Gluon Jet Function at three loops in QCD

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Factorisation

* Differential cross section probing jet invariant mass takes the factorised form

$$\frac{d\sigma}{d\tau} = H(Q) \times [B_a \otimes B_b \otimes J_{i_1} \otimes ... \otimes J_{i_N} \otimes S](\tau)$$

H(Q) - Hard function

B_a - Beam function

S -Soft function

J_{i_j}- Jet function

* This factorisation is at LO in τ/Q and to all orders in α_s

$$A(\tau) \otimes B(\tau) \equiv \int d\tau' A(\tau - \tau')B(\tau')$$

Status

* quark jet functions are known to three loops.

[Bauer et al. (0312109), Bosch et al. (0402094), Becher et al. (0603140)]

[Bruser et al. (1804.09722)]

* gluon jet function is also known up to two-loop order.

[Becher et al. (0911.0681, 1008.1936)]

* All these results have been obtained through direct computation from formal SCET definition.

DIS in QCD improved parton model

Deep Inelastic Scattering:

$$\sigma^I(x,Q^2) = \sigma_B^I(\mu_R^2) \sum_{a=q,\bar{q},g} \int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z},\mu_F^2\right) \Delta_a^I(a_s,z,Q^2,\mu_R^2,\mu_F^2)$$

Scaling variables

hadronic $x = \frac{Q^2}{2P.q}$ partonic $z = \frac{Q^2}{2p.q}$

$$\Delta_a^I(a_s,z,Q^2,\mu_F^2,\mu_R^2) = \Delta_a^{I,\text{hard}}(a_s,z,Q^2,\mu_F^2,\mu_R^2) + \Delta^{I,\text{SV}}(a_s,z,Q^2,\mu_F^2,\mu_R^2)$$

Distributions

$\delta(1-z)$ $\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$

Mass factorization

$$\sigma_a = |\hat{F}_I(Q^2,\epsilon)|^2 Z_I^2(\mu_R,\epsilon)\sigma_a^{S+H}(z,Q^2)$$

$$\sigma_a^{S+H} = \mathcal{C}e^{2\Phi^I_{SJ}}$$

Bloch and Nordsiek:

Final state Soft and Collinear singularities cancel

KLN theorem:

Initial state collinear singularities are removed by MASS FACTORISATION

Factorise the collinear part of Partonic cross section:

$$\sigma_a(z) = \sum_b \Gamma_{ab}\left(\mu_F, \frac{1}{\epsilon}\right) \otimes \Delta_b(\mu_F)$$

$$f_a(z,\mu_F) = \Gamma\left(z,\mu_F,\frac{1}{\epsilon}\right) \otimes f_a^B(z)$$

$$\Delta^{I,\mathrm{SV}} = \left(Z^I\right)^2 |\hat{\mathcal{F}}^I|^2 \delta(1-z) \otimes \mathcal{C}e^{2\Phi^I_{\mathrm{SJ}}} \otimes \Gamma_{II}^{-1}$$

Building blocks

- $\ln \left[Z^I \right]^2$
- $\ln |\hat{F}^I|^2$
- $\ln \Gamma_{I\,I}$
- $\Phi^I_{S\,J}$

J_I -Jet function

I = q,g

K+G equation

$$\mathcal{F}^{\lambda}_{\beta} = < \beta|\mathcal{O}^{\lambda}|\beta >$$

[Sen, Sterman,Magnea]

$$d = 4 + \varepsilon$$

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2}\left[K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) + G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon)\right]$$

poles

No poles

RG invariance

$$\mu_R^2 \frac{d}{d\mu_R^2} K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) = -\mu_R^2 \frac{d}{d\mu_R^2} G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon) = -A^{\lambda}_{\beta}(a_s(\mu_R^2))$$

Cusp (soft) Anomalous dim.

Casimir Duality

$$A_q = \frac{C_F}{C_A} A_g$$

Upto 3 loops

Collinear and UV RGE

[Moch, Vogt, Vermaseren]

DGLAP kernel Γ satisfies RGE

Γ is available up to $\mathcal{O}(a_s^4)$

$$\mu_F^2 \frac{d}{d\mu_F^2}\Gamma(z,\mu_F^2,\varepsilon) = \frac{1}{2} P\left(z,\mu_F^2\right) \otimes \Gamma\left(z,\mu_F^2,\varepsilon\right)$$

$$P_{II}^{(i)}(z) = 2\left[B_{i+1}^I \delta(1-z) + A_{i+1}^I \mathcal{D}_0\right] + P_{reg,II}^{(i)}(z)$$

RGE for Z :

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2)\gamma_{i-1}^I$$

$\mathcal{O}(a_s^3)$ for $I = b, g$ and $\gamma^q = 0 (q \neq b)$

Where we are

- [x] $\ln \left[Z^I \right]^2$
- [x] $\ln |\hat{F}^I|^2$
- [x] $\ln \Gamma_{I\,I}$
- [ ] $\Phi^I_{S\,J}$

