Infrared structure of $SU(N) \times U(1)$ gauge theory to three loops

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Introduction

- The study of Higgs production in $b\bar{b}$-annihilation channel for mixed QCD-QED gives an insight to the underlying IR structure in mixed gauge theory till NNLO.
- Till NNLO, there exists a nice relation between the color factors of QCD , charge factors of QED and charge-color factors of mixed QCD-QED.
- What happens at N3LO!

Introduction

- We extend our study of IR structure of QCD-QED to third order
- We begin with the renormalisation of the coupling constants when both the interactions are simultaneously present.

$$\frac{\hat{a}_c}{\left(\mu^2\right)^{\frac{\varepsilon}{2}}} S_\varepsilon = \frac{a_c(\mu_R^2)}{\left(\mu_R^2\right)^{\frac{\varepsilon}{2}}} \, Z_{a_c} \left(a_s(\mu_R^2), a_e(\mu_R^2), \varepsilon \right) \, .$$

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z_{a_c} = \frac{\varepsilon}{4} + \beta_{a_c}(a_s(\mu_R^2), a_e(\mu_R^2)) \, .$$

UV renormalization

- Solving RG equation we get the renormalization constants till third order

$$Z_{a_s} = 1 + a_s\left(\frac{2\beta_{00}}{\varepsilon}\right) + a_s a_e\left(\frac{\beta_{01}}{\varepsilon}\right) + a_s a_e^2\left(\frac{2\beta'_{00}\beta_{01}}{3\varepsilon^2} + \frac{2\beta_{02}}{3\varepsilon}\right) + a_s^2\left(\frac{4\beta_{00}^2}{\varepsilon^2} + \frac{\beta_{10}}{\varepsilon}\right) + a_s^2 a_e\left(\frac{4\beta_{00}\beta_{01}}{\varepsilon^2} + \frac{2\beta_{11}}{3\varepsilon}\right) + a_s^3\left(\frac{8\beta_{00}^3}{\varepsilon^3} + \frac{14\beta_{00}\beta_{10}}{3\varepsilon^2} + \frac{2\beta_{20}}{3\varepsilon}\right) + \cdots,$$

$$Z_{a_e} = 1 + a_e\left(\frac{2\beta'_{00}}{\varepsilon}\right) + a_e a_s\left(\frac{\beta'_{10}}{\varepsilon}\right) + a_e a_s^2\left(\frac{2\beta_{00}\beta'_{10}}{3\varepsilon^2} + \frac{2\beta'_{20}}{3\varepsilon}\right) + a_e^2\left(\frac{4\beta'^2_{00}}{\varepsilon^2} + \frac{\beta'_{01}}{\varepsilon}\right) + a_e^2 a_s\left(\frac{4\beta'_{00}\beta'_{10}}{\varepsilon^2} + \frac{2\beta'_{11}}{3\varepsilon}\right) + a_e^3\left(\frac{8\beta'^3_{00}}{\varepsilon^3} + \frac{14\beta'_{00}\beta'_{01}}{3\varepsilon^2} + \frac{2\beta'_{02}}{3\varepsilon}\right) + \cdots.$$

UV renormalization

- The renormalization for yukawa coupling,

$$
\begin{aligned}
Z_\lambda^b(a_s, a_e, \varepsilon) &= 1 + \cdots \\
&+ a_e^3 \Big\{ \frac{1}{\varepsilon^3}\Big(\frac{4}{3}\big(\gamma_b^{(0,1)}\big)^3 + 4\beta'_{00}\big(\gamma_b^{(0,1)}\big)^2 + \frac{8}{3}\beta'^2_{00}\big(\gamma_b^{(0,1)}\big)\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(2\gamma_b^{(0,1)}\gamma_b^{(0,2)} + \frac{4}{3}\beta'_{01}\gamma_b^{(0,1)} + \frac{4}{3}\beta'_{00}\gamma_b^{(0,2)}\Big) + \frac{1}{\varepsilon}\Big(\frac{2}{3}\gamma_b^{(0,3)}\Big)\Big\} \\
&+ a_s a_e^2 \Big\{ \frac{1}{\varepsilon^3}\Big(4\gamma_b^{(1,0)}\big(\gamma_b^{(0,1)}\big)^2 + 4\beta'_{00}\gamma_b^{(1,0)}\gamma_b^{(0,1)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(2\gamma_b^{(0,1)}\gamma_b^{(1,1)} + 2\gamma_b^{(1,0)}\gamma_b^{(0,2)} + \frac{4}{3}\beta'_{10}\gamma_b^{(0,1)} + \frac{2}{3}\beta'_{00}\gamma_b^{(1,1)}\Big) + \frac{1}{\varepsilon}\Big(\frac{2}{3}\gamma_b^{(1,2)}\Big)\Big\} \\
&+ a_s^2 a_e \Big\{ \frac{1}{\varepsilon^3}\Big(4\big(\gamma_b^{(1,0)}\big)^2\gamma_b^{(0,1)} + 4\beta_{00}\gamma_b^{(1,0)}\gamma_b^{(0,1)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(2\gamma_b^{(0,1)}\gamma_b^{(2,0)} + 2\gamma_b^{(1,0)}\gamma_b^{(1,1)} + \frac{4}{3}\beta_{01}\gamma_b^{(1,0)} + \frac{2}{3}\beta_{00}\gamma_b^{(1,1)}\Big) + \frac{1}{\varepsilon}\Big(\frac{2}{3}\gamma_b^{(2,1)}\Big)\Big\} \\
&+ \cdots .
\end{aligned}
$$

- Z_λ^b factorizes UV singularities. But QCD and QED singularities mix from two loop onwards

Form factor

- Aim is to calculate β_{ij} and $\gamma^{(i,j)}$ and anomalous dimensions.
- Follow the same methedology as NNLO - exploit the IR structure of FF using sudakov $K + G$ eq.
- Solving RGE for K and G, we get the structure of FF at third order as

$$\ln \hat{F}_I = \sum_{i,j} \hat{a}_s^i \hat{a}_e^j \Big(\frac{Q^2}{\mu^2}\Big)^{(i+j)\frac{\varepsilon}{2}} S_\varepsilon^{(i+j)} \hat{\mathcal{L}}_{F_I}^{(i,j)}(\varepsilon)\,,$$

Form factor

$$
\begin{aligned}
\hat{\mathcal{L}}_{F_I}^{(3,0)} &= \frac{1}{\varepsilon^4}\Big(-\frac{8}{9}\beta_{00}^2 A_I^{(1,0)}\Big) + \frac{1}{\varepsilon^3}\Big(\frac{8}{9}\beta_{00} A_I^{(2,0)} + \frac{2}{9}\beta_{10} A_I^{(1,0)} + \frac{4}{3}\beta_{00}^2 G_I^{(1,0)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(-\frac{2}{9} A_I^{(3,0)} - \frac{1}{3}\beta_{10} G_I^{(1,0)} - \frac{4}{3}\beta_{00} G_I^{(2,0)}\Big) + \frac{1}{3\varepsilon}\Big(G_I^{(3,0)}\Big)\,, \\
\hat{\mathcal{L}}_{F_I}^{(1,2)} &= \frac{1}{\varepsilon^3}\Big(\frac{4}{9}\beta'_{00} A_I^{(1,1)} + \frac{2}{9}\beta'_{10} A_I^{(0,1)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(-\frac{2}{9} A_I^{(1,2)} - \frac{1}{3}\beta'_{10} G_I^{(0,1)} - \frac{2}{3}\beta'_{00} G_I^{(1,1)}\Big) + \frac{1}{3\varepsilon}\Big(G_I^{(1,2)}\Big) \\
\hat{\mathcal{L}}_{F_I}^{(2,1)} &= \frac{1}{\varepsilon^3}\Big(\frac{4}{9}\beta_{00} A_I^{(1,1)} + \frac{2}{9}\beta_{01} A_I^{(1,0)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(-\frac{2}{9} A_I^{(2,1)} - \frac{1}{3}\beta_{01} G_I^{(1,0)} - \frac{2}{3}\beta_{00} G_I^{(1,1)}\Big) + \frac{1}{3\varepsilon}\Big(G_I^{(2,1)}\Big) \\
\hat{\mathcal{L}}_{F_I}^{(0,3)} &= \frac{1}{\varepsilon^4}\Big(-\frac{8}{9}\beta'^2_{00} A_I^{(0,1)}\Big) + \frac{1}{\varepsilon^3}\Big(\frac{8}{9}\beta'_{00} A_I^{(0,2)} + \frac{2}{9}\beta'_{01} A_I^{(0,1)} + \frac{4}{3}\beta'^2_{00} G_I^{(0,1)}\Big) \\
&\quad + \frac{1}{\varepsilon^2}\Big(-\frac{2}{9} A_I^{(0,3)} - \frac{1}{3}\beta'_{01} G_I^{(0,1)} - \frac{4}{3}\beta'_{00} G_I^{(0,2)}\Big) + \frac{1}{3\varepsilon}\Big(G_I^{(0,3)}\Big)\,.
\end{aligned}
$$

- From explicit calculation and comparing the scalar and vector FF, we get $A^{(i,j)}$, $\gamma^{(i,j)}$, β_{ij}, β'_{ij} and $2B^{(i,j)} + f^{(i,j)}$ till third order.

$$G_I^{(i,j)}(\varepsilon) = 2(B_I^{(i,j)} - \gamma_I^{(i,j)}) + f_I^{(i,j)} + \sum_{k=0} \varepsilon^k g_{I,ij}^k \,.$$

- How to extract the $f^{(i,j)}$ and $B^{(i,j)}$ from virtual corrections!
- Can we use the abelizanization procedure which we observed for NNLO?

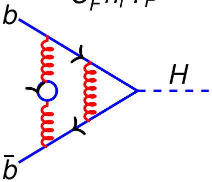
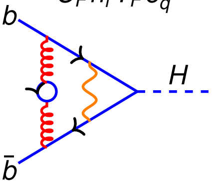
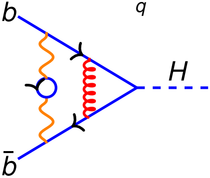
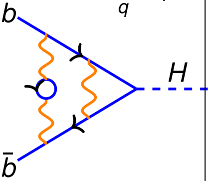
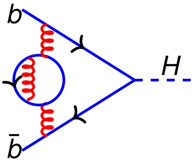
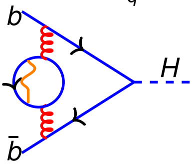
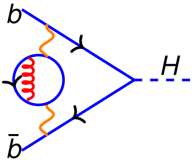
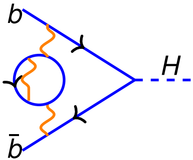
Form factor

- For FF, the abelianization rule are a s follows,

QCD (a_s^3)	QCD-QED ($a_s^2 a_e$)	QCD-QED ($a_s a_e^2$)	QED (a_e^3)
C_F^3	$3C_F^2 e_l^2$	$3C_F e_l^4$	e_l^6
$C_A C_F^2$	$C_A C_F e_l^2$	0	0
$C_F^2 n_f T_F$	$F_1\ C_F n_f T_F e_l^2 +$ $F_2\ C_F T_F \left(\sum_q e_q^2 + \sum_l e_l^2 \right)$	$C_F e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2 \right)$	$F_1\ e_l^4 \left(N \sum_q e_q^2 + \sum_l e_l^2 \right) +$ $F_2\ e_l^2 \left(N \sum_q e_q^4 + \sum_l e_l^4 \right)$
$C_F C_A^2$	0	0	0
$C_F C_a n_f T_F$	0	0	0
$C_F n_f^2 T_F^2$	0	0	$e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2 \right)^2$

$$F_1 = \frac{1}{\varepsilon^3}\left(-\frac{32}{9}\right) + \frac{1}{\varepsilon^2}\left(\frac{656}{27} - \frac{128}{9}\zeta_3\right) + \frac{1}{\varepsilon}\left(-\frac{8012}{81} - 4\zeta_2 + \frac{64}{15}\zeta_2^2 + \frac{1472}{27}\zeta_3\right) + \left(\frac{76781}{243} + \frac{82}{3}\zeta_2 - \frac{736}{45}\zeta_2^2 - \frac{14180}{81}\zeta_3 - 16\zeta_2\zeta_3 - \frac{224}{9}\zeta_5\right)$$

$$F_2 = \frac{1}{\varepsilon^5}\left(\frac{128}{3}\right) + \frac{1}{\varepsilon^4}\left(-\frac{1184}{9}\right) + \frac{1}{\varepsilon^3}\left(\frac{8816}{27} + \frac{16}{3}\zeta_2\right) + \frac{1}{\varepsilon^2}\left(-\frac{53960}{81} - \frac{532}{9}\zeta_2 + 144\zeta_3\right) + \frac{1}{\varepsilon}\left(\frac{301766}{243} + \frac{5806}{27}\zeta_2 - \frac{1877}{45}\zeta_2^2 - \frac{1300}{3}\zeta_3\right) + \left(-\frac{3192859}{1458} - \frac{47449}{81}\zeta_2 + \frac{49577}{540}\zeta_2^2 + \frac{12910}{9}\zeta_3 - \frac{542}{9}\zeta_2\zeta_3 + \frac{1676}{45}\zeta_5\right)$$

QCD (a_s^2)	QCD-QED ($a_s^2 a_e$)	QCD-QED ($a_s a_e^2$)	QED (a_e^2)
$C_F^2 n_f T_F$ 	$C_F n_f T_F e_q^2$ 	$C_F e_b^2 (N \sum_q e_q^2)$ 	$e_b^4 (N \sum_q e_q^2)$ 
$C_F^2 n_f T_F$ 	$C_F T_F (\sum_q e_q^2)$ 	$C_F e_b^2 (N \sum_q e_q^2)$ 	$e_b^2 (N \sum_q e_q^4)$ 

Cusp anomalous dim.

- From explicit FF results, we obtain $A^{(i,j)}$,

$$A_I^{(3,0)} = C_A^2 C_F \Big(\frac{490}{3} - \frac{1072}{9}\zeta_2 + \frac{176}{5}\zeta_2^2 + \frac{88}{3}\zeta_3\Big) + C_A C_F n_f T_F \Big(-\frac{1672}{27} + \frac{320}{9}\zeta_2 - \frac{224}{3}\zeta_3\Big) + C_F^2 n_f T_F \Big(-\frac{220}{3} + 64\zeta_3\Big) + C_F n_f^2 T_F^2 \Big(-\frac{64}{27}\Big),$$

$$A_I^{(1,2)} = C_F e_I^2 \Big(N \sum_q e_q^2 + \sum_l e_l^2\Big)\Big(-\frac{220}{3} + 64\zeta_3\Big)$$

$$A_I^{(2,1)} = C_F T_F \Big(\sum_q e_q^2 + \sum_l e_l^2\Big)\Big(-\frac{220}{3} + 64\zeta_3\Big),$$

$$A_I^{(0,3)} = e_I^2 \Big(N \sum_q e_q^4 + \sum_l e_l^4\Big)\Big(-\frac{220}{3} + 64\zeta_3\Big) + e_I^2 \Big(N \sum_q e_q^2 + \sum_l e_l^2\Big)^2\Big(-\frac{64}{27}\Big).$$

- $A^{(i,j)}$ using abelianization rules of FF: Apply the rules to $A^{(3,0)}$.
- Both results doesn't match!

Cusp anomalous dim.

- Abelianization rule for $A^{(i,j)}$ is different than FF for $C_F^2 n_f T_F$.
- There is $1-1$ map bw QCD, QCD-QED and QED for $A^{(i,j)}$!

QCD (a_s^3)	QCD-QED ($a_s^2 a_e$)	QCD-QED ($a_s a_e^2$)	QED (a_e^3)
C_F^3	$3C_F^2 e_l^2$	$3C_F e_l^4$	e_l^6
$C_A C_F^2$	$C_A C_F e_l^2$	0	0
$C_F^2 n_f T_F$	$C_F T_F \left(\sum_q e_q^2 + \sum_l e_l^2\right)$	$C_F e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2\right)$	$e_l^2 \left(N \sum_q e_q^4 + \sum_l e_l^4\right)$
$C_F C_A^2$	0	0	0
$C_F C_a n_f T_F$	0	0	0
$C_F n_f^2 T_F^2$	0	0	$e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2\right)^2$

UV anomalous dim.

$$\gamma_b^{(3,0)} = C_A^2 C_F\Big(\frac{11413}{108}\Big) + C_A C_F^2\Big(-\frac{129}{4}\Big) + C_A C_F n_f T_F\Big(-\frac{556}{27} + 48\zeta_3\Big) + C_F^3\Big(\frac{129}{2}\Big)$$
$$+ C_F n_f^2 T_F^2\Big(-\frac{140}{27}\Big) + C_F^2 n_f T_F\Big(-46 + 48\zeta_3\Big),$$

$$\gamma_b^{(1,2)} = 3C_F e_b^4\Big(\frac{129}{2}\Big) + C_F e_b^2\Big(N\sum_q e_q^2 + \sum_l e_l^2\Big)\Big(-46 + 48\zeta_3\Big),$$

$$\gamma_b^{(2,1)} = C_A C_F e_b^2\Big(-\frac{129}{4}\Big) + 3C_F^2 e_q^2\Big(\frac{129}{2}\Big) + C_F n_f T_F e_q^2\Big(-1\Big)$$
$$+ C_F T_F\Big(\sum_q e_q^2 + \sum_l e_l^2\Big)\Big(-45 + 48\zeta_3\Big),$$

$$\gamma_b^{(0,3)} = e_b^6\Big(\frac{129}{2}\Big) + e_b^2\Big(N\sum_q e_q^2 + \sum_l e_l^2\Big)^2\Big(-\frac{140}{27}\Big) + e_b^4\Big(N\sum_q e_q^2 + \sum_l e_l^2\Big)\Big(-1\Big)$$
$$+ e_b^2\Big(N\sum_q e_q^4 + \sum_l e_l^4\Big)\Big(-45 + 48\zeta_3\Big).$$

- The abelianization rules again changes for $\gamma(i,j)$ for $C_F^2 n_f T_F$!

UV anomalous dimension

- The abelianization rule for $\gamma_{(i,j)}$

QCD (a_s^3)	QCD-QED ($a_s^2 a_e$)	QCD-QED ($a_s a_e^2$)	QED (a_e^3)
C_F^3	$3C_F^2 e_l^2$	$3C_F e_l^4$	e_l^6
$C_A C_F^2$	$C_A C_F e_l^2$	0	0
$C_F^2 n_f T_F$	$C_1\ C_F n_f T_F e_l^2 + C_2\ C_F T_F \left(\sum_q e_q^2 + \sum_l e_l^2\right)$	$C_F e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2\right)$	$C_1\ e_l^4 \left(N \sum_q e_q^2 + \sum_l e_l^2\right) + C_2\ e_l^2 \left(N \sum_q e_q^4 + \sum_l e_l^4\right)$
$C_F C_A^2$	0	0	0
$C_F C_a n_f T_F$	0	0	0
$C_F n_f^2 T_F^2$	0	0	$e_l^2 \left(N \sum_q e_q^2 + \sum_l e_l^2\right)^2$

Soft and collinear anomalous dim.

- How we proceed to get $f^{(i,j)}$ and $B^{(i,j)}$?
- Claim : $f^{(i,j)}$ satisfies the same abelianization rule as $A^{(i,j)}$

$$
\begin{aligned}
f_I^{(3,0)} &= C_A^2 C_f \Big(\frac{136781}{729} - \frac{12650}{81}\zeta_2 + \frac{352}{5}\zeta_2^2 - \frac{1316}{3}\zeta 3_3 + \frac{176}{3}\zeta_2\zeta_3 + 192\zeta_5 \Big) \\
&+ C_A C_F n_f T_F \Big(-\frac{23684}{729} + \frac{5656}{81}\zeta_2 - \frac{192}{5}\zeta_2^2 + \frac{1456}{27}\zeta_3 \Big) + C_F n_f^2 T_F^2 \Big(\\
&- \frac{8320}{729} - \frac{160}{27}\zeta_2 + \frac{448}{27}\zeta_3 \Big) + C_F^2 n_f T_F \Big(-\frac{3422}{27} + 8\zeta_2 + \frac{64}{5}\zeta_2^2 + \frac{608}{9}\zeta_3 \Big) , \\
f_I^{(1,2)} &= C_f e_q^2 \Big(N \sum_q e_q^2 + \sum_l e_l^2 \Big) \Big(-\frac{3422}{27} + 8\zeta_2 + \frac{64}{5}\zeta_2^2 + \frac{608}{9}\zeta_3 \Big) , \\
f_I^{(2,1)} &= C_F T_F \Big(\sum_q e_q^2 + \sum_l e_l^2 \Big) \Big(-\frac{3422}{27} + 8\zeta_2 + \frac{64}{5}\zeta_2^2 + \frac{609}{9}\zeta_3 \Big) , \\
f_I^{(0,3)} &= e_q^2 \Big(N \sum_q e_q^2 + \sum_l e_l^2 \Big)^2 \Big(-\frac{8320}{729} - \frac{160}{27}\zeta_2 + \frac{448}{27}\zeta_3 \Big) \\
&+ e_q^2 \Big(N \sum_q e_q^4 + \sum_l e_l^4 \Big) \Big(-\frac{3422}{27} + 8\zeta_2 + \frac{64}{5}\zeta_2^2 + \frac{608}{9}\zeta_3 \Big) .
\end{aligned}
$$

$$
\begin{aligned}
B_l^{(3,0)} &= C_A^2 C_F\Big(- \frac{1657}{36} + \frac{4496}{27}\zeta_2 - 2\zeta_2^2 - \frac{1552}{9}\zeta_3 + 40\zeta_5\Big) \\
&+ C_A C_F n_f T_F\Big(40 - \frac{2672}{27}\zeta_2 + \frac{8}{5}\zeta_2^2 + \frac{400}{9}\zeta_3\Big) + C_f n_f^2 T_F^2\Big(- \frac{68}{9} + \frac{320}{27}\zeta_2 - \frac{64}{9}\zeta_3\Big) \\
&+ C_A C_f^2\Big(\frac{151}{4} - \frac{410}{3}\zeta_2 - \frac{988}{15}\zeta_2^2 + \frac{844}{3}\zeta_3 + 16\zeta_2\zeta_3 + 120\zeta_5\Big) \\
&+ C_f^3\Big(\frac{29}{2} + 18\zeta_2 + \frac{288}{5}\zeta_2^2 + 68\zeta_3 - 32\zeta_2\zeta_3 - 240\zeta_5\Big) \\
&+ C_F^2 n_f T_F\Big(- 46 + \frac{40}{3}\zeta_2 + \frac{464}{15}\zeta_2^2 - \frac{272}{3}\zeta_3\Big)\,, \\
B_l^{(1,2)} &= 3C_F e_l^4\Big(\frac{29}{2} + 18\zeta_2 + \frac{288}{5}\zeta_2^2 + 68\zeta_3 - 32\zeta_2\zeta_3 - 240\zeta_5\Big) + C_F e_l^2\Big(\sum_q e_q^2 + \sum_l e_l^2\Big) \\
&\quad \Big(- 46 + \frac{40}{3}\zeta_2 + \frac{464}{15}\zeta_2^2 - \frac{272}{3}\zeta_3\Big)\,, \\
B_l^{(2,1)} &= C_F C_A e_l^2\Big(\frac{151}{4} - \frac{410}{3}\zeta_2 - \frac{988}{15}\zeta_2^2 + \frac{844}{3}\zeta_3 + 16\zeta_2\zeta_3 + 120\zeta_5\Big) \\
&+ 3C_F^2 e_l^2\Big(\frac{29}{2} + 18\zeta_2 + \frac{288}{5}\zeta_2^2 + 68\zeta_3 - 32\zeta_2\zeta_3 - 240\zeta_5\Big) + C_F T_F\Big(\sum_q e_q^2 + \sum_l e_l^2\Big) \\
&\Big(- 37 - 16\zeta_2 + 48\zeta_3\Big) + C_F n_f T_F e_l^2\Big(- 9 + \frac{88}{3}\zeta_2 + \frac{464}{15}\zeta_2^2 - \frac{416}{3}\zeta_3\Big)\,, \\
B_l^{(0,3)} &= e_l^2\Big(N\sum_q e_q^2 + \sum_l e_l^2\Big)^2\Big(- \frac{68}{9} + \frac{320}{27}\zeta_2 - \frac{64}{9}\zeta_3\Big) + e_l^6\Big(\frac{29}{2} + 18\zeta_2 + \frac{288}{5}\zeta_2^2 + 68\zeta_3 \\
&- 32\zeta_2\zeta_3 - 240\zeta_5\Big) + e_l^2\Big(N\sum_q e_q^4 + \sum_l e_l^4\Big)\Big(- 37 - 16\zeta_2 + 48\zeta_3\Big) \\
&+ e_l^4\Big(N\sum_q e_q^2 + \sum_l e_l^2\Big)\Big(- 9 + \frac{88}{3}\zeta_2 + \frac{464}{15}\zeta_2^2 - \frac{416}{3}\zeta_3\Big)\,.
\end{aligned}
$$

Conclusion

- We studied the IR structure of mixed gauge theory at third order and obtain the universal IR anomalous dimensions.
- From the explicit computations, we see that there is no simple abelianization rule, as in NNLO, which connects QCD, QCD-QED and QED at third order- there is no fermion loop induced diagrams at second order for mixed QCD-QED.
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Thank You

$$\beta_{00} = \frac{11}{3} C_A - \frac{4}{3} n_f T_F \,, \quad \beta'_{00} = -\frac{4}{3}\Big(N \sum_q e_q^2 + \sum_l e_l^2 \Big) \,,$$

$$\beta_{01} = -2 \left(\sum_q e_q^2 + \sum_l e_l^2 \right) \,, \quad \beta'_{01} = -4 \left(N \sum_q e_q^4 + \sum_l e_l^4 \right) \,,$$

$$\beta_{10} = \left(\frac{34}{3} C_A^2 - \frac{20}{3} C_A n_f T_F - 4 C_F n_f T_F \right) \,, \qquad \beta'_{10} = -4 C_F \left(N \sum_q e_q^2 + \sum_l e_l^2 \right)$$