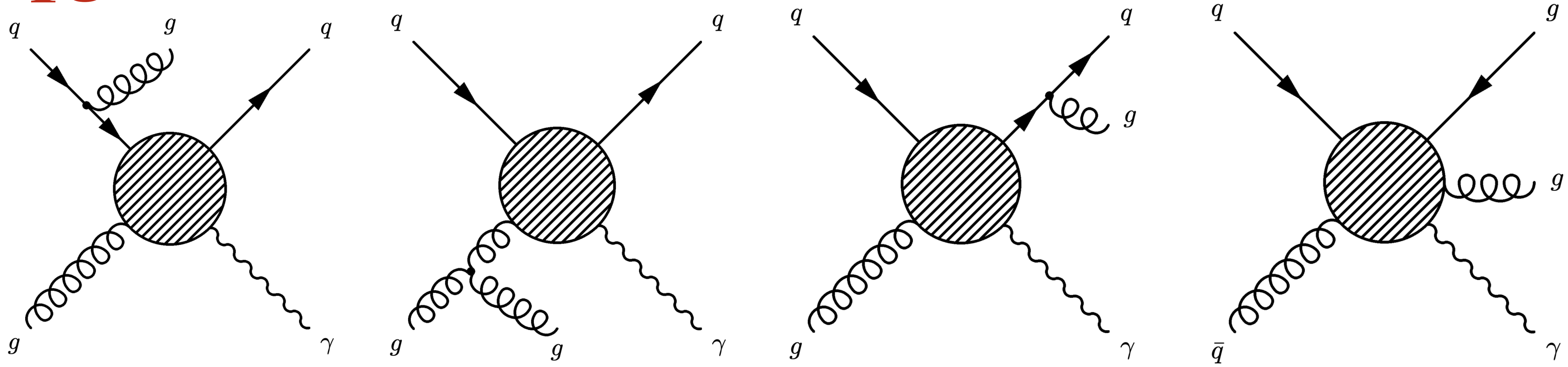
Prompt photon production $pp \to \gamma + X$





qg channel

$$g(p_1)q(p_2) \to \gamma(p_\gamma)g(k)q(p_R)$$



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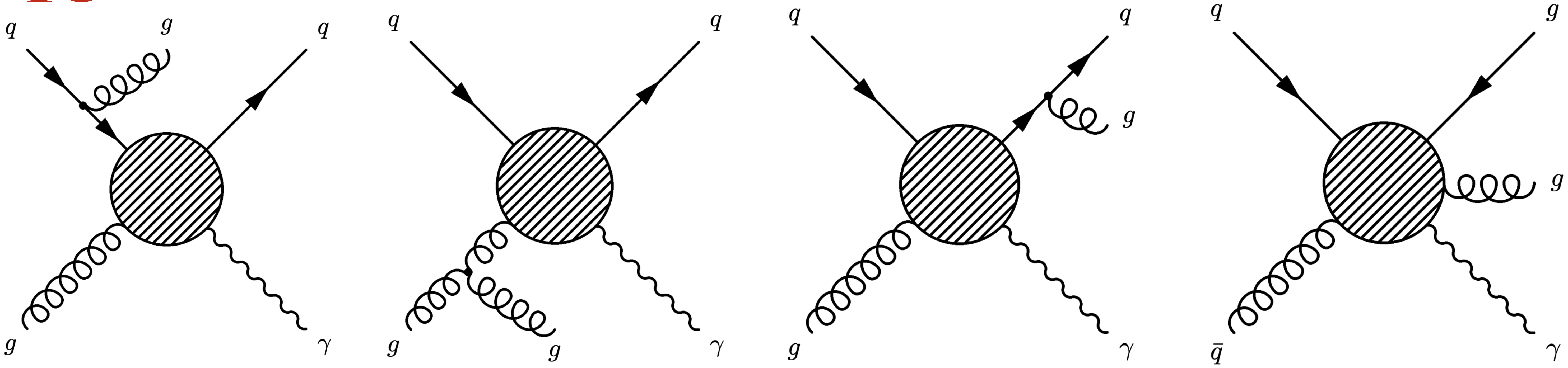
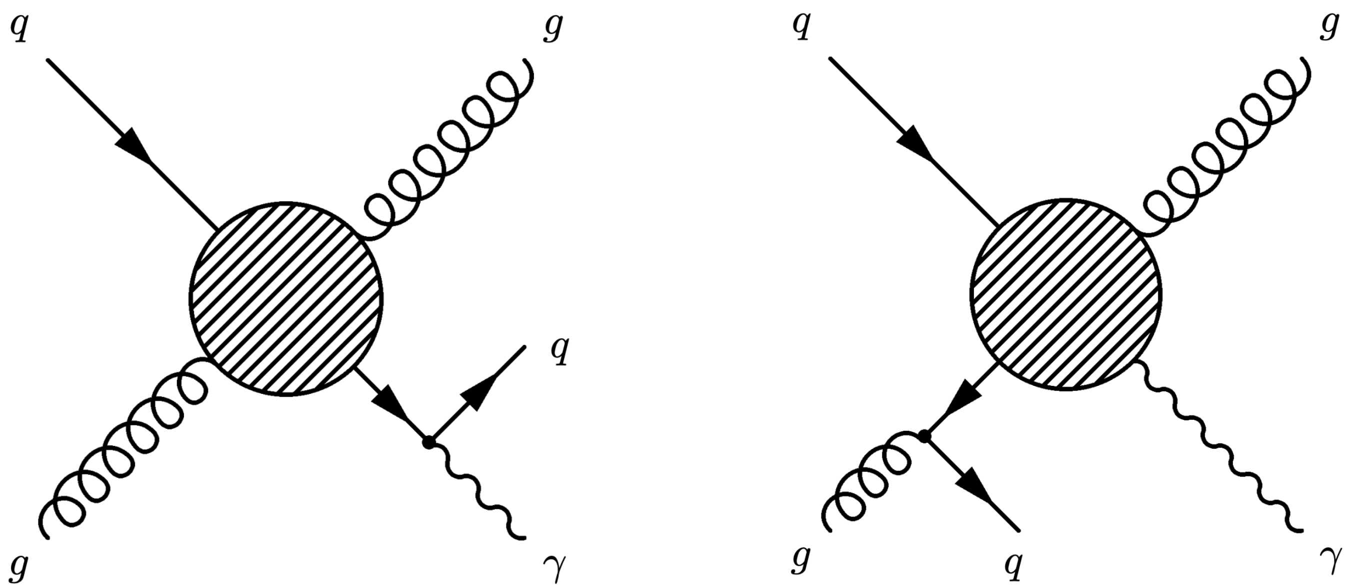
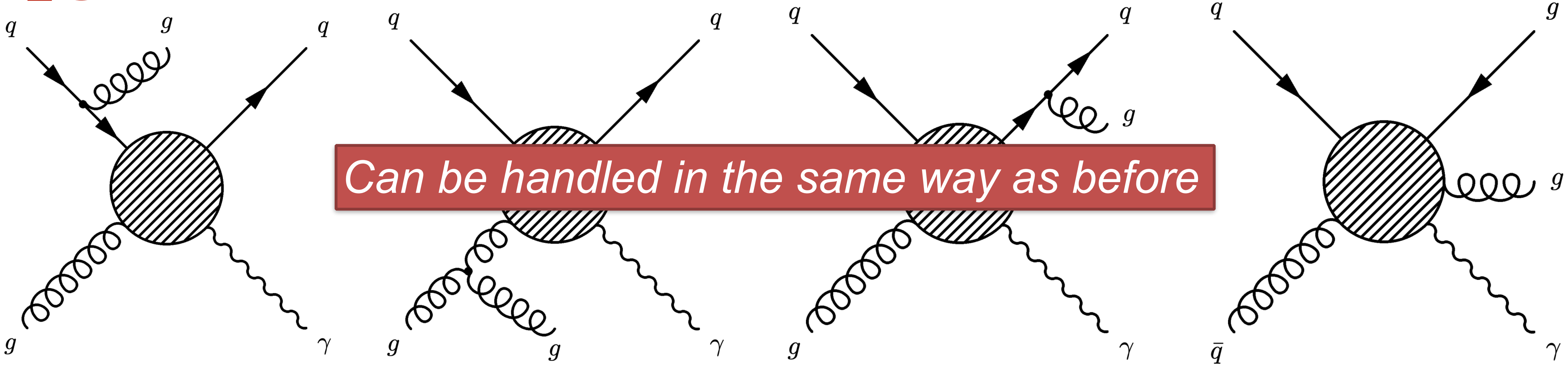


but also:



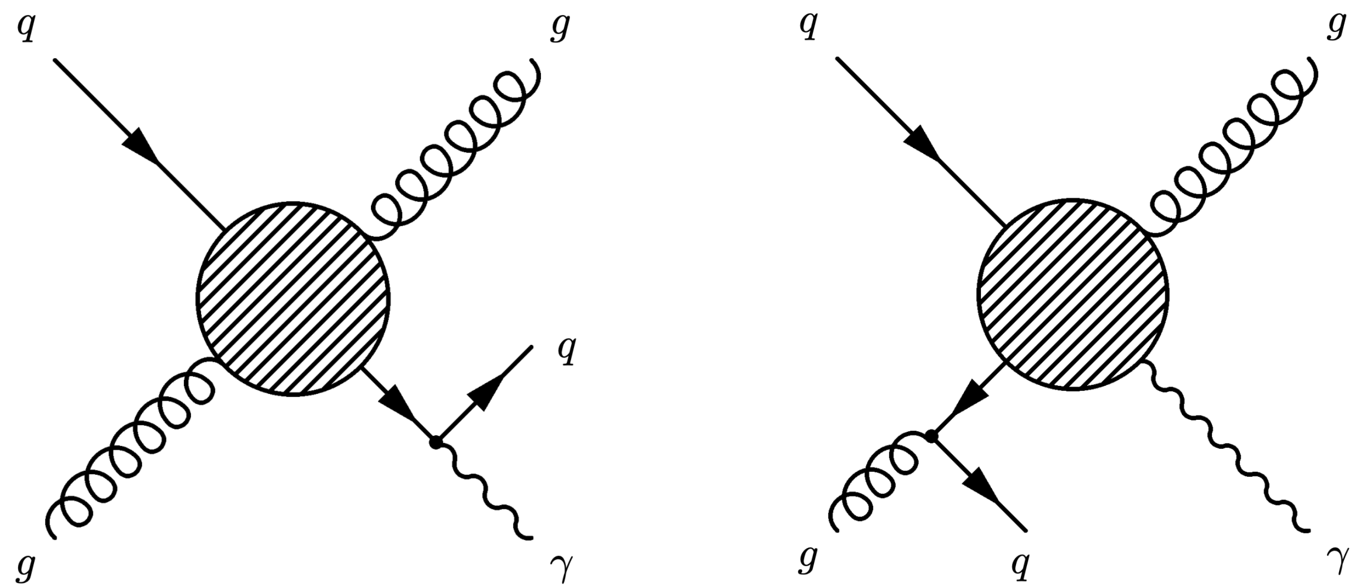
qg channel

$$g(p_1)q(p_2) \to \gamma(p_\gamma)g(k)q(p_R)$$

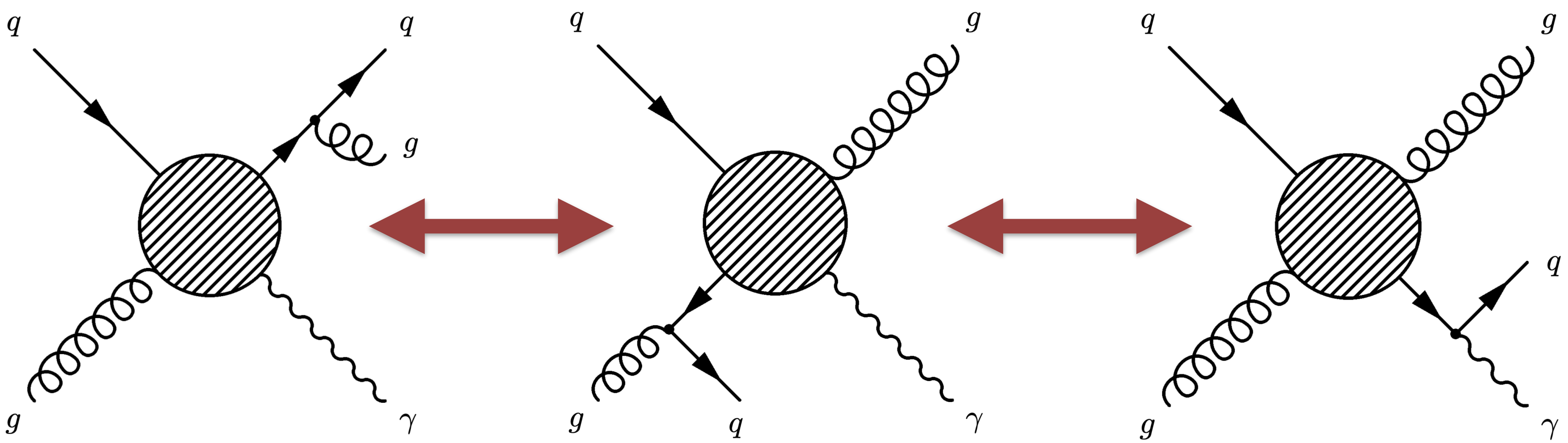


Can be handled in the same way as before

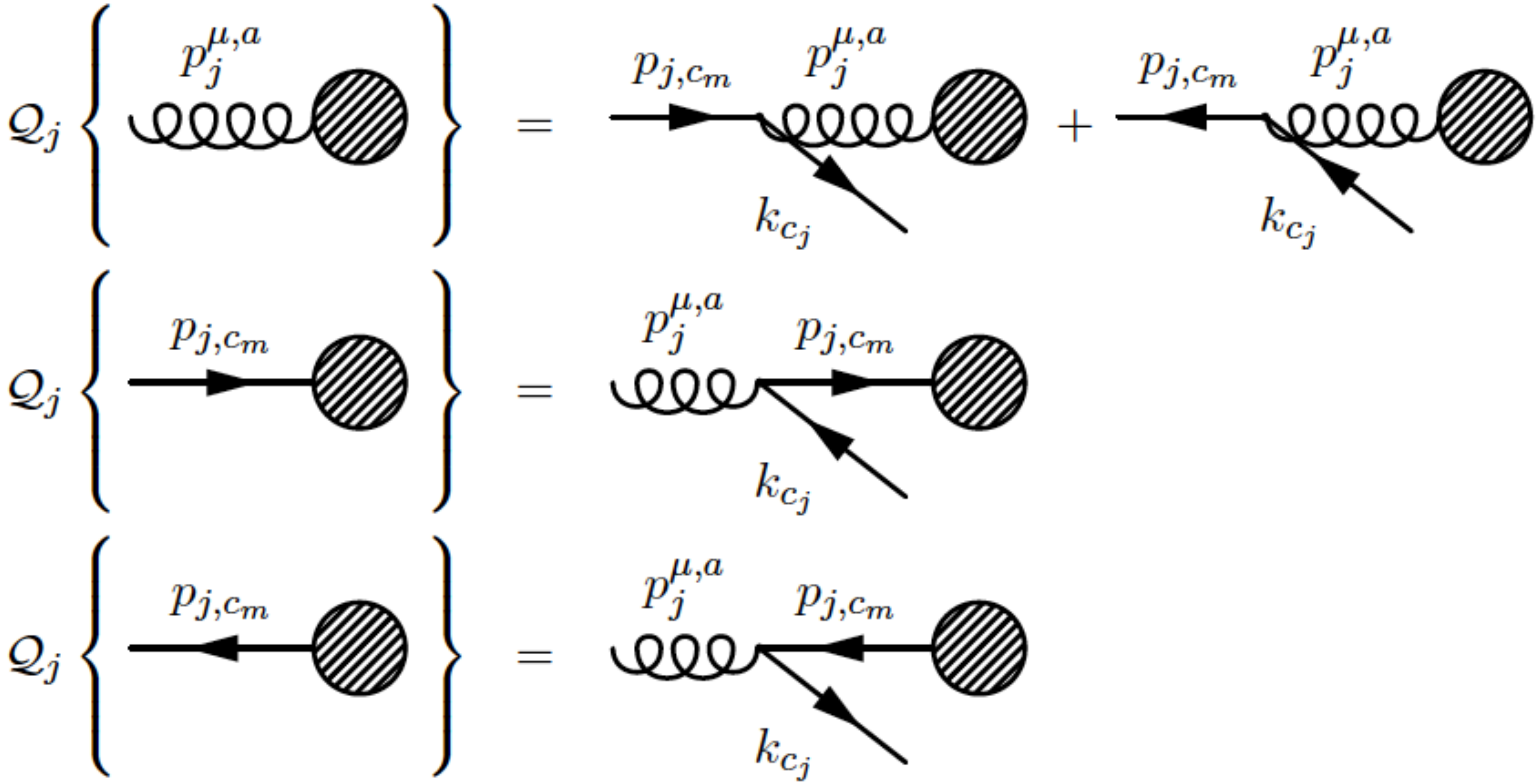
but also:



And the soft quarks interfere!



Quark emission operator (schematically)



Full NLP NLO amplitude

$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}$$

Soft gluon contribution

$$+ \sum_{i=1}^{m} \mathbf{T}_i \frac{1}{2p_i \cdot k} \mathscr{Q}_i \otimes \mathscr{M}_{i,\rm LO}$$

Soft quark contribution

Leading-logarithmic terms at LP and NLP

- By combining the soft quark and gluon amplitude, all LP + NLP LL terms are obtained
- All 7 NLO prompt photon channels [Gordon, Vogelsang, 1993] can be correctly described up to LL NLP
- Also works for DIS and e+e- to jets

Leading-logarithmic terms at LP and NLP

- By combining the soft quark and gluon amplitude, all LP + NLP LL terms are obtained
- All 7 NLO prompt photon channels [Gordon, Vogelsang, 1993] can be correctly described up to LL NLP
- Also works for DIS and e+e- to jets

Take-home message:

Soft quarks and gluons generate all NLP LL contributions at NLO

Open questions:

1. How does this extend to multiple emissions?

2. What if loops are involved?

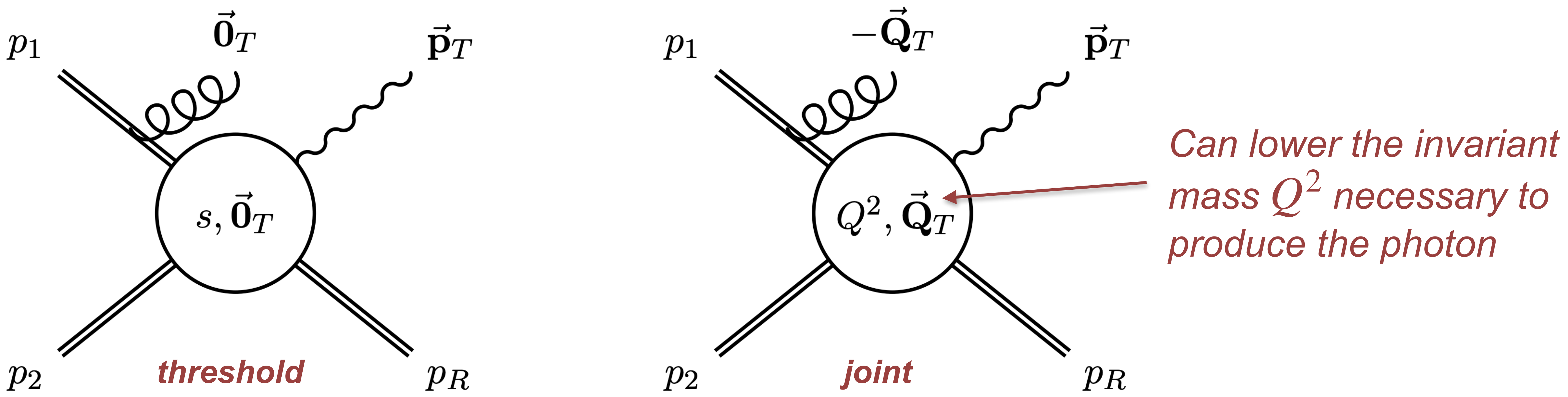
3. How to handle collinear contributions?

2. What is their numerical impact?

[1905.11771]

NLP resummation of prompt photon

- Threshold resummation of powers of $\ln(1 - x_T^2)$ with $x_T^2 = \dfrac{4p_T^2}{s}$
- We consider joint resummation of threshold and recoil to NLL, $\tilde{x}_T^2 = \dfrac{4p_T'^2}{Q^2}$



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Resummation in Mellin space

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

$$\int_0^1 \mathrm{d}z \left(\frac{\ln(1-z)}{1-z} \right)_+ z^{N-1} = \frac{1}{12} \left(6\psi^{(0)}(N)\left(\psi^{(0)}(N) + 2\gamma_E\right) - 6\psi^{(1)}(N) + \pi^2 + 6\gamma_E^2 \right)$$

Resummation in Mellin space

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

$$\int_0^1 \mathrm{d}z \left(\frac{\ln(1-z)}{1-z} \right)_+ z^{N-1} = \frac{1}{12} \left(6\psi^{(0)}(N) \left(\psi^{(0)}(N) + 2\gamma_E \right) - 6\psi^{(1)}(N) + \pi^2 + 6\gamma_E^2 \right)$$

$$\underset{N \to \infty}{\simeq} \frac{1}{2} \left(\ln^2 \bar{N} + \zeta(2) - \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N} \right) \right) \qquad \left(\bar{N} = N e^{\gamma_E} \right)$$

Resummation in Mellin space

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

$$\int_0^1 \mathrm{d}z \left(\frac{\ln(1-z)}{1-z} \right)_+ z^{N-1} \simeq \frac{1}{2} \left(\ln^2 \bar{N} + \zeta(2) - \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N} \right) \right)$$

$$\int_0^1 \mathrm{d}z \, \ln(1-z)\, z^{N-1} \simeq \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N^2} \right)$$

Resummation in Mellin space

$$\frac{d\sigma}{dz} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

$$\int_0^1 dz \left(\frac{\ln(1-z)}{1-z} \right)_+ z^{N-1} \simeq \frac{1}{2} \left(\ln^2 \bar{N} + \zeta(2) - \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N}\right) \right)$$

LP LL in Mellin space

$$\int_0^1 dz \, \ln(1-z)\, z^{N-1} \simeq \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Resummation in Mellin space

$$\frac{d\sigma}{dz} \propto \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} \left[c_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c'_{nm} \ln^m(1-z) \right] + d_n \delta(1-z) + f'_n$$

$$\int_0^1 dz \left(\frac{\ln(1-z)}{1-z} \right)_+ z^{N-1} \simeq \frac{1}{2} \left(\ln^2 \bar{N} + \zeta(2) - \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N} \right) \right)$$

NLP LL in Mellin space

$$\int_0^1 dz \, \ln(1-z) \, z^{N-1} \simeq \frac{\ln \bar{N}}{N} + \mathcal{O}\left(\frac{1}{N^2} \right)$$

