NNLO QCD$\oplus$QED corrections to Higgs production in bottom quark annihilation

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PLAN OF THE PRESENTATION

- Motivation.
- Theorectical Framework.
- Methodology.
- UV and IR structures in QED and QCDQED.
- Abelianization procedure.

Motivation

- The state-of-the-art for bottom quark annihilation, has reached upto N^3LO as well as N^3LO+N^3LL accuracy.
  [Gehrmann(2014),Mistlberger(2019),Ravindran(2019)]
- We expect the QED corrections to be comparable to the fixed and resummed results solely from third order in perturbative QCD.
- We also make an attempt to study the very approach of Abelianization, through explicit computation of pure QED and QCD$\times$QED corrections to inclusive production of the Higgs boson in bottom quark annihilation up to NNLO level.
  [D. de Florian(2018)]

Theorectical Framework

- The inclusive production of a colorless state in hadronic collision is given by

$$\sigma(S,q^2) = \sigma_0(\mu_R^2) \sum_{cd} \int dx_1 dx_2 f_c(x_1,\mu_F^2) f_d(x_2,\mu_F^2) \times \Delta_{cd}(s,q^2,\mu_F^2,\mu_R^2)\,,$$

  where σ_0 is the Born cross section and $f_a(x_i,\mu_F^2)$ are the parton distribution functions (PDFs) for $a = q, \overline{q}, g$.

- Mass factorised partonic cross-section Δ_{cd} can be expanded as

$$\Delta_{cd}(z,q^2) = \sum_{i,j=0}^{\infty} a_s^i a_e^j \Delta_{cd}^{(i,j)}(z,q^2)\,,$$

UV and IR structures in QED and QCDQED

- $Z_{a_c}, c = s, e$ for the QCD and QED coupling constants

$$\frac{\hat{a}_c}{(\mu^2)^{\frac{\varepsilon}{2}}} S_\varepsilon = \frac{a_c(\mu_R^2)}{(\mu_R^2)^{\frac{\varepsilon}{2}}} \; Z_{a_c} \left(a_s(\mu_R^2), a_e(\mu_R^2), \varepsilon \right) \; ,$$

- The renormalization constant Z_{a_c} up to two-loops are given by

$$Z_{a_s} = 1 + a_s \left(\frac{2\beta_{00}}{\varepsilon} \right) + a_s a_e \left(\frac{\beta_{01}}{\varepsilon} \right) + a_s^2 \left(\frac{4\beta_{00}^2}{\varepsilon^2} + \frac{\beta_{10}}{\varepsilon} \right)$$

$$Z_{a_e} = 1 + a_e \left(\frac{2\beta'_{00}}{\varepsilon} \right) + a_e a_s \left(\frac{\beta'_{10}}{\varepsilon} \right) + a_e^2 \left(\frac{4\beta'^2_{00}}{\varepsilon^2} + \frac{\beta'_{01}}{\varepsilon} \right)$$

- The renormalization constant $Z_\lambda^b(a_s, a_e)$ is given as:

$$Z_{\lambda_b}(a_s, a_e, \varepsilon) = 1 + \cdots + a_e^2 \Big\{ \frac{1}{\varepsilon^2} \Big(2 \big(\gamma_b^{(0,1)} \big)^2 + 2\beta'_{00} \gamma_b^{(0,1)} \Big) + \frac{1}{\varepsilon} \gamma_b^{(0,2)} \Big\} + a_s a_e \Big\{ \frac{1}{\varepsilon^2} \big(4\gamma_b^{(1,0)} \gamma_b^{(0,1)} \big) + \frac{1}{\varepsilon} \big(\gamma_b^{(1,1)} \big) \Big\}$$

Form Factor

- The bare form factors $\hat{F}_I(\hat{a}_s, \hat{a}_e, Q^2, \mu^2)$, $I = q, b$ takes the following form:

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}_I = \frac{1}{2}\left[K_I\left(\{\hat{a}_c\}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G_I\left(\{\hat{a}_c\}, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right],$$

[Sudakov(1956),Sen(1981),Magnea(1990)]

- where the RG invariance of $\hat{F}_I$ implies

$$\mu_R^2 \frac{d}{d\mu_R^2} K_I\left(\{\hat{a}_c\}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) = -A_I(\{a_c(\mu_R^2)\}),$$

$$\mu_R^2 \frac{d}{d\mu_R^2} G_I\left(\{\hat{a}_c\}, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) = A_I(\{a_c(\mu_R^2)\}),$$

where A_I are the cusp anomalous dimensions.