J_I -Jet function

I = q,g

Soft + Jet function (SJ)

$$Q^2 \frac{d}{dQ^2} \Phi^I_{\rm SJ} = \frac{1}{2} \Big[\bar{K}^I \Big(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \epsilon\Big) + \bar{G}^I_{\rm SJ}\Big(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \epsilon\Big) \Big]$$

All order Solution:

$$\Phi^I_{\rm SJ} = \sum_{i=1}^{\infty} \hat{a}_s^i S_\epsilon^i \left(\frac{Q^2(1-z)}{\mu^2} \right)^{i\frac{\epsilon}{2}} \frac{i\epsilon}{2(1-z)} \hat{\phi}^{I,(i)}_{\rm SJ}(\epsilon)$$

where

$$\hat{\phi}^{I,(i)}_{\rm SJ}(\epsilon) = \frac{1}{i\epsilon} \left[\bar{K}^{I,(i)}(\epsilon) + \bar{G}^{I,(i)}_{\rm SJ}(\epsilon) \right]$$

$$\sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q_z^2}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_\epsilon^i \bar{G}^{I,(i)}_{\rm SJ}(\epsilon) = \sum_{i=1}^{\infty} a_s^i(Q_z^2) \bar{\mathcal{G}}^I_{i,\rm SJ}(\epsilon)$$

$$\bar{\mathcal{G}}^I_{i,\rm SJ} = -(B_i^I + f_i^I) + C_i^I + \sum_{k=1}^{\infty} \epsilon^k \bar{\mathcal{G}}^{I,k}_{i,\rm SJ}$$

from N3LO DIS

Factorisation

Factorization

$$\mathcal{C}e^{2\Phi^{I}_{\mathrm{SJ}}} = \mathcal{Z}^{I} \otimes \mathcal{C}e^{2\Phi^{I,\mathrm{fin}}_{\mathrm{SJ}}}$$

IR singular constant:

$$\mathcal{Z}^{I} = \delta(1-z) + \sum_{i=1}^{n} \sum_{j=1}^{2\,i} a_s^i \, \frac{\mathcal{Z}^{I}_{i\,j}}{\epsilon^j}$$

Jet function:

$$\mathcal{C}e^{2\Phi^{I,\mathrm{fin}}_{\mathrm{SJ}}} = \delta(1-z) + \sum_{i=1}^{\infty} a_s^i J_i^I \Big|_k$$

DIS results at N3LO level

* Deep Inelastic Scattering involving large off-shell photons is sensitive to Quark Jet function

* Deep Inelastic Scattering involving large off-shell Higgs boson is sensitive to Gluon Jet function

* Both are available at N3LL level from Vermaseren, Moch, Vogt and Soar

* We reproduce three loop Quark Jet function

Gluon jet function

[Banerjee, PKD, Ravindran (1805.02637)]

* Delta function coefficient of the gluon jet function is obtained from DIS with Higgs boson.

$$J_3^g\big|_\delta = C_A^3\left[\frac{55853711}{26244} - 44\zeta_5 - \frac{452770}{243}\zeta_3 + \frac{1600}{9}\zeta_3^2 - \frac{2055109}{4374}\pi^2 + \frac{1364}{9}\pi^2\zeta_3 + \frac{53633}{1620}\pi^4 - \frac{16309}{20412}\pi^6\right]$$

$$+C_A^2 n_f\left[-\frac{17323633}{26244} + \frac{208}{9}\zeta_5 + \frac{2734}{9}\zeta_3 + \frac{330062}{2187}\pi^2 - \frac{88}{9}\pi^2\zeta_3 - \frac{18727}{2430}\pi^4\right] + C_F^2 n_f\left[\frac{143}{9} - 80\zeta_5 + \frac{148}{3}\zeta_3\right]$$

$$+C_A n_f^2\left[\frac{1613639}{26244} - \frac{1004}{243}\zeta_3 - \frac{3656}{243}\pi^2 + \frac{506}{1215}\pi^4\right] + C_F n_f^2\left[\frac{7001}{162} - \frac{104}{3}\zeta_3 - \frac{10}{9}\pi^2\right]$$

$$+C_A C_F n_f\left[-\frac{389369}{972} + \frac{584}{9}\zeta_5 + \frac{21200}{81}\zeta_3 + \frac{712}{27}\pi^2 - \frac{160}{9}\pi^2\zeta_3 + \frac{76}{405}\pi^4\right] + n_f^3\left[-\frac{1000}{729} + \frac{40}{81}\pi^2\right]$$

* Renormalisation Group Equation:

$$\mu_R^2 \frac{d}{d\mu_R^2} J^I = \gamma_J^I \otimes J^I$$

$$\gamma_J^I = \left[B^I + f^I - A^I \log\left(\frac{Q^2}{\mu_R^2}\right)\right]\delta(1-z) - A^I \mathcal{D}_0$$

Results

* N3LO results for DIS with photon and higgs boson from Vermaseren, Moch, Vogt and Soar
* Deep Inelastic Scattering involving large off-shell photons can give Quark Jet function directly.
* We reproduce three loop Quark Jