Radiative EW corrections to the Drell-Yan process

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Intro

- The precision era at LHC has already started with the Run II phase collecting an unprecedented amount of statistics (147 fb^{-1})
- LHC operation is planned to continue for 20 years (up to 2038) collecting 20 times the statistics accumulated so far
- Moreover, the design of Future Linear and Circular colliders is proceeding fast and their approval would extend the program for more than 40 years from now
- Although no signal of physics beyond the Standard Model has been observed so far
- In this scenario it is clear that Electroweak corrections will play an important role
- In particular mixed QCD-EW corrections are already required for several processes in the most recent Les Houches wish list [Brooijmans et al. 2018]
- In the following I'll focus mainly on the Drell-Yan process

Intro

- From template fits to the reconstructed distributions of the charged lepton transverse momentum and of the W boson transverse mass in electron and muon decay channel, the ATLAS collaboration finds:

  [ATLAS coll. 1701.0724]

$$m_W = 80370 \pm 7(\text{stat.}) \pm 11(\text{exp.syst.}) \pm 14(\text{mod.syst.})\text{MeV} = 80370 \pm 19\text{MeV}$$

- 14MeV comes mainly from pdf uncertainty

- Uncertainty from QCD scale variations relatively small (although could be underestimated [Duhr, Dulat, Mistlberger 2001.07717]

- Overall uncertainty coming from the lack of EW higher orders considered under control

Intro

- Mixed QCD-EW corrections described by NLO QCD corrections matched to (multiple) parton shower programs

- Approximated fixed order computation shows that the leading part of the mixed correction is given by QCD corrections in the production times QED corrections in the decay. Δmw=-14MeV [Dittmaier, Huss, Schwinn 1511.08016]

- mW shift induced by proper account of mixed corrections quite relevant and with a certain dependence on the way the computation is performed:
  ➡ matching NLO corrections with PS [Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841] find Δmw=-16MeV

- In view of the precision attainable at the LHC it is desirable to evaluate exactly the mixed QCD-EW contribution

  ➡ its knowledge could open the way to the simulation of the mixed contribution at NNLOPS level

Outline

- The exact computation of full mixed QCD-EW corrections is on its way
- I will discuss about some steps in this direction
  - ✦ qT subtraction scheme suitable for such NNLO computation
  - ✦ NLO corrections

Intro

The neutral current DY process represents a first step in this direction

Recent developments (selected)

- Mixed transverse momentum resummation for on-shell Z
  [Cieri, Ferrera, Sborlini 2018]

- Inclusive mixed corrections for on-shell Z
  [de Florian, Der, Fabre 2018]

- Differential mixed corrections for on-shell Z production + NLO EW to decay
  [Delto, Jaquier, Melnikov, Rontsch 2019]

- NNNLO QCD corrections for DY lepton pair production via virtual photon
  [Duhr, Dulat, Mistelberger 2020]

Automation of virtual one loop computation

- How it works in GoSam
  - Construction of 1-loop amplitude adopting the DRED scheme
  - Required the computation of all the renormalisation constants in DRED

Coupling and gauge fields

$$\delta Z_{AZ}^{\mathrm{DRED}} = \delta Z_{AZ}^{\mathrm{CDR}} + \frac{\alpha}{4\pi}\frac{4}{3}\frac{c_W}{s_W}$$

$$\delta Z_{W}^{\mathrm{DRED}} = \delta Z_{W}^{\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{2}{3s_W^2}$$

$$\delta Z_{e}^{\mathrm{DRED}} = \delta Z_{e}^{\mathrm{CDR}} + \frac{\alpha}{4\pi}\frac{1}{3}, \qquad \delta Z_{ZA}^{\mathrm{DRED}} = \delta Z_{ZA}^{\mathrm{CDR}},$$

$$\delta Z_{AA}^{\mathrm{DRED}} = \delta Z_{AA}^{\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{2}{3}, \qquad \delta Z_{ZZ}^{\mathrm{DRED}} = \delta Z_{ZZ}^{\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{2}{3}\frac{c_W^2}{s_W^2}, \qquad \delta Z_{W}^{\mathrm{DRED}}$$

$$\delta M_W^{2\,\mathrm{DRED}} = \delta M_W^{2\,\mathrm{CDR}} + \frac{\alpha}{4\pi}\frac{2}{3}\frac{M_W^2}{s_W^2}, \qquad \delta M_Z^{2\,\mathrm{DRED}} = \delta M_Z^{2\,\mathrm{CDR}} + \frac{\alpha}{4\pi}\frac{2}{3}\frac{M_Z^2 c_W^2}{s_W^2}.$$

$$\delta Z_{H}^{\mathrm{DRED}} = \delta Z_{H}^{\mathrm{CDR}}, \qquad \delta M_H^{2\,\mathrm{DRED}} = \delta M_H^{2\,\mathrm{CDR}} + \frac{\alpha}{4\pi}\frac{3(2c_W^2 M_W^2 + M_Z^2)}{2c_W^2 s_W^2}.$$

Massless light quarks

$$\delta Z_u^{L\,\mathrm{DRED}} = \delta Z_u^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{(3-4s_W^2)^2 + 18c_W^2}{36c_W^2 s_W^2},$$

$$\delta Z_u^{R\,\mathrm{DRED}} = \delta Z_u^{R\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{4s_W^2}{9c_W^2};$$

$$\delta Z_d^{L\,\mathrm{DRED}} = \delta Z_d^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{(3-2s_W^2)^2 + 18c_W^2}{36c_W^2 s_W^2},$$

$$\delta Z_d^{R\,\mathrm{DRED}} = \delta Z_d^{R\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{s_W^2}{9c_W^2}.$$

Massive light quarks

$$\delta m_u^{\mathrm{DRED}} = \delta m_u^{\mathrm{CDR}} + \frac{\alpha}{4\pi} m_u \frac{(3-4s_W^2)^2 + 2c_W^2(9+8s_W^2)}{72c_W^2 s_W^2},$$

$$\delta Z_u^{L\,\mathrm{DRED}} = \delta Z_u^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{(3-4s_W^2)^2 + 2c_W^2(9+8s_W^2)}{36c_W^2 s_W^2},$$

$$\delta Z_u^{R\,\mathrm{DRED}} = \delta Z_u^{R\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{4}{9c_W^2};$$

$$\delta m_d^{\mathrm{DRED}} = \delta m_d^{\mathrm{CDR}} + \frac{\alpha}{4\pi} m_d \frac{9 + 4s_W^4 + 2c_W^2(9+2s_W^2)}{72c_W^2 s_W^2},$$

$$\delta Z_d^{L\,\mathrm{DRED}} = \delta Z_d^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{(3-2s_W^2)^2 + 2c_W^2(9+2s_W^2)}{36c_W^2 s_W^2},$$

$$\delta Z_d^{R\,\mathrm{DRED}} = \delta Z_d^{R\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{1}{9c_W^2};$$

$$\delta m_e^{\mathrm{DRED}} = \delta m_e^{\mathrm{CDR}} + \frac{\alpha}{4\pi} m_e \frac{1 + 4s_W^2(s_W^2 - 2) + c_W^2(2+4s_W^2)}{8c_W^2 s_W^2},$$

$$\delta Z_e^{L\,\mathrm{DRED}} = \delta Z_e^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{1+2c_W^2}{4c_W^2 s_W^2},$$

$$\delta Z_e^{R\,\mathrm{DRED}} = \delta Z_e^{R\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{1}{c_W^2},$$

$$\delta Z_\nu^{L\,\mathrm{DRED}} = \delta Z_\nu^{L\,\mathrm{CDR}} - \frac{\alpha}{4\pi}\frac{1+2c_W^2}{4c_W^2 s_W^2}.$$

Automation of virtual one loop computation

- How it works in GoSam

  - Once the amplitude in DRED is renormalised (see next slide) we convert it to CDR to match the conventions of integrators like POWHEG, SHERPA and others

  - following unitarity argument [Catani, Seymour, Trocsanyi 1997], shifts are easily derived (in general) for example from the shifts in the integrated Catani Seymour dipoles involved in a specific computation

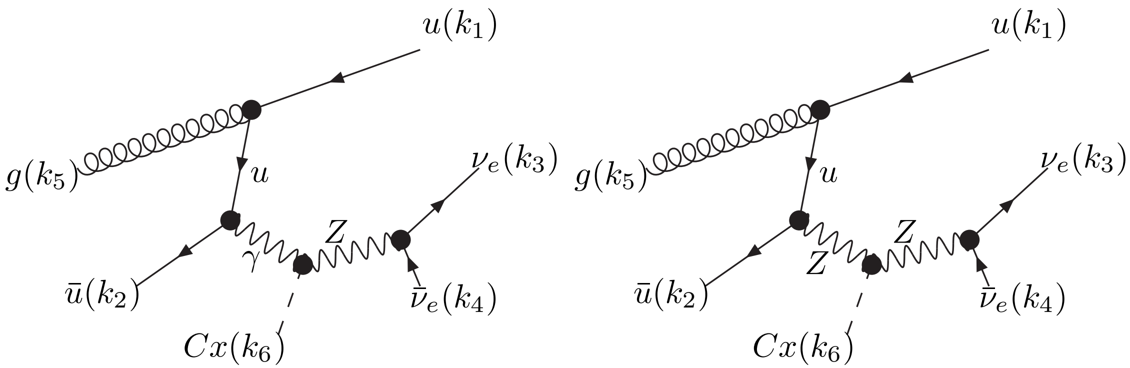
✓ QED radiation: shift is equal to the underlining tree level interference times the sum of factors:

$$\delta_{RS} = -\frac{1}{2} q_i \sigma_i q_k \sigma_k$$

for each pair of emitter (i) and spectator (k), sigma being 1 (-1) for an incoming fermion and outgoing anti-fermions (viceversa)

Automation of virtual one loop computation

- How it works in GoSam
  - We added the generation of the counter term diagrams that contracted with the Born amplitude provides the renormalisation
  - Technical strategy: introduce an extra dummy particle (Cx) with no momentum and mount all the counter term defining appropriate vertices including the Cx particle



$i\, g^{\mu\nu}\, (Z_1^{AZ}\, p^2 + Z_2^{AZ})$ $\qquad$ $i\, g^{\mu\nu}\, (Z_1^{ZZ}\, p^2 + Z_2^{ZZ})$

Z_{i} expressed in terms of the renormalisation constants of the previous slide

Automation of virtual one loop computation

- example: $u\bar{u} \to \nu\bar{\nu}g$ plus EW corr. (144 diagrams @1L, 4 helicities)

  $u\bar{u} \to \nu\bar{\nu}\gamma$ plus QCD corr. (24 diagrams @1L, 4 helicities)

- setup:

$$m_W = 80.376GeV \qquad \alpha = 0.0072973525376$$
$$\Gamma_W = 2.124GeV \qquad m_t = 171.1GeV$$
$$m_Z = 91.1876GeV \qquad m_H = 125GeV$$
$$\Gamma_Z = 2.4592GeV \qquad m_\tau = 1.77684GeV$$

- phase space point ($\mu = 500GeV$):

250.00000000000000	0.0000000000000000	0.0000000000000000	250.00000000000000
250.00000000000000	0.0000000000000000	0.0000000000000000	-250.00000000000000
82.919464801749541	60.472760266272836	-26.391765847389888	50.221087246723535
222.63574675290613	-179.51152719782607	-126.69164588330415	-35.937643199125638
194.44478844534430	119.03876693155314	153.08341173069402	-14.283444047597936

- results:

$u\bar{u} \to \nu\bar{\nu}g$ @NLOEW

GoSam	RECOLA
# LO: 0.4625281418689202E-06	
# NLO, finite part: -27.66024207452922	-27.66024207453221
# NLO, single pole: -1.3333333333333026	
# NLO, double pole: -0.8888888888888884	
# IR, single pole: -1.3333333333333333	
# IR, double pole: -0.8888888888888887	

timing: 2.5ms

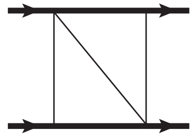
$u\bar{u} \to \nu\bar{\nu}\gamma$ @NLOQCD

GoSam	RECOLA
# LO: 0.1413813417377276E-07	
# NLO, finite part: 2.59890042341055	2.59890042341051
# NLO, single pole: -3.99999999999999	
# NLO, double pole: -2.66666666666665	
# IR, single pole: -4.00000000000000	
# IR, double pole: -2.66666666666666	

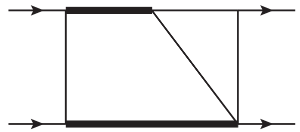
timing: 0.3ms

Two loop amplitudes

diagrams with which have problematic
root-valued leading singularities



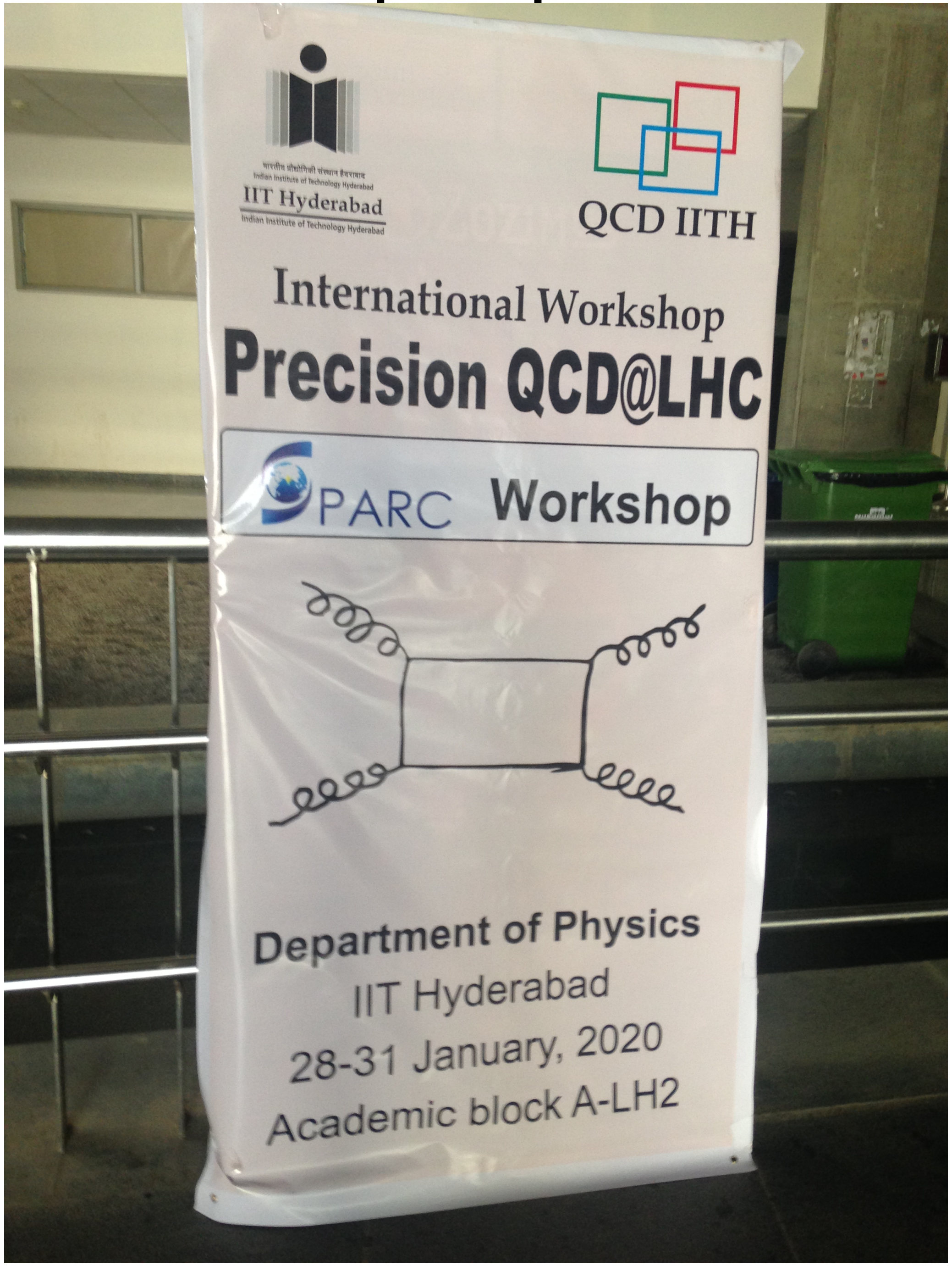
(s, t, m^2)



(s, t, m^2)

- Several groups are working on that

Two loop amplitudes



this building

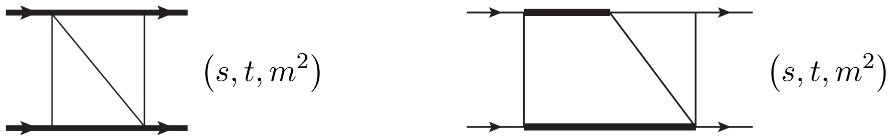
Two loop amplitudes

just outside Lorenzo's office in the next bdg

this building

Two loop amplitudes

diagrams with which have problematic root-valued leading singularities



- Several groups are working on that

- Master integrals computed by two groups:

  [Bonciani, Di Vita, Mastrolia, Schubert 1604.08581]

  [Heller, von Manteuffel, Schabinger 1907.00491]

- Note that the presence of two internal masses, mw and mz, is critical

$$\frac{1}{p^2 - m_Z^2} = \frac{1}{p^2 - m_W^2 - \Delta m^2} \approx \frac{1}{p^2 - m_W^2} + \frac{m_Z^2}{(p^2 - m_W^2)^2}\,\xi + \ldots$$

$$\xi = \frac{\Delta m^2}{m_Z^2} = \frac{m_Z^2 - m_W^2}{m_Z^2} \sim \frac{1}{4} \qquad \mathbf{(0.223)}$$

- Full amplitude still not built, but is now certainly doable

qT subtraction in a nutshell

- let X be a physical infrared safe variable that can separate the leading singularities from the rest

$$\frac{d\sigma}{d\Phi_n dX} = \frac{d\sigma^{\text{reg}}}{d\Phi_n dX} + \frac{d\sigma^{\text{sing}}}{d\Phi_n dX}$$

- the singular part can be expanded in powers of the strong coupling

$$\frac{d\sigma^{\text{sing}}}{d\Phi_n dX}(\Phi_N) = \mathcal{H}(\Phi_n)\delta(X) + \sum_{k\geq 0} \mathcal{C}_k(\Phi_n)\left[\theta(X)\frac{\ln^k X}{X}\right]_+ \qquad \mathcal{H} = \sum_{m\geq 0}\alpha_s^m \mathcal{H}^{(m)}, \quad \mathcal{C}_k = \sum_{m\geq 0}\alpha_s^m \mathcal{C}_k^{(m)}$$

$$\frac{d\sigma^{\text{sing}}}{d\Phi_n dX}(\Phi_N) = \sum_{m\geq 0}\alpha_s^m \left[\mathcal{H}^{(m)}(\Phi_n)\delta(X) + \sum_{k=0}^{2m-1}\mathcal{C}_k^{(m)}(\Phi_n)\left[\theta(X)\frac{\ln^k X}{X}\right]_+\right]$$

- considering as X the transverse momentum of a colourless system produced in hadron collisions (qT), the coefficient functions can be obtained by comparing the fixed order with the expansion of the resummation formula:

$$\frac{d\sigma_F^{\text{sing}}}{d^2\mathbf{q}_T dM^2 dy d\mathbf{\Omega}}(P_1,P_2;\mathbf{q}_T,M,y,\mathbf{\Omega}) = \frac{M^2}{S}\sum_{c=q,\bar{q},g}\frac{d\hat{\sigma}^{(0)}_{c\bar{c},F}}{dM^2 d\mathbf{\Omega}}(P_1,P_2;M,\mathbf{\Omega})$$
$$\times \int \frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\cdot\mathbf{q}_T}S_c(M,b)\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}[H^F C_1 C_2]_{c\bar{c};a_1,a_2} f_{a_1/h_1}(x_1,b_0^2/b^2) f_{a_2/h_2}(x_2,b_0^2/b^2)$$

qT subtraction

- Recent progress for NNLO subtraction schemes for all proposed solutions!

- Here we consider the qT subtraction formalism [Catani, Grazzini 2007]

$$d\sigma_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma_{LO} + \left[d\sigma^{F+jet}_{(N)LO} - d\sigma^{CT} \right]$$

- In the original formulation, the final state F of the Born process had to be a colourless system

- All genuine NNLO singularities manifest themselves in the limit of vanishing transverse momentum of the Born final state system (qT)

- Recently, the method has been successfully extended to treat radiation from the final state in case of massive particles as top pair production in hadron collisions

  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli 2019]

- The method developed for NNLO QCD corrections contains all the ingredients to address mixed QCD-EW corrections to the Drell-Yan process
  - ✦ indeed, it contains much more and the downgrade is not as trivial as it might appear

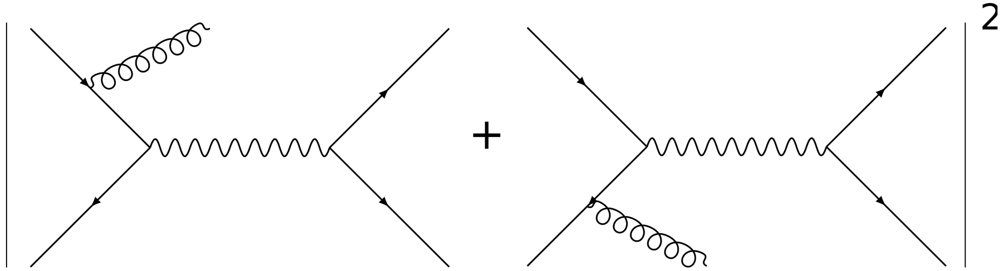
Method

- the counter term is non local by construction

$$d\sigma^{CT} = d\sigma_{L0} \otimes \Sigma^F \left(\frac{q_T}{M}\right) d^2 q_T$$

- At NLO: initial state

$\frac{q_T}{M} > r_{cut}$



$$\Sigma^{F(1;2)}_{c\bar{c}\to ab}(z)\tilde{I}_2\left(\frac{q_T}{M}\right) + \Sigma^{F(1;1)}_{c\bar{c}\to ab}(z)\tilde{I}_1\left(\frac{q_T}{M}\right)$$

$$\ln^2 r_{\rm cut} \sim \frac{1}{\epsilon^2} \qquad \ln r_{\rm cut} \sim \frac{1}{\epsilon}$$

In Mellin space:

$\Sigma^{F(1;2)}_{c\bar{c}\to ab,N} = -\frac{1}{2}A^{(1)}_c \delta_{ca}\delta_{\bar{c}b}$ soft-collinear, proportional to the color charges C_F, C_A

$\Sigma^{F(1;1)}_{c\bar{c}\to ab,N} = -[\delta_{ca}\delta_{\bar{c}b}B^{(1)}_c + \delta_{ca}\gamma^{(1)}_{\bar{c}b,N} + \delta_{\bar{c}b}\gamma^{(1)}_{ca,N}]$

soft: $-\frac{3}{2}C_F, -\frac{1}{6}(11C_A - 2n_f)$

collinear: convolution with AP kernels $\int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z}\right) P_{ca}(z)$

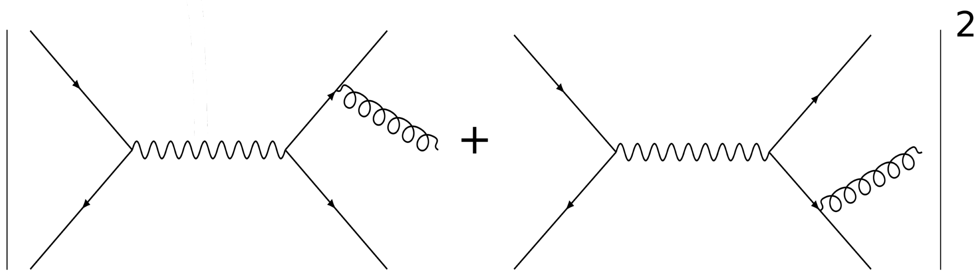
Method

- the counter term is non local by construction

$$d\sigma^{CT} = d\sigma_{L0} \otimes \Sigma^F \left(\frac{q_T}{M}\right) d^2 q_T$$

- At NLO: final state

$$\frac{q_T}{M} > r_{cut}$$



New contributions to the single pole part with non-trivial color structure

$$\Sigma^{Q\overline{Q}}_{c\overline{c}\to ab,N} = \Sigma^{F(1;1)}_{c\overline{c}\to ab,N} - \delta_{ca}\delta_{\overline{c}b} \frac{< \tilde{\mathcal{M}}^{(0)}_{c\overline{c}\to Q\overline{Q}} |(\Gamma_t^{c\overline{c}(1)} + \Gamma_t^{c\overline{c}(1)\dagger})| \tilde{\mathcal{M}}^{(0)}_{c\overline{c}\to Q\overline{Q}} >}{|\tilde{\mathcal{M}}^{(0)}_{c\overline{c}\to Q\overline{Q}}|^2}$$

$$\Gamma_t^{(1)} = -\frac{1}{4}\Bigg\{(\mathbf{T}_3^2+\mathbf{T}_4^2)(1-i\pi) + \sum_{i=1,2;j=3,4} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{(2p_i \cdot p_j)^2}{M^2 m^2} + 2\mathbf{T}_3 \cdot \mathbf{T}_4 \left[\frac{1}{2v} \ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v}+1\right)\right]\Bigg\}$$

Method: power corrections

$$d\sigma_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma_{LO} + \left[d\sigma^{F+jet}_{(N)LO} - d\sigma^{CT} \right]$$

- Real emission cross section and the counter term are integrated separately, giving rise to logs in r_{cut}
- Large global cancellation between logs (may affect numerical stability)
- The slicing is exact in the zero r_{cut} limit. For finite r_{cut}, it introduces spurious power suppressed terms
- Let's have a closer look at the counter term:

$$\tilde{I}_1(r) = -\frac{1}{r^2} + \frac{b_0^2}{4}(1 - 2\ln r) + O(r^2),$$

$$\tilde{I}_2(r) = \frac{4\ln r}{r^2} + \frac{b_0^2}{2}\left(-1 + 2\ln^2 r\right) + O(r^2)$$

$$d\hat{\sigma}^{CT}_{ab}(r_{\rm cut}) = \sum_{c=q,\bar{q},\gamma} \int_{r_{\rm cut}}^{\infty} 2r dr \, \frac{\alpha_{\rm S}}{\pi} \Sigma^{(1)}_{c\bar{c}\leftarrow ab} \otimes d\hat{\sigma}^{l^+l^-}_{LO\, c\bar{c}}$$

$$\frac{d\hat{\sigma}^{CT}_{ab}(r_{\rm cut})}{dr_{\rm cut}} = -2r_{\rm cut}\frac{\alpha_{\rm S}}{\pi}\left(\Sigma^{(1,2)}_{c\bar{c}\leftarrow ab}\tilde{I}_2(r_{\rm cut}) + \Sigma^{(1,1)}_{c\bar{c}\leftarrow ab}\tilde{I}_1(r_{\rm cut})\right) \otimes d\hat{\sigma}^{l^+l^-}_{LO\, c\bar{c}}$$

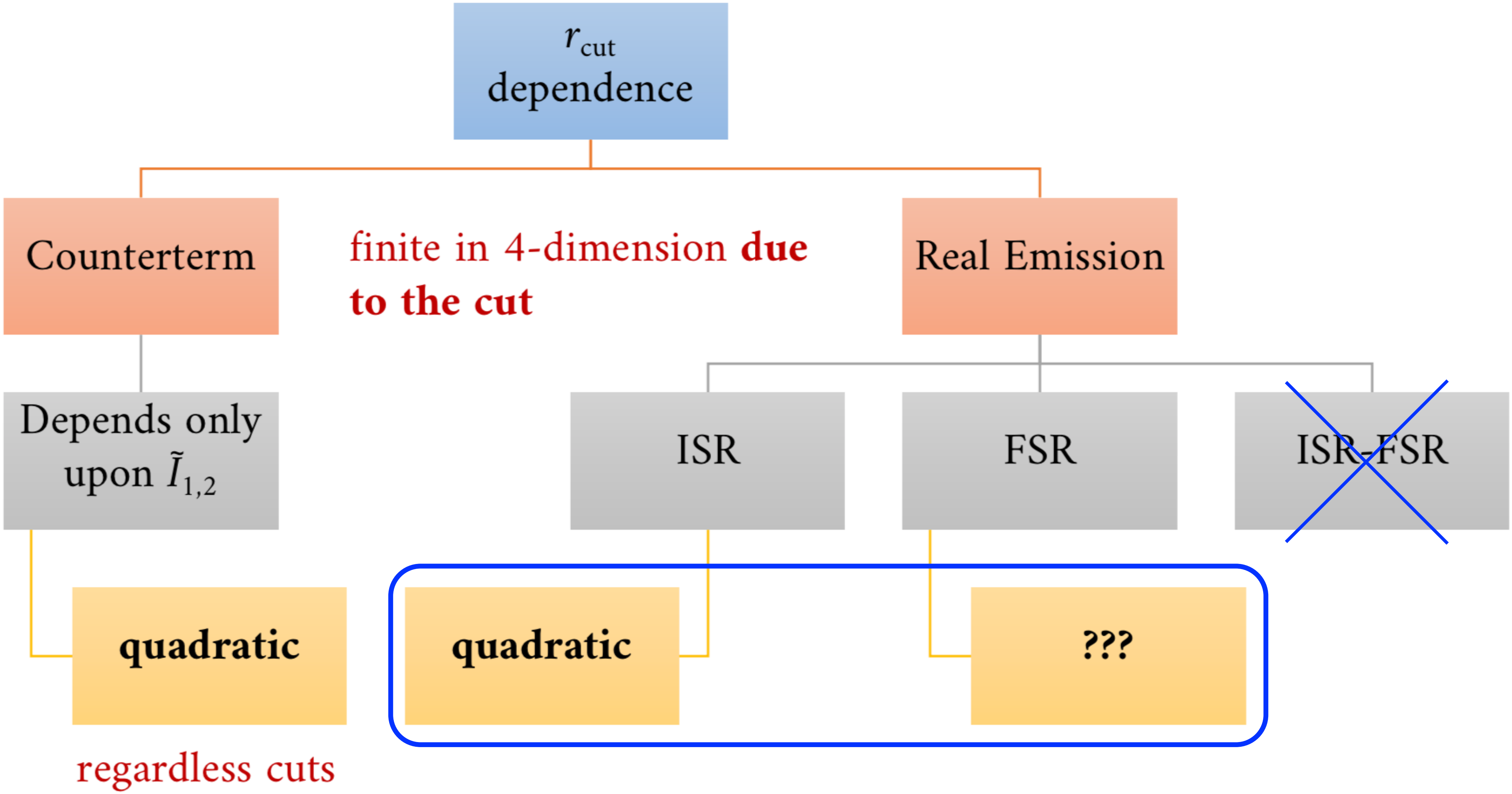
$$b_0 = 2\, e^{-\gamma_E}$$

➡ Counter term develops quadratic dependence on r_{cut} (valid also at NNLO!)

Overview

We focus on pure QED case (laboratory process):

- IS, FS and their interference are gauge invariant subsets and can be treated separately



Questions

1. set up the framework for initial state mixed corrections

2. Understand how the method performs with very small masses like muon mass

3. in view of the challenging integration, could be helpful to keep under control the cut dependence

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We start our exploration considering hadroproduction of a Z boson decaying to neutrinos

EW q_T-subtraction: abelianisation procedure

- full EW for the virtual part and only photons in the real emission

  → the subtraction sees only the QED part

- There is no need to compute it from scratch. Recycle QCD computation

- Exploit well established abelianisation procedure

  [de Florian, Sborlini, Rodrigo 2016]

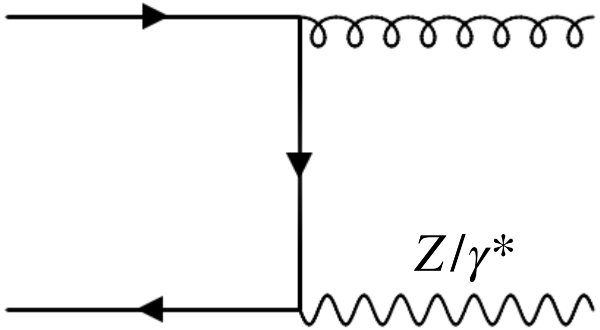
- FSR soft radiation:

$$\mathbf{T}_i^2 \to e_i^2 \mathbf{I}$$
$$\mathbf{T}_i \cdot \mathbf{T}_j \to e_i e_j \mathbf{I}$$

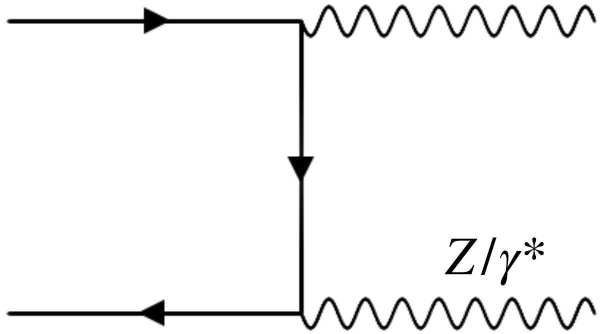
trivial color structure

Cross-checks: it reproduces the analytical structure in the eikonal limit

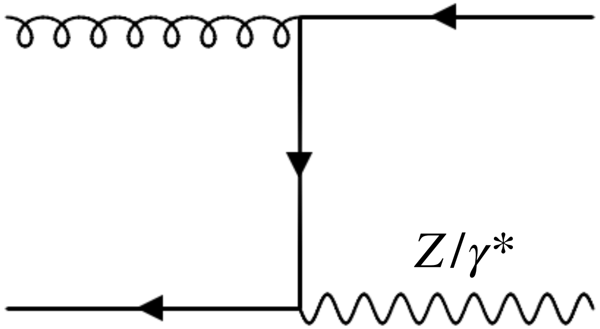
EW q_T-subtraction: abelianisation at NLO (ISR)



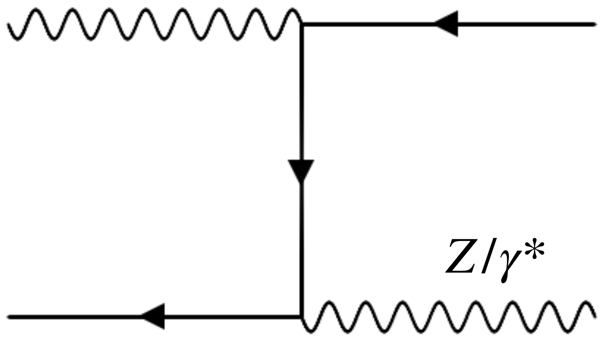
$$\frac{1}{N_C^2}\text{Tr}[T^aT^a] = \frac{C_F}{N_C}$$



$$\frac{1}{N_C^2}N_C e_f^2 = \frac{e_f^2}{N_C}$$



$$\frac{1}{N_C}\frac{1}{N_C^2-1}\text{Tr}[T^aT^a] = \frac{T_R}{N_C}$$



$$\frac{1}{N_C}N_C e_f^2 = e_f^2$$

EW q_T-subtraction: abelianisation at NLO (ISR)

Altarelli-Parisi splitting kernel and hard collinear functions for NLO EW

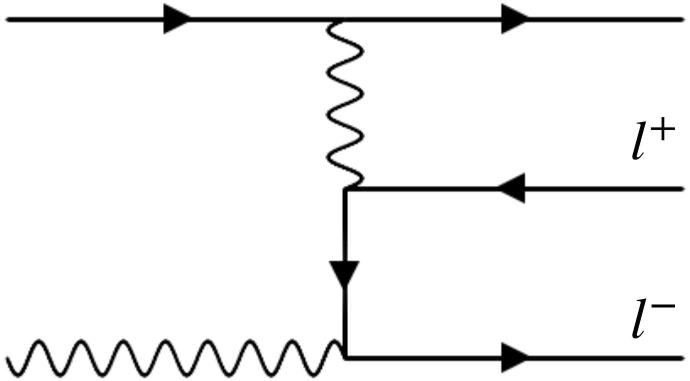
- splitting:

$$P_{qq}^{(1,0)} \to P_{qq}^{(0,1)} = \frac{e_f^2}{C_F} P_{qq}^{(1,0)} \qquad C_{qq}^{(1,0)} \to C_{qq}^{(0,1)} = \frac{e_f^2}{C_F} C_{qq}^{(1,0)}$$

- splitting:

$$P_{qg}^{(1,0)} \to P_{q\gamma}^{(0,1)} = \frac{e_f^2 N_C}{T_R} P_{qg}^{(1,0)} \qquad C_{qg}^{(1,0)} \to C_{q\gamma}^{(0,1)} = \frac{e_f^2 N_C}{T_R} C_{qg}^{(1,0)}$$

Later we will also need the splitting (for the corrections to the Born):



- splitting:

$$P_{gq}^{(1,0)} \to P_{\gamma q}^{(0,1)} = \frac{e_f^2}{C_F} P_{gq}^{(1,0)}$$

$$C_{gq}^{(1,0)} \to C_{\gamma q}^{(0,1)} = \frac{e_f^2}{C_F} C_{gq}^{(1,0)}$$

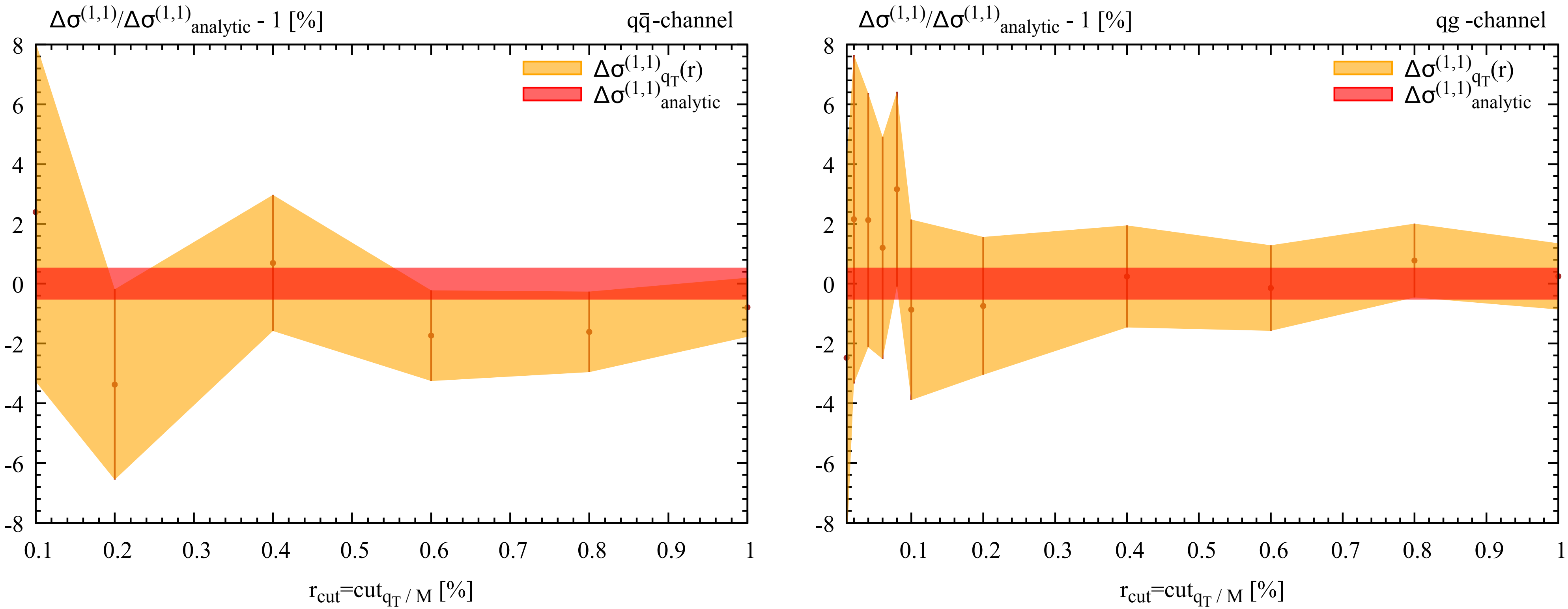
QCDxQED corrections for pp → Z(νν) with q_T-subtraction

$$\Delta_r^{(i,j)} = \frac{\Delta^{(i,j)}}{\sigma_{ref}} \qquad \sigma_{ref} = \sigma^{(0,0)} + \sigma^{(1,0)}$$

first index for QCD, second for QED

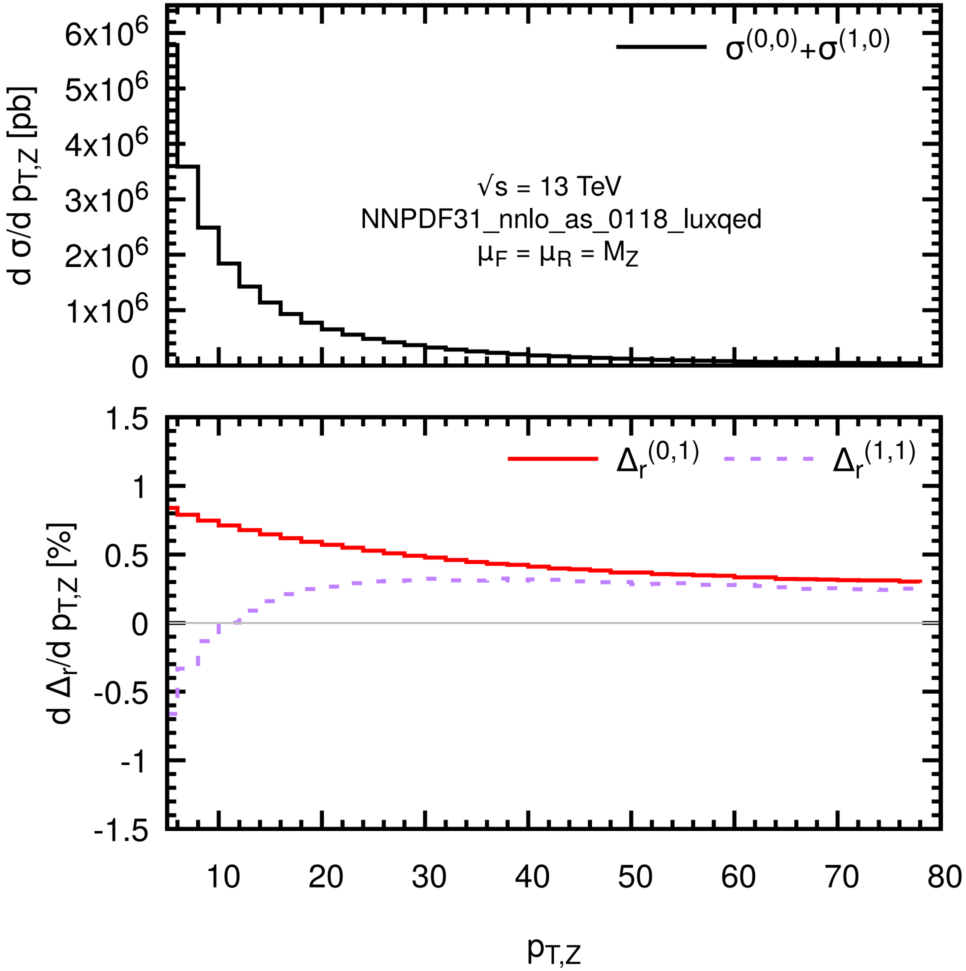
$$\Delta_r^{(0,1)} = (3.228 \pm 0.004) \times 10^{-3}, \quad \Delta_r^{(0,1)} = (6.34 \pm 0.14) \times 10^{-2}, \quad \Delta_r^{(1,1)} = (3.0 \pm 0.1) \times 10^{-4}$$

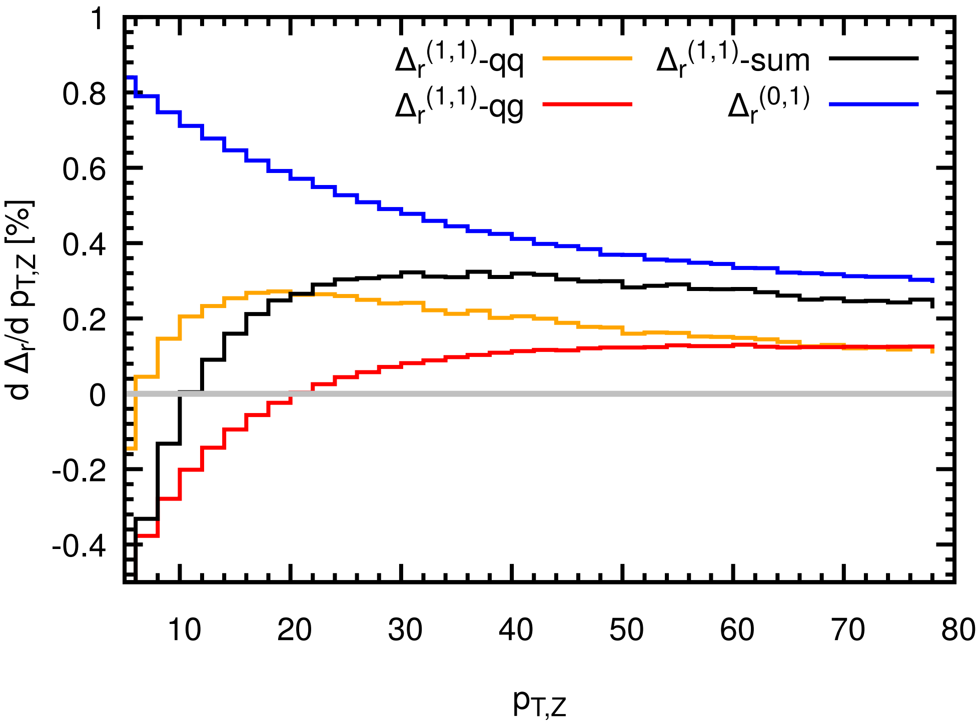
in good agreement with [Delto, Jaquier, Melnikov, Röntsch 2019]



QCDxQED corrections for pp → Z(νν) with q_T-subtraction

	$\Delta^{(1,1)}_{q\bar{q}}$ [pb]	$\Delta^{(1,1)}_{Qg}$ [pb]	$\Delta^{(1,1)}_{Q\gamma}$ [pb]	$\Delta^{(1,1)}_{g\gamma}$ [pb]	$\Delta^{(1,1)}_{qq+\bar{q}\bar{q}}$ [pb]
[de Florian, Der, Fabre 2018] analytic	57.46 ± 0.02	-39.5 ± 0.2	-1.576 ± 0.009	0.6496 ± 0.0016	0.594 ± 0.001
q_T subtraction	56.9 ± 0.6	-39.8 ± 0.5	-1.575 ± 0.013	0.646 ± 0.008	0.594 ± 0.003





Questions

1. set up the framework for initial state mixed corrections

2. Understand how the method performs with very small masses like muon mass

3. in view of the challenging integration, could be helpful to keep under control the cut dependence

NLO EW correction to Z/W boson decaying to massive leptons

EW corrections to the Drell-Yan process

Relevant literature

- Baur, Wackeroth et al., PRD 65 (2002) 033007, PRD 70 (2004) 073015
- Dittmaier, Kramer, PRD 65 (2002) 073007
- Jadach, Płaczek, EPJC 29 325 (2003), D. Bardin et al., Acta Phys. Polon. B40 (2009) 75
- Carloni Calame et al., PRD 69 (2004) 037301, JHEP 0612 (2006) 016, JHEP 0710 (2007) 109
- Arbuzov et al., EPJC 46, 407 (2006), EPJC 54 (2008) 451
- Dittmaier, Huber, JHEP 1001 (2010) 060
- Barze' et al., EPJC 73 (2013) no.6, 2474

not meant to be exhaustive!

Tools

- Z/WGRAD, NLO EW to CC and NC DY
- SANC, NLO EW to CC and NC DY
- WINHAC, NLO EW + multiple photon to CC DY
- HORACE, NLO EW + matched multiple photon emission to CC and NC DY
- RADY, NLO EW + MSSM to NC DY
- POWHEG, factorised QCDxEW matched to parton shower

EW corrections to the Drell-Yan process

Inhouse implementation

Framework: dynnlo fortran code

Matrix elements:

- all tree-level amplitudes computed using helicity amplitudes (FORM)
- EW on loop amplitude generated with GoSam and Recola for cross check

Subtraction scheme:

- qT subtraction, abelianised version of the heavy quark case
- Catani-Seymour for cross check

EW renormalization scheme: complex mass + Gmu-scheme

Work in progress: port into MATRIX (in collaboration with S. Kallweit)

NLO EW: physical case with muons

Benchmark setup similar to [Dittmaier, Huber 2010]:

Physical Parameters (G_μ-scheme):

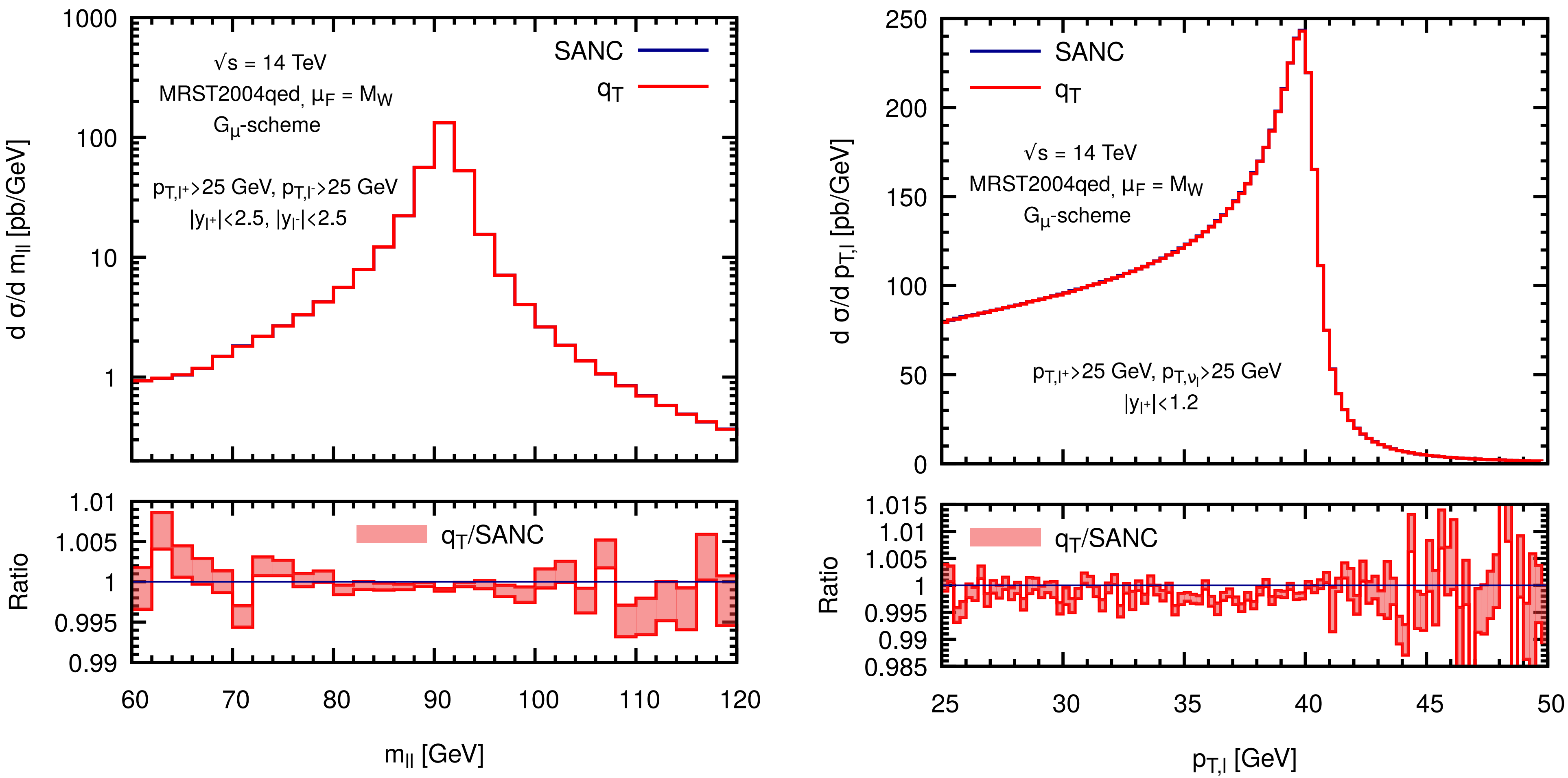
- G_F = 1.16637 x10^{-5} GeV^{-2} — α_0 = 1/137.03599911
- M_W = 80.403 GeV — M_Z = 91.1876 GeV
- Γ_W = 2.141 GeV — Γ_Z = 2.4952 GeV
- $m_l = m_\mu$ =105.658369 MeV — M_H = 115 GeV

Fiducial cuts:

- $M_{ll} > 50$ GeV $p_{T,l} > 25$ GeV $|y_l| < 2.5$
- *no lepton-photon recombination*

	q_T + GoSam	Sanc
$\Delta\sigma_{q\bar{q}} + \Delta\sigma_{q\gamma}$ (pb)	-29.95 ± 0.04	-29.99 ± 0.02

NLO EW: differential distributions



- ✤ efficiency of the method strongly depends on the usage of Monte Carlo techniques
  - ➡ importance sampling and multi channel integration

Questions

1. set up the framework for initial state mixed corrections

2. Understand how the method performs with very small masses like muon mass

3. in view of the challenging integration, could be helpful to keep under control the cut dependence

Let’s consider specifically hadroproduction of a Z boson decaying to charged leptons

NLO EW: case for regulator study

Benchmark setup similar to [Dittmaier, Huber 2010]:

Physical Parameters (G_μ-scheme):

- G_F = 1.16637 x10^{-5} GeV^{-2} α_0 = 1/137.03599911
- M_W = 80.403 GeV M_Z = 91.1876 GeV
- Γ_W = 2.141 GeV Γ_Z = 2.4952 GeV
- m_l = 10 GeV M_H = 115 GeV

Fiducial cuts:

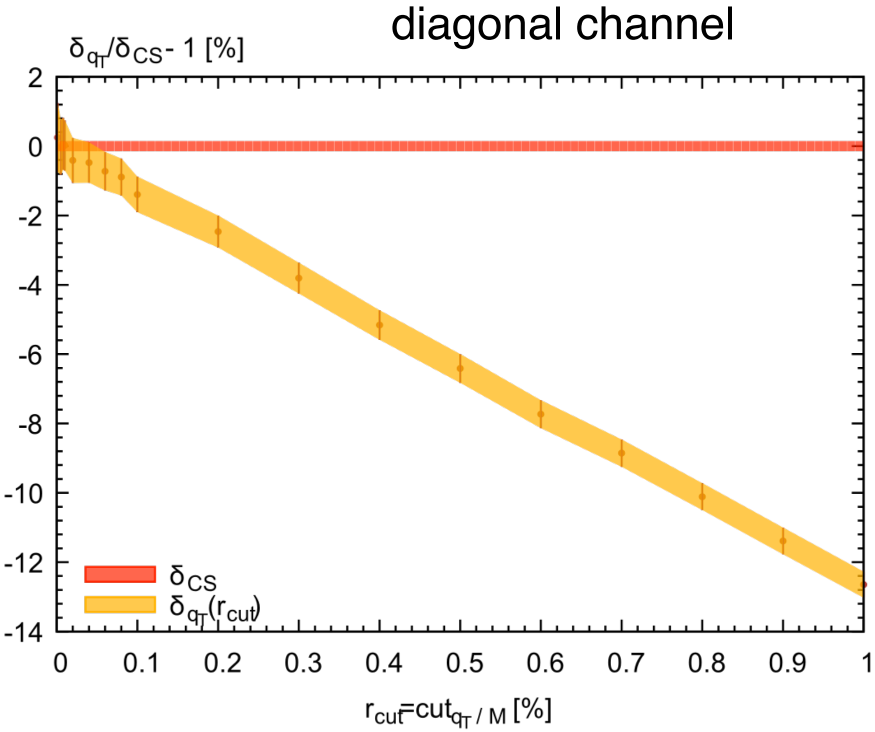
- $M_{ll} > 50$ GeV $p_{T,l} > 25$ GeV $|y_l| < 2.5$
- *no lepton-photon recombination*

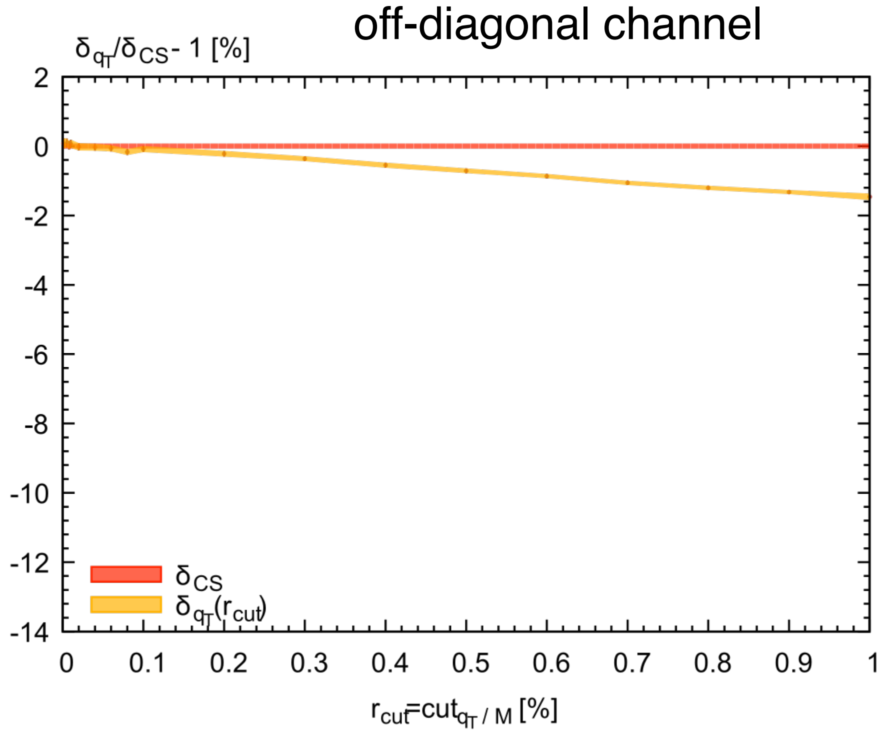
	q_T + GoSam	CS+RECOLA
$\sigma^{q\bar{q}}_{LO}$ (pb)	683.53 ± 0.03	
$\Delta\sigma_{q\bar{q}}$ (pb)	$-5.920 \pm 0.034^*$	-5.919 ± 0.008
$\sigma^{\gamma\gamma}_{LO}$ (pb)	1.1524 ± 0.0004	
$\Delta\sigma_{q\gamma}$ (pb)	-0.6694 ± 0.0008	-0.6690 ± 0.0005

* uncertainty dominated by the real-ct contribution and extrapolation at r_{cut} =0

NLO EW: case for regulator study

Dependence of the NLO corrections on the r_{cut} regulator for the **fiducial cross section**



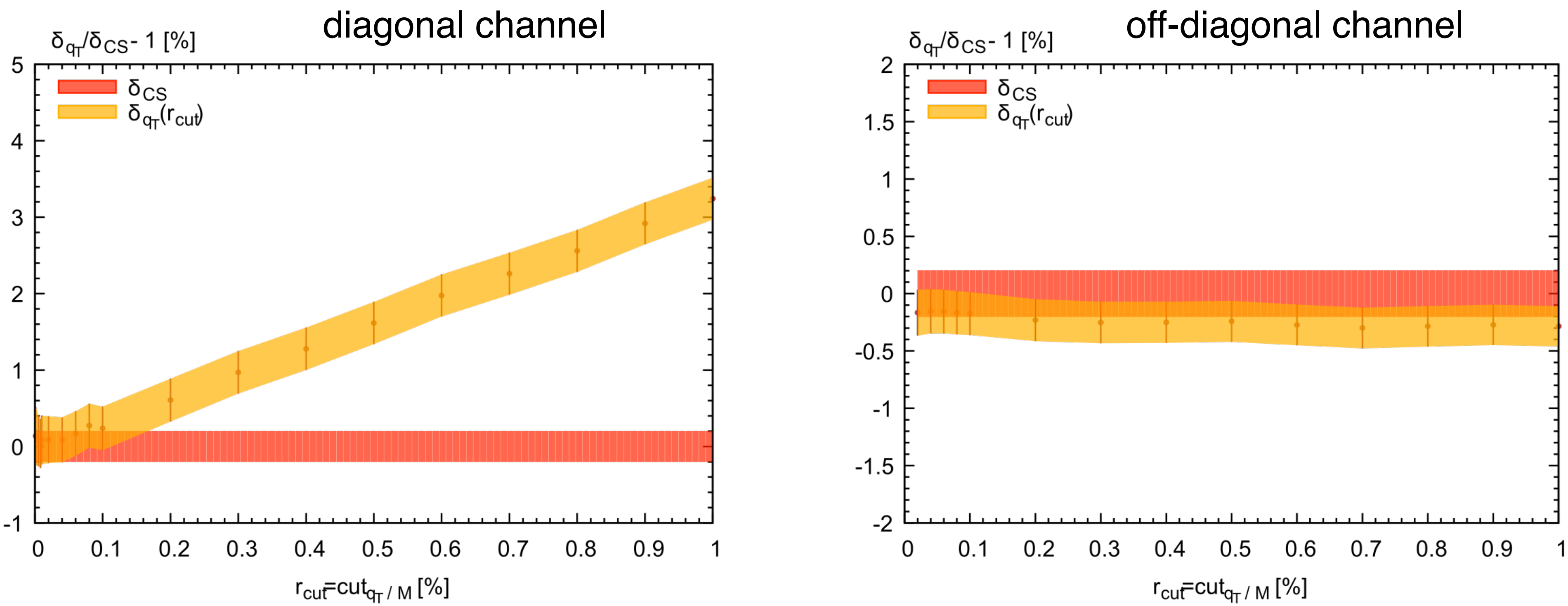


The q_T prediction has been obtained with a **linear extrapolation**.

Remark: Sizeable -dependence also in the channel.
Symmetric cuts on the p_T of the leptons **worsen** the dependence already for color singlet production (no final state radiation)
[Grazzini, Kallweit, Wiesemann 2018]

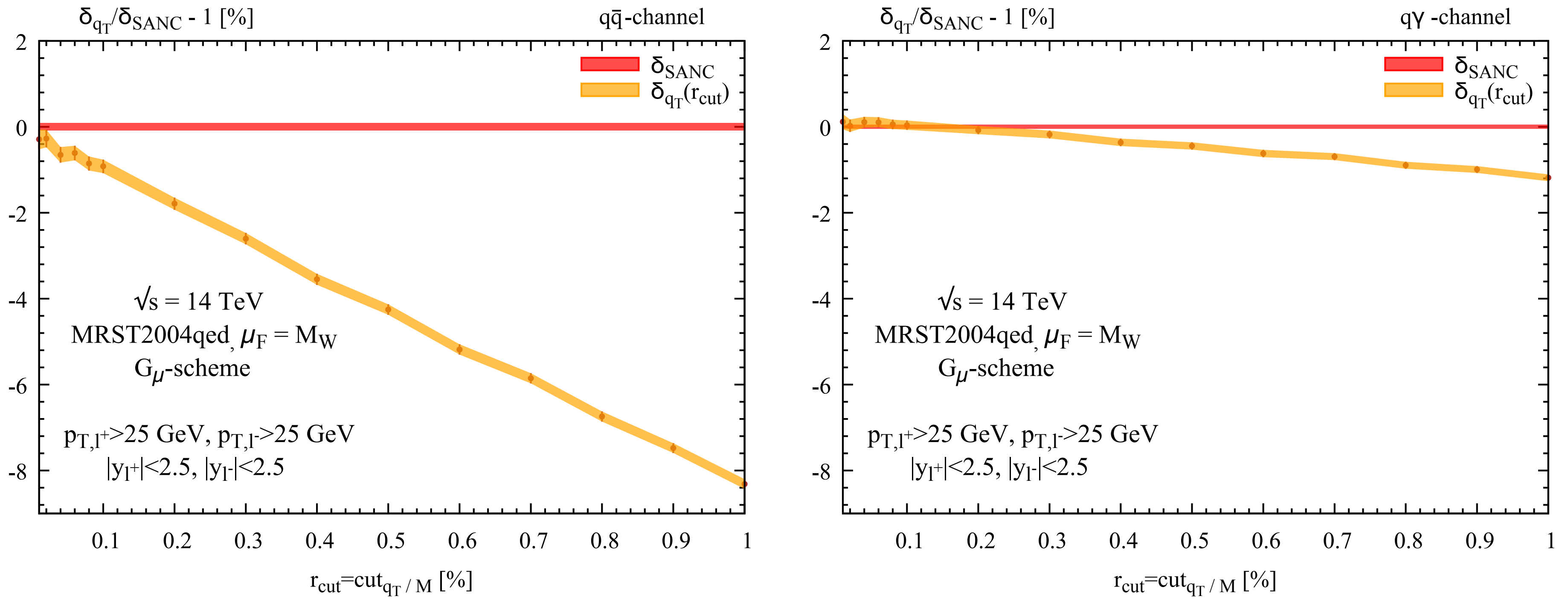
NLO EW: case for regulator study

Dependence of the NLO corrections on the r_{cut} regulator for the **inclusive cross section**



- **Flat dependence in the qγ off-diagonal channel**, as it occurs in color singlet [Grazzini, Kallweit, Wiesemann 2018]
- Distinct **linear behavior** in the **qq diagonal channel** as in heavy quark production, genuine effect of **the emission off massive final state**

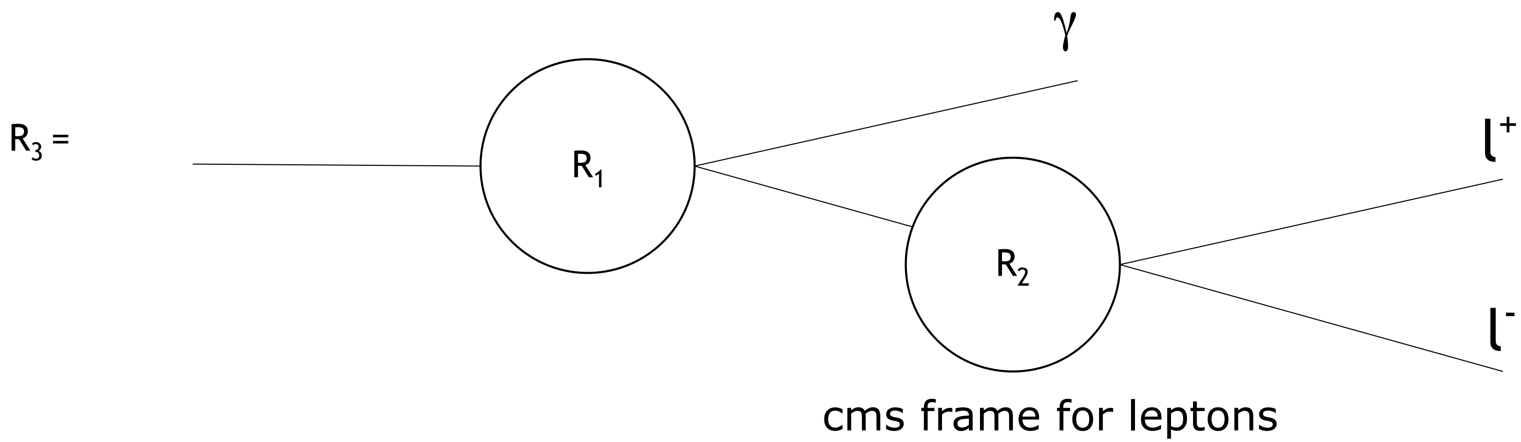
NLO EW: physical case, comparison with SANC



Analytic computation of Power Corrections

- $q\bar{q} \to l^+ l^- \gamma$ at tree level simple enough to study rcut dependence

- phase space parametrisation



$$\sigma = \frac{1}{16}\frac{1}{(2\pi)^4}\int dM^2 dq_T^2 \frac{1}{\sqrt{(s-M^2)^2 - 4q_T^2 s}}\sqrt{1-\frac{4m^2}{M^2}}\int d\Omega |\mathcal{M}|^2$$

- q_T appears explicitly among the integration variables.
  It allows a simplified treatment of the cut in the integration.

Analytic computation of Power Corrections

- $q\bar{q} \to l^+ l^- \gamma$ at tree level simple enough to study rcut dependence

- phase space parametrisation

$$\frac{q_T}{M} > r_{cut}$$

$$\frac{d\sigma}{dr_{\rm cut}^2} = -\frac{1}{32}\frac{1}{(2\pi)^4}\int_{z_{\rm min}}^{z_{\rm max}} \frac{z\,dz}{\sqrt{(1-z)^2 - 4z r_{\rm cut}^2}}\sqrt{1-\frac{4m^2}{zs}}\int d\Omega |\mathcal{M}|^2$$

$$z = \frac{M^2}{s}, \quad z_{\rm min} = \frac{4m^2}{s}, \quad z_{\rm max} = 1 - 2r_{\rm cut}\sqrt{1+r_{\rm cut}^2} + 2r_{\rm cut}^2$$

Analytic computation of Power Corrections

- First, integrate over angular variables exploiting known results

[W. Beenakker, H. Kuijf, W. L. van Neerven, PRD1989]

$$I^{(k,l)} = \int_0^\pi \sin\vartheta_1 d\vartheta_1 \int_0^\pi d\vartheta_2 (a + b\cos\vartheta_1)^{-k}(A + B\cos\vartheta_1 + C\sin\vartheta_1\cos\vartheta_2)^{-j}$$

$$\frac{d\sigma^{\rm FS}}{dr_{\rm cut}^2} = \frac{4\alpha^3 e_q^2}{3s}\int_{z_{\rm min}}^{z_{\rm max}} dz \left[\frac{K_1(z;m^2/s)}{(1-z)^2\sqrt{(1-z)^2-4zr_{\rm cut}^2}} + \frac{K_2(z;m^2/s)r_{\rm cut}^2}{(1-z)^4\sqrt{(1-z)^2-4zr_{\rm cut}^2}}\right]$$

$$\frac{d\sigma^{\rm IS}}{dr_{\rm cut}^2} = -\frac{4\alpha^3 e_q^4}{9s}\int_{z_{\rm min}}^{z_{\rm max}} dz \left[\frac{K_3(z;m^2/s)}{r_{\rm cut}^2\sqrt{(1-z)^2-4zr_{\rm cut}^2}} + \frac{K_4(z;m^2/s)}{\sqrt{(1-z)^2-4zr_{\rm cut}^2}}\right]$$

- coefficient functions K_i regular at z=1
- to get an expansion in rcut we treat the singular factors as distributions

Expansions

We expand all the relevant distributions

FS

$$\frac{\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^2\sqrt{(1-z)^2-4zr_{\rm cut}^2}} = \frac{1}{4}\delta(1-z)\frac{1}{r_{\rm cut}^2} + \frac{\pi}{8}\left[\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\rm cut}} + \mathcal{O}(1)$$

contribution to linear NLP

$$\frac{r_{cut}^2\,\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^4\sqrt{(1-z)^2-4zr_{\rm cut}^2}} = \frac{1}{24}\delta(1-z)\frac{1}{r_{\rm cut}^2} + \frac{\pi}{64}\left[3\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\rm cut}} + \mathcal{O}(1)$$

Remark: up to the considered order, no dependence on the lower limit

IS

$$\begin{aligned} T(z, r_{\rm cut}, z_{\min}) &= \frac{\Theta(z-z_{\min})\Theta(z_{\max}-z)}{\sqrt{(1-z)^2-4zr_{\rm cut}^2}} \\ &= T^{(0,1)}(z,a)\ln r_{cut}^2 + T^{(0,0)}(z,a) \\ &+ T^{(2,1)}(z,a)r_{cut}^2\ln r_{cut}^2 + T^{(2,0)}(z,a)r_{cut}^2 \end{aligned}$$

Results

Born cross section

$$\sigma_0(s) = \frac{2\pi}{9s}\alpha^2 e_q^2 \beta(3-\beta^2) \qquad \beta = \sqrt{1-\frac{4m^2}{s}}$$

pure linear NLP (no logs!)

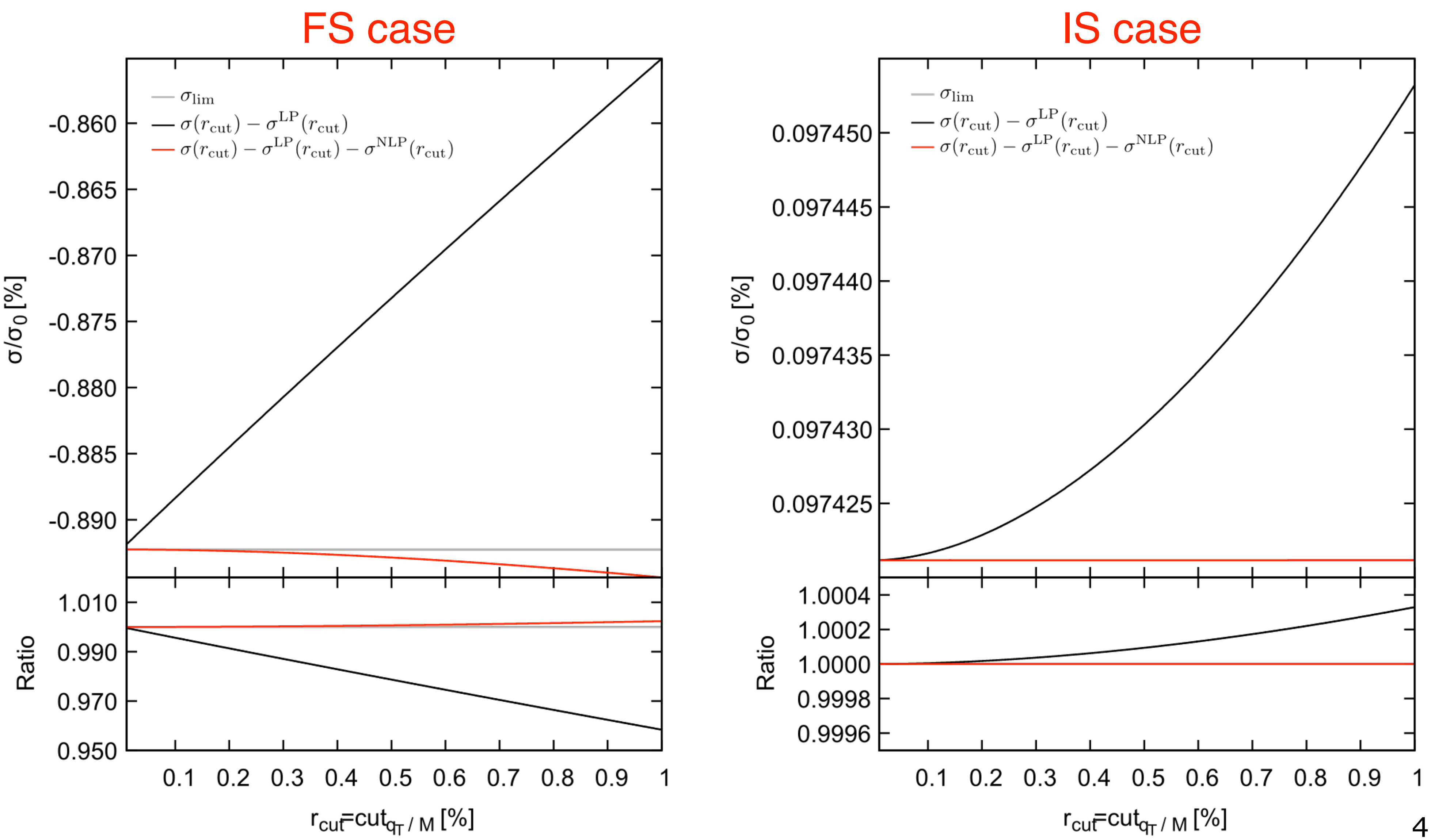
$$\begin{aligned}\sigma^{\rm FS}(s;r_{\rm cut}) - \sigma_{\rm lim} = \sigma_0(s)\frac{\alpha}{2\pi}\Bigg\{ &\left[2-\frac{(1+\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta}\right]\log\left(r_{\rm cut}^2\right) \\ &-\frac{3\pi}{8}\left[\frac{6(5-\beta^2)}{3-\beta^2}+\frac{(-47+8\beta^2+3\beta^4)}{\beta(3-\beta^2)}\log\frac{1+\beta}{1-\beta}\right]r_{\rm cut}\Bigg\} + O(r_{\rm cut}^2) \\ &\equiv \sigma^{\rm LP}(r_{\rm cut}) + \sigma^{\rm NLP}(r_{\rm cut}) + O(r_{\rm cut}^2)\end{aligned}$$

$$\begin{aligned}\sigma^{\rm IS}(s;r_{\rm cut}) - \sigma_{\rm lim} = \sigma_0(s)\frac{\alpha}{2\pi}e_q^2\Bigg\{ &\ln^2 r_{\rm cut}^2 - 4\left(2\ln 2 - \frac{4}{3} - \ln\frac{1-\beta^2}{\beta^2} - \frac{1}{\beta(3-\beta^2)}\ln\frac{1+\beta}{1-\beta}\right)\ln r_{\rm cut}^2 \\ &-\frac{3}{2}\frac{(1+\beta^2)(1-\beta^2)^2}{\beta^4(3-\beta^2)}r_{\rm cut}^2\ln r_{\rm cut}^2 - \frac{3}{2}\frac{(1+\beta^2)(1-\beta^2)^2}{\beta^4(3-\beta^2)}\left(1-4\ln 2+2\ln\frac{1-\beta^2}{\beta^2}\right)r_{\rm cut}^2\Bigg\} + \ldots. \\ &\equiv \sigma^{\rm LP}(r_{\rm cut}) + \sigma^{\rm NLP}(r_{\rm cut}) + \ldots\ldots.\end{aligned}$$

- **Check**: as byproduct we re-derive the result for the production of a color-singlet system of fixed mass [Cieri, Oleari, Rocco 2019]
- **Remark:** Our partonic result is a smooth function of β, at variance with what happens for the on-mass shell color singlet production

Validation: numerical checks

Dependence of the **real emission partonic cross section** on the regulator



Hadronic cross section

- In principle the convolution integrals might change the cut dependence

$$z_{\max} = 1 - 2r_{\rm cut} + \mathcal{O}(r_{\rm cut}^2)$$

$$\sigma(S, r_{\rm cut}) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab}(s, r_{\rm cut}) \delta(x_1 x_2 S - s)$$

$$s > \frac{4m^2}{z_{\max}}$$

- By using: $x_1 = \sqrt{\frac{z_0}{z}} e^y, \qquad x_2 = \sqrt{\frac{z_0}{z}} e^{-y}, \qquad z_0 \equiv \frac{4m^2}{S}$

$$\begin{aligned}
\sigma(S, r_{\rm cut}) &= \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \Theta\left(x_1 x_2 S - \frac{4m^2}{z_{\max}}\right) \hat{\sigma}_{ab}\left(s = x_1 x_2 S, r_{\rm cut}\right) \\
&= z_0 \sum_{a,b} \int_{z_0}^{z_{\max}} \frac{dz}{z^2} \int_{\ln\sqrt{z_0/z}}^{-\ln\sqrt{z_0/z}} dy f_a\left(\sqrt{\frac{z_0}{z}} e^y, \mu_F\right) f_b\left(\sqrt{\frac{z_0}{z}} e^{-y}, \mu_F\right) \hat{\sigma}_{ab}\left(s = \frac{4m^2}{z}, r_{\rm cut}\right) \\
&\equiv \sum_{a,b} \int_{z_0}^{z_{\max}} dz\, \mathcal{L}_{ab}(z, z_0; \mu_F)\, \hat{\sigma}_{ab}\left(s = \frac{4m^2}{z}, r_{\rm cut}\right)
\end{aligned}$$

However: $\hat{\sigma}_{ab}\left(s = \frac{4m^2}{z_{\max}}, r_{\rm cut}\right) = 0$ a sufficient conditions to prevent the appearance of a further linear term upon integration

Questions:

- can we remove the linear rcut dependence?

- can we do it at differential level?

Remarks:

✦ this is just an academic exercise specific for qT subtraction at NLO:
Catani Seymour dipoles or FKS local schemes do not have such problem

✦ the aim is to improve the efficiency of the method for NNLO @ differential level eventually

Simple solution

- Note that qT counter term is integrated over the full qT range (from qT=0)

$$d\hat{\sigma}^F_{NLO} = \mathcal{H}^F_{NLO} \otimes d\hat{\sigma}^F_{LO} + \left[d\hat{\sigma}^{F+\text{jet}}_{LO} - d\hat{\sigma}^{F,CT}_{NLO} \right] \Theta\left(\frac{q_T}{M} - r_{\text{cut}} \right)$$

- perform integration of real matrix element below rcut (only FSR for the moment) and use:
  - ✤ normal qt subtraction above rcut
  - ✤ another auxiliary cross section below rcut with the only requirement that it does not generate linear rcut dependance upon integration

$$d\hat{\sigma}^F_{NLO} = \mathcal{H}^F_{NLO} \otimes d\hat{\sigma}^F_{LO} + \left[d\hat{\sigma}^{F+\text{jet}}_{LO} - d\hat{\sigma}^{F,CT}_{NLO} \right] \Theta\left(\frac{q_T}{M} - r_{\text{cut}} \right) + \left[d\hat{\sigma}^{F+\text{jet}}_{FS,LO} - d\hat{\sigma}^{F,CT}_{S,NLO} \right] \Theta\left(r_{\text{cut}} - \frac{q_T}{M} \right)$$

➡ one can chose also a local counter term

(no need to make any analytic integration)

➡ we have chosen a local mapping and a massive FKS subtraction

$$d\hat{\sigma}^{CT}_{S} = d\hat{\sigma}_{LO}(\Phi_B) \times \frac{e^2}{4\pi^3 s} \frac{d\xi}{\xi} dy d\phi \left[\frac{s - 2m^2}{(1-\beta y_{\text{phy}})(1+\beta y_{\text{phy}})} - \frac{m^2}{(1-\beta y_{\text{phy}})^2} - \frac{m^2}{(1+\beta y_{\text{phy}})^2} \right]$$

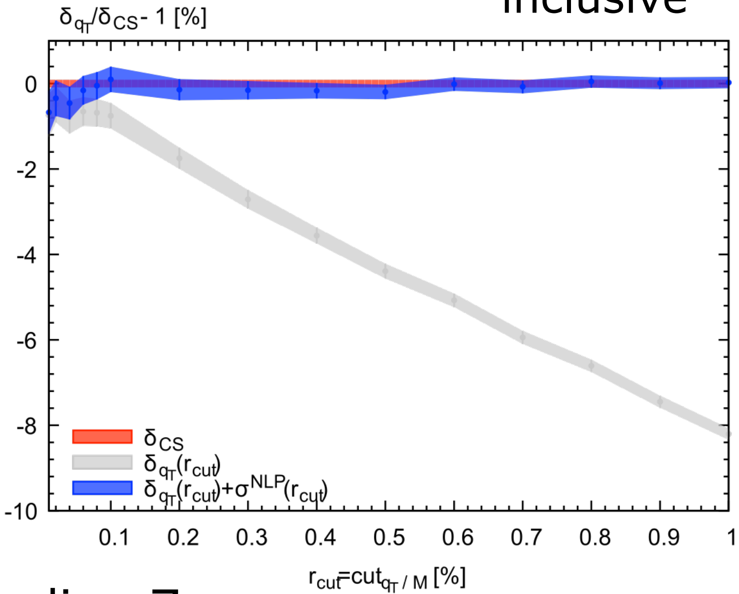
[Buonocore MSc thesis 2016]

Going beyond inclusive predictions

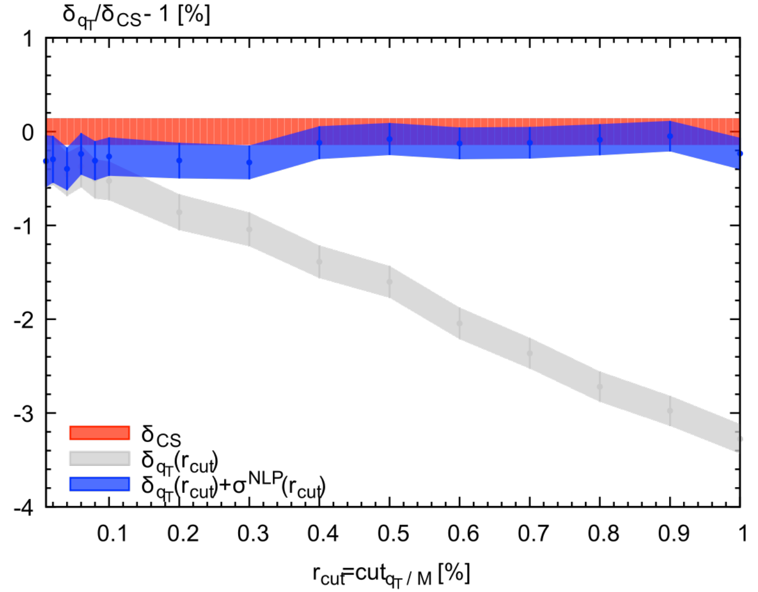
asymmetric cuts

Pure QED case

inclusive

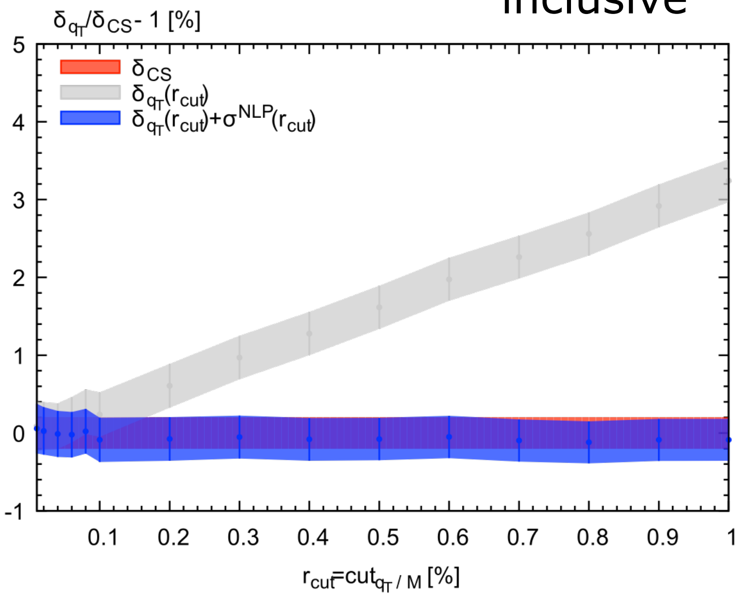


$|y_l| < 2.5, \quad p_{T,l^-} > 25\,\mathrm{GeV}, \quad p_{T,l^+} > 20\,\mathrm{GeV}$

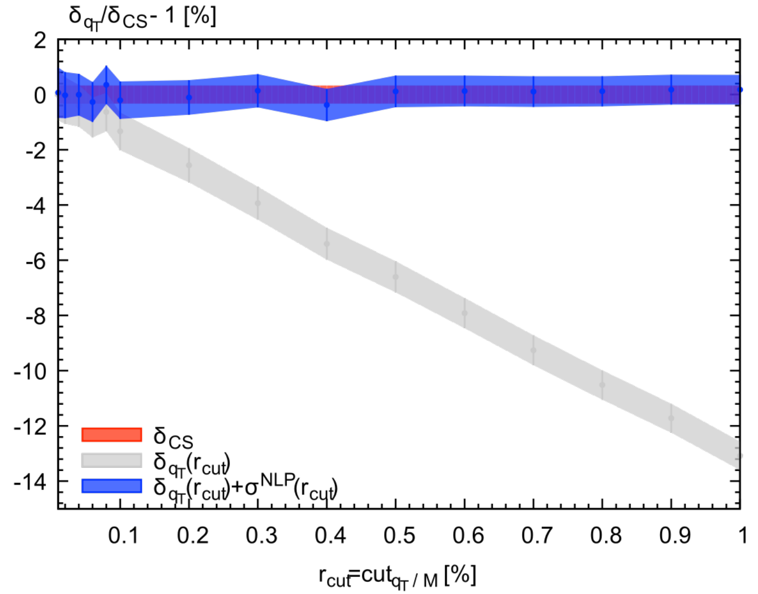


Including Z resonance

inclusive



$|y_l| < 2.5, \quad p_{T,l^-} > 25\,\mathrm{GeV}, \quad p_{T,l^+} > 20\,\mathrm{GeV}$



Simple solution

- it remains to understand if it works also at NNLO

  ✤ possibly an optimal double soft and a soft-collinear mappings and counter terms might be enough

  ✤ we stress again that one can choose them without worrying of their analytic integration, because it is not needed

Conclusion

- Current experiments at CERN and future plans urgently need of the computation of higher order corrections
- mixed QCD-EW NNLO corrections among them
- a campaign is ongoing in the community approaching the problem form different perspectives
- subtraction schemes are in good shape, I have discussed a bit about qT subtraction for EW(QED) and mixed QCDxEW(QED) corrections
- can be easily be extended to compute mixed corrections to tt production in pp and also in e+e- collisions
- bottleneck is probably double virtual computation, but things are proceeding fast!

Stay tuned!