Joint resummation

$$p_T^3 \frac{\mathrm{d}\sigma^{(\mathrm{direct,joint})}_{AB\to\gamma+X}(x_T^2)}{\mathrm{d}p_T} = \frac{p_T^4}{8\pi S^2} \sum_{a,b} \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i} f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F)$$

$$\times \int \frac{\mathrm{d}^2\mathbf{Q}_T}{(2\pi)^2} \left(\frac{S}{4|\mathbf{p}_T - \mathbf{Q}_T/2|^2} \right)^{N+1} \int_0^1 \mathrm{d}\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|\mathcal{M}_{ab\to\gamma d}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} C_\delta^{(ab\to\gamma d)}(\alpha_s, \tilde{x}_T^2)$$

$$\times \int \mathrm{d}^2\mathbf{b}\ \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\ P_{abd}(N, b, Q, \mu_F, \mu).$$

[0010080, 1701.01464]

Joint resummation

$$p_T^3 \frac{\mathrm{d}\sigma^{(\mathrm{direct,joint})}_{AB\to\gamma+X}(x_T^2)}{\mathrm{d}p_T} = \frac{p_T^4}{8\pi S^2} \sum_{a,b} \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i} f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F)$$

$$\times \int \frac{\mathrm{d}^2\mathbf{Q}_T}{(2\pi)^2} \left(\frac{S}{4|\mathbf{p}_T - \mathbf{Q}_T/2|^2}\right)^{N+1} \int_0^1 \mathrm{d}\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|\mathcal{M}_{ab\to\gamma d}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} C_\delta^{(ab\to\gamma d)}(\alpha_s, \tilde{x}_T^2)$$

$$\times \int \mathrm{d}^2\mathbf{b}\ \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\, P_{abd}(N, b, Q, \mu_F, \mu).$$

Resummed exponent

Resummed exponent

$$P_{abd}(N, b, Q, \mu_F, \mu) =$$
$$\exp\left[E_a^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + E_b^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + F_d(N, Q, \mu) + g_{abd}(N, \mu)\right]$$

initial state *initial state* *final state* *interference*

Resummed exponent

$$P_{abd}(N, b, Q, \mu_F, \mu) =$$
$$\exp\left[E_a^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + E_b^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + F_d(N, Q, \mu) + g_{abd}(N, \mu)\right]$$

initial state *initial state* *final state* *interference*

$$E_a^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) = \int_0^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \left[J_0(bk_T) K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right)\right]$$
$$- \ln \bar{N} \int_{\mu_F^2}^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2))$$

Resummed exponent

$$P_{abd}(N, b, Q, \mu_F, \mu) = \exp\left[E_a^{\rm PT}(N, b, Q, \mu_F, \mu) + E_b^{\rm PT}(N, b, Q, \mu_F, \mu) + F_d(N, Q, \mu) + g_{abd}(N, \mu)\right]$$

initial state *initial state* *final state* *interference*

$$E_a^{\rm PT}(N, b, Q, \mu_F, \mu) = \int_0^{Q^2} \frac{{\rm d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \left[J_0(bk_T)K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right)\right] - \ln\bar{N} \int_{\mu_F^2}^{Q^2} \frac{{\rm d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2))$$

There is no general NLP resummation framework, but can we make an educated guess?

Extension to NLP

$$P_{abd}(N, b, Q, \mu_F, \mu) = \exp\left[E_a^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + E_b^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) + F_d(N, Q, \mu) + g_{abd}(N, \mu)\right]$$

initial state *initial state* *final state* *interference*

$$E_a^{\mathrm{PT}}(N, b, Q, \mu_F, \mu) = \int_0^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \left[J_0(bk_T) K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right)\right] - \ln \bar{N} \int_{\mu_F^2}^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2))$$

Joint resummation:

- Recoil can be separated from threshold resummation
- Gives LP NNLL and NLP LL contribution

Threshold resummation at NLP

Isolate pure threshold behavior in:

$$\begin{aligned} E_a^{\rm thres}(N,Q,\mu_F,\mu) &= -\int_{Q^2}^{Q^2/\bar{N}^2} \frac{{\rm d}k_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right) \ln\left(\frac{\bar{N}k_T}{Q}\right) \\ &\quad - \ln\bar{N}\int_{\mu_F^2}^{Q^2}\frac{{\rm d}k_T^2}{k_T^2}A_a\left(\alpha_s(k_T^2)\right). \\ &= \int_0^1 {\rm d}z\,\frac{z^{N-1}-1}{1-z}\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{{\rm d}k_T^2}{k_T^2}A_a\left(\alpha_s(k_T^2)\right) \end{aligned}$$

How to 'dress' this with NLP contributions?

Threshold resummation at NLP

$$E_a^{\text{thres}}(N,Q,\mu_F,\mu) = -\int_{Q^2}^{Q^2/\bar{N}^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right) \ln\left(\frac{\bar{N}k_T}{Q}\right) - \ln\bar{N}\int_{\mu_F^2}^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right).$$

$$= \int_0^1 \mathrm{d}z\, \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{(1-z)^2Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right)$$

Option 1: $\frac{z^{N-1}-1}{1-z}A_a^{(1)} \rightarrow \left(\frac{z^{N-1}-1}{1-z} - z^{N-1}\right)A_a^{(1)}$

[9611272, 0704.3180]

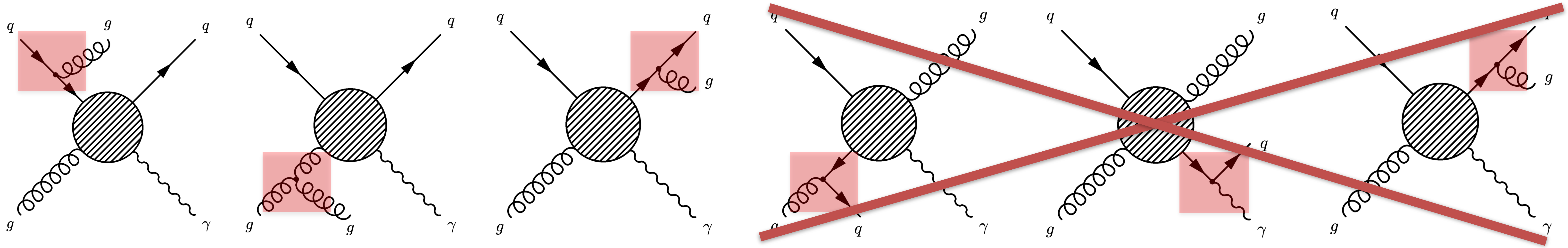
Threshold resummation at NLP

$$E_a^{\text{thres}}(N, Q, \mu_F, \mu) = -\int_{Q^2}^{Q^2/\bar{N}^2} \frac{dk_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right) \ln\left(\frac{\bar{N} k_T}{Q}\right)$$
$$- \ln \bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a\left(\alpha_s(k_T^2)\right).$$

Option 2: $= -\int_{\mu_F^2}^{Q^2/\bar{N}^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \ln \bar{N} \rightarrow -\int_{\mu_F^2}^{Q^2/\bar{N}^2} \frac{dk_T^2}{k_T^2} P_{aa}\left(\alpha_s(k_T^2)\right)$

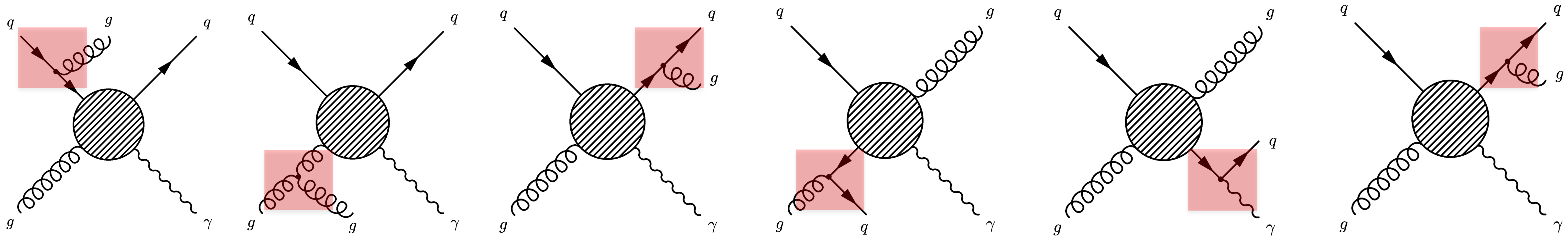
[0202251, 0309264]

Splitting functions



Option 2a: Include only diagonal contributions up to NLP

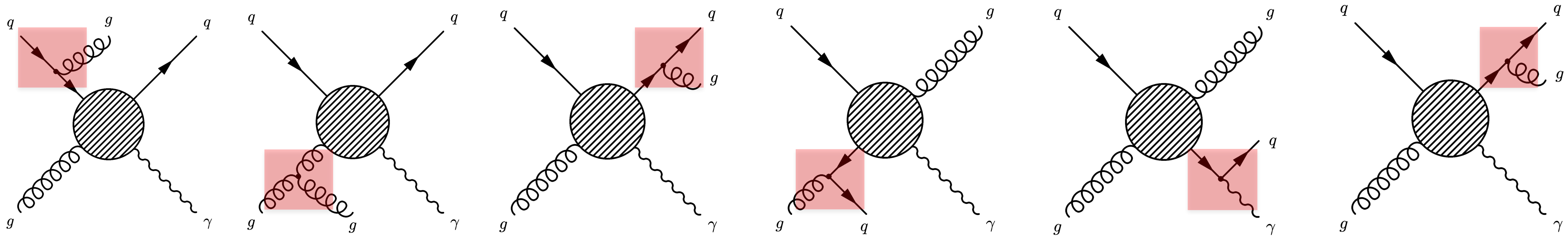
Splitting functions



Option 2a: Include only diagonal contributions up to NLP

Option 2b: Include also off-diagonal contributions up to NLP

Splitting functions



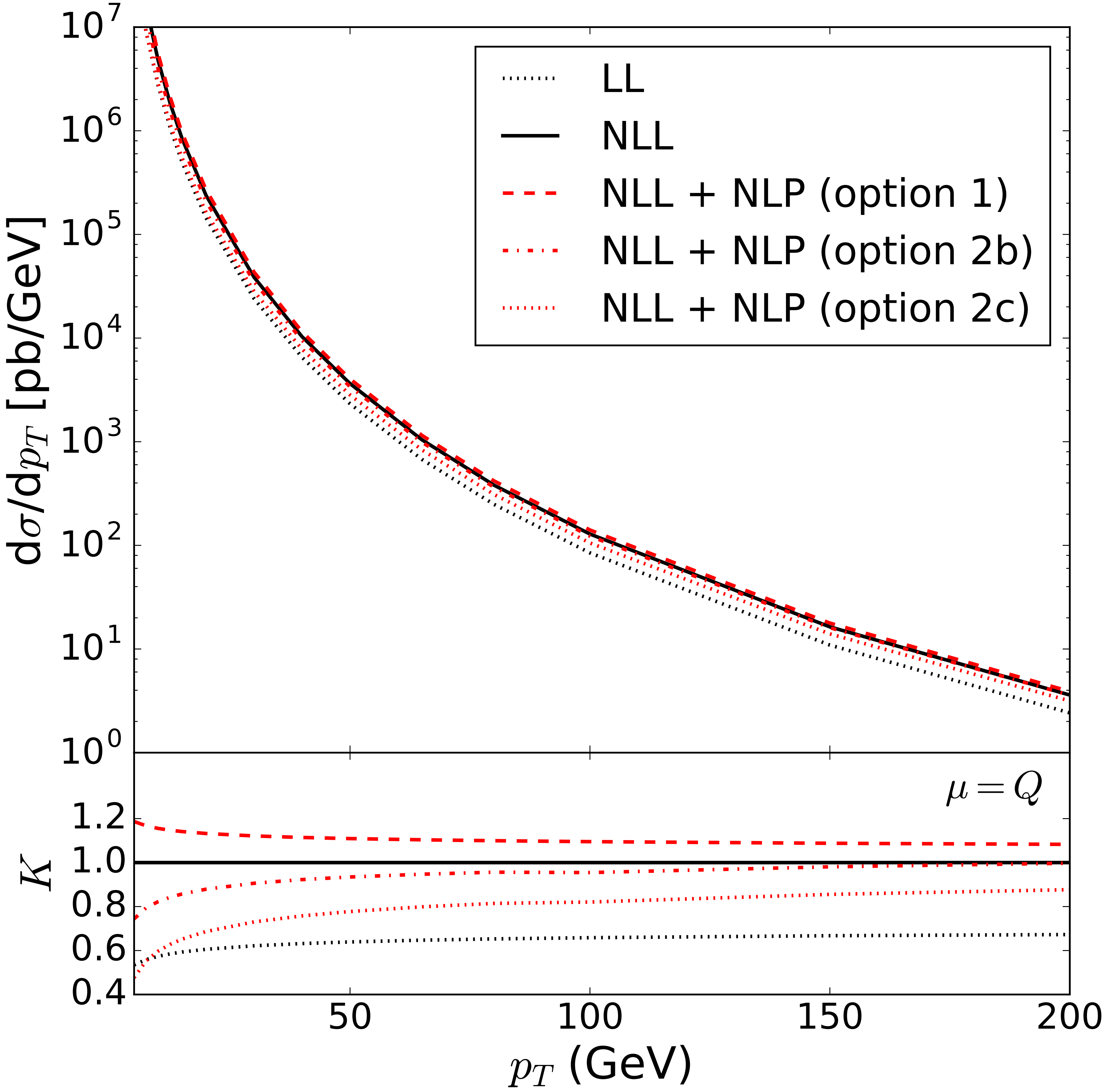
Option 2a: Include only diagonal contributions up to NLP