- $$\ln \hat{F}_I = \sum_{i,j} \hat{a}_s^i \hat{a}_e^j \left(\frac{Q^2}{\mu^2}\right)^{(i+j)\frac{\varepsilon}{2}} S_\varepsilon^{(i+j)} \hat{\mathcal{L}}_{F_I}^{(i,j)}(\varepsilon),$$

Form Factor

- The analytic structure of Form Factor is matched against the computed ones:

$$\hat{\mathcal{L}}_{F_I}^{(1,0)} = \frac{1}{\varepsilon^2}\Big(- 2A_I^{(1,0)}\Big) + \frac{1}{\varepsilon}\Big(G_I^{(1,0)}(\varepsilon)\Big)\,.$$

$$\hat{\mathcal{L}}_{F_I}^{(0,1)} = \frac{1}{\varepsilon^2}\Big(- 2A_I^{(0,1)}\Big) + \frac{1}{\varepsilon}\Big(G_I^{(0,1)}(\varepsilon)\Big)\,.$$

$$\hat{\mathcal{L}}_{F_I}^{(2,0)} = \frac{1}{\varepsilon^3}\Big(\beta_{00} A_I^{(1,0)}\Big) + \frac{1}{\varepsilon^2}\Big(- \frac{1}{2}A_I^{(2,0)} - \beta_{00}\, G_I^{(1,0)}(\varepsilon)\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(2,0)}(\varepsilon)\Big)\,.$$

$$\hat{\mathcal{L}}_{F_I}^{(0,2)} = \frac{1}{\varepsilon^3}\Big(\beta'_{00} A_I^{(0,1)}\Big) + \frac{1}{\varepsilon^2}\Big(- \frac{1}{2}A_I^{(0,2)} - \beta'_{00}\, G_I^{(0,1)}(\varepsilon)\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(0,2)}(\varepsilon)\Big)\,.$$

$$\hat{\mathcal{L}}_{F_I}^{(1,1)} = \frac{1}{\varepsilon^2}\Big(- \frac{1}{2}A_I^{(1,1)}\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(1,1)}(\varepsilon)\Big)\,.$$

- The coefficient $G_I^{(i,j)}$ inspired from the Pure QCD $G_I^{(i,0)}$ takes the following form:

$$G_I^{(i,j)}(\varepsilon) = 2(B_I^{(i,j)} - \gamma_I^{(i,j)}) + f_I^{(i,j)} + \sum_{k=0} \varepsilon^k g_{I,ij}^k$$

[Smith,Van Neervan,Ravindran]

Form Factor

- We find $A_I^{(i,j)}$ up to two loops as

$$A_I^{(0,1)} = 4e_I^2 \,. \qquad \gamma_b^{(0,1)} = 3e_b^2 \,,$$

$$A_I^{(0,2)} = 8e_I^2\Big(N\sum_{k=1}^{n_f} e_k^2\Big)\Big(-\frac{10}{9}\Big) \,. \quad \gamma_b^{(0,2)} = \frac{3}{2}e_b^4 - \frac{10}{3}e_b^2\Big(N\sum_{k\in Q} e_k^2\Big) \,.$$

$$A_I^{(1,1)} = 0 \,. \qquad \gamma_b^{(1,1)} = 3C_F e_b^2$$

[Ajjath,Mukherjee,Ravindran(2019)]

- The soft distribution functions denoted by Φ_I, in terms of cusp (A_I) and soft anomalous dimensions f_I are obtained from,

$$\mathcal{C}\exp\Big(2\Phi_I(z)\Big) = \frac{\hat{\sigma}_{I\bar{I}}(z)}{Z_I^2\big|\hat{F}_I\big|^2}$$

Soft distribution Function

- Due to the similar IR structure, Φ_I satisfies Sukakov $K+G$ equation, analogous to the form factor

$$q^2 \frac{d}{dq^2}\Phi_I = \frac{1}{2}\Big[\overline{K}_I\Big(\{\hat{a}_c\}, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\Big) + \overline{G}_I\Big(\{\hat{a}_c\}, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\Big)\Big],$$

- The same anomalous dimensions ($A_I^{(i,j)}$) govern the evolution of both $\overline{K}_I$ and $\overline{G}_I$.
- The solution to the above eq. is found to be :

$$\Phi_I(\{\hat{a}_c\}, q^2, \mu^2, \varepsilon, z) = \sum_{i,j} \hat{a}_s^i \hat{a}_e^j \Big(\frac{q^2(1-z)^2}{\mu^2}\Big)^{(i+j)\frac{\varepsilon}{2}} \times S_\varepsilon^{(i+j)}\Big(\frac{(i+j)\varepsilon}{1-z}\Big)\hat{\phi}_I^{(i,j)}(\varepsilon)$$

Here $\hat{\phi}_I^{(i,j)}$ has the same structure as $\hat{\mathcal{L}}_{F_I}^{(i,j)}$ except for $G_I^{(i,j)}$ replaced by

$$\overline{G}_I^{(i,j)}(\varepsilon) = -f_I^{(i,j)} + \sum_{k=0} \varepsilon^k \overline{\mathcal{G}}_{I,ij}^{(k)}$$

Abelianization procedure

- With the explicit computations at hand we deduce the following set of rules:

  Rule 1 : *quark-quark initiated cases*

QCD	QCD×QED	QED
C_F^2	$2C_F e_b^2$	e_b^4
$C_F C_A$	0	0
$C_F n_f T_F$	0	$e_b^2 \left(N \sum_q e_q^2\right)$
$C_F T_F$	0	$N e_b^2 e_q^{2*}$

- These rules are found to be true at the mass-factorised partonic cross-section level also.
- The anamolous dimensions found earlier are also found to obey the transformation rules.
- Whether there exists such rules at the three loop level remains to be answered until the next talk.