Two Loop QCD amplitudes for di-pseudo scalar Higgs Production

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Plan of Talk

- Effective Field Theory with pseudo-scalar
- operator renormalisation and mixing
- Universal IR structure
- Summary

Motivation

- Di-Higgs is an important channel that can probe the Higgs self coupling, which in turn describes the shape of the Higgs potential
- To go beyond NNLO in di-Higgs production channel, relevant two-loop amplitudes have recently been computed in the gluon fusion process
- BSM scenarios with pseudo-scalar Higgs boson exists which can lead to rich phenomenology
- Dominant production channel for single and double pseudo-scalar Higgs boson is gluon fusion, through top quark loop

Motivation

- Effective Field Theory approach can be used to study these processes in the large top quark mass limit
- Unlike the scalar Higgs boson , in EFT, the pseudo-scalar couples to gluons through two composite CP odd operators , gluonic and fermionic , dimension four operators
- The production cross section is comparable to the di-Higgs cross section. Hence, it is important to have predictions at the same level of precision for any phenomenological study

Effective Lagrangian

Interaction of a pseudo-scalar field $\Phi^A(x)$ with the gauge field $G^{a\mu\nu}$ and the fermion ψ:

$$\mathcal{L}^A_{eff} = \Phi^A(x)\left[-\frac{1}{8}C_G O_G(x) - \frac{1}{2}C_J O_J(x)\right] .$$

- $O_G(x)$ pseudo-scalar gluonic operator
- $O_J(x)$ pseudo-scalar fermionic operator
- C_G and C_J are the wilsons coefficients as a result of integrating out the heavy top quark degrees of freedom

Operators

- pseudo-scalar gluonic and fermionic operator

$$O_G(x) = G^{a\mu\nu}\tilde{G}^a_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma}, \quad O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma^5\psi\right) .$$

gluonic Field strength tensor

$$G^{a\mu\nu} = \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{abc}G^\mu_b G^\nu_c,$$

- wilsons coefficient

$$C_G\left(a_s\right) = -a_s 2^{\frac{5}{4}} G_F^{\frac{1}{2}} \cot\beta\,,$$
$$C_J\left(a_s\right) = -\left[a_s C_F\left(\frac{3}{2} - 3\ln\frac{\mu_R^2}{m_t^2}\right) + a_s^2 C_J^{(2)} + ...\right] C_G\,,$$

- G_F is the Fermi constant, $\cot\beta$ – the ratio of the two Higgs doublets' vacuum expectation values in a CP conserving two-Higgs doublet model
- Adler-Bardeen Theorem ensures , there are no QCD corrections to C_G beyond one loop , while C_J begins at two loop order

Tensor decomposition of amplitudes

- Production of a pair of pseudo-scalar Higgs boson A of mass m_A

$$g(p_1) + g(p_2) \to A(p_3) + A(p_4)\,,$$

- Amplitude can be decomposed in terms of two second rank Lorentz tensors $\mathcal{T}_i^{\mu\nu}$ $(i = 1, 2)$:

$$\mathcal{M}_{ab}^{\mu\nu}\epsilon_\mu(p_1)\epsilon_\nu(p_2) = \delta_{ab}\left(\mathcal{T}_1^{\mu\nu}\ \mathcal{M}_1 + \mathcal{T}_2^{\mu\nu}\ \mathcal{M}_2\right)\epsilon_\mu(p_1)\epsilon_\nu(p_2)\,,$$

- The second rank tensors are given by

$$\begin{aligned}
\mathcal{T}_1^{\mu\nu} &= g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1\cdot p_2}\,,\\
\mathcal{T}_2^{\mu\nu} &= g^{\mu\nu} + \frac{1}{p_1\cdot p_2\ p_T^2}\Big(m_A^2\ p_2^\mu p_1^\nu - 2p_1\cdot p_3\ p_2^\mu p_3^\nu - 2p_2\cdot p_3\ p_3^\mu p_1^\nu\\
&\quad +2p_1\cdot p_2\ p_3^\mu p_3^\nu\Big)\,,
\end{aligned}$$

with $p_T^2 = (tu - m_A^4)/s$ and the the Mandelstam variables

$$s = (p_1 + p_2)^2,\quad t = (p_1 - p_3)^2,\quad u = (p_2 - p_3)^2\,,$$

which satisfy $s + t + u = 2m_A^2$.

Projectors

- Scalar functions $\mathcal{M}_{1,2}$ can be obtained from $\mathcal{M}_{ab}^{\mu\nu}$, by using appropriate d-dimensional projectors

$$
\begin{aligned}
P_{1,ab}^{\mu\nu} &= \frac{\delta_{ab}}{N^2-1}\left(\frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_1^{\mu\nu} - \frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_2^{\mu\nu}\right), \\
P_{2,ab}^{\mu\nu} &= \frac{\delta_{ab}}{N^2-1}\left(-\frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_1^{\mu\nu} + \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_2^{\mu\nu}\right),
\end{aligned}
$$

- Using projector method one can compute amplitudes which are much easier to handle analytically and numerically
- Use these amplitudes in other pseudo-scalar processes

Feynman Diagrams

Two types of diagrams contribute:

- Type-1:
  - one AAgg effective vertex
  - one Agg vertex and one AAA vertex
- Type-2:
  - Two Agg effective vertex: 2 tree level, 35 one loop and 789 two loop diagrams
  - one Agg effective vertex and one $Aq\bar{q}$ effective vertex : 8 one loop diagrams

AAgg vertex proportional to C_G^2 of $\mathcal{O}(a_s^2)$
Agg vertex proportional to C_G of $\mathcal{O}(a_s)$
$Aq\bar{q}$ vertex proportional to C_J of $\mathcal{O}(a_s^2)$.

Type-I Form factor type

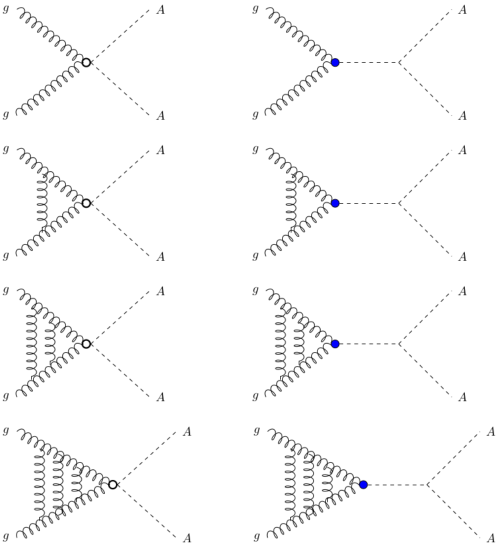


Figure: Type-1 tree level,one loop and two loop diagrams

Type-2 C_G^2

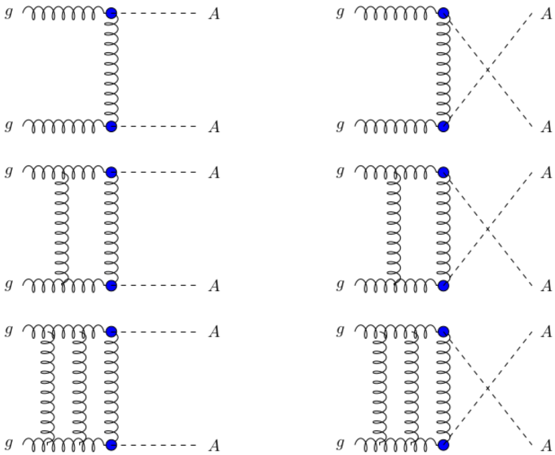
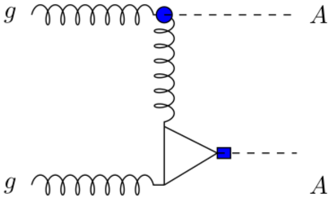


Figure: Type-2a tree,one loop,two loop diagrams



- Due to axial anomaly, pseudo-scalar operator for the gluonic field strength mixes with the diverences of the singlet axial vector current
- Effective $Aq\bar{q}$ vertex, proportional to C_J begins at $\mathcal{O}(a_s^2)$
- Effective Agg vertex, proportional to C_G begins at $\mathcal{O}(a_s)$
- Two loop contribution starts at $\mathcal{O}(a_s^4)$

$$\mathcal{M}_i = \mathcal{M}_i^{\mathrm{I}} + \mathcal{M}_i^{\mathrm{II}}, \qquad i = 1, 2$$

where $\mathcal{M}_i^{\mathrm{I}}$ and $\mathcal{M}_i^{\mathrm{II}}$ are Type-I and Type-II amplitudes contributing upto $\mathcal{O}(a_s^4)$

γ_5 : *dimensional Regularisation*

- Computing higher order corrections with chiral quantities involve inherently $d = 4$ dimensional objects like γ_5 and $\epsilon_{\mu\nu\rho\sigma}$ warrants a prescription in going to $4 + \epsilon$
- Prescription

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4} ,$$

  where Levi-Civita tensor is purely 4-dimensional, while the Lorentz indices on the γ^{μ_i} are in $d = 4 + \epsilon$ dimensions
- To maintain the anti-commuting nature of γ_5 with d-dimensional γ^{μ_i}, the symmetrical form of the axial current has to be used

$$J^5_\mu = \frac{1}{2}\bar{\psi}(\gamma_\mu\gamma_5 - \gamma_5\gamma_\mu)\psi ,$$

- The O_G and O_J operators now take the form

$$O_G(x) = \epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma}, \qquad O_J(x) = \frac{i}{3!}\epsilon_{\mu\nu_1\nu_2\nu_3}\partial^\mu\left(\bar{\psi}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\psi\right) .$$

- Contraction of two Levi-Civita tensors that result from either O_G operator or the mixing of O_G and O_J operators is given by

$$\epsilon_{\mu_1\nu_1\rho_1\sigma_1}\epsilon^{\mu_2\nu_2\rho_2\sigma_2} = \begin{vmatrix} \delta^{\mu_2}_{\mu_1} & \delta^{\nu_2}_{\mu_1} & \delta^{\rho_2}_{\mu_1} & \delta^{\sigma_2}_{\mu_1} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\nu_2}_{\nu_1} & \delta^{\rho_2}_{\nu_1} & \delta^{\sigma_2}_{\nu_1} \\ \delta^{\mu_2}_{\rho_1} & \delta^{\nu_2}_{\rho_1} & \delta^{\rho_2}_{\rho_1} & \delta^{\sigma_2}_{\rho_1} \\ \delta^{\mu_2}_{\sigma_1} & \delta^{\nu_2}_{\sigma_1} & \delta^{\rho_2}_{\sigma_1} & \delta^{\sigma_2}_{\sigma_1} \end{vmatrix},$$

- Lorentz indices in this determinant, could now be considered as d-dimensional and the consequence would be, addition of only the inessential $\mathcal{O}(\epsilon)$ terms to the renormalisated quantity
- A finite renormalisation of the axial vector current is required in order to fulfill the chiral Ward identities and the Adler-Bardeen theorem

UV renormalisation in $d = 4 + \epsilon$

- In $d = 4 + \epsilon$, the bare strong coupling constant $\hat{a}_s$ is related to its renormalized coupling a_s by

$$\hat{a}_s S_\epsilon = \left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon/2} Z_{a_s} a_s \,,$$

  with $S_\epsilon = \exp\left[(\gamma_E - \ln 4\pi)\epsilon/2\right]$ with $\gamma_E \approx 0.5772...$ the Euler-Mascheroni constant and μ is the scale introduced to keep the strong coupling constant dimensionless in $d = 4 + \epsilon$ space-time dimensions.

- The renormalisation constant Z_{a_s} is given by

$$Z_{a_s} = 1 + a_s \left[\frac{2}{\epsilon}\beta_0\right] + a_s^2 \left[\frac{4}{\epsilon^2}\beta_0^2 + \frac{1}{\epsilon}\beta_1\right] + a_s^3 \left[\frac{8}{\epsilon^3}\beta_0^3 + \frac{14}{3\epsilon^2}\beta_0\beta_1 + \frac{2}{3\epsilon}\beta_2\right] ,$$

Operator renormalisation

- The amplitudes require the renormalisation of vertices resulting from the composite operators O_G and O_J of the effective Lagrangian
- The renormalisation of O_J is related to the renormalisation of the singlet axial vector current J_5^μ which needs the standard overall UV renormalisation constant $Z^s_{\overline{MS}}$
- Even with $Z^s_{\overline{MS}}$ the 1-loop character of the operator relation of axial anomaly can not be maintained

$$[\partial_\mu J_5^\mu] = a_s \frac{n_f}{2} \left[G\tilde{G}\right] , \qquad \text{i.e.} \quad [O_J] = a_s \frac{n_f}{2} [O_G] ,$$

  which is true in Pauli-Villars, a 4-dimensional regularisation
- To preserve this in $4+\epsilon$ dimensions, the multiplicative finite renormalisation constant Z_5^s is required.

$$[O_J] = Z_5^s \; Z^s_{\overline{MS}} \, O_J ,$$

Operator Mixing

- bare pseudo-scalar gluon operator O_G mixes with fermionic operator O_J under the renormalisation through

$$[O_G] = Z_{GG}\, O_G + Z_{GJ}\, O_J\,,$$

- Mixing matrix

$$[O_i] = Z_{ij}\, O_j\,, \qquad \text{where} \quad i,j = \{G,J\}\,,$$

$$O \equiv \begin{pmatrix} O_G \\ O_J \end{pmatrix} \qquad \text{and} \qquad Z \equiv \begin{pmatrix} Z_{GG} & Z_{GJ} \\ Z_{JG} & Z_{JJ} \end{pmatrix}\,,$$

where $Z_{JG} = 0$ to all orders in perturbation theory and $Z_{JJ} \equiv Z_5^s Z_{\overline{MS}}^s$. The renormalisation constants required for above equation are available up to $\mathcal{O}(a_s^3)$

- The renormalisation constants are

$$Z_5^s = 1 + a_s \{-4C_F\} + a_s^2 \left\{22C_F^2 - \frac{107}{9}C_A C_F + \frac{31}{18}C_F n_f\right\}.$$

$$Z_{GG} = 1 + a_s \left[\frac{22}{3\epsilon}C_A - \frac{4}{3\epsilon}n_f\right] + a_s^2 \left[\frac{1}{\epsilon^2}\left\{\frac{484}{9}C_A^2 - \frac{176}{9}C_A n_f + \frac{16}{9}n_f^2\right\} + \frac{1}{\epsilon}\left\{\frac{34}{3}C_A^2 - \frac{10}{3}C_A n_f - 2C_F n_f\right\}\right],$$

$$Z_{GJ} = a_s \left[-\frac{24}{\epsilon}C_F\right] + a_s^2 \left[\frac{1}{\epsilon^2}\left\{-176C_A C_F + 32C_F n_f\right\} + \frac{1}{\epsilon}\left\{-\frac{284}{3}C_A C_F + 84C_F^2 + \frac{8}{3}C_F n_f\right\}\right],$$

$$Z_{JJ} = 1 + a_s [-4C_F] + a_s^2 \left[-\frac{44}{3\epsilon}C_A C_F - \frac{10}{3\epsilon}C_F n_f + 22C_F^2 - \frac{107}{9}C_A C_F + \frac{31}{18}C_F n_f\right].$$

$g + g \rightarrow A + A$ *to* $\mathcal{O}(a_s^4)$

- amplitude can be obtained *via* the insertion of two renormalised operators $[O_G]$ and $[O_J]$ for each A

$$\langle g|[O_G O_G]|g\rangle; \quad \langle g|[O_G O_J]|g\rangle \quad \text{and} \quad \langle g|[O_J O_J]|g\rangle\,.$$

- renormalisated operators are related to bare ones.The gluonic operator O_G couples to gluons at LO ($\mathcal{O}(a_s)$) and the fermionic operator O_J couples to quarks at LO ($\mathcal{O}(a_s^2)$)

$$\begin{aligned} [O_G O_G] &= Z_{GG}^2\, O_G O_G + 2 Z_{GG} Z_{GJ}\, O_G O_J + Z_{GJ}^2\, O_J O_J\,, \\ [O_G O_J] &= Z_{GG} Z_{JJ}\, O_G O_J + Z_{GJ} Z_{JJ}\, O_J O_J\,. \end{aligned}$$

- The O_G^2 starts to contribute at tree level at $\mathcal{O}(a_s^2)$, $O_G O_J$ begins to contribute at one-loop level and at $\mathcal{O}(a_s^4)$ while O_J^2 starts to contribute at $\mathcal{O}(a_s^6)$

$$\begin{aligned} \mathcal{M}^{\rm II}_{GG,g} &= Z_{GG}^2 \Big(\hat{\mathcal{M}}^{{\rm II}(0)}_{GG,g} + \hat{a}_s \hat{\mathcal{M}}^{{\rm II},(1)}_{GG,g} + \hat{a}_s^2 \hat{\mathcal{M}}^{{\rm II}(2)}_{GG,g} \Big) \\ &\quad + 2 Z_{GG} Z_{GJ} \Big(\hat{a}_s \hat{\mathcal{M}}^{{\rm II}(1)}_{GJ,g} + \hat{a}_s^2 \hat{\mathcal{M}}^{{\rm II}(2)}_{GJ,g} \Big) \\ &\quad + Z_{GJ}^2 \Big(\hat{a}_s \hat{\mathcal{M}}^{{\rm II}(1)}_{JJ,g} + \hat{a}_s^2 \hat{\mathcal{M}}^{{\rm II}(2)}_{JJ,g} \Big)\,, \end{aligned}$$

- Relation between UV finite amplitude and bare amplitudes

$$\mathcal{M}^{\text{II}}_{GG,g} = \mathcal{M}^{\text{II}(0)}_{GG,g} + a_s \mathcal{M}^{\text{II}(1)}_{GG,g} + a_s^2 \mathcal{M}^{\text{II}(2)}_{GG,g} + \mathcal{O}\left(a_s^3\right) \ .$$

$$\begin{aligned}
\mathcal{M}^{\text{II}(0)}_{GG,g} &= \hat{\mathcal{M}}^{\text{II}(0)}_{GG,g} \,, \\
\mathcal{M}^{\text{II}(1)}_{GG,g} &= \frac{1}{\mu_R^{\epsilon}} \hat{\mathcal{M}}^{\text{II}(1)}_{GG,g} + 2 Z^{(1)}_{GG} \hat{\mathcal{M}}^{\text{II}(0)}_{GG,g} \,, \\
\mathcal{M}^{\text{II}(2)}_{GG,g} &= \frac{1}{\mu_R^{2\epsilon}} \hat{\mathcal{M}}^{\text{II}(2)}_{GG,g} + \frac{1}{\mu_R^{\epsilon}} \left(\frac{2\beta_0}{\epsilon} \hat{\mathcal{M}}^{\text{II}(1)}_{GG,g} + 2 Z^{(1)}_{GJ} \hat{\mathcal{M}}^{\text{II}(1)}_{GJ,g} + 2 Z^{(1)}_{GG} \hat{\mathcal{M}}^{\text{II}(1}_{GG} \right. \\
&\quad + \left(2 Z^{(2)}_{GG} + (Z^{(1)}_{GG})^2 \right) \hat{\mathcal{M}}^{\text{II}(0)}_{GG,g} \,.
\end{aligned}$$

Calculation of amplitudes to $\mathcal{O}(a_s^4)$

- Feynman diagrams are generated using QGRAF
- Type-IIa : tree leve : 2 diagrams
  1 loop : 35 diagrams
  2 loop : 789 diagrams
- Type-IIb : 1 loop : 8 diagrams
- We use REDUZE2 package to identify the momentum shifts required to express each of these diagrams in terms of auxiliary topology
- IBP and LI identities via LiteRed package used to tensor reduction of integrals to Master Integrals.
- Master integrals are substituted to get the analytical result for the amplitude.

- **The IBP identities**
  - **In dimensional regularization, the integral of the total derivative with respect to any loop momenta vanishes, that is**

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \int \frac{d^d k_L}{(2\pi)^d} \frac{\partial}{\partial k_i} \cdot \left(v_j \frac{1}{\prod_l D_l^{n_l}} \right) = 0 \tag{1}$$

where $l = 1, 2, 3 \cdots$ and $n_l \in Z$, L is the number of loops and D_ls are propagators which depend on the loop and external momenta and also masses. The four vector v_j^μ can be both loop and external momenta.

  - **Differentiating the left hand side and expressing the scalar products of k_i and p_j linearly in terms of D_l's, one obtains the IBP identities given by**

$$\sum_i a_i J(b_{i,1} + n_1, ..., b_{i,N} + n_N) = 0 \tag{2}$$

where

$$J(\vec{m}) = J(m_1, \cdots, m_N) = \int \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_L}{(2\pi)^d} \frac{1}{\prod_l D_l^{m_l}} \tag{3}$$

with $b_{i,j} \in \{-1, 0, 1\}$ and a_i are polynomial in n_j.

- **The LI identities**
  - **Invariance of the loop integrals under Lorentz transformations of the external momenta leads to LI Identities**

$$p_i^{\mu} p_j^{\nu} \left(\sum_k p_{k[\nu} \frac{\partial}{\partial p_k^{\mu]}} \right) J(\vec{n}) = 0. \tag{4}$$

- **The Master Integrals**
  - **All Feynman Integrals appearing in all the Feynman diagrams can be expressed in terms of a small number of integral families.**
  - **Using IBPs, LIs and symmetry relations all Feynman integrals described by these integral families can be reduced to a simpler minimal subset of integrals, the master Integrals**
  - **Finally one has to evaluate the MIs only.**

- UV finite amplitudes contain only diverences of IR origin, appear as poles in ϵ
- amplitudes beyond LO: very rich universal structure in the IR

$$
\begin{aligned}
\mathcal{M}_i^{\mathrm{II},(0)} &= \mathcal{M}_i^{\mathrm{II},(0)}, \\
\mathcal{M}_i^{\mathrm{II},(1)} &= 2\mathbf{I}_g^{(1)}(\epsilon)\mathcal{M}_i^{\mathrm{II},(0)} + \mathcal{M}_i^{\mathrm{II},(1),\mathrm{fin}}, \\
\mathcal{M}_i^{\mathrm{II},(2)} &= 4\mathbf{I}_g^{(2)}(\epsilon)\mathcal{M}_i^{\mathrm{II},(0)} + 2\mathbf{I}_g^{(1)}(\epsilon)\mathcal{M}_i^{\mathrm{II},(1)} + \mathcal{M}_i^{B,(2),fin},
\end{aligned}
$$

where $\mathbf{I}_g^{(1)}(\epsilon)$, $\mathbf{I}_g^{(2)}(\epsilon)$ are the IR singularity operators given by

$$
\boldsymbol{I}_g^{(1)}(\epsilon) = -\frac{e^{-\frac{\epsilon}{2}\gamma_E}}{\Gamma\left(1+\frac{\epsilon}{2}\right)}\left(\frac{4C_A}{\epsilon^2} - \frac{\beta_0}{\epsilon}\right)\left(-\frac{s}{\mu_R^2}\right)^{\frac{\epsilon}{2}},
$$

$$
\boldsymbol{I}_g^{(2)}(\epsilon) = -\frac{1}{2}\boldsymbol{I}_g^{(1)}(\epsilon)\left[\boldsymbol{I}_g^{(1)}(\epsilon) - \frac{2\beta_0}{\epsilon}\right] + \frac{e^{\frac{\epsilon}{2}\gamma_E}\Gamma(1+\epsilon)}{\Gamma\left(1+\frac{\epsilon}{2}\right)}\left[-\frac{\beta_0}{\epsilon} + K\right]\boldsymbol{I}_g^{(1)}(2
$$

Universal Infrared Divergence Structure

- Two loop case a fully analytical comparision was possible for poles ϵ^{-i} with $i = 2 - 4$
- Due to the large file size for the ϵ^{-1} pole term comparision only at the numerical level
- The finite part is extracted

Summary

- Computed all the virtual amplitudes to $\mathcal{O}(a_s^4)$ for di pseudo-scalar production at the LHC
- The computation is done in the EFT where the top quark degrees of freedom is integrated out
- Larin prescription for γ_5 and Levi-Civita tensor in $d = 4 + \epsilon$ dimension
- UV renormalisation with operator mixing and finite renormalisation, with no counter term
- Agrees with the universal IR structure

Thank You