Option 2b: Include also off-diagonal contributions up to NLP

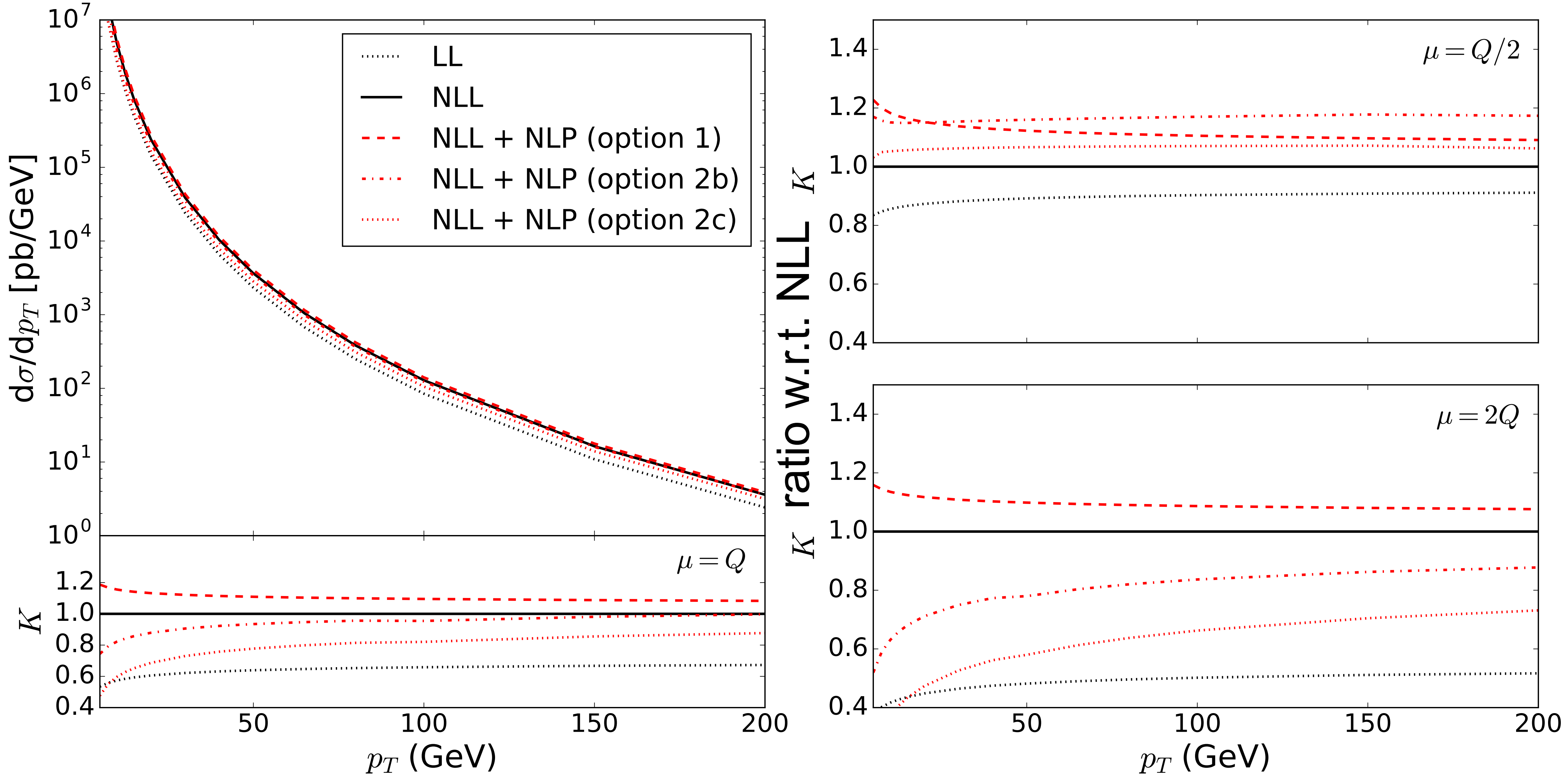
Option 2c: Keep the full form of the splitting functions

Numerical results

- Results for LHC@13 TeV, MMHT PDF set, $\mu_F = \mu = Q = 2p_T$
- NLP effects smaller than LL$\rightarrow$ NLL
- The NLP effects of option 1 (=2a) give a 5-10% correction



Scale dependence



Numerical correction of option 2b and 2c depends on the scale

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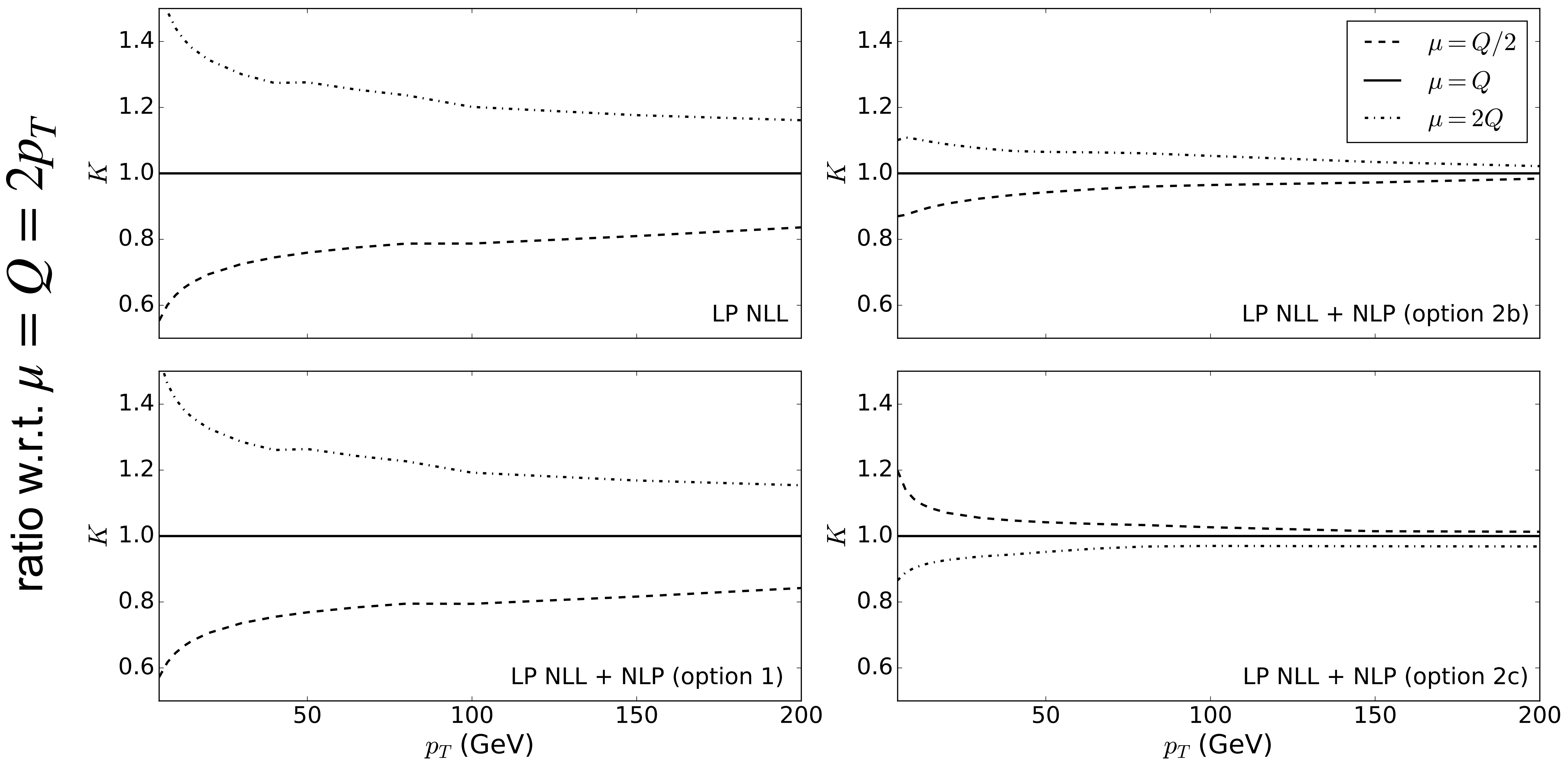
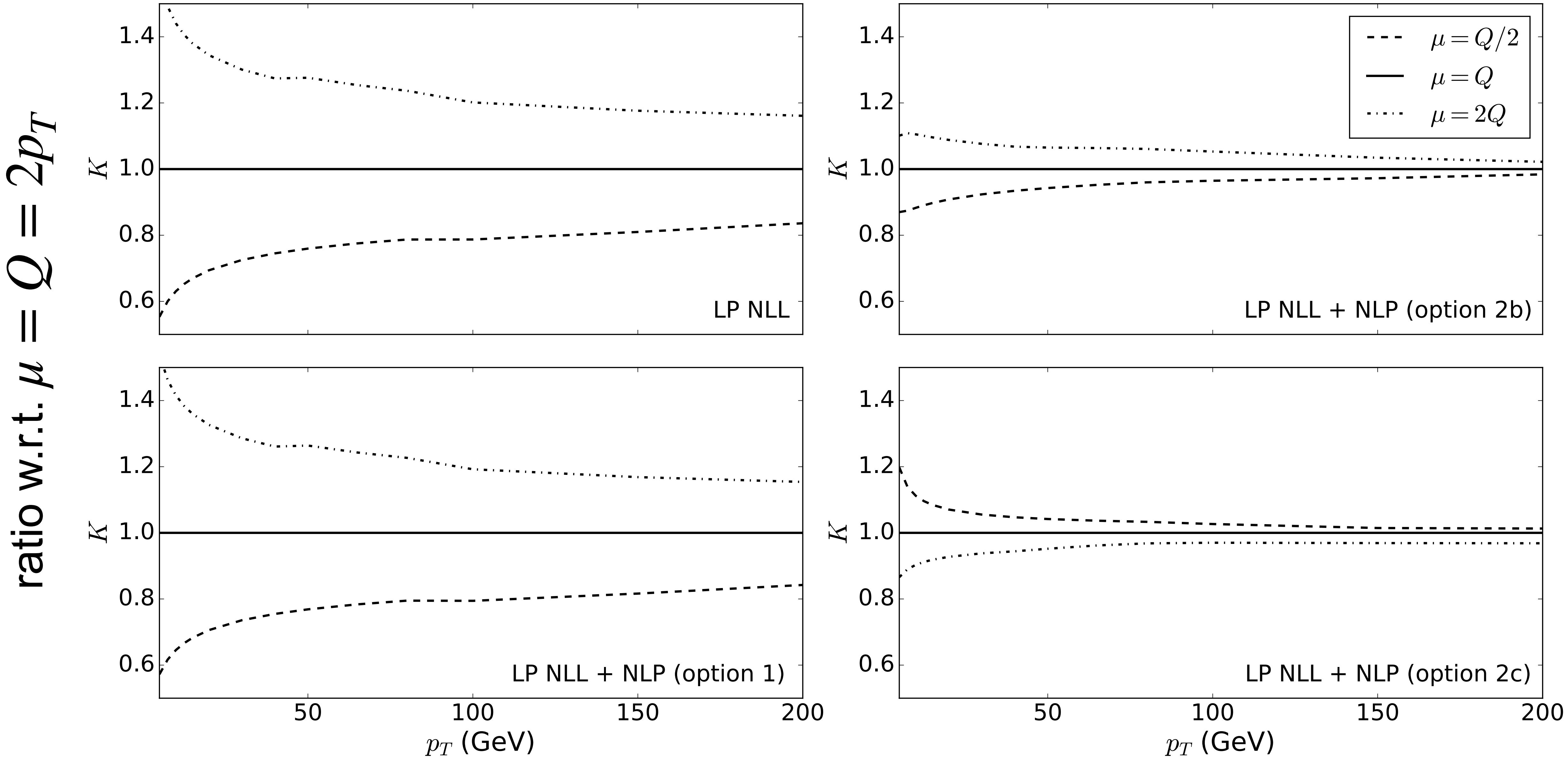
Scale dependence



Cause of scale dependence: the LP NLL expression

Scale dependence

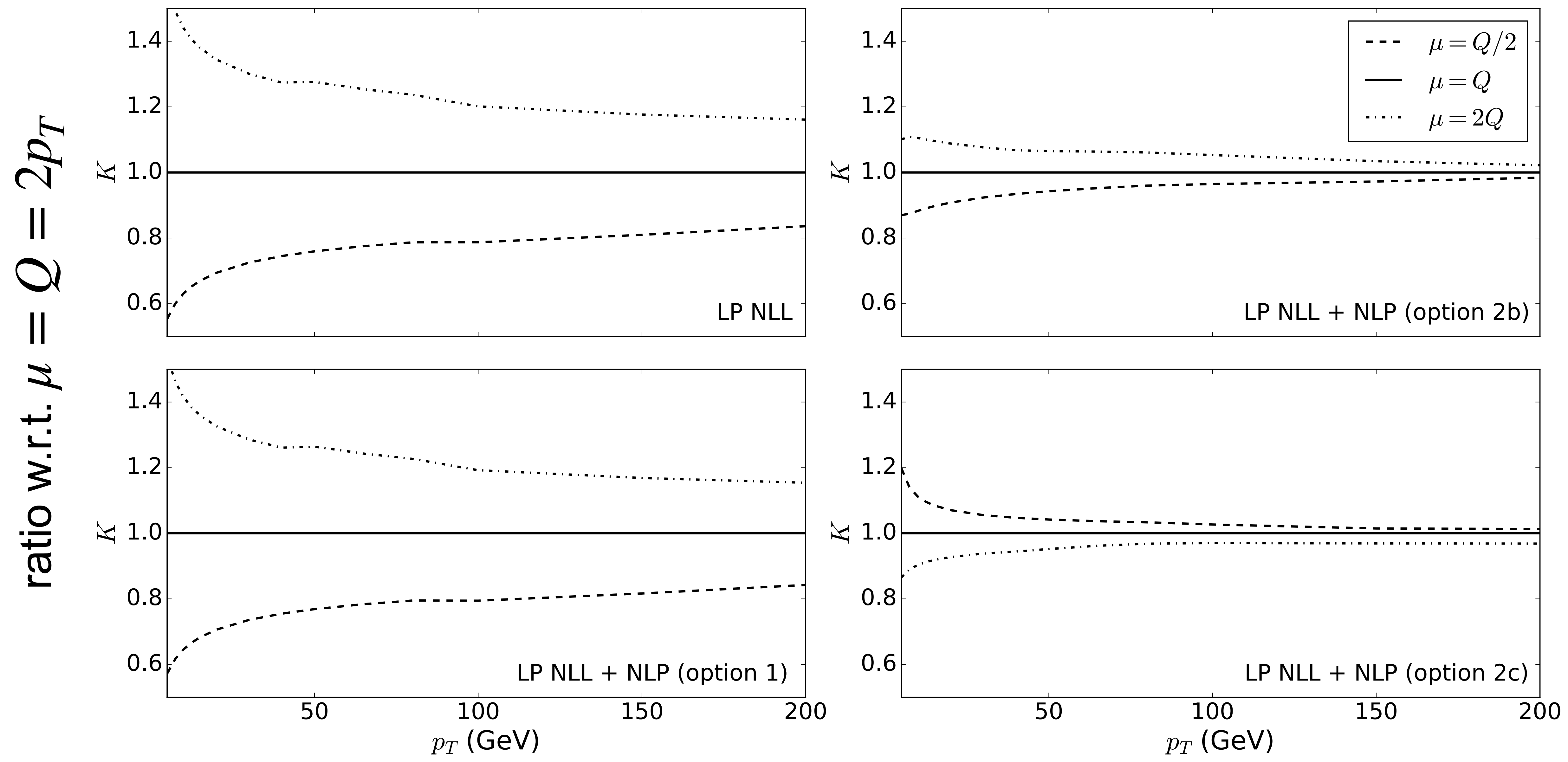


Cause of scale dependence: the LP NLL expression

Take-home message:

Scale dependence hugely decreased by including off-diagonal contributions of the splitting functions

Scale dependence



Both approaches only include NLP effects of collinear origin

So not all LL NLP contributions at NLO are covered!

Questions:

- *What happens when all NLP contributions are included?*
- *How do other processes behave?*

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Conclusions

- NLP amplitude for soft gluons is universal and creates a shift to the Born matrix element
  - But note that the emission of soft quarks is needed too!
- For processes with final state partons we recover all LL NLP contributions at NLO
  - For NLL NLP contributions we need to understand (next-to-)collinear emissions
- Gluon NLP terms give a 5-10% correction to the NLL distribution for prompt photon
- Including quark emissions can significantly decrease the scale dependence

Extra slides

Form of the shifts

$$|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 = \frac{C_F}{C_A}\left[C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\right.$$
$$+\frac{1}{2}C_A \frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2$$
$$+\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2$$
$$\left.-\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\right]$$

$$\delta p^{\alpha}_{i;j} \equiv -\frac{1}{2}\left(k^{\alpha} + \frac{p_j\cdot k}{p_i\cdot p_j}p_i^{\alpha} - \frac{p_i\cdot k}{p_i\cdot p_j}p_j^{\alpha}\right)$$

Role of the cut-off

$$p_T^3 \frac{\mathrm{d}\sigma^{(\text{direct,joint})}_{AB\to\gamma+X}(x_T^2)}{\mathrm{d}p_T} = \frac{p_T^4}{8\pi S^2} \sum_{a,b} \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i} f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F)$$

$$\times \int \frac{\mathrm{d}^2\mathbf{Q}_T}{(2\pi)^2} \left(\frac{S}{4|\mathbf{p}_T - \mathbf{Q}_T/2|^2} \right)^{N+1} \int_0^1 \mathrm{d}\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|\mathcal{M}_{ab\to\gamma d}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} C_\delta^{(ab\to\gamma d)}(\alpha_s, \tilde{x}_T^2)$$

$$\times \int \mathrm{d}^2\mathbf{b}\ \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\ \theta\left(\bar{\mu} - |\mathbf{Q}_T|\right) P_{abd}(N, b, Q, \mu_F, \mu).$$

Cut-off

Approximation of kinematic function

$$\left(\frac{S}{4(\mathbf{p}_T - \mathbf{Q}_T/2)^2}\right)^{N+1} = \left(\frac{4p_T^2}{S}\right)^{-N-1}\left(1 - \frac{\mathbf{p}_T\cdot\mathbf{Q}_T}{p_T^2} + \frac{Q_T^2}{4p_T^2}\right)^{-N-1}$$

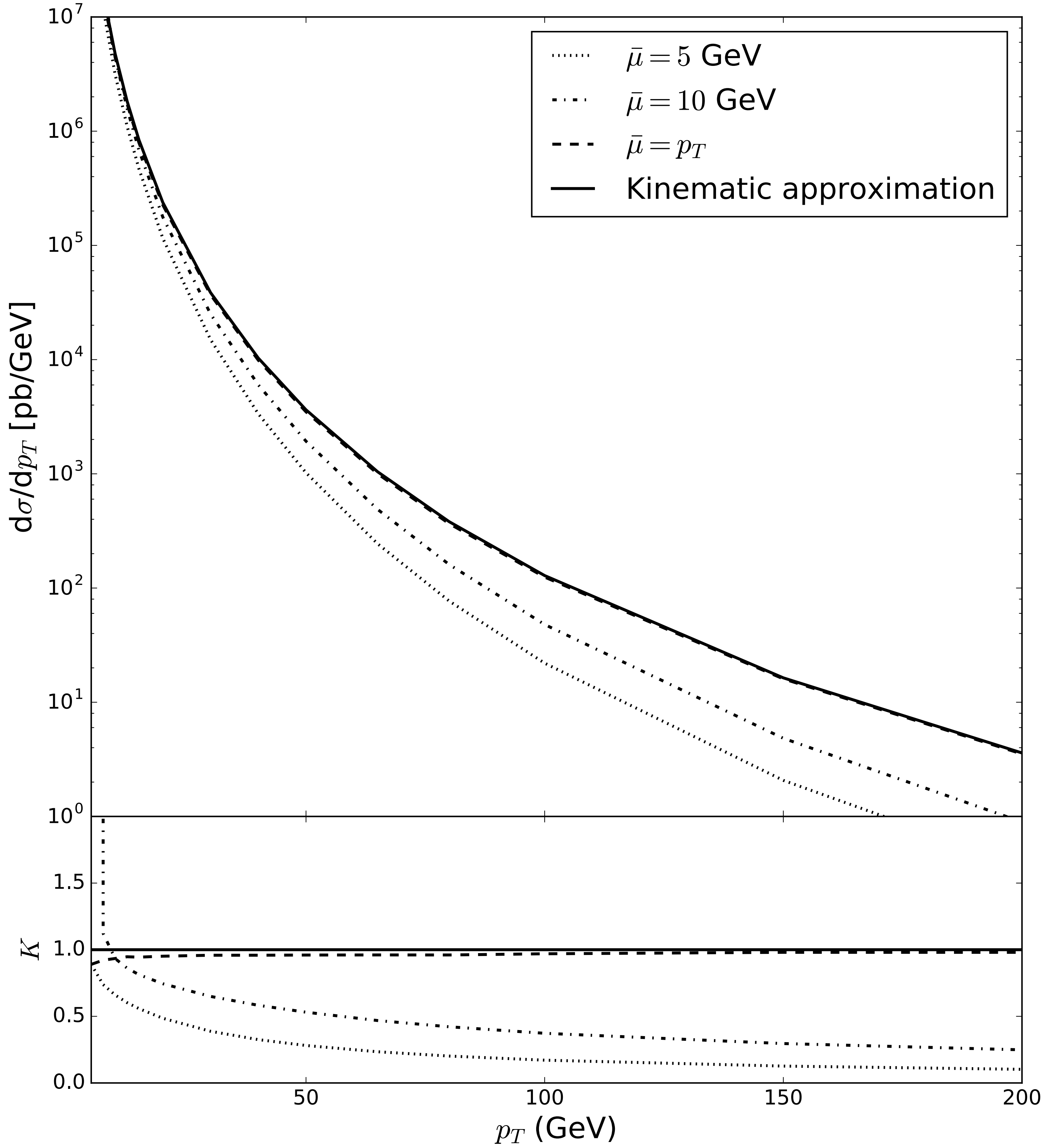
$$\simeq (x_T^2)^{-N-1}\exp\left[(N+1)\frac{\mathbf{p}_T\cdot\mathbf{Q}_T}{p_T^2}\left[1+\mathcal{O}\left(Q_T/p_T\right)\right]\right]$$

Produces $\delta\left(\mathbf{b} - i(N+1)\mathbf{p}_T/p_T^2\right)$ *when integrated over* $\int\frac{\mathrm{d}^2\mathbf{Q}_T}{(2\pi)^2}$

[0409234]

Approximation

- Reduces the 5D integral to 1D
- Numerically more stable
- Converges to the result obtained by setting $\bar{\mu} = p_T = Q_T/2$



Nikhef

Radboud University

Recoil NLP correction

$$E_a^{\mathrm{recoil}}(N,Q,\mu) = 2A_a^{(1)}\frac{\alpha_s}{\pi}\int_0^{2N}\frac{\mathrm{d}x}{x}\left(1+2\alpha_s b_0 \ln\frac{x}{2N}\right)\Big[\left(I_0(x)-1\right)K_0(x) + \frac{x}{N}I_1(x)K_0(x)\Big] + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\simeq A_a^{(1)}\frac{\alpha_s}{2\pi}\left(\frac{\zeta(2)}{1-2\lambda}+\frac{\ln\bar{N}}{N}\right) \equiv h^{(1)}_{a,\mathrm{recoil}}(\lambda,\alpha_s).$$

Can be regarded as a wide angle contribution, as it is only there for non-zero k_T

Isolating threshold behavior

$$E_a^{\text{joint}}\left(N, b = i\frac{N+1}{p_T}, Q, \mu\right) = \int_0^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \left[K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right)\right]$$

$$+ \int_0^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} A_a(\alpha_s(k_T^2)) \left[I_0\left(\frac{(N+1)k_T}{p_T}\right) - 1\right] K_0\left(\frac{Nk_T}{p_T}\right) .$$

$$\equiv E_a^{\text{leading}}(N, Q, \mu) + E_a^{\text{recoil}}(N, Q, \mu) .$$

Extended evolution

$$P_{aa}(N) = -A_a(\alpha_s)\ln\bar{N} - B_a(\alpha_s) + \mathcal{O}(1/N)$$

Non-singlet evolution is diagonal:

Same term as in option 1

$$\exp\left[\frac{P_{\rm NS}^{(0)}(N)}{2\pi b_0}\ln(1-2\lambda)\right] = \exp\left[\frac{1}{2\pi b_0}\left(-2A_q^{(1)}\ln\bar{N} - 2B_q^{(1)} - \frac{A_q^{(1)}}{N}\right)\ln(1-2\lambda)\right]$$

Stems from evolution $\alpha_s(k_T^2)$ from μ_F^2 to $Q^2/\bar{N}^2$

Singlet: 2x2 matrix, off-diagonal terms correspond to flavor changes (quark emission)

Similar approach for the fragmentation function

Scale dependence of direct vs fragmentation components