Higgs+n-gluon amplitude at one loop

Satyajit Seth

International Workshop
Precision QCD @LHC

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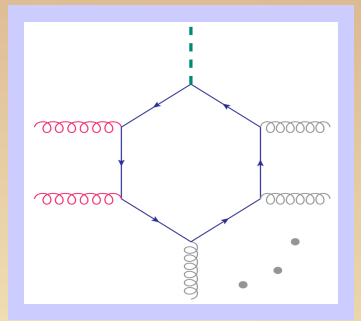
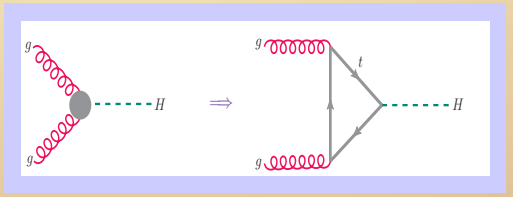


- Higgs boson: Primary focus of Run 3 and high luminosity LHC
- Large top mass (m_t) limit: one reduced loop at every order

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\left\{1 - \frac{\alpha_s}{3\pi}\frac{H}{v}\left(1 + \frac{\alpha_s}{\pi}\frac{11}{4}\right)\right\} G^A_{\mu\nu} G^{A\mu\nu}$$

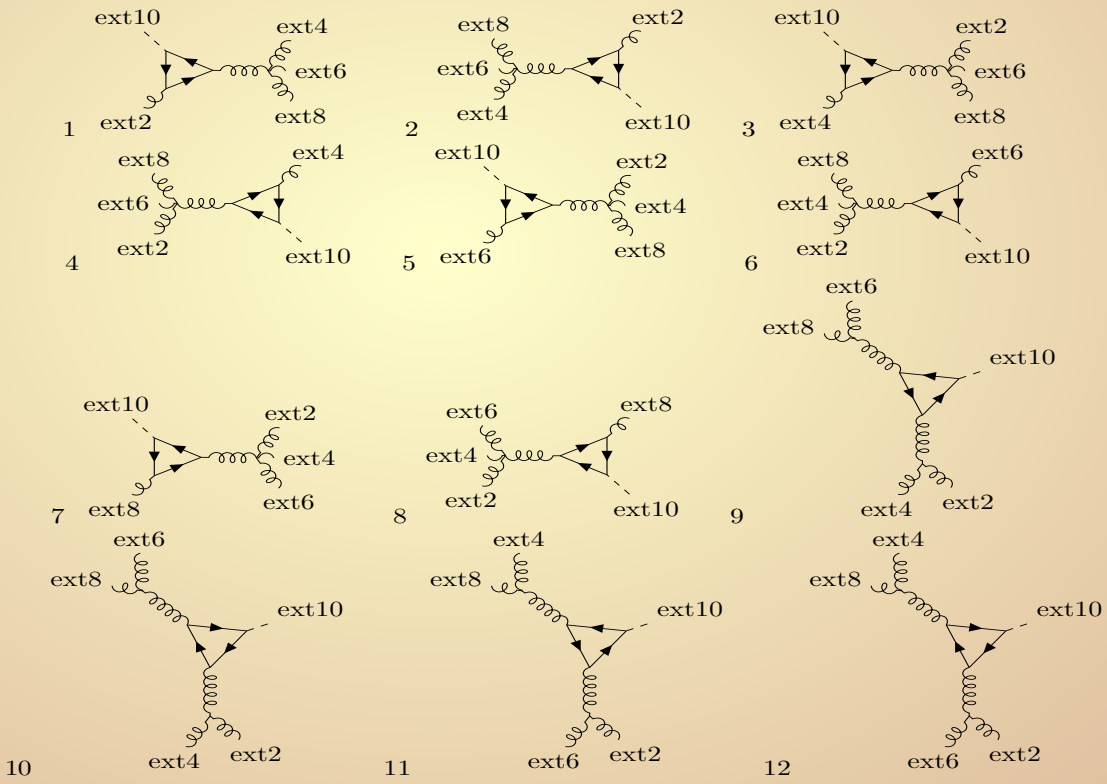
- Intermediate details become visible at high energies

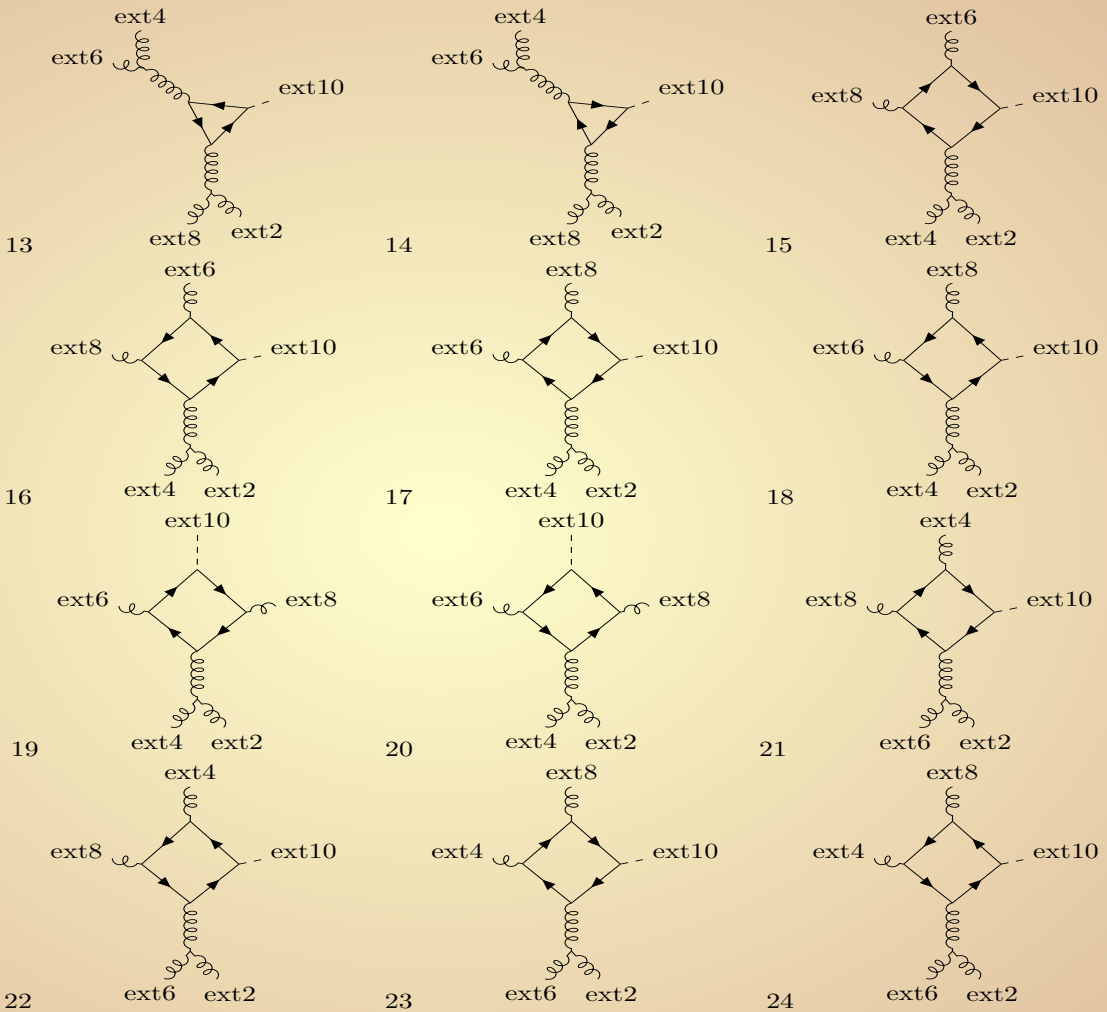


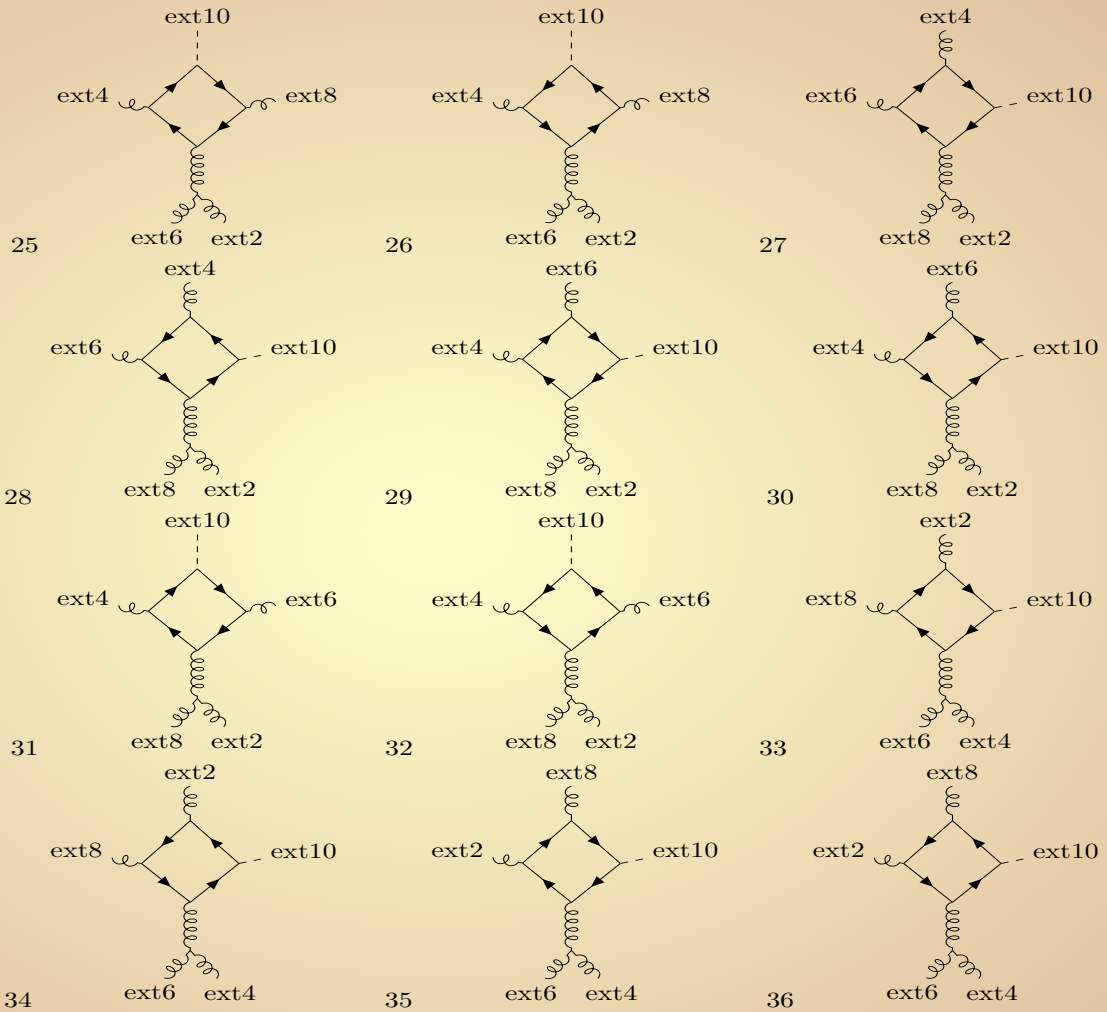
- Analytical results ⇒ CPU friendly

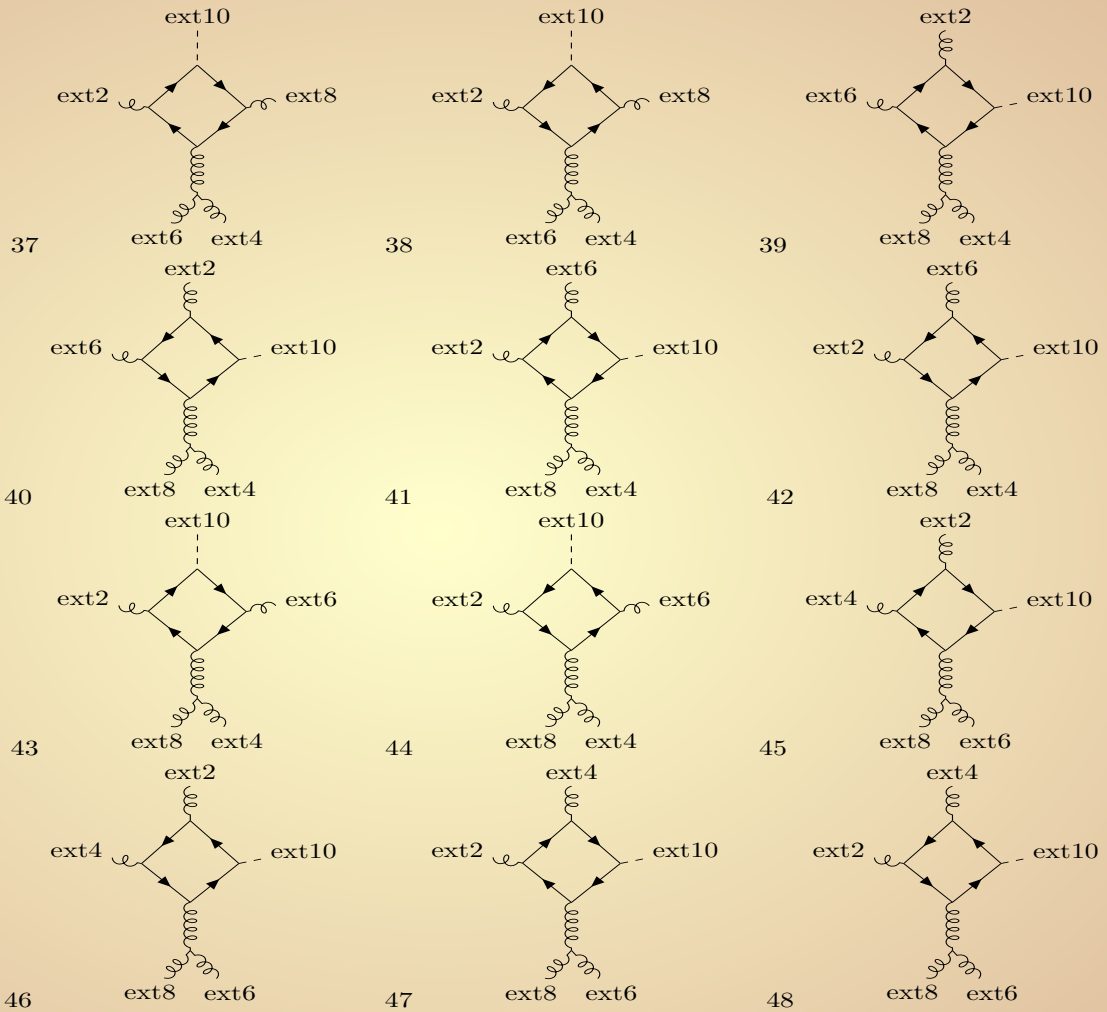
Diagrams

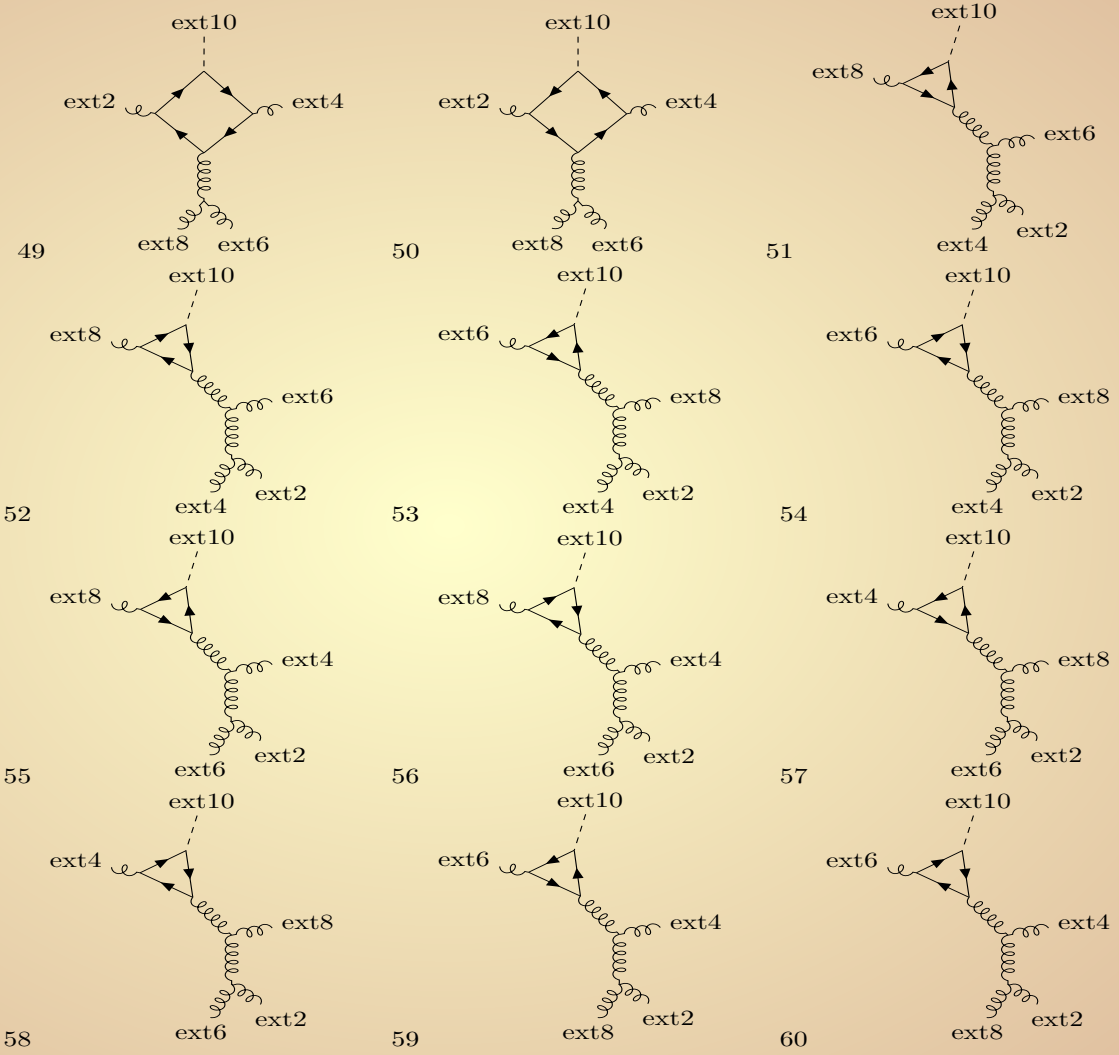
January 21, 2020

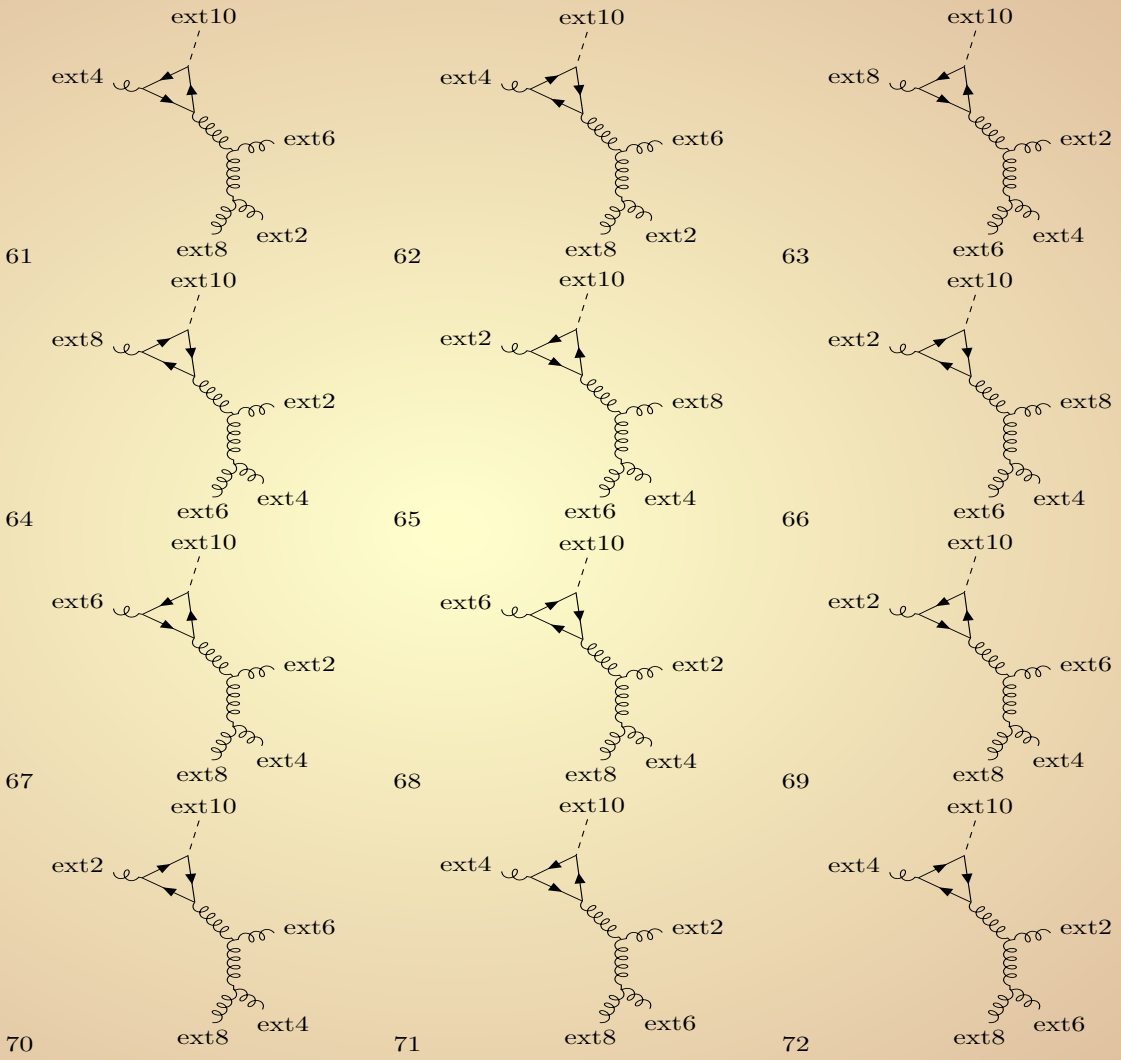


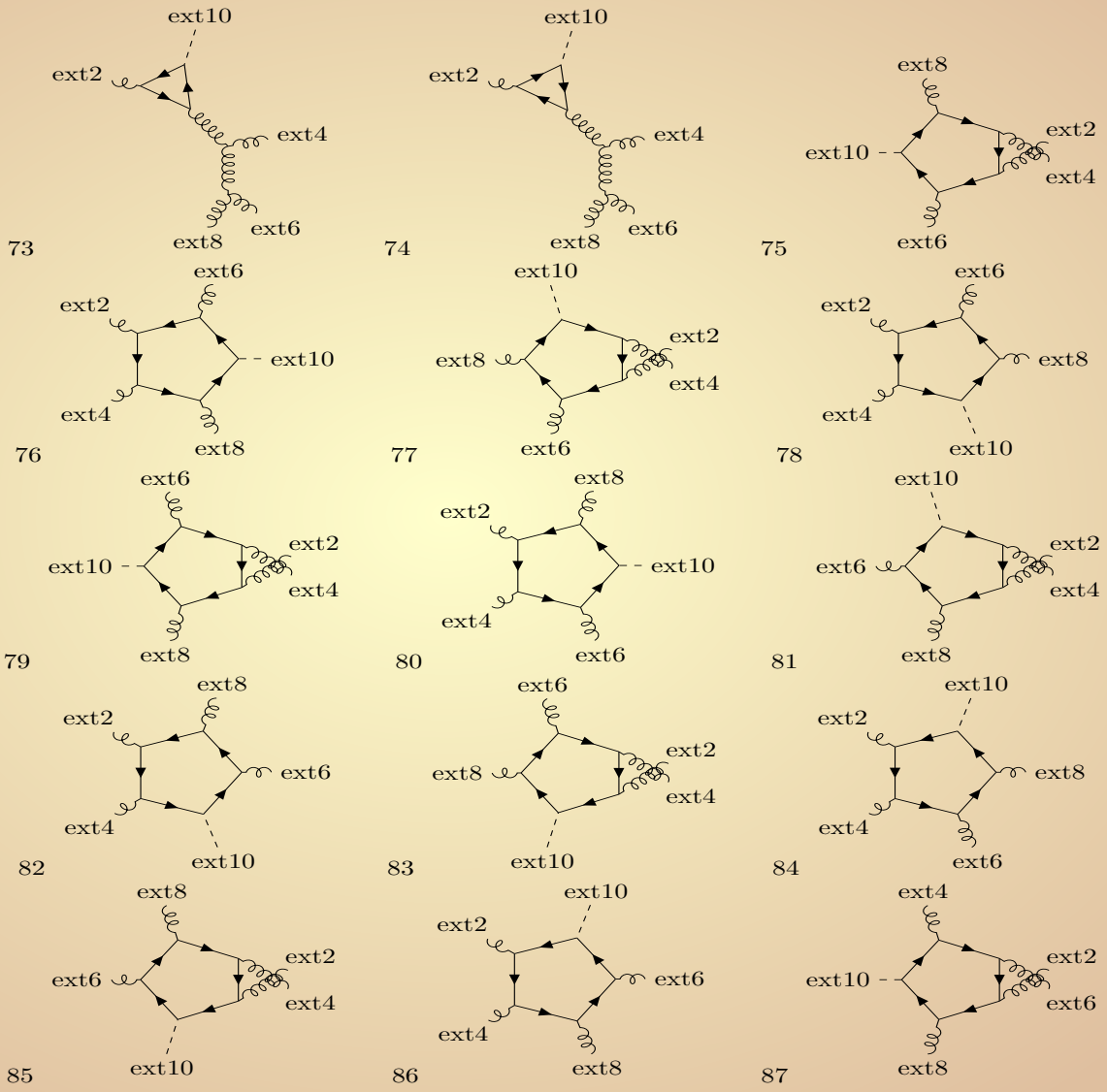


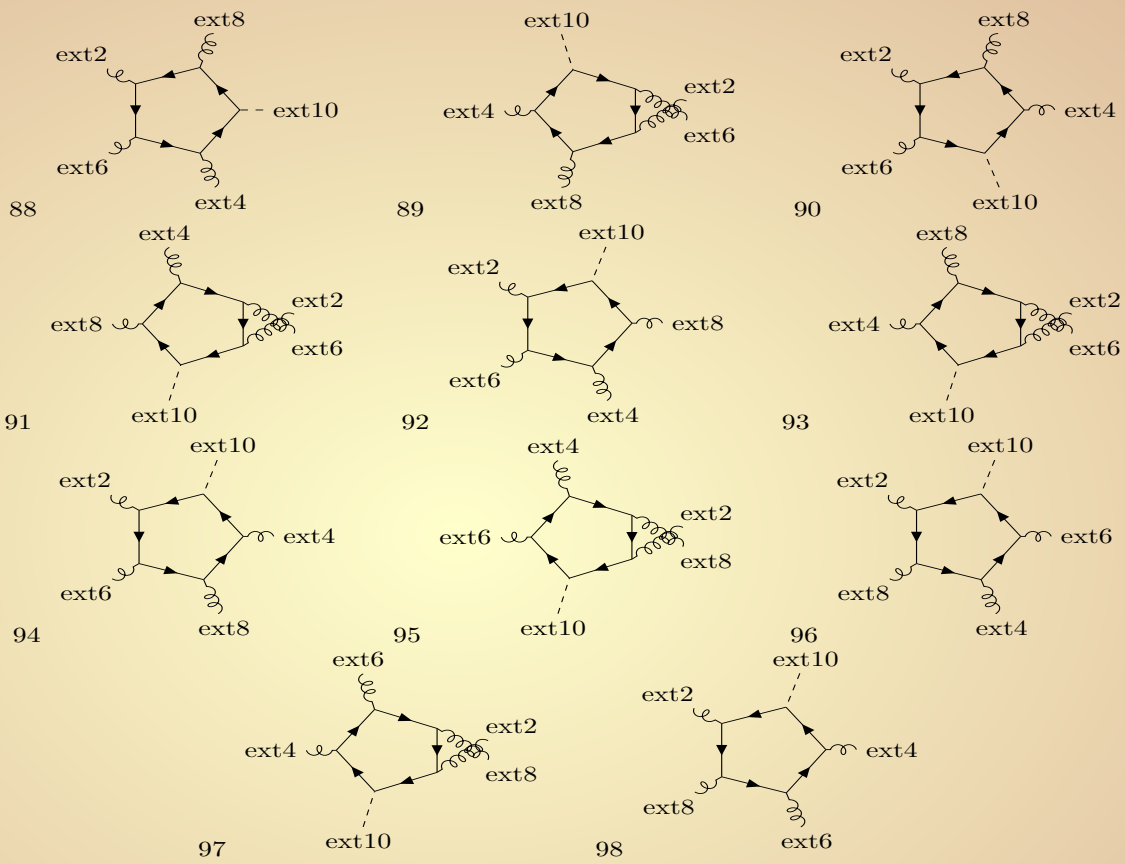








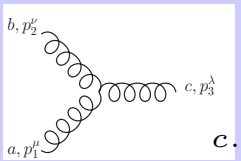




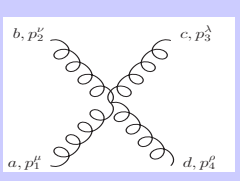
Colour Decomposition

$$
\begin{aligned}
[T^a, T^b] &= i\sqrt{2}\, f^{abc}\, T^c \\
Tr(T^a T^b) &= \delta^{ab} \\
T^a_{ij} T^a_{kl} &= \delta^{il}\delta^{jk} - \frac{1}{N}\delta^{ij}\delta^{kl}
\end{aligned}
$$

$$
\begin{aligned}
f^{abc} &= -\frac{i}{\sqrt{2}} Tr\{T^a [T^b, T^c]\} \\
Tr(T^a S_1) Tr(T^a S_2) &= Tr(S_1 S_2) - \frac{1}{N} Tr(S_1) Tr(S_2) \\
Tr(T^a S_1 T^a S_2) &= Tr(S_1) Tr(S_2) - \frac{1}{N} Tr(S_1 S_2)
\end{aligned}
$$



$$
\begin{aligned}
&= g_s f^{abc} \left[g^{\mu\nu}(p_1 - p_2)^\lambda + g^{\nu\lambda}(p_2 - p_3)^\mu + g^{\lambda\mu}(p_3 - p_1)^\nu \right] \\
&\overset{c.s.\ FR}{=} i \left[g^{\mu\nu}(p_1 - p_2)^\lambda + g^{\nu\lambda}(p_2 - p_3)^\mu + g^{\lambda\mu}(p_3 - p_1)^\nu \right]
\end{aligned}
$$



$$
\begin{aligned}
&= -i\, g_s^2 \left[f^{abe} f^{cde} (g^{\mu\lambda} g^{\nu\rho} - g^{\mu\rho} g^{\nu\lambda}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\lambda} - g^{\mu\rho} g^{\nu\lambda}) \right. \\
&\qquad \left. + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\lambda} - g^{\mu\lambda} g^{\nu\rho}) \right] \\
&\overset{c.s.\ FR}{=} i \left(2 g^{\mu\lambda} g^{\nu\rho} - g^{\mu\rho} g^{\nu\lambda} - g^{\mu\nu} g^{\lambda\rho} \right)
\end{aligned}
$$

Colour Decomposition (contd...)

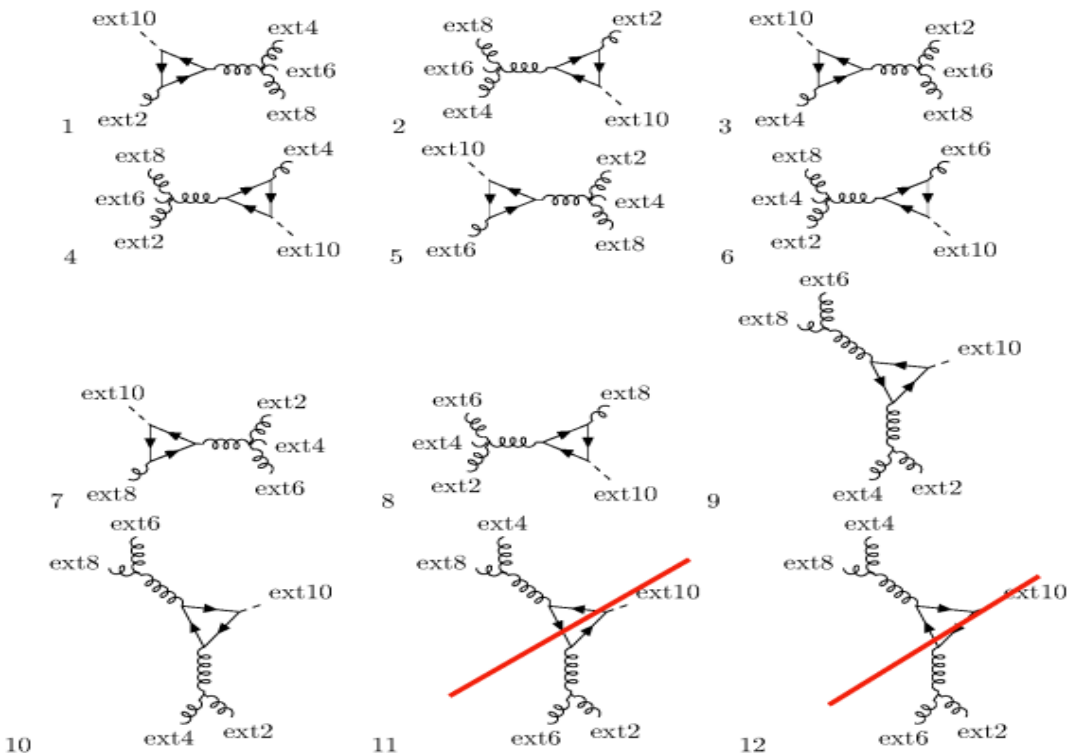
$$\mathcal{A}^{\text{tree}}(\{p_i,h_i,a_i\}) = g_s^{n-2} \sum_{\sigma\in S_n/Z_n} Tr(T^{a_{\sigma 1}},T^{a_{\sigma 2}},\ldots,T^{a_{\sigma n}})\, A_n(\sigma_1^{\lambda_1},\sigma_2^{\lambda_2},\ldots,\sigma_n^{\lambda_n})$$

Bern, Kosower 1991

$$\begin{aligned}\mathcal{A}^{1-\text{loop}}(\{p_i,h_i,a_i\}) \;=\; g_s^n \Bigg[& N_c \sum_{\sigma\in S_n/Z_n} Tr(T^{a_{\sigma 1}},T^{a_{\sigma 2}},\ldots,T^{a_{\sigma n}})\, A^{[1]}_{n;1}(\sigma_1^{h_1},\sigma_2^{h_2},\ldots,\sigma_n^{h_n}) \\ & + \sum_{c=2}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} Tr(T^{a_{\sigma 1}},T^{a_{\sigma 2}},\ldots,T^{a_{\sigma c-1}}) \\ & \qquad Tr(T^{a_{\sigma 1}},T^{a_{\sigma c}},\ldots,T^{a_{\sigma n}})\, A^{[1]}_{n;c}(\sigma_1^{h_1},\sigma_2^{h_2},\ldots,\sigma_n^{h_n}) \\ & + n_f \sum_{\sigma\in S_n/Z_n} Tr(T^{a_{\sigma 1}},T^{a_{\sigma 2}},\ldots,T^{a_{\sigma n}})\, A^{[1/2]}_{n;1}(\sigma_1^{h_1},\sigma_2^{h_2},\ldots,\sigma_n^{h_n}) \Bigg]\end{aligned}$$

$$\mathcal{A}^{\text{n-gluon+H}}(\{p_i,h_i,a_i\}) = \frac{g_s^n}{v} \sum_{\sigma\in S_n/Z_n} Tr(T^{a_{\sigma 1}},T^{a_{\sigma 2}},\ldots,T^{a_{\sigma n}})\, A_n(\sigma_1^{h_1},\sigma_2^{h_2},\ldots,\sigma_n^{h_n};H)$$

Diagrams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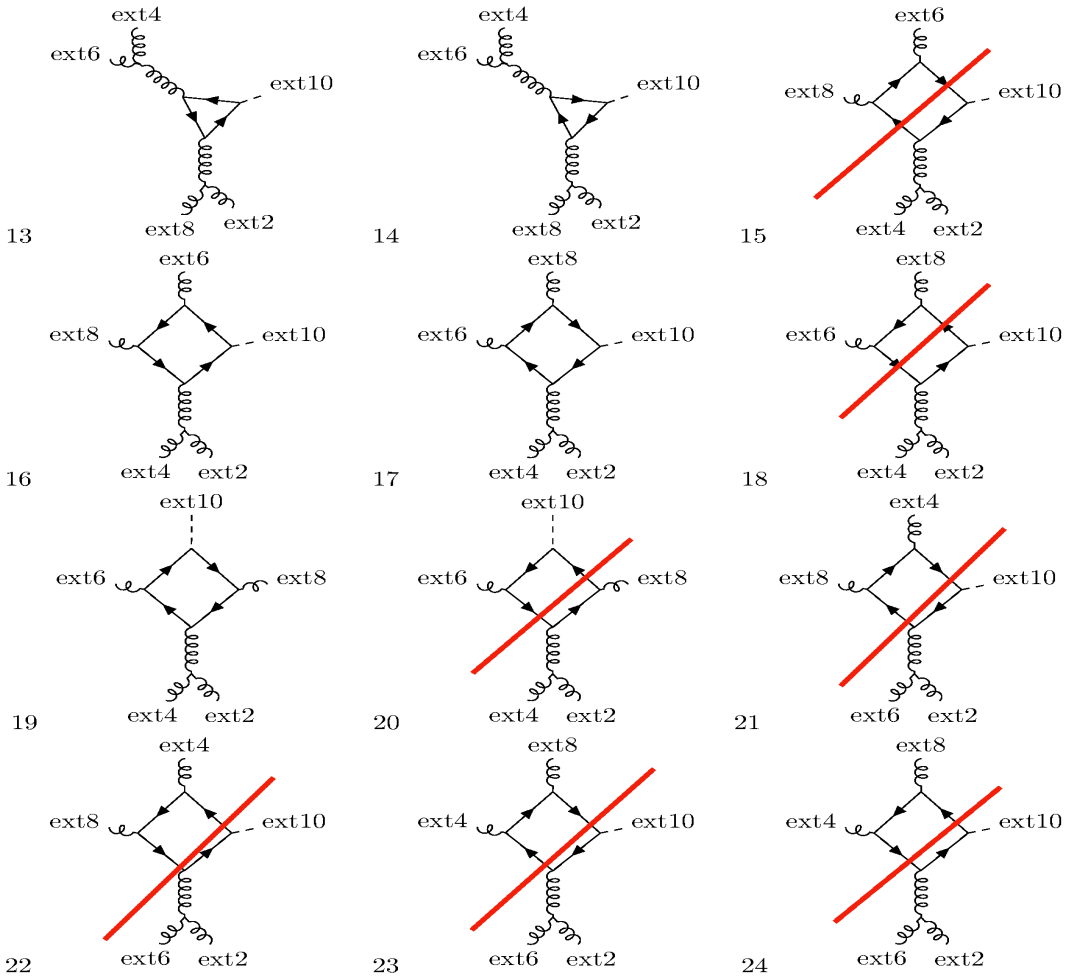
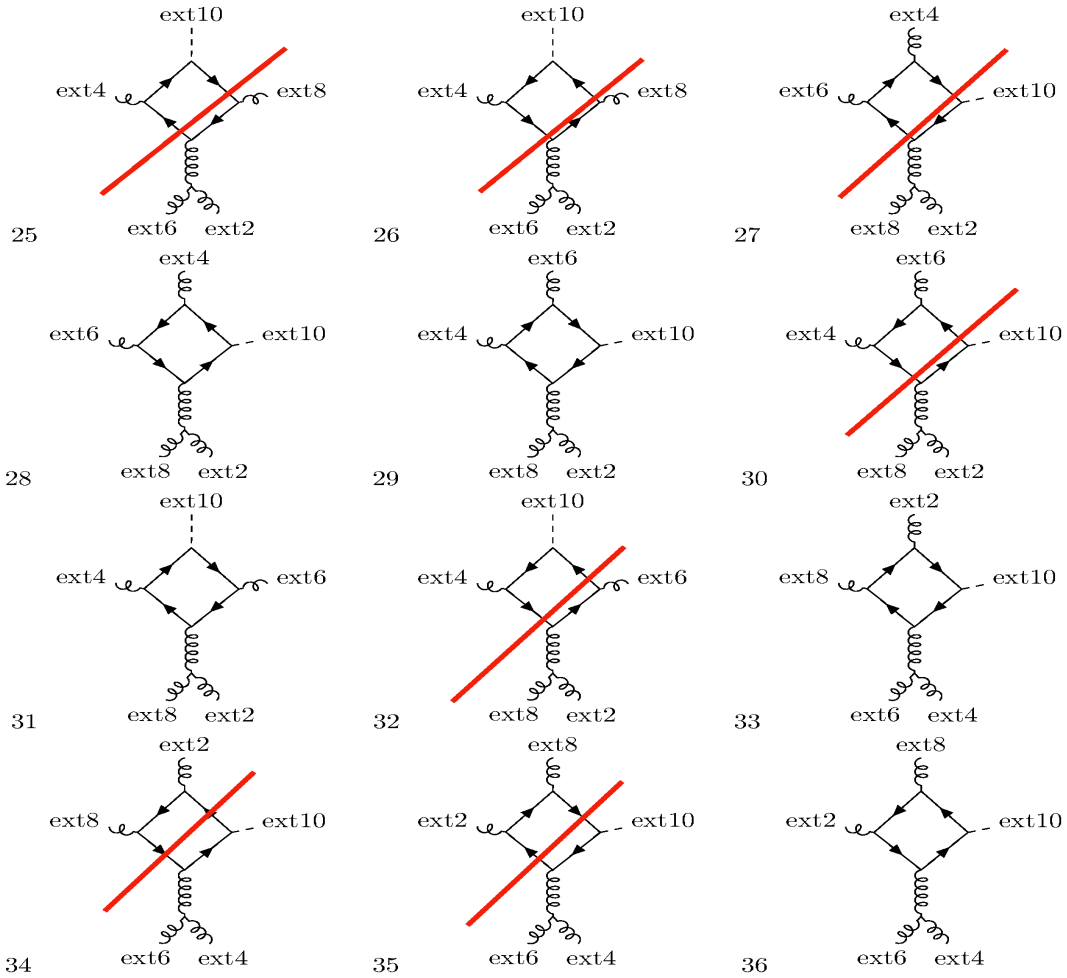
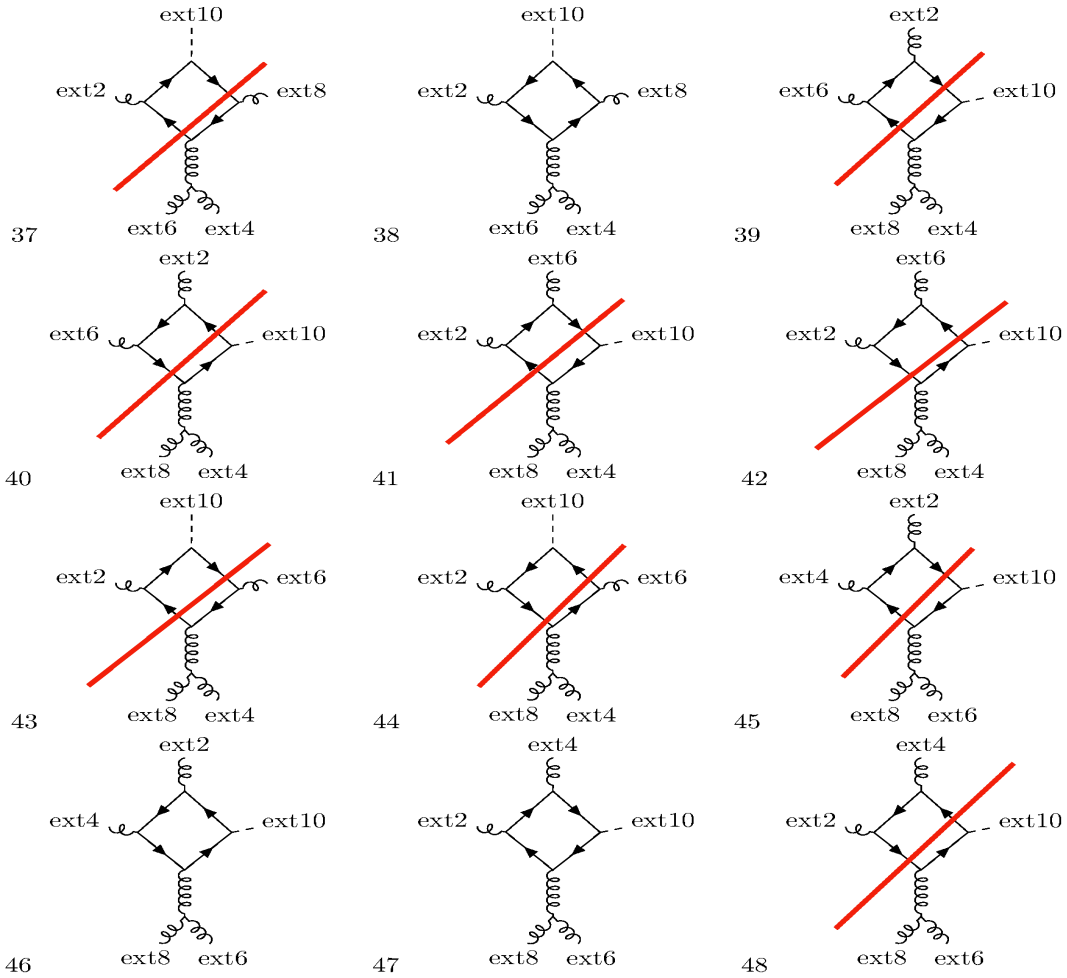
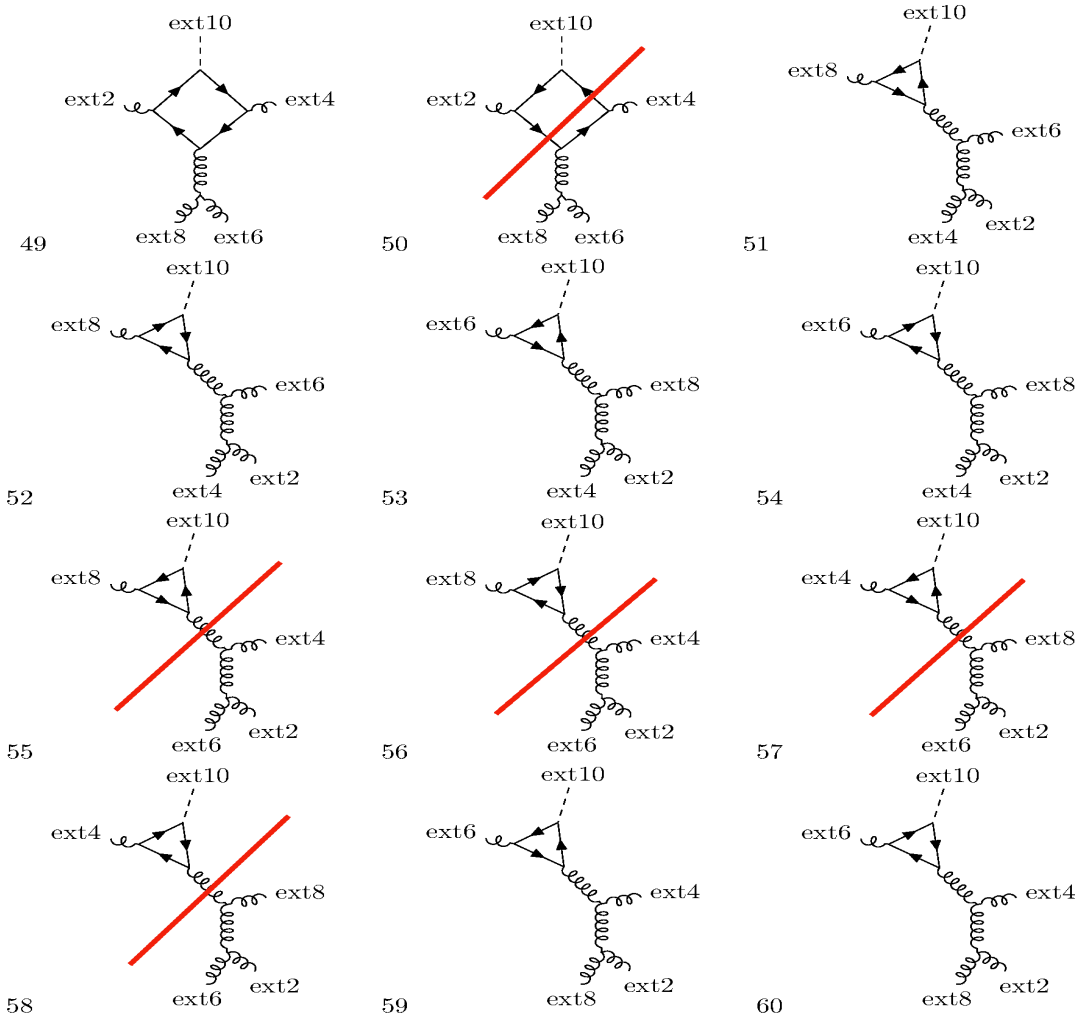
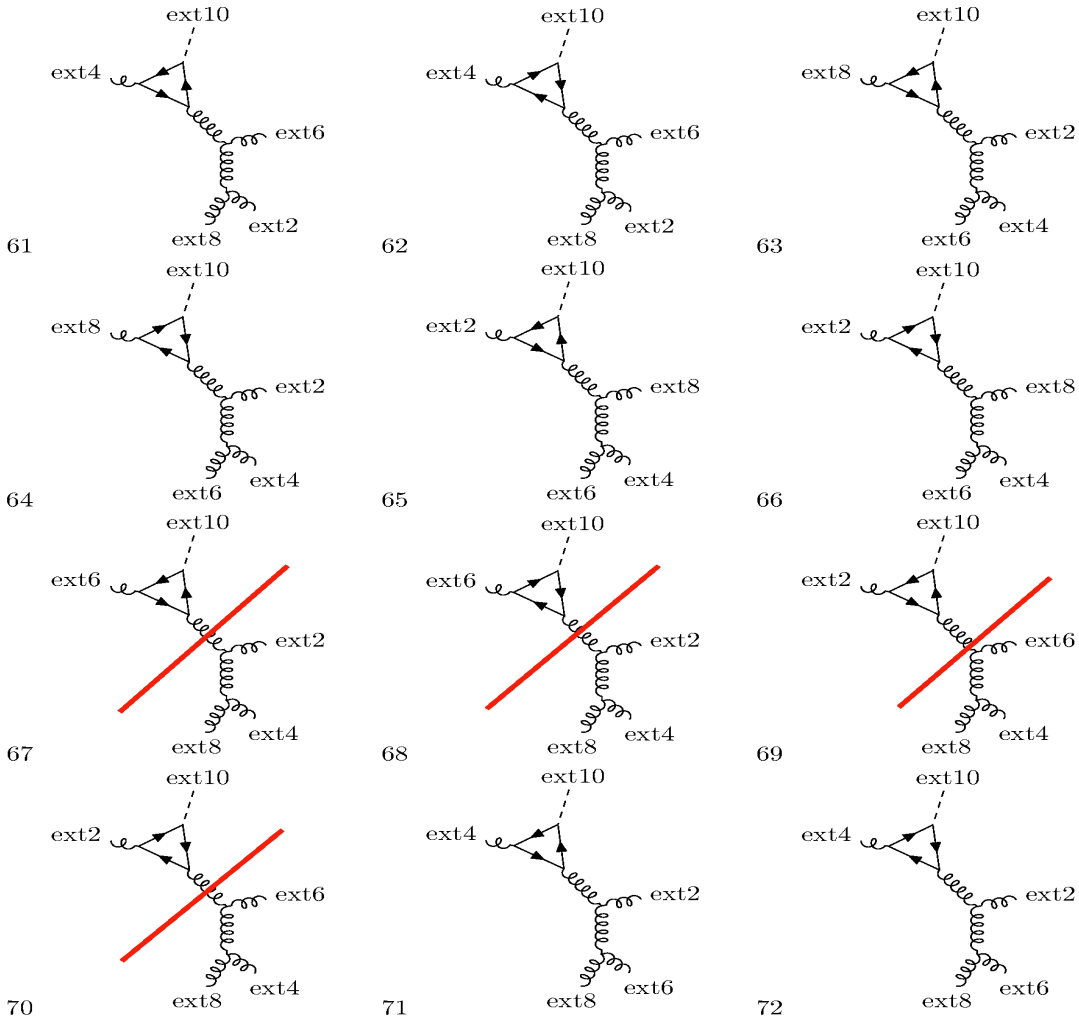
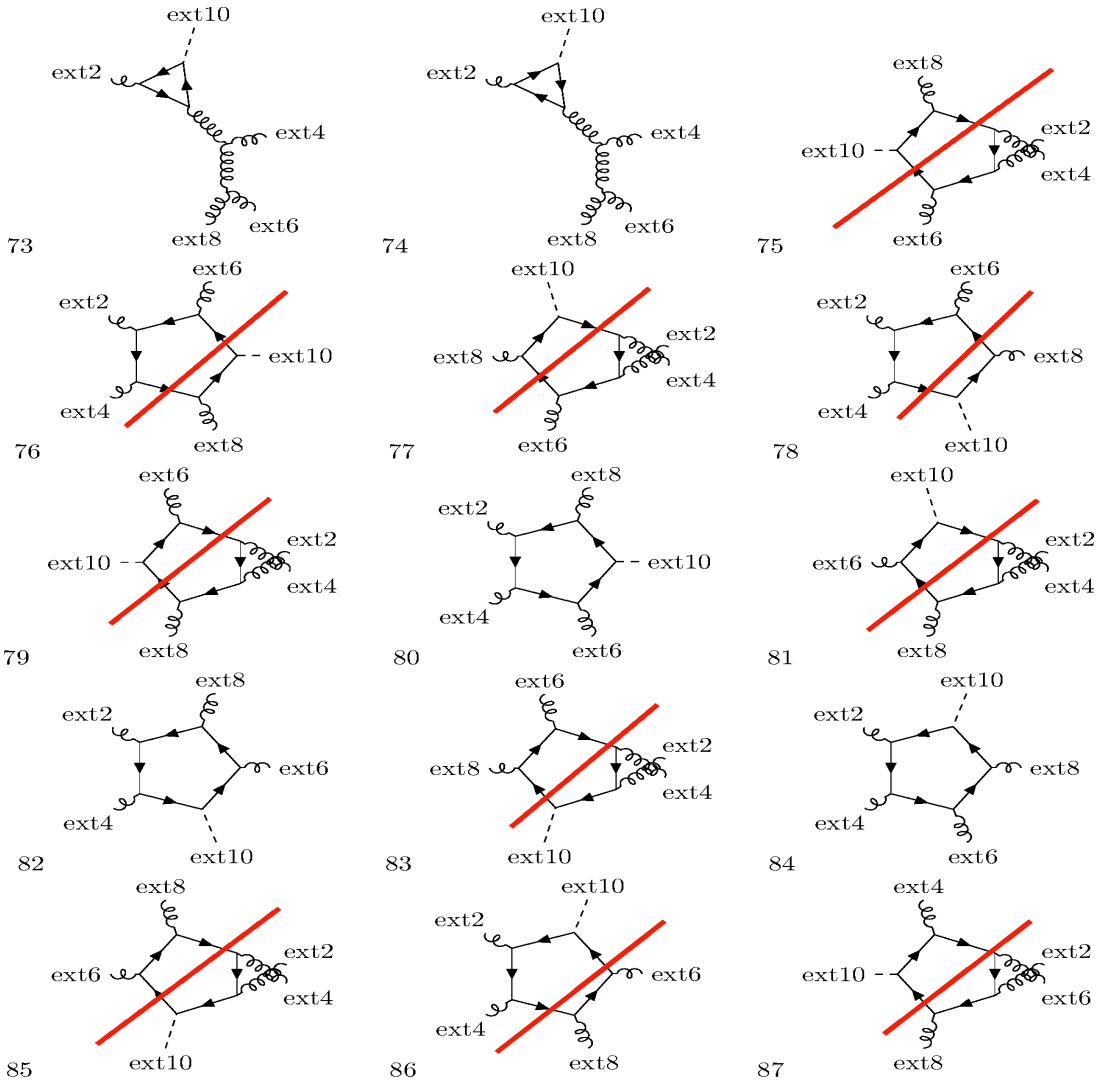


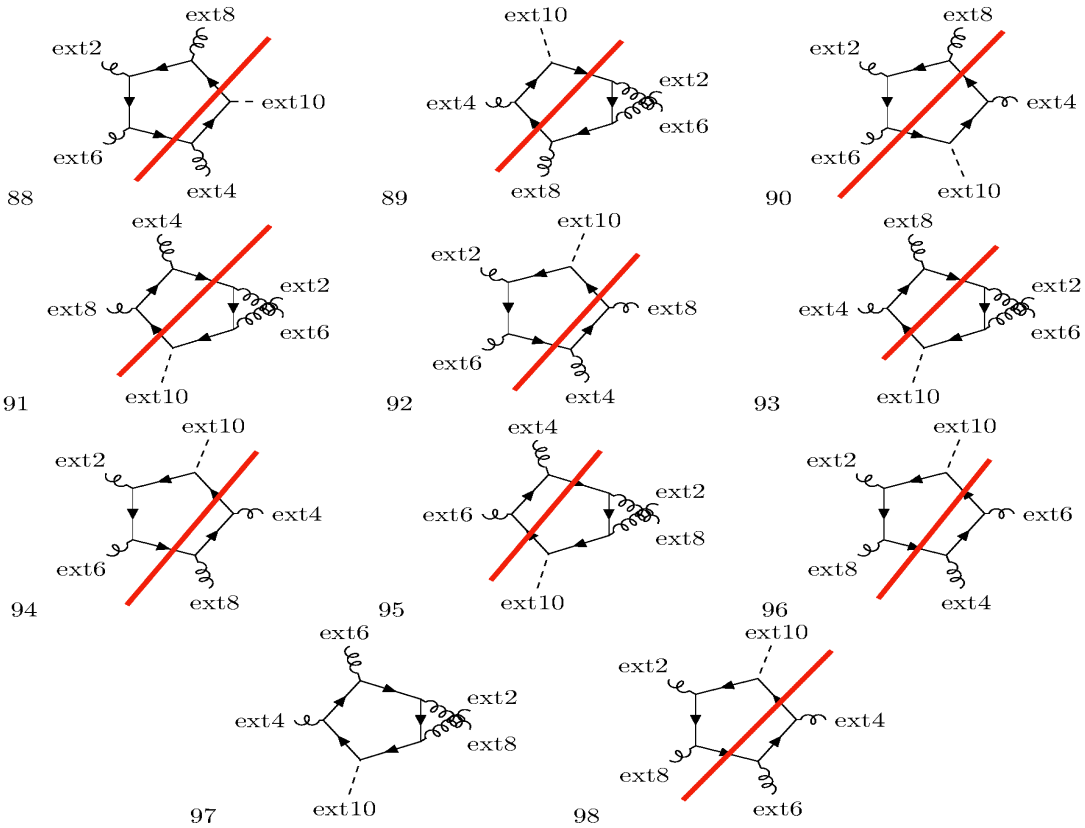












Spinor Helicity

$$\boxed{\gamma_{R/L} = \frac{1}{2}\left(1 \pm \gamma_5\right)}$$

$$u_+(p) = \begin{bmatrix} \sqrt{p^+} \\ \sqrt{p^-}e^{i\phi_p} \\ 0 \\ 0 \end{bmatrix}, \quad u_-(p) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{p^-}e^{-i\phi_p} \\ -\sqrt{p^+} \end{bmatrix}$$

$$p^{\pm} = E \pm p^z, \quad e^{\pm i\phi_p} = \frac{p^x \pm ip^y}{\sqrt{(p^x)^2 + (p^y)^2}} = \frac{p^x \pm ip^y}{\sqrt{p^+p^-}}$$

$$\begin{aligned} u_+(p_i) = v_-(p_i) &= |i+\rangle = |i\rangle \\ u_-(p_i) = v_+(p_i) &= |i-\rangle = |i] \\ \overline{u_+}(p_i) = \overline{v_-}(p_i) &= \langle i+| = [i| \\ \overline{u_-}(p_i) = \overline{v_+}(p_i) &= \langle i-| = \langle i| \end{aligned}$$

Spinor Helicity (contd...)

❑ Basic Identities

$$\langle ij] = [ij\rangle = 0$$
$$\langle ij\rangle = -\langle ji\rangle$$
$$[ij] = -[ji]$$
$$\langle ii\rangle = [jj] = 0$$
$$\langle ij\rangle[ij] = -s_{ij}$$

❑ Special Identities

$$[a|\gamma_\mu|b\rangle\langle c|\gamma^\mu|d] = 2\,[ad]\,\langle cb\rangle$$
$$\langle ab\rangle\langle cd\rangle = \langle ad\rangle\langle cb\rangle + \langle ac\rangle\langle bd\rangle$$
$$[a|\gamma_\mu|b\rangle\gamma^\mu = 2\,(\,|a]\,\langle b| + |b\rangle\,[a|\,)$$
$$\langle a|\gamma_\mu|b]\,\gamma^\mu = 2\,(|a\rangle\,[b| + |b]\,\langle a|)$$

❑ Polarisation

$$\varepsilon^+_\mu(k,q) = \frac{[k|\gamma_\mu|q\rangle}{\sqrt{2}\langle qk\rangle}, \quad \not{\varepsilon}^+(k,q) = \frac{\sqrt{2}(\,|k]\,\langle q| + |q\rangle\,[k|\,)}{\langle qk\rangle}$$

$$\varepsilon^-_\mu(k,q) = \frac{\langle k|\gamma_\mu|q]}{\sqrt{2}[kq]}, \quad \not{\varepsilon}^-(k,q) = \frac{\sqrt{2}(|k\rangle\,[q| + |q]\,\langle k|)}{[kq]}$$

Notation

$$
\begin{aligned}
\mathrm{tr}_5\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_5 \not{p}_1 \not{p}_2 \cdots \not{p}_n\} \\
&\equiv \mathrm{tr}_+\{1\,2\ldots n\} - \mathrm{tr}_-\{1\,2\ldots n\} \\
\mathrm{tr}_+\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_R \not{p}_1 \not{p}_2 \cdots \not{p}_n\} \\
\mathrm{tr}_-\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_L \not{p}_1 \not{p}_2 \cdots \not{p}_n\}
\end{aligned}
$$

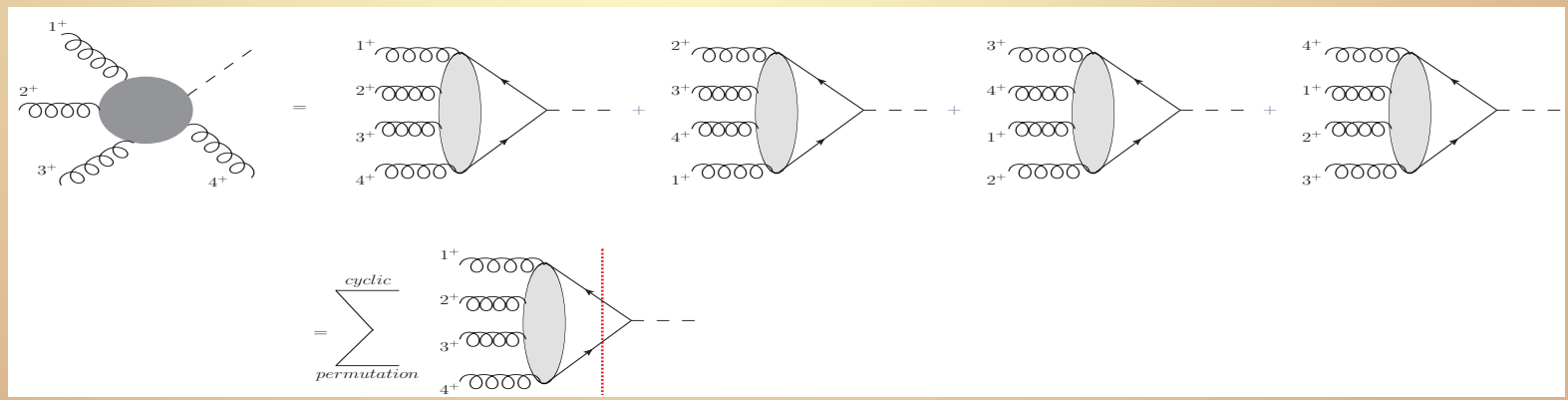
$$\boxed{\gamma_{R/L} = (1 \pm \gamma_5)/2}$$

$$
\begin{aligned}
&\mathrm{tr}(\gamma_5 \not{p}_1 \not{p}_2 \not{p}_3 \not{p}_4 \not{p}_5 \not{p}_6) \\
&\Downarrow \text{massless} \\
&\big([1\,2]\ \langle 2\,3\rangle\ [3\,4]\ \langle 4\,5\rangle\ [5\,6]\ \langle 6\,1\rangle \\
&- \langle 1\,2\rangle\ [2\,3]\ \langle 3\,4\rangle\ [4\,5]\ \langle 5\,6\rangle\ [6\,1] \big) \\
&\Downarrow\ p_6^2 \neq 0 \\
&\big([1\,2]\ \langle 2\,3\rangle\ [3\,4]\ \langle 4\,5\rangle\ [5|6|1\rangle \\
&- \langle 1\,2\rangle\ [2\,3]\ \langle 3\,4\rangle\ [4\,5]\ \langle 5|6|1] \big)
\end{aligned}
$$

Independent Helicity Structures

- $++++H$
- $+++-H$
- $++--H$
- $+-+-H$

Cyclic permutation: $++++H$



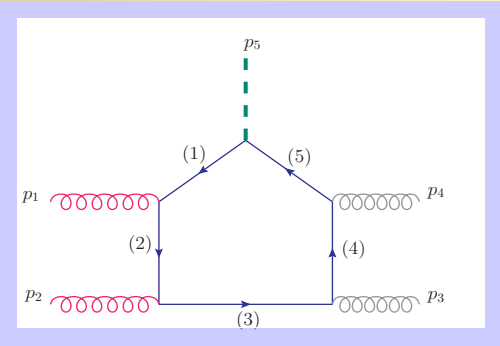
One Loop Amplitude

$$\mathcal{A}^{1-\text{loop}} = \sum_i d_i I^i_{\text{Box}} + \sum_i c_i I^i_{\text{Triangle}} + \sum_i b_i I^i_{\text{Bubble}} + \mathcal{R}$$

Pentagon to Boxes

van Neerven, Vermaseren 1984
Bern, Dixon, Kosower 1993

$$S_{ij} = m^2 - \frac{1}{2} p_{ij}^2$$



$$p_{ii} = 0; \quad p_{ij} = p_{ji} = p_i + p_{i+1} + \cdots + p_{j-1} \quad \text{for } i \leq j$$

$$c_i = -\frac{1}{2} \sum_{j=1}^{5} S^{-1}_{ij}$$

$$E_0 = \sum_{i=1}^{5} c_i D_0^{(i)}$$

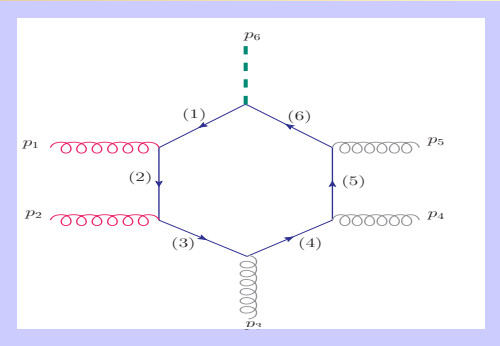
One Loop Amplitude

$$\mathcal{A}^{1-\text{loop}} = \sum_i d_i I^i_{\text{Box}} + \sum_i c_i I^i_{\text{Triangle}} + \sum_i b_i I^i_{\text{Bubble}} + \mathcal{R}$$

Hexagon to Pentagons

van Neerven, Vermaseren 1984
Bern, Dixon, Kosower 1993

$$S_{ij} = m^2 - \frac{1}{2} p_{ij}^2$$



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$$c_i = -\frac{1}{2} \sum_{j=1}^{5} S^{-1}_{ij}$$

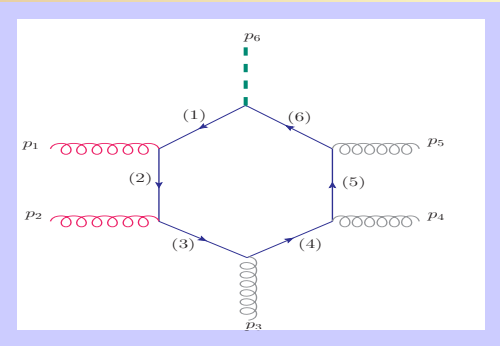
$$F_0 = \sum_{i=1}^{6} c_i E_0^{(i)}$$

One Loop Amplitude

$$\mathcal{A}^{1-\text{loop}} = \sum_i d_i I^i_{\text{Box}} + \sum_i c_i I^i_{\text{Triangle}} + \sum_i b_i I^i_{\text{Bubble}} + \mathcal{R}$$

Hexagon to Pentagons

van Neerven, Vermaseren 1984
Bern, Dixon, Kosower 1993

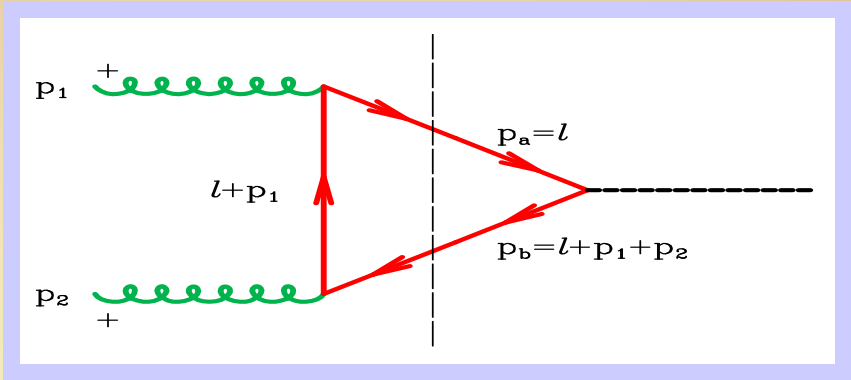


Ellis, *Seth* 2018

$$\begin{cases} c^{(1)}_{12345} = & +\text{tr}_5\{2\,3\,4\,5\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \\ c^{(2)}_{12345} = & -\text{tr}_5\{(1+2)\,3\,4\,5\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \\ c^{(3)}_{12345} = & +\text{tr}_5\{1\,(2+3)\,4\,5\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \\ c^{(4)}_{12345} = & -\text{tr}_5\{1\,2\,(3+4)\,5\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \\ c^{(5)}_{12345} = & +\text{tr}_5\{1\,2\,3\,(4+5)\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \\ c^{(6)}_{12345} = & -\text{tr}_5\{1\,2\,3\,4\}/\text{tr}_5\{1\,2\,3\,4\,5\,6\} \end{cases}$$

H+n-gluons; n=2

Wilczek 1977
Georgi, Glashow, Machacek, Nanopoulos 1978
Bern, Morgan 1996

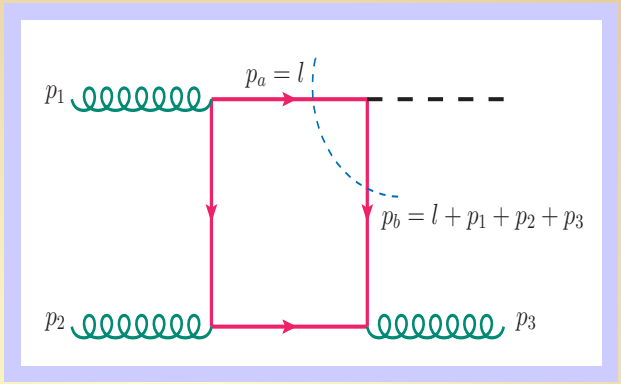


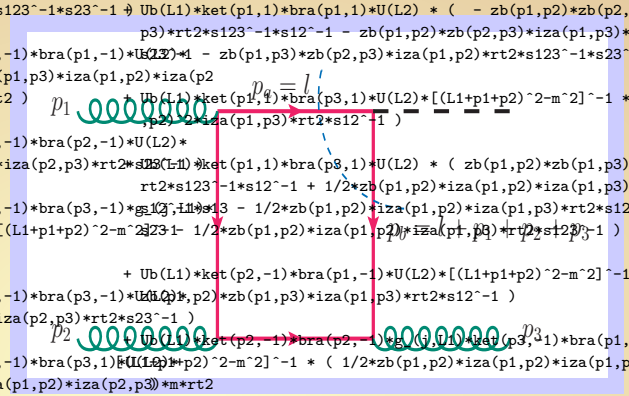
$$\text{LHS} = G_2^{\text{tree}}(a,1^+,2^+,b) = \frac{[1\,2]}{\langle 1\,2\rangle}\frac{\bar{u}(p_a)\gamma_R(m)u(p_b)}{(s_{a1}-m^2)}$$

$$\text{RHS} = H_0^{\text{tree}} = m\bar{u}(p_b)u(p_a)$$

$$\text{LHS}\bullet\text{RHS} = m^2\frac{[1\,2]}{\langle 1\,2\rangle}\frac{\text{Tr}\{\gamma_R(\not{p}_b+m)(\not{p}_a+m)\}}{(s_{a1}-m^2)} = m^2\frac{[1\,2]}{\langle 1\,2\rangle}\frac{(4m^2-M_h^2)}{(s_{a1}-m^2)}$$

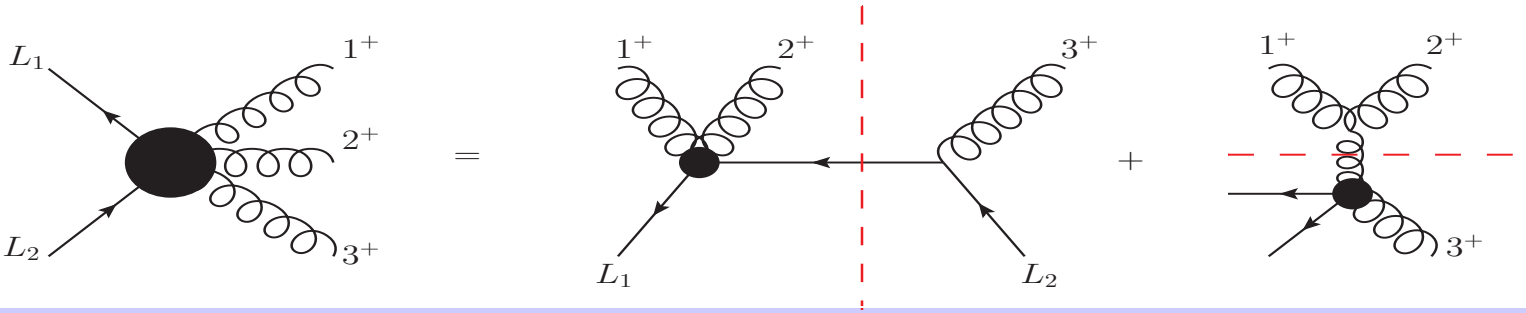
$$\text{Full Result} = A_2(1_g^+,2_g^+;H) = 2m^2\frac{[1\,2]}{\langle 1\,2\rangle}\left[(4m^2-M_h^2)C_0(p_1,p_2;m)+2\right]$$




$p_4 = l$
p_1
p_2
p_3

$$G_3(a,1^+,2^+,3^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2 + (s_{a1}-m^2)\right)|3]}{(s_{a1}-m^2)(s_{a12}-m^2)\ \langle 1\,2\rangle\,\langle 2\,3\rangle}$$

Britto, Cachazo, Feng, Witten 2005



$$\begin{aligned} |\hat{2}] &= |2] - z|3] \\ |\hat{3}\rangle &= |3\rangle + z|2\rangle \end{aligned}$$

$$z_1 = -\frac{[3|L_2|3\rangle}{[3|L_2|2\rangle}, \quad z_2 = \frac{[12]}{[13]}$$

$$\begin{aligned} A(L_1, 1^+, 2^+, 3^+, L_2) &= A(L_1, 1^+, \hat{2}^+, -P)\frac{1}{(L_1+p_1+p_2)^2 - m^2}A(P, \hat{3}^+, L_2) \\ &\quad + A(1^+, \hat{2}^+, -P)\frac{1}{(p_1+p_2)^2 - m^2}A(L_1, \hat{3}^+, L_2) \\ &= m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2 + (s_{a1} - m^2)\right)|3]}{(s_{a1} - m^2)(s_{a12} - m^2)\ \langle 1\,2\rangle\,\langle 2\,3\rangle} \end{aligned}$$

H+n-gluons; $n>2$

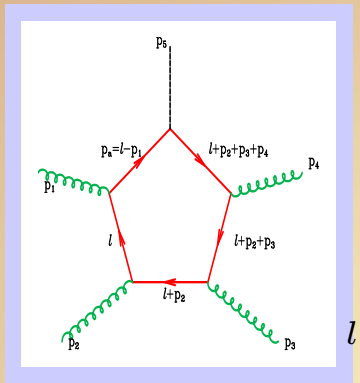
$$G_3(a,1^+,2^+,3^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2 + (s_{a1}-m^2)\right)|3]}{(s_{a1}-m^2)(s_{a12}-m^2)\ \langle 1\,2\rangle\,\langle 2\,3\rangle}$$

$$G_4(a,1^+,2^+,3^+,4^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2 + (s_{a1}-m^2)\right)\left(\not{p}_{a12}\not{p}_3 + (s_{a12}-m^2)\right)|4]}{(s_{a1}-m^2)(s_{a12}-m^2)(s_{a123}-m^2)\ \langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle}$$

Ochirov 2018

$$G_n(a,1^+,2^+,\ldots,n^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\prod_{j=1}^{n-2}\left\{\not{p}_{a\ldots j}\not{p}_{j+1} + (s_{a1\ldots j}-m^2)\right\}|n]}{(s_{a1}-m^2)(s_{a12}-m^2)\ldots(s_{a1\ldots(n-1)}-m^2)\,\langle 12\rangle\langle 23\rangle\ldots\langle n-1|n\rangle}$$

Pentagon coefficient of H+4-gluons



$$l^\nu = \alpha\, p_1^\nu + \beta\, p_2^\nu + \frac{\gamma}{2}\,\langle 1|\gamma^\nu|2] + \frac{\delta}{2}\,\langle 2|\gamma^\nu|1] + l_\epsilon^\nu$$

$$\begin{aligned}
l^2 - m^2 &= 0, \quad \rightarrow \quad -\gamma\delta\,\langle 1\,2\rangle\,[2\,1] - m^2 - \mu^2 = 0 \\
(l-p_1)^2 - m^2 &= 0, \quad \rightarrow \quad \beta = 0 \\
(l+p_2)^2 - m^2 &= 0, \quad \rightarrow \quad \alpha = 0 \\
(l+p_2+p_3)^2 - m^2 &= 0, \quad \rightarrow \quad \gamma\,\langle 1\,3\rangle\,[3\,2]) + \delta\,\langle 2\,3\rangle\,[3\,1] + s_{23} = 0 \\
(l+p_2+p_3+p_4)^2 - m^2 &= 0, \quad \rightarrow \quad \gamma\,\langle 1\,4\rangle\,[4\,2] + \delta\,\langle 2\,4\rangle\,[4\,1] + s_{234} - s_{23} = 0
\end{aligned}$$

$$G_4(a,1^+,2^+,3^+,4^+,b) = m\,\frac{\bar{u}(a)\gamma_R u(b)\;[1|\left(\not{p}_{a1}\not{p}_2 + (s_{a1}-m^2)\right)\left(\not{p}_{a12}\not{p}_3 + (s_{a12}-m^2)\right)|4]}{(s_{a1}-m^2)(s_{a12}-m^2)(s_{a123}-m^2)\;\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle}$$

$$m^2(4m^2 - M_h^2)\,\frac{[1|\not{l}\not{p}_2\,(\not{l}+\not{p}_2)\not{p}_3|4]}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle} = -m^4(4m^2 - M_h^2)\,\frac{\mathrm{tr}_+\{1\,2\,3\,4\}}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}$$

H+n-gluons; n=2

Wilczek 1977
Georgi, Glashow, Machacek, Nanopoulos 1978

$$A_2(1_g^+, 2_g^+; H) = 2m^2 \frac{[1\,2]}{\langle 1\,2\rangle}\left[(4m^2 - M_h^2)C_0(p_1, p_2; m) + 2\right]$$

H+n-gluons; n=3

Ellis, Hinchliffe, Soldate, van der Bij 1988

$$\begin{aligned} A_3(1_g^+, 2_g^+, 3_g^+; H) \quad &= \quad m^2\left[\left\{\frac{4m^2 - M_h^2}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,1\rangle}\left[-\frac{1}{2}s_{12}s_{23}D_0(p_1, p_2, p_3; m)\right.\right.\right. \\ &- \quad \left.\left.\left.(s_{12} + s_{13})C_0(p_1, p_{23}; m)\right] - 2\frac{s_{12} + s_{13}}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,1\rangle}\right\} \\ &+ \quad \left\{2 \text{ cyclic permutations}\right\}\Bigg] \end{aligned}$$

H+n-gluons; n=4

Neumann, Williams 2017
Ellis, *Seth*

$$
\begin{aligned}
A_4(1_g^+, 2_g^+, 3_g^+, 4_g^+; H) \;=\; & m^2 \Bigg[\Bigg\{ \frac{4m^2 - M_h^2}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle} \Big[-\mathrm{tr}_+\{1\,2\,3\,4\} m^2 E_0(p_1, p_2, p_3, p_4; m) \\
& + \frac{1}{2}((s_{12}+s_{13})(s_{24}+s_{34}) - s_{14}s_{23}) D_0(p_1, p_{23}, p_4; m) \\
& + \frac{1}{2} s_{12}s_{23} D_0(p_1, p_2, p_3; m) \\
& + (s_{12}+s_{13}+s_{14}) C_0(p_1, p_{234}; m) \Big] + 2\frac{s_{12}+s_{13}+s_{14}}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle} \Bigg\} \\
& + \Big\{ 3 \text{ cyclic permutations} \Big\} \Bigg]
\end{aligned}
$$


```
 double complex function hjetmass_triangle_pppp_0_s234_mhsq_dp
&     (i1,i2,i3,i4,za,zb,mt)
 implicit double complex (t)
     integer i1,i2,i3,i4
     include 'types.f'
     include 'mxpart.f'
     include 'constants.f'
     include 'zprods_decl.f'
     double complex ret
     double precision mt
     double precision cg

 t1 = za(i2, i4)
 t2 = zb(i2, i1)
 t3 = zb(i4, i1)
 t4 = zb(i4, i2)
 t5 = za(i2, i3)
 t6 = za(i3, i4)
 t7 = zb(i3, i2)
 t8 = zb(i4, i3)
 t9 = t5 * t7
 t10 = t1 * t4
 t11 = t6 * t8
 t12 = t9 + t10 + t11
 t13 = zb(i3, i1)
 t14 = za(i1, i2)
 t15 = za(i1, i3)
 t16 = t1 * t3
 t17 = t13 * t5 + t16
 t18 = za(i1, i4)
 t19 = t14 * t2
 t20 = t15 * t13
 t21 = t18 * t3
 t22 = t21 + t19 + t20
 if ( dreal(t22) > 0d0) then; cg = 1d0; else; cg = -1d0; end if
 t22 = cg * cdsqrt(t22 ** 2) + t19 + t20 + t21
 t23 = t10 + t9
 t24 = 0.1D1 / t22
 t25 = -2 * t19 * t12 * t24 + t23
 t26 = -2 * t14
 t27 = t1 * t8
 t28 = t26 * t13 * t12 * t24 + t27
 t26 = t26 * t3 * t12 * t24 - t5 * t8
 t29 = t15 * t28
 t30 = t18 * t26
 t31 = t2 * (t29 + t30)
 t32 = t14 * t25
 t33 = t2 * (t29 + t32)
 t34 = -2 * t15
 t35 = t34 * t3 * t12 * t24 + t4 * t5
 t36 = -2 * t18
 t37 = t36 * t13 * t12 * t24 + t1 * t7
 t38 = t6 * t4
 t39 = t34 * t2 * t12 * t24 + t38
 t40 = t36 * t2 * t12 * t24 - t6 * t7
 t41 = t2 * t25
 t42 = t13 * t39
 t43 = t14 * (t41 + t42)
 t44 = t3 * t40
 t45 = t14 * (t44 + t42)
 t34 = t34 * t13 * t12 * t24 + t11 + t9
 t46 = t10 + t11
 t36 = t36 * t3 * t12 * t24 + t46
 t47 = 0.1D1 / t40
 t48 = 0.1D1 / t14
 t31 = 0.1D1 / t31
 t49 = 0.1D1 / t12
 t50 = 0.1D1 / t18
 t22 = 0.1D1 / t22
 t51 = 0.1D1 / t5
 t52 = 0.1D1 / t25
 t33 = 0.1D1 / t33
 t53 = 0.1D1 / t26
```


```
 t43 = 0.1D1 / t43
 t45 = 0.1D1 / t45
 t54 = 0.1D1 / t6
 t55 = 0.1D1 / t1
 t56 = t15 * t36
 t57 = t18 * t35
 t58 = t57 + t56
 t59 = t4 * t51
 t60 = t7 * t55
 t61 = t60 + t59
 t62 = t4 * t54
 t63 = t8 * t55
 t64 = t63 + t62
 t65 = t14 * t50
 t66 = t65 * t51
 t67 = t66 + t54
 t68 = t10 * t54
 t69 = t68 + t8
 t70 = t40 ** 2
 t71 = t48 ** 2
 t72 = t48 * t71
 t73 = mt ** 2
 t74 = t18 ** 2
 t75 = t33 ** 2
 t76 = t33 * t75
 t77 = t5 ** 2
 t78 = t4 ** 2
 t79 = t54 ** 2
 t80 = t8 ** 2
 t81 = t52 ** 2
 t82 = t7 ** 2
 t83 = t2 ** 2
 t84 = t2 * t83
 t85 = t51 ** 2
 t86 = t31 ** 2
 t87 = t31 * t86
 t88 = t6 ** 2
 t89 = t50 ** 2
 t90 = t50 * t89
 t91 = t18 * t48
 t92 = t39 * t47
 t93 = t92 * t32
 t94 = t53 * t3
 t95 = t3 * t51
 t96 = t95 * t19
 t97 = t40 * t52
 t98 = t97 * t54
 t99 = t42 * t3
 t100 = t5 * t54
 t101 = t100 * t82
 t102 = t1 * t78
 t103 = t29 * t84
 t104 = t103 * t25
 t105 = t53 * t85
 t106 = t25 * t54
 t107 = t106 * t21
 t108 = t29 * t83
 t109 = t108 * t7
 t110 = t62 * t29
 t111 = t63 * t6
 t112 = t111 + t4
 t113 = t60 * t5
 t114 = t113 + t4
 t115 = t14 ** 2
 t116 = t53 ** 2
 t117 = t17 * t4
 t118 = t8 * t25
 t119 = t73 * t83
 t120 = t13 * t49
 t121 = t120 * t73
 t122 = t2 * t8
 t123 = t15 * t80
 t124 = t17 * t18
 t56 = -t118 * t117 * t108 * t65 * t86 * t53 - t119 * t17 * t33 * (
#t57 + t56) * t79 * t24 - t121 * t4 * t50 - t122 * t73 * t49 * t50
#+ t63 * t73 * t17 * t49 * t24 * (t21 + t20) * t48 + t124 * t123 *
#t84 * t25 * t26 * t76
 t57 = t2 * t26
```


```
 t125 = t20 * t57
 t126 = t105 * t89
 t127 = t57 * t17
 t128 = t127 * t79
 t129 = t113 * t3 * t17 * t79
 t130 = t129 * t52
 t131 = t20 * t117 * t18
 t132 = t94 * t73
 t133 = t73 * t3
 t134 = t62 * t74
 t135 = t7 * t54
 t136 = t135 * t48
 t137 = (t2 * (t60 * t19 * t17 * t54 * t31 + t126 * t112 * t17 * t1
#15 + t125 * t51 * t33 * t54) + (t128 * t114 * t81 - t130) * t71 *
#t74 + t131 * t71 * t79 * t52) * t22 * t12
 t138 = t82 * t26
 t139 = t14 * t17
 t140 = t139 * (t14 * (t132 * t40 * t24 * t55 * t89 + t38 * t118 *
#t116 * t90) - t123 * t84 * t6 * t25 * t26 * t87) * t85
 t56 = t48 * (t103 * t74 * t17 * t80 * t26 * t55 * t76 - t120 * t62
# * t73) + t51 * t56 + t17 * (t14 * (-t107 * t12 * t22 * t61 * t86
#* t83 + t104 * (-t55 * (t65 * t88 * t80 * t85 + t101) - t102 * t51
# * t67) * t87 + t105 * t22 * t89 * t12 * (t94 * t29 * t4 + t63 * (
#-t32 * t94 - t20) * t6)) + t26 * (t48 * (t103 * t77 * t82 * t74 *
#t79 * t55 * t76 + t109 * t18 * t52 * t64 * t75) + t110 * t83 * t74
# * t52 * t69 * t75 * t71) + (t55 * (-t94 * t14 * t15 * t37 * t89 *
# t85 + (t33 * (-t51 * t58 * t54 - t91 * t58 * t79) + t93 * t85 * t
#45) * t13 * t2) + t98 * (-t96 * t39 * t43 * t54 + t60 * (-t20 - t1
#9) * t50 * t49) - t99 * t14 * t70 * t81 * t43 * t79 * t55) * t24 *
# t73) - t136 * (t134 * t5 * t17 * t26 * t71 * t81 + t133 * t49) +
#t137 - t138 * t32 * t15 * t84 * t17 * t54 * t87 + t140
 t58 = t15 ** 2
 t137 = t31 * t51
 t140 = t74 * t26
 t141 = t140 * t72 * t81 * t33 * t79
 t142 = t115 * t25
 t143 = t142 * t31
 t144 = t4 * t28
 t145 = t13 * t55
 t146 = t95 * t31
 t147 = t14 * t39
 t148 = t15 * t25
 t149 = t94 * t25
 t150 = t65 * t85
 t151 = t150 * t55
 t152 = t2 * t54
 t153 = t2 * t79
 t154 = t152 * t51
 t155 = t2 * t15
 t156 = t13 * t25
 t157 = t156 * t31
 t158 = t63 * t17
 t159 = t158 * t74
 t160 = t2 * t28
 t161 = t57 * t18
 t162 = t161 * t75
 t163 = t26 * t13
 t164 = t163 * t83
 t165 = t63 * t18
 t166 = t3 * t25
 t167 = t166 * t31
 t168 = t95 * t17
 t169 = t83 * t115
 t170 = t137 * t169
 t171 = t29 - t30
 t172 = t21 - t20
 t173 = t1 ** 2
 t174 = t145 * t50
 t175 = t152 * t18
 t176 = t138 * t5 * t33 * t55
 t177 = t62 * t12 * t22
 t178 = t138 * t108 * t77 * t17 * t74 * t52 * t75 * t79 * t55 + t15
#2 * t124 * t29 * (-t177 + t176) * t81
 t179 = t26 * t51
 t177 = t179 * t148 * t173 * t84 * t78 * t17 * t79 * t76 + t179 * t
#139 * t63 * t84 * t12 * t22 * t75 - t177 * t168 * t2 * t33
 t180 = t28 * t55
 t181 = t8 * t51
 t182 = t181 * t144
 t183 = t179 * t106
 t184 = t60 * t2
```


```
 t185 = t184 * t31
 t186 = t139 * t51 * (t6 * (-t180 * t84 * t25 * t87 * t80 - t182 *
#t90 * t116) - t183 * t173 * t84 * t78 * t87) * t15
 t187 = t73 * t54
 t188 = t187 * t17 * (t55 * (t175 * t171 * t52 * t71 + t41 * t20 *
#t51 * t45 + t98 * t172 * t48) + t97 * t3 * (t15 * (-t174 * t52 * t
#54 * t70 - t154 * t50 * t40) - t42 * t51 * t55) * t43 * t14) * t24
 t189 = t9 * t124 * t29 * t79 * (t161 * t113 * t33 + t4) * t81 * t7
#2
 t190 = t142 * t109 * t86 * t50 * (t59 * t50 * t53 + t185) * t17
 t191 = t159 * t12 * t22 * t84 * t26 * t54 * t75
 t56 = t18 * t177 + t71 * t178 + t17 * ((t154 * (t147 * (t145 * (t4
#5 + t31) + t146) + t148 * t146) + t153 * t91 * t36 * t52 * t55 + t
#151 * (t42 * t19 * t31 - t149)) * t24 * t73 + t155 * (t144 * (-t13
#7 * t32 * t8 * t89 * t116 + t30 * t135 * t71 * t81 * t33) + t138 *
# t18 * t5 * t83 * t25 * t79 * t76 + t102 * (-t143 * t90 * t85 * t1
#16 + t141) * t28)) + t56 + (-t168 * (t59 * t115 * t25 * t89 * t116
# + t165 * t2 * t33) + t170 * t60 * t17 * (-t167 + t50) + t152 * (t
#3 * (t160 * t156 * t58 * t51 * t75 + (t157 + (t30 * t86 + t32 * t7
#5) * t28 * t83) * t51 * t15 - t159 * t48 * t33) + t117 * t19 * t51
# * (t162 + t31) + t164 * t58 * t28 * t86 * t51)) * t22 * t12 + t18
#6 + t95 * t68 * t2 * t49 + t188 + t108 * t101 * t91 * t17 * t26 *
#t55 * t52 * t75 + t189 - t190 + t191
 t146 = t66 * t6
 t177 = 1 + t146
 t178 = t91 * t54
 t186 = t178 + t51
 t188 = t25 ** 2
 t189 = t6 * t51
 t190 = t60 * t8
 t191 = t189 * t80
 t192 = t102 * t54
 t193 = t2 * t50
 t194 = t25 * t47
 t195 = t3 * t85
 t196 = t55 * t24
 t197 = t196 * t17
 t198 = t29 * t17
 t199 = t198 * t55
 t200 = t181 + t135
 t201 = t140 * t100
 t202 = t189 * t142 * t86
 t203 = t3 * t49
 t204 = t4 * t14
 t205 = t74 * t3
 t206 = t51 * t17
 t207 = t206 * ((-t20 * t65 * t70 * t52 * t43 * t55 + (t147 + t148)
# * t45 * t51 * t2) * t54 * t3 + t137 * t20 * t19 * t25 * t50 * t55
#) * t24 * t73
 t208 = t17 * (t20 * t113 * t18 * t71 * t79 * t52 - t205 * t4 * t71
# * t79 * t52 - t204 * t126 * t20) * t22 * t12
 t56 = t2 * (t203 * t200 + t198 * t122 * t60 * (t201 * t71 * t52 *
#t75 - t202 * t89 * t53)) + t26 * (t124 * t103 * (t55 * (t101 + t19
#1) + t192 * t186) * t76 - t181 * t117 * t18 * t83 * t75) + t28 * (
#t148 * t65 * t86 * t17 * t83 * (-t189 * t177 * t55 * t80 - t150 *
#t102 - t190) * t53 - t137 * t123 * t19 * t6 * t17 * t25 * t89 * t5
#5 * t177 * t116) + t56 + t197 * t73 * (-t2 * (t65 + t194) + t20 *
#(t2 * t188 * t47 * t45 + t50)) * t85 + t199 * (t195 * t11 * t14 *
#t89 * t116 - t9 * t153 * t18 * t71 * t81) * t22 * t12 + t73 * (t14
#5 * (t155 * t106 * t17 * t31 * t24 - t11 * t49 * t50) - t68 * t49
#* (t3 * t48 + t193)) * t51 + t135 * (t117 * t32 * t83 * t86 - t121
# * t5 * t48 * t55) + t197 * t133 * t70 * t79 * t81 + t207 + t208
 t133 = t3 ** 2
 t177 = t17 ** 2
 t207 = t63 * t28
 t208 = t7 * t50
 t209 = t18 * t54
 t210 = t51 * t14
 t211 = t106 * t1
 t212 = t211 * t133
 t213 = t66 * t31
 t214 = t7 * t26
 t215 = t60 * t28
 t216 = t124 * t63
 t217 = t57 * t28
 t218 = t180 * t19
```


```
 t219 = t91 * t75
 t220 = t13 * t83
 t221 = t179 + t180
 t222 = t13 ** 2
 t223 = t106 * t51
 t224 = t1 * t83
 t225 = t222 * t55
 t226 = t8 * t48
 t227 = t1 * t26
 t228 = t83 * t26
 t229 = t228 * t31
 t230 = t229 * t54
 t186 = t155 * (t33 * (t206 * t62 * t13 + t13 * (-t180 * t186 - t22
#3) * t3 + t179 * (t224 * t79 + t225)) + t3 * t2 * (-t145 * t32 * t
#28 * t51 + t209 * (-t156 * t221 + t206 * (t144 + t210 * (-t57 + t1
#66)))) * t86 - t213 * t60 * t13 * t17) * t22 * t12
 t231 = t156 * t51 * t86 * t83 * (-t145 * t65 * t28 + t95 * (t54 *
#(t139 - t227) - t65 * t28)) * t22 * t12 * t58
 t107 = t55 * (-t54 * (t138 * t77 * t17 * t74 * t72 * t81 * t54 + t
#119 * t28 * t31) - t121 * (t226 + t208)) + t190 * t83 * t17 * (-t2
#01 * t48 * t75 + t202 * t50) + (t83 * (t160 * t145 * (-t209 - t210
#) * t75 - t212 * t18 * t86 * t85) * t26 + t206 * (t84 * (-t180 * t
#95 * t86 * t115 + t75 * (t54 * (-t144 + t107) - t207) * t14) + t17
#4 * t170 + t207 * t3 * t50 * t116 - t208 * t96 * t31) + t210 * t63
# * t83 * (-t175 * t75 + t213 * t53) * t177) * t22 * t12 * t15 + t2
#20 * (-t219 * (t165 * t177 + t217) * t79 + t150 * t17 * t86 * (t21
#4 - t218) + t17 * (t86 * (t51 * (t28 * (-t19 * t55 + t4) + t214) +
# t215) - t51 * (t144 + t216) * t75) * t54) * t22 * t12 * t58 - t73
# * (t226 * t203 + t230) * t51 + t186 + t231 - t193 * t135 * t73 *
#t49
 t119 = t8 * t17
 t121 = t3 * t28
 t165 = -t121 + t119
 t170 = t28 ** 2
 t175 = t60 * t54
 t186 = t175 * t177
 t201 = t181 * t50
 t136 = t136 * t52
 t202 = t17 * t55
 t203 = t18 * t177
 t231 = t117 * t14
 t232 = t6 * t80
 t233 = t59 * t25
 t234 = t134 * t26
 t235 = t214 * t52
 t236 = t91 * t55
 t237 = t122 * t17
 t238 = t124 * t54
 t239 = t145 * t28
 t240 = t145 * t54
 t241 = t106 * t33
 t242 = t20 * t26
 t243 = t95 * t54
 t244 = t163 * t51
 t245 = t145 * t51
 t246 = t202 * t74 * t52
 t247 = t95 * t1
 t248 = t28 * t26
 t249 = t75 * t54
 t250 = t57 * t10
 t251 = t140 * t113
 t252 = t17 * t54
 t150 = t15 * (t2 * (t31 * (-t240 * t28 + t150 * (-t163 - t119)) +
#t242 * (-t178 * (t135 * t17 + t239) + t13 * (t55 * (-t238 - t28) -
# t106) * t51) * t75 - t243 * (t137 * t26 + t241) * t1) + t157 * (t
#65 * (t195 + t245) + t240) - t248 * t75 * t54 * (t246 * t54 + t210
#) * t84 - t137 * t62 * t17 * (t247 + t13) + t249 * (-t124 * t106 *
# t8 - t244 * (t29 + t32) - t186 * t140 * t52) * t83) * t22 * t12 +
# t252 * (-t250 * t15 * t7 * t51 * t86 - t196 * t187 * t91 * t34 *
#t52 - t251 * t8 * t71 * t52 * t33)
 t253 = t20 + t19
 t254 = t3 * t7
 t255 = t254 + t122
 t256 = t29 * t3
 t257 = t28 * t18
 t258 = t29 * t2
 t259 = t258 * t33
 t260 = t153 * t75
```


```
 t261 = t227 * t21
 t262 = t139 * t55
 t263 = t215 * t17
 t264 = t21 * t54
 t212 = t155 * (t51 * (t158 * t13 * t33 * (-t259 + 1) + t106 * t2 *
# (-t119 * t20 * t75 - t256 * t13 * t86 - t257 * t133 * t86) + t260
# * (-t127 * t18 * t253 + t16 * t253 * t188)) + t238 * t180 * t83 *
# t75 * (t264 - t8) + t54 * ((t225 * t26 + t252 * t255) * t33 * t18
# - t263 * t81) * t48 + t31 * (t17 * (t254 * t14 * (-t262 + t26) +
#t62 * (-t139 * t20 + t261)) * t31 * t2 + t229 * (t29 * t65 * t13 +
# t261 * t54) + t212) * t85) * t22 * t12
 t229 = t178 + t51
 t238 = t206 * t54
 t253 = t202 * t142
 t261 = t75 * t55
 t265 = t117 * t18
 t266 = t181 * t32
 t267 = t1 * t25
 t268 = t174 * t73
 t269 = t7 * t17
 t270 = t79 * t81
 t271 = t270 * t17
 t272 = t26 * t75
 t72 = (t20 * (t3 * (t28 * (t253 * t105 * t86 * t50 + (t25 * t48 *
#t79 + t238 * t55) * t75 * t18) + t261 * t229 * t170) + t206 * (t11
#8 * t66 * t86 - t265 * t75 * t79)) + t238 * (t214 * t21 * t86 + t6
#8 * t33)) * t22 * t12 * t83 + t271 * (-t268 * t70 * t24 + t257 * (
#t77 * t82 * t55 + t102) * t72) + t33 * (-t267 * t179 * t20 * t33 *
# t79 + t121 * t33 * (t18 * t25 * t79 + t180 * (t210 + t209)) - t25
#2 * (t180 * t140 * t48 * t81 * t54 + t266 * t33) - t203 * t59 * t1
#4 * t33 * t79) * t22 * t12 * t84 + t152 * (-t241 * t91 * t3 * t13
#+ t269 * (-t145 - t95) * t31) * t22 * t12 - t272 * t79 * (t32 * t1
# * t51 + t257) * t22 * t12 * t83 ** 2
 t72 = t15 * t72 + t2 * t150 + t83 * (t233 * (t208 * t115 * t17 * t
#86 + (t165 * t75 * t15 + t231 * t86) * t54 * t1) - t179 * t124 * t
#75 * (t232 * t55 + t192) - t234 * t226 * t17 * t75) + t107 + t197
#* t132 * t50 * t85 * (t14 * t34 + t29) + t237 * (-t234 * t71 * t52
# * t33 - t236 * (t235 + t187) * t33 + t142 * t137 * t60 * t6 * t89
# * t53) + t15 * (t13 * (t202 * (-t201 * t53 + t136) + (-t139 * t26
# * t85 * t54 + (t66 + t54) * t55 * t170) * t86 * t15 * t84 + t51 *
# (t14 * (t145 * t106 * t17 - t186) - t60 * t115 * t177 * t50 * t51
# - t163 * t106) * t86 * t15 * t83) + t3 * (-t203 * t62 * t83 * t14
# * t86 * t85 - t153 * t145 * t17 * t48 * t33 * t74)) * t22 * t12 +
# t262 * (-t29 * t88 * t80 * t90 * t85 * t116 + t101 * t83 * t25 *
#t86) + t212
 t77 = t60 + t59
 t107 = t2 * t4
 t132 = t60 * t50
 t150 = t142 * t50
 t186 = t54 * t14
 t192 = t261 * t83
 t197 = t50 * t31
 t212 = t197 * t19
 t234 = t145 + t95
 t238 = t121 - t119
 t241 = t62 * t7
 t262 = t197 * t180
 t273 = t223 * t75 * t18
 t274 = t54 * t22 * t12
 t275 = t10 * t25
 t276 = t275 * t79
 t277 = t3 * t170
 t174 = t2 * (t15 * (t115 * (t202 * t149 * t12 * t22 * t238 * t85 *
# t86 - t262 * t94 * t17 * t12 * t22 * t85) + t14 * (t86 * (-t133 *
# t25 * t28 * t12 * t22 * t85 + t241 * t177 * t51) - t174 * t137 *
#t28 * t12 * t22) + t163 * t91 * t12 * t22 * t33 * t79) + t274 * t1
#3 * (-t239 * t25 * t86 + (t249 * t180 * t48 * t74 + t273) * t17 *
#t3) * t58) + t73 * (t252 * t31 * t61 + t51 * (t17 * (-t63 - t62) +
# t3 * (-t180 - t106)) * t33 - t106 * t234 * t45) + ((t264 * t170 *
# t86 * t55 + t210 * (t277 * t86 * t55 + (-t240 * t30 - t276) * t75
# * t17)) * t22 * t12 - t248 * t86 * (t181 + t135)) * t15 * t83
 t239 = t75 - t86
 t278 = t258 * t75
 t279 = t20 * t50
```


```
 t280 = t91 * t8
 t281 = t10 * t51
 t282 = t73 * t55
 t283 = -t282 * (t252 * t42 * t40 * t24 * t81 + t160 * t91 * t3 * t
#33) + t59 * t83 * (-t217 * t1 * t86 + t203 * t8 * t75) * t15
 t284 = t155 * (t154 * (t119 * t33 + t121 * (-t31 - t33) - t163 * t
#31) + t202 * (t209 * (t96 * (t160 * t239 - t269 * t86) + t226 * (-
#t278 + t33) * t13 + t254 * t160 * t86) + t19 * t7 * t28 * t86 * t5
#1 * (t279 + t3) + t205 * t160 * t48 * t52 * t33 * t79) - t260 * t2
#0 * t17 * (t280 + t281) * t25) * t22 * t12
 t88 = t253 * t88 * t80 * t90 * t85 * t116
 t253 = t57 * t13
 t285 = t3 * t79
 t286 = t228 * t85
 t287 = t155 * t22 * t12
 t143 = t287 * (t202 * (t3 * (t223 * t19 * t13 * t18 * t86 - t157 *
# t105 * t50 * t115) + t253 * t74 * t48 * t52 * t33 * t79 * (-t259
#+ 1) + t143 * t133 * t28 * t50 * t85 * t116) + (-t197 * t118 * t10
#5 * t115 * (t20 * t2 * t31 + t94) + (t52 * (-t125 * t79 * t75 + t2
#85 * t33) - t270 * t57 * t33) * t48 * t74 * t7) * t55 * t177 + t28
#6 * t86 * (t242 * t1 * t54 + t121 * t14))
 t44 = t143 + t14 * (t73 * t51 * (t185 * t17 + t44 * (-t145 - t152)
# * t43) * t50 - t198 * t102 * t85 * t116 * t90) + t54 * t283 + t2
#* t174 + t17 * (t150 * (t102 * t89 * t85 * t116 + t107 * t208 * t1
#37 * t53 + t83 * t82 * t86 * t55) + t108 * t4 * (-t135 * t86 + t18
#1 * t75) + t118 * t19 * t31 * (t77 * t31 * t2 + t132 * t53) + t196
# * (-t155 * t37 * t48 * t52 * t79 + (t148 * t48 + t39) * t85 * t47
# * t13) * t73) + t3 * (-t108 * t200 * t75 * t25 + t186 * t13 * (-t
#73 * t70 * t50 * t52 * t43 * t55 + (-t43 - t45) * t51 * t39 * t2))
# + t72 + t252 * t83 * t15 * (t242 * t10 * t86 * t85 - t152 * t159
#* t75) * t22 * t12 - t228 * t124 * t7 * t75 * (t63 + t62) - t73 *
#t2 * t51 * t55 * (t13 * t188 * t47 * t45 + t212 * t28) - t140 * t1
#7 * t48 * (t270 * t102 * t71 + t192 * t80) + t284 + t88
 t70 = t156 + t160
 t72 = -t178 - t51
 t88 = t54 * t15
 t90 = t144 * t2
 t143 = t160 * t60
 t157 = t63 * t51
 t159 = t126 * t28
 t174 = t31 * t2
 t185 = t181 * t6
 t200 = t28 * t50
 t242 = t26 * (t160 * t52 - t13) - t121
 t283 = t179 + t180
 t284 = -t60 - t59
 t288 = t20 * t48
 t289 = t288 + t2
 t290 = t279 + t3
 t291 = t179 * t10 * t83
 t292 = t19 * t85
 t293 = t137 * t83
 t294 = t97 * t49
 t295 = t106 * t7
 t296 = t26 * t15
 t241 = t296 * t84 * (t266 * t7 * t87 + (t28 * (t284 * t8 - t241) -
# t295 * t8) * t76 * t18 - t9 * t144 * t74 * t48 * t79 * t76) + t28
#7 * (t74 * (t217 * t113 * t48 * t52 * t289 * t79 * t75 + t113 * t5
#2 * t71 * t242 * t79 * t33) + t292 * t86 * t8 * (-t180 * t65 * t6
#* t25 * t53 * t290 + t57) + (t253 * t175 * t29 * t48 * t52 - t291
#* t79) * t75 * t18) + (t54 * (-t293 * t15 * t283 + (-t59 * t20 + t
#21 * t284) * t49 * t48) + t294 * (-t95 * t62 + t63 * (t152 - t95))
#) * t24 * t73
 t266 = t6 * t28
 t297 = t64 * t28
 t298 = t261 * t108 * t35
 t146 = t148 * t84 * (-t30 * t10 * t7 * t79 * t76 + (t10 * t8 * t26
# * t85 + t297 * t7 + t182) * t87 * t14) + t287 * (t260 * t48 * t4
#* t242 * t74 + t2 * (-t297 * t95 + t135 * (-t106 * t3 + t180 * (t5
#7 * t52 - t3))) * t75 * t18 + t213 * t63 * (t25 * (-t121 * t116 +
#t13 * (-t258 * t31 + t146) * t53) + t266 * t293 * t14)) + (t13 * (
#-t298 * t48 * t79 * t74 - t298 * t209 * t51) + t49 * (t153 * t113
#* t97 + t195 * (-t111 - t4)) + (t95 * t106 * t83 * t37 * t55 * t86
```


```
# - t262 * t83 * t85) * t15 * t14) * t24 * t73
 t182 = t180 * t91
 t213 = t25 * t51
 t242 = t213 + t182
 t260 = t3 * t37
 t34 = t34 * t13
 t196 = t196 * t73
 t262 = t22 * t12
 t293 = t15 * t50
 t298 = t33 * t18
 t299 = t298 * t15
 t300 = t8 * t15
 t301 = t300 * t249
 t302 = t4 * t13
 t303 = t95 * t73
 t304 = t33 * t15
 t305 = (t113 + t4) * t28
 t36 = t115 * (t201 * t60 * t104 * t6 * t87 + t293 * t51 * t83 * (t
#143 * t12 * t22 + t213 * (t200 * t38 * t8 * t53 + t196 * (t34 + t2
#60))) * t86 + t107 * t148 * t126 * (t200 * t11 * t53 + t262 * t13)
# * t31) + t2 * (t187 * t49 * t24 * t61 + t304 * (-t302 * t140 * t1
#2 * t71 * t22 * t52 + t303 * t40 * t24) * t79) + t83 * (-t301 * t2
#62 * t21 * t242 + t299 * (t28 * (t71 * (-t9 * t30 * t4 * t33 * t52
# + t262 * t30 * t4 * t81) + t196 * t36 * t52 * t48) - t262 * t275
#* t95 * t33) * t79) + t186 * t85 * t15 * t83 * (t250 * t12 * t22 +
# t73 * t25 * t24 * (t13 * t35 + t36 * t3)) * t86 - t303 * t60 * t2
#4 * t49 + t301 * t30 * (-t305 * t91 * t33 - t262 * t51) * t84
 t38 = t121 + t163
 t40 = t111 * t13
 t104 = t111 * t2
 t187 = t166 * t68
 t201 = t209 * t33
 t250 = t149 * t115
 t301 = t91 * t33
 t303 = t262 * t15
 t306 = t17 * t2
 t34 = t306 * (-t9 * t141 * t29 * t4 + (t294 * t4 * t79 + (t250 * t
#37 * t31 * t89 * t85 + t32 * (-t197 * t94 * t28 * t85 + t34 * t154
# * t86) + t301 * t79 * (-t217 * t91 * t52 + t260)) * t55 * t15) *
#t24 * t73 + t303 * (t115 * (t159 * (-t149 * t4 + t104) * t31 + t41
# * t50 * (t40 + t144 * (-t279 - t3) * t53) * t85 * t86) + t14 * (t
#86 * (-t157 * t160 * t149 + t187 * t2 * t85) + t137 * t63 * t50 *
#t53 * t70) + t201 * (t60 * (t48 * (t52 * (-t121 - t163) + t217 * t
#81) - t100 * t91 * t2 * t33 * t38 - t253 * t33) + t257 * t62 * t71
# * t52 * (t125 * t33 - t3))))
 t125 = t269 + t160
 t141 = -t121 + t119
 t154 = t144 * t50
 t257 = t117 * t7
 t260 = t228 * t52
 t279 = t144 * t3
 t294 = -t302 - t254
 t307 = t180 * t6
 t308 = t60 * t13
 t309 = t2 * t170
 t310 = t152 * t29
 t311 = t118 * t3
 t93 = t18 * (t304 * t100 * t7 * (-t277 * t52 * t55 + t252 * t8) *
#t71 + t310 * t75 * (t163 * t4 + t57 * (t10 + t9) * t54 + t119 * t1
#13) * t48) + t54 * (-t195 * t10 * t15 * t17 + t25 * t294) * t31 +
#t51 * (t4 * (t121 - t163) + t311) * t33 - t278 * t129 * t74 * t48
#- t225 * t93 * t51 * t45 + t155 * (-t252 * t173 * t78 * t26 * t85
#+ t139 * (-t232 * t25 * t50 * t85 + t181 * (-t145 * t25 + t208 * (
#-t307 - t25)) + t106 * (t294 * t51 - t308)) + t179 * t156 * t68 -
#t309 * t64) * t86
 t129 = t163 + t119
 t68 = -t190 * t139 * t6 * t89 * t53 - t212 * t158 * t20 * t25 * t5
#3 + t272 * t103 * t68 + t17 * (t7 * (t150 * t158 * t86 * t53 - t14
#4 * t65 * t86) + t202 * t82 * t115 * t86 * t50 + t249 * (t102 * t2
#8 - t118 * t21)) * t15 * t83
 t103 = t2 * (t18 * (t88 * t33 * (t228 * t8 * t28 * t33 + t145 * t1
#65) + t304 * t128 * t7 * t52) + t74 * (t158 * t3 * t33 * t54 - t26
#2 * t285 * t202 * t52) + t60 * t29 * t52 * t33 * t129) * t48
 t99 = t292 * (t15 * (t197 * t3 * (t200 * t25 * t53 * t46 - t269) +
# t306 * (t257 * t65 - t311) * t86) - t194 * t99 * t45)
```


```
 t194 = t269 * t52
 t212 = t262 * t202
 t228 = t212 * t91
 t183 = t155 * (t115 * (t193 * t111 * t17 * t85 * t125 * t86 + t126
# * t17 * (t6 * (t202 * t25 * t53 * t80 + t190 * t17) - t149 * t144
#) * t31) + t14 * (-t306 * t85 * (t208 * t11 * t26 + t187) * t86 +
#t160 * t137 * t50 * (-t281 * t50 - t145 - t11 * t50 * (t180 * t53
#+ t51))) + t2 * (t30 * t28 * t71 * t52 * t79 * (t265 * t52 + t9) *
# t33 + t28 * (-t205 * t117 * t48 * t79 + t91 * (t3 * (t54 * (-t305
# - t118) - t207) + t202 * t80) + t111 * t51 * t141) * t75) + t228
#* t79 * t52 * (-t194 + t13) + t224 * t183 * t69 * t75)
 t205 = t117 - t166
 t224 = t7 * t89
 t151 = t262 * (t202 * t195 * t115 * t50 * t205 * t116 - t151 * t20
# * t94 * t17)
 t40 = t155 * (t51 * (t2 * (t26 * (t40 * t28 * t75 + t211 * (t254 *
# t86 + t302 * t75)) + t124 * t75 * (-t62 * t3 * (t211 + t28) + t63
# * (t163 - t121))) + t65 * t31 * (-t308 * t17 + t160 * (t112 * t13
# + t254) * t31 * t25)) + t219 * t122 * (t17 * (t54 * (-t180 * t21
#+ t214) + t9 * t25 * t79) + t248 * t145) + t139 * t31 * (-t224 * t
#11 + t10 * (t230 - t224)) * t85 + t143 * (-t252 * t21 * t75 + t156
# * (t100 + t65) * t86) + t235 * t124 * t10 * t71 * t79 * t33)
 t40 = t183 + t51 * t68 + t2 * t93 + t15 * (t18 * (t48 * (t152 * t6
#0 * t17 * t52 * t141 * t33 + t215 * t164 * t100 * t75) + (t128 * t
#5 * t82 * t52 + t237 * t62 * t28 * t52) * t33 * t71) + t74 * ((t26
#0 * t158 * t125 * t54 + t260 * t17 * (t202 * t5 * t82 + t257 + t90
#) * t79) * t75 * t48 + t153 * t52 * t17 * (t269 * (t4 * t26 * t52
#+ t63 * t5) - t279) * t33 * t71) + t66 * (t83 * (t158 * t28 * t53
#* t31 + t233 * (t8 * (t14 * t53 * t177 - t1 * t17) + t16 * t28) *
#t86) + t262 * t168 * t55 * t116 * t238 + t174 * t53 * (t25 * (t8 *
# (t66 * t4 * t53 * t177 - t168) - t191 * t17 * t50 + t154 * t13) +
# t269 * (-t154 + t158)))) + t310 * t21 * t33 * (-t144 * t52 - t23
#* t54) * t71 + t103 + t99 + t40 + t151
 t68 = t214 + t144
 t93 = t62 * t26
 t99 = t86 * t15
 t103 = -t269 + t156
 t125 = t269 + t160
 t128 = t60 * t14 * t177
 t151 = t83 * (t51 * (-t111 * t19 * t170 * t50 + t263 * (-t149 + t2
#) * t50 * t115 + t8 * t26 * t103) + t135 * (t156 * t26 + t128) + t
#139 * (t8 * (t117 + t57) + t65 * (t232 * t202 - t279) * t53 * t25)
# * t85) * t86 + t193 * t53 * (t8 * t28 * (t103 * t55 + t95 * t25)
#+ t117 * t85 * t125 * t50 * t115) * t31
 t154 = t7 * t133 * t51 * t49
 t164 = t145 * (-t179 * t122 * t6 * t33 - t98 * t73 * t50)
 t40 = t15 * t151 + t83 * (-t247 * t147 * t43 * t79 - t8 * t49 * t5
#4) + t83 * (t1 * (-t198 * t86 * t51 * t54 * t78 + t93 * (t201 * t2
#9 * t71 * t52 + t137)) + t99 * (-t180 * t101 * t17 + t65 * (t266 *
# t195 * t118 - t221 * t17 * t82)) + t93 * t195 * t148 * t173 * t86
#) + t40 - t228 * t270 * t83 * t171 + t15 * (t200 * t117 * t115 * t
#86 * t85 + (t17 * (t54 * (t51 * t68 + t215) + t207 * t51) - t248 *
# t85 * t46 * t50) * t86 * t14 + t179 * t8 * t28 * t75) * t84 + t12
#0 * t111 * t95 + t282 * (t51 * (t119 * t50 * t53 - t156 * t47 * t4
#8) - t136 * t17) + t117 * (t209 * t8 * t71 * t52 - t224 * t210 * t
#53) + t161 * t110 * t13 * t71 * t52 * t33 + t301 * t122 * (t306 *
#t110 * t33 - t145 * t26) + t246 * t153 * (t272 * t109 * t5 + t262
#* t117 * t52) * t48 - t154 + t164
 t78 = t26 ** 2
 t93 = t205 * t51
 t109 = t20 * t51
 t110 = t203 * t7
 t147 = t118 + t144
 t151 = t302 + t254
 t154 = t83 * t15
 t164 = t154 * t75
 t168 = t164 * t7
 t171 = t16 * t188
 t173 = t154 * (t119 * (t51 * (-t144 * t86 + t216 * t75) + t215 * t
#239) + t163 * t75 * (t118 * t51 + t215))
```


```
 t183 = t193 * t31 * (-t51 * (t119 * t7 + t96 * t28) * t15 - t32 *
#t7 * (t145 + t95))
 t40 = t54 * (t108 * t25 * t151 * t86 + t174 * (-t25 * (t95 * t10 +
# t113 * t13) + (-t151 * t51 - t308) * t17 * t15) + t257 * t29 * t7
#1 * t81 + t168 * (t147 * t17 + t163 * (t124 * t55 + t25))) - t51 *
# (t19 * t1 * t133 * t39 * t51 * t54 * t45 + t117 * t29 * t8 * t89
#* t116) + t40 - t88 * t1 * t84 * t78 * t51 * (t281 + t7) * t86 + t
#83 * ((-t206 * t7 * t115 - t300 * t170 * t53) * t55 * t50 - t88 *
#t227 * t195) * t31 + t83 * t4 * t15 * (t110 * t79 + t244 * t28) *
#t75 + t2 * (t54 * (t15 * (t7 * t48 * t165 + t93 * t13) + t21 * (t1
#17 * t51 + t118 * t48) + t275 * t95) + t55 * (t109 * t165 + t8 * t
#3 * (t51 * (t266 + t124) + t91 * t28)) + t155 * (t93 * t1 + t91 *
#t165) * t79) * t33 + t173 + t168 * (-t171 + t57 * (t267 + t124)) *
# t79 + t183
 t93 = t214 * t17
 t108 = t124 * t52
 t151 = t1 * t2
 t130 = t155 * (t2 * (t51 * (t31 * (t54 * (-t121 - t163) - t200 * t
#8) + t75 * (t119 * (t161 * t54 + t118) - t277 * t4) + t28 * (-t208
# * t19 * t26 + t187) * t86) - t227 * t8 * t86 * t205 * t85 - t180
#* (t13 * t31 * t54 + t254 * t28 * t75) + t217 * t135 * t75) + t71
#* (-t130 * t74 * t28 * t33 + t270 * t176 * t177 * t74) + t178 * t1
#63 * t60 * t17 * t52 * t33)
 t165 = t5 * t71
 t80 = t2 * (t120 * t4 + t174 * t68) + t80 * (t192 * t15 * t177 * t
#48 * t74 + t219 * t148 * t83 * t17) + t165 * t199 * t82 * t81 + t1
#90 * t124 * (t164 * t17 + t165 * t52) - t145 * t73 * t48
 t124 = t133 * t51
 t23 = t83 * (-t23 * t49 + (t226 * t4 * t177 * t74 + t281 * (t4 * (
#t267 * t17 + t203) - t171) + t269 * t25 * (-t21 + t10)) * t75 * t1
#5) * t79
 t164 = t122 * (-t57 * t51 * t33 + t120 * t55)
 t158 = t185 * (t158 * t29 * t89 * t116 + t124 * t49)
 t23 = t3 * (-t155 * t60 * t170 * t48 * t52 * t33 + t120 * t77) + t
#54 * t80 + t2 * (t54 * (t2 * (t33 * (t26 * (-t281 - t280) + t51 *
#t141 * t15) + t179 * t131 * t75 + t99 * t17 * (t139 * t102 * t85 -
# t138)) + t113 * (-t99 * t83 * t170 + t120)) + t299 * (t291 * t17
#* t33 + t117 * t8 * t71 * (t108 + t1) + t48 * (t9 * (t17 * (t214 +
# t216) - t166 * t28) + t10 * (t141 * t25 + t93)) * t33 * t2) * t79
# + t99 * t122 * (t51 * (-t151 * t78 * t51 + t128) + t234 * t28 * t
#25) + t293 * t19 * t86 * (-t93 * t10 * t85 + t309 * t284)) + t40 +
# t42 * t14 * (-t152 * t145 * t45 + (t97 * (-t153 - t240) - t245) *
# t43 * t3) + t130 + t83 * t31 * (t54 * (-t210 * t117 + t60 * (t28
#* t5 - t139)) + t65 * t7 * t221) - t10 * t133 * t85 * t49 + t23 +
#t164 - t158
 t40 = t137 * t19
 t23 = t115 * (t202 * t193 * t105 * (t262 * t3 + t122 * t29 * (-t16
#6 * t86 + t197) * t6) - t104 * t198 * t167 * t89 * t85 * t116) + t
#23 - t268 * t51 + t298 * t184 * t100 * t29 * t71 * t52 * t129 + t1
#54 * (t33 * (t271 * t251 * t28 * t71 + t248 * t136) - t27 * t243 *
# t188 * t75) + t40 * t89 * t15 * (-t4 * (t206 * t118 * t1 + t309)
#+ t111 * t28 * t103) * t53
 t43 = t210 * t290
 t45 = t222 * t58
 t65 = t19 * t51
 t77 = t30 * t79
 t78 = t166 * t87
 t80 = t303 * t83 * t17
 t38 = t80 * (t2 * (-t140 * t144 * t76 * t79 * t289 + t30 * (t14 *
#(-t160 * t62 * t76 * t51 + t78 * t69 * t85) - t175 * t29 * t13 * t
#76) + t175 * t32 * t29 * t13 * t87) + t262 * (-t186 * t195 * t31 +
# t261 * t79 * (-t160 * t3 - t253 + t296 * t48 * (t259 - 1) * t222)
# * t74 - t45 * t213 * t218 * t87 * t67 + t209 * t51 * (t261 * (t24
#8 * t84 * t115 * t33 + t45 * t217 * t33 - t163 * t19 - t20 * t38)
#+ t33 * (-t57 * t33 * (t20 + t19) + t3) * t54 + t96 * t26 * t86)))
 t69 = t30 * t76
 t93 = t180 * t57 * t76
 t96 = t160 * t55
 t99 = t210 * t54
 t96 = t80 * (t2 * (t30 * t14 * (t295 * t95 * t87 + (-t181 * t106 +
```


```
# t180 * (-t181 - t135)) * t76 * t2) - t93 * t54 * (t100 * t7 + t8)
# * t74 + t215 * t142 * t95 * t87 + t109 * (t150 * t144 * t87 * t51
# + t32 * (t54 * (t281 + t7) * t26 + t297) * t87 + t69 * (t54 * (-t
#118 - t144) - t207))) + t262 * t21 * (-t96 * t195 * t142 * t87 - t
#20 * t75 * t79 * t242 + t99 * (t25 * (t86 * (-t137 * t57 * t21 + t
#145) - t152 * t75) + t96 * (t86 - t75))))
 t26 = t303 * t17 * t84 * (t32 * (-t281 * t161 * t79 * t76 + t121 *
# (t209 * t61 + t157 * (t189 * t14 + t18)) * t87 - t124 * t274 * t1
#80 * t74 * t87) + t20 * (-t207 * t140 * t48 * t76 * t54 + t150 * t
#215 * t87 * t51 - t69 * t276 * t51)) + t96 + t38 + t287 * t17 * (t
#83 * (t32 * (t195 * t144 * t14 * t87 - t161 * t7 * t79 * t76) + t2
#0 * (t8 * (t307 * t150 * t87 * t85 + t32 * t26 * t87 * t85) - t77
#* t7 * t76 * (t182 * t5 + t25))) + t262 * (-t45 * t286 * t32 * t87
# * t54 + t65 * (t2 * (t179 * t20 * t54 + t180 * (t20 * t54 + t43))
# + t25 * (t54 * (t124 * t18 + t225 * t15 + t20 * t95) + t43 * t145
#)) * t86 - t40 * t145 * t67 + t240 * t21 * t229 * t33 + t77 * t83
#* (t180 * t83 * t14 * t18 + t213 * (t45 + t169)) * t76))
 t38 = t17 * (t196 * t49 * (t122 * t51 - t254 * t98) + t262 * t20 *
# t83 * (t32 * t54 * t61 * t86 + t30 * t75 * (-t62 * t51 + t63 * t7
#2) + t132 * t142 * t86 * t51))
 t40 = t118 + t144
 t43 = t214 + t144
 t45 = t212 * (t91 * (-t152 * t43 * t81 + (t302 + t254) * t54 * t52
#) + t66 * t53 * (t94 * t40 - t122 - t302))
 t61 = t19 - t20
 t1 = t20 * t1
 t62 = (t300 - t204) * t17
 t66 = t54 * t52
 t67 = t66 * t48 * (t121 * t91 * t114 + t252 * (t5 * t82 * t15 * t4
#8 + t18 * (t48 * (t258 * t52 + t21) - t2) * t4 - t91 * t102 + t2 *
# (t5 * (-t140 * t48 * t52 * t55 + t236 * t29 * t52) + t15) * t7) +
# t236 * t101 * t15 * t177 * t52)
 t66 = t48 * t54 * (t48 * (-t134 * t127 * t81 + t66 * (t10 * (t125
#* t15 - t161) + t9 * t18 * (-t17 * (t20 * t55 + t4) - t57))) + t25
#4 + t302)
 t46 = t126 * (t14 * (t117 * (t15 * (t119 * t53 + t13) + t11) - t16
#6 * t46) + t256 * t46 - t119 * t10 * t15 + t250 * t17 * t112)
 t5 = t2 * (t3 * (t97 + t91) - t42 * t52 + t48 * (t9 * t28 * t48 *
#t52 - t13) * t15) + t110 * t4 * t71 * t81 * (-t5 * t18 * t55 + t15
#) + t108 * t71 * (t113 * t21 - t20 * t4)
 t21 = t91 * t52 * (-t163 * t114 * t48 + t202 * t255) + t288 * t55
#* (t194 - t13) - t225 * t39 * t52
 t28 = t195 * (t47 * (t41 - t42) + t50 * t61 - t231 * t29 * t89 * t
#116)
 t1 = t51 * (t144 * t19 * t89 * t53 + t225 * (-t92 - t293)) + t21 *
# t54 + t79 * t5 + t89 * (t85 * (t17 * (t6 * (-t63 * t14 * t61 - t1
#23) + t204 * (t10 - t19)) * t53 + t139 * t111 * (-t256 + t62) * t1
#16) + t210 * (-t156 * t4 + t111 * (t160 - t156)) * t53) + t48 * (t
#55 * ((-t163 + t119 + t121) * t52 * t7 + t264 * t13) - t152 * t52
#* t68) + t50 * (t55 * ((t160 - t269 - t156) * t53 * t8 + t65 * t13
#) - t94 * t51 * t147) + t145 * (t41 * t51 * t47 + t98 * t3) + t79
#* (t4 * (t18 * (-t203 * t4 * t81 + t16) - t1) + t9 * t172) * t71 +
# t85 * (t4 * (t14 * (-t204 * t177 * t116 + t151) - t1) + t11 * t61
#) * t89 - t62 * t94 * t85 * t50 + t67 + (t302 + t122 + t202 * (-t1
#4 * t255 - t20 * t8) * t53) * t51 * t50 + t66 + t46 + t28
 t3 = t17 * (t209 * (t113 * t43 + t4 * t43) * t81 * t71 + t210 * t1
#16 * t89 * (-t111 * t40 - t4 * t40))
 t5 = t22 ** 2 * t12 ** 2 * t17 * t58 * t13 * t84 * (t18 * (-t179 *
# t19 * t25 * t76 * t79 + t99 * (t78 * t283 - t93)) + t195 * t180 *
# t142 * t87 - t93 * t74 * t79)
 t6 = 32
 ret = -8 * t44 - 16 * t56 + t6 * (t34 + t17 * t36 + t17 * t146 + t
#17 * t241 + t17 * ((t97 * t51 * t49 * (t19 * t64 + t20 * t64) * t5
#0 + t88 * (t95 * t2 * t37 * t33 * t55 + t30 * (t180 * t72 - t223)
#* t75 * t84 - t273 * t220 * t35 - t120 * t60 * t48)) * t24 * t73 +
# t59 * t148 * t84 * (t227 * t135 * t14 * t87 - t27 * t30 * t54 * t
```


```
#76 + t200 * (t185 + t7) * t87 * t115) + t19 * t31 * t15 * (t14 * (
#t193 * t4 * t31 * t85 * t70 + t159 * (-t149 * t111 + t107)) + t174
# * (t54 * (t51 * (t7 * (t57 + t166) + t90) + t143) + t157 * t70))
#* t22 * t12)) - 4 * t23 - 64 * t26 + 48 * t38 + 12 * t45 + 6 * t3
#+ 128 * t5 + 24 * t80 * (t195 * t32 * t8 * t86 - t162 * t7 * t79)
#- 2 * t1

 hjetmass_triangle_pppp_0_s234_mhsq_dp = ret/32d0/(0,1d0)
 return

 end function
```

H+n-gluons; n=5

Ellis, *Seth*

$$
\begin{aligned}
A_5(1_g^+, 2_g^+, 3_g^+, 4_g^+, 5_g^+; H) \;=\; & m^2 \Bigg[\Bigg\{ \frac{(4m^2 - M_h^2)}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \Big[\sum_{i=1}^{6} e_{(i)} E_{(i)} \\
& - \frac{1}{2} s_{12} s_{23} D_0(p_1, p_2, p_3; m) \\
& - \frac{1}{2} \left[(s_{12} + s_{13})(s_{24} + s_{34}) - s_{14} s_{23} \right] D_0(p_1, p_{23}, p_4; m) \\
& - \frac{1}{2} \left[(s_{12} + s_{13} + s_{14})(s_{25} + s_{35} + s_{45}) \right. \\
& \qquad \left. - s_{15}(s_{23} + s_{24} + s_{34}) \right] D_0(p_1, p_{234}, p_5; m) \\
& - (s_{12} + s_{13} + s_{14} + s_{15})\, C_0(p_1, p_{2345}; m) \Big] \\
& - \frac{2(s_{12} + s_{13} + s_{14} + s_{15})}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \Bigg\} \\
& + \Big\{ 4 \text{ cyclic permutations} \Big\} \Bigg]
\end{aligned}
$$

Coefficients of Pentagons in n=5

Ellis, *Seth*

$$\mathbf{E_{2\times3\times4\times5}}: \quad e_{(1)} = m^2\left[\frac{1}{2}\mathrm{tr}_-\{2\,3\,4\,5\} + \frac{s_{23}s_{34}s_{45}(\mathrm{tr}_-\{2\,6\,5\,1\} + s_{51}s_{12})}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}\right]$$

$$\mathbf{E_{12\times3\times4\times5}}: \quad e_{(2)} = -m^2 s_{45}s_{34}\frac{\mathrm{tr}_-\{5\,1\,2\,3\,(1+2)\,6\}}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}$$

$$\mathbf{E_{1\times23\times4\times5}}: \quad e_{(3)} = -m^2\frac{\mathrm{tr}_+\{5\,4\,(2+3)\,1\}\,\mathrm{tr}_-\{1\,2\,3\,4\,5\,6\}}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}$$

$$\mathbf{E_{1\times2\times34\times5}}: \quad e_{(4)} = -m^2\frac{\mathrm{tr}_+\{1\,2\,(3+4)\,5\}\,\mathrm{tr}_-\{5\,4\,3\,2\,1\,6\}}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}$$

$$\mathbf{E_{1\times2\times3\times45}}: \quad e_{(5)} = -m^2 s_{12}s_{23}\frac{\mathrm{tr}_-\{1\,5\,4\,3\,(4+5)\,6\}}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}$$

$$\mathbf{E_{1\times2\times3\times4}}: \quad e_{(6)} = m^2\left[\frac{1}{2}\mathrm{tr}_-\{4\,3\,2\,1\} + \frac{s_{12}s_{23}s_{34}(\mathrm{tr}_-\{4\,6\,1\,5\} + s_{45}s_{51})}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}\right]$$

Soft Higgs limit

$$\frac{m}{v}\frac{d}{dm}\left(\frac{1}{\not{p}-m}\right)=\frac{1}{\not{p}-m}\frac{m}{v}\frac{1}{\not{p}-m}$$

$$\begin{aligned}\text{Insertion Operator} &= \frac{m}{v}\frac{d}{dm}\\ &= \frac{1}{v}2m^2\frac{d}{dm^2}\end{aligned}$$

$$\left[A_n(1_g^+,2_g^+,\ldots,n_g^+;H)\right]_{p_H\to 0}\Rightarrow\frac{1}{v}2m^2\frac{d}{dm^2}\left[A_n(1_g^+,2_g^+,\ldots,n_g^+)\right]$$

Soft Higgs limit: H+n-gluons; n=4

Bern, Morgan 1996

$$\boxed{A_4(1^+,2^+,3^+,4^+) = -2\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\left[m^4 D_0(p_1,p_2,p_3;m) - \frac{1}{6}\right]}$$

Ellis, *Seth*

$$\begin{aligned} A_4(1_g^+,2_g^+,3_g^+,4_g^+;H) &= m^2\Bigg[\Bigg\{\frac{4m^2-M_h^2}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\Big[-\mathrm{tr}_+\{1\,2\,3\,4\}m^2 E_0(p_1,p_2,p_3,p_4;m) \\ &+ \frac{1}{2}((s_{12}+s_{13})(s_{24}+s_{34})-s_{14}s_{23})D_0(p_1,p_{23},p_4;m) \\ &+ \frac{1}{2}s_{12}s_{23}D_0(p_1,p_2,p_3;m) \\ &+ (s_{12}+s_{13}+s_{14})C_0(p_1,p_{234};m)\Big] + 2\frac{s_{12}+s_{13}+s_{14}}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\Bigg\} \\ &+ \Big\{3 \text{ cyclic permutations}\Big\}\Bigg] \end{aligned}$$

$$\boxed{\left[A_4(1_g^+,2_g^+,3_g^+,4_g^+;H)\right]_{p_H\to 0} \Rightarrow \frac{1}{v}2m^2\frac{d}{dm^2}\left[A_4(1_g^+,2_g^+,3_g^+,4_g^+)\right]}$$

Large m_t limit

Dixon, Glover, Khoze 2004

$$\boxed{A_4(1^+, 2^+, \ldots, n^+, H) \propto \frac{M_h^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n-1|n\rangle}}$$

$$\begin{aligned}
C_0(p_1, p_2; m) &= -\frac{1}{2m^2} - \frac{(p_1^2 + p_2^2 + p_{12}^2)}{24m^4} + O\left(\frac{1}{m^6}\right) \\
D_0(p_1, p_2, p_3; m) &= \frac{1}{6m^4} + \frac{(s_{23} + s_{12} + p_1^2 + p_2^2 + p_3^2 + p_{123}^2)}{60m^6} + O\left(\frac{1}{m^8}\right) \\
E_0(p_1, p_2, p_3, p_4; m) &= -\frac{1}{12m^6} + O\left(\frac{1}{m^8}\right)
\end{aligned}$$

$$\begin{aligned}
A_2(1_g^+, 2_g^+; H) &= +\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle\,\langle 2\,1\rangle} \\
A_3(1_g^+, 2_g^+, 3_g^+; H) &= -\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,1\rangle} \\
A_4(1_g^+, 2_g^+, 3_g^+, 4_g^+; H) &= +\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \\
A_5(1_g^+, 2_g^+, 3_g^+, 4_g^+, 5_g^+; H) &= -\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}
\end{aligned}$$

Other Helicity Configurations

Budge, Campbell, de Laurentis, Ellis, *Seth*

1. Momentum twistor
   - All spinor quantities are not independent
   - Apply independent twistor variables after removing phase factor
2. Analytic reconstruction via numerical analysis
   - Study limiting behaviour of all possible spinor quantities and kinematic variables
   - Get an idea of the possible denominator structure
3. Equivalent scalar theory

$$\mathcal{L} = (D_\mu \phi^\dagger)_i (D^\mu \phi)_i - \lambda \phi_i^\dagger \phi_i H$$

6.2.3 $c^{(0)}_{12\times34}, c^{(2)}_{12\times34}$

This coefficient is defined in terms of the corresponding coefficient with a scalar loop, $\tilde{c}^{(0)}_{12\times34}$

$$\begin{aligned} c^{(0)}_{12\times34}(1^+,2^-,3^+,4^-) &= \tilde{c}^{(0)}_{12\times34}(1^+,2^-,3^+,4^-) \\ &+ \left\{\frac{\langle 2\,1\rangle^2\,[1\,3]^2\,\langle 3\,4\rangle^2 - \langle 2\,4\rangle^2\,\langle 1|(3+4)|1]\,\langle 4|(1+2)|4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle\,\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]}\right\} \\ &+ \Big\{1\leftrightarrow 3, 2\leftrightarrow 4\Big\} + \Big\{1\leftrightarrow 2, 3\leftrightarrow 4, \langle\,\rangle\leftrightarrow[\,]\Big\} + \Big\{1\leftrightarrow 4, 2\leftrightarrow 3, \langle\,\rangle\leftrightarrow[\,]\Big\} \end{aligned} \tag{6.10}$$

$$\begin{aligned} c^{(2)}_{12\times34}(1^+,2^-,3^+,4^-) &= \left\{4\frac{\langle 2|(3+4)|1]}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]}\left[\frac{[2\,3]^2\,(s_{23}-s_{14})}{[1\,2]\,[3\,4]\,\langle 1|(3+4)|2]} + \frac{3}{2}\frac{[1\,3]\,[2\,3]}{[1\,2]\,[3\,4]}\right.\right. \\ &+ \left.\left.\langle 2\,4\rangle\,\frac{(\langle 3|(1+2)|3] - \langle 4|(1+2)|4])}{\Delta(1,2,3,4)}\left([2\,3] - \frac{\langle 1\,4\rangle\,p_{12}\cdot p_{34}}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\right)\right]\right\} \\ &+ \Big\{1\leftrightarrow 3, 2\leftrightarrow 4\Big\} + \Big\{1\leftrightarrow 2, 3\leftrightarrow 4, \langle\,\rangle\leftrightarrow[\,]\Big\} + \Big\{1\leftrightarrow 4, 2\leftrightarrow 3, \langle\,\rangle\leftrightarrow[\,]\Big\} \end{aligned} \tag{6.11}$$

where Δ is given by Eq. (B.2).

6.2.4 $\tilde{c}^{(0)}_{12\times 34}$

$$
\begin{aligned}
\tilde{c}^{(0)}_{12\times 34}(1^+,2^-,3^+,4^-) = \Big\{ & 2\,\frac{\langle 2\,3\rangle^3\,[3\,4]\,\langle 3|(1+2)|3]\,(\langle 3|(1+2)|3]\,[2\,3] - [1\,2]\,\langle 1\,4\rangle\,[3\,4])}{\langle 1\,2\rangle\,\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]^3} \\
& + 2\,\frac{\langle 2\,3\rangle^2\,[3\,4]\,\langle 4|(1+2)|3]\,(-2\,s_{23}-s_{24})}{\langle 1\,2\rangle\,\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]^2} \\
& + 2\,\frac{[1\,2]\,\langle 2\,3\rangle^2\,[3\,4]\,(2\,s_{12}\,(s_{23}-s_{14}-s_{34})+2\,s_{13}\,s_{23}+2\,s_{23}^2+s_{14}\,s_{34}-s_{23}\,s_{34}+2\,[1\,2]\,\langle 1\,3\rangle\,\langle 2\,4\rangle\,[3\,4])}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]^3} \\
& + 2\,\frac{[1\,4]^2\,(\langle 1\,4\rangle\,\langle 2\,4\rangle\,(2\,(s_{13}-s_{24})-3\,(s_{34}+s_{14})-4\,(s_{12}+s_{23}))-2\,\langle 1\,3\rangle\,[2\,3]\,\langle 2\,4\rangle^2+3\,[1\,3]\,\langle 1\,4\rangle^2\,\langle 2\,3\rangle)}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]^2} \\
& + \frac{s_{14}^2\,s_{12}\,(6\,s_{13}-2\,s_{14}+2\,s_{23}+2\,s_{24})-s_{14}^4+s_{14}^2\,s_{23}^2}{\langle 1|(3+4)|2]^2\,\langle 3|(1+2)|4]^2} - 8\,\frac{s_{12}\,s_{13}\,s_{14}\,s_{23}}{\langle 1|(3+4)|2]^2\,\langle 3|(1+2)|4]^2} \\
& + 4\,\frac{s_{14}\,s_{1234}\,\langle 2|(3+4)|1]\,\langle 4|(1+2)|3]}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]\,\Delta(1,2,3,4)} + 4\,\frac{\langle 1\,2\rangle\,[1\,3]\,\langle 2|(3+4)|1]\,\langle 4|(1+2)|3]\,\langle 3|(1+4)|2]}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]\,\Delta(1,2,3,4)} \\
& + \frac{\langle 1|(2+3)|4]\,\langle 2|(3+4)|1]\,\langle 3|(1+4)|2]\,\langle 4|(1+2)|3]\,(\Pi(4,3,2,1)\,\Pi(1,2,3,4)+\Delta(1,2,3,4))}{\langle 1|(3+4)|2]^2\,\langle 3|(1+2)|4]^2\,\Delta(1,2,3,4)} \\
& - 3\,\frac{s_{1234}\,\langle 2|(3+4)|1]\,\langle 4|(1+2)|3]\,\Pi(4,3,2,1)\,\Pi(1,2,3,4)\,(s_{13}+s_{14}+s_{23}+s_{24})}{2\,\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]\,\Delta(1,2,3,4)^2} \\
& + 5\,\frac{s_{1234}\,\langle 2|(3+4)|1]\,\langle 4|(1+2)|3]\,(s_{13}+s_{14}+s_{23}+s_{24})}{2\,\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]\,\Delta(1,2,3,4)} \\
& - 4\,\frac{\langle 2|(3+4)|1]\,\langle 4|(1+2)|3]}{\langle 1|(3+4)|2]\,\langle 3|(1+2)|4]}\Big\} \\
& + \Big\{1\leftrightarrow 3, 2\leftrightarrow 4\Big\} + \Big\{1\leftrightarrow 2, 3\leftrightarrow 4, \langle\,\rangle\leftrightarrow[\,]\Big\} + \Big\{1\leftrightarrow 4, 2\leftrightarrow 3, \langle\,\rangle\leftrightarrow[\,]\Big\}
\end{aligned}
\tag{6.12}
$$

where

$$
\Pi(i,j,k,l) = s_{ik} + s_{jk} - s_{il} - s_{jl} \tag{6.13}
$$

Summary

1. We have compact analytic results for Higgs+4-gluon
2. Stable in the corners of phase space
3. Advantageous to take various limits
4. Modern methods of simplifying amplitudes play crucial role

Thank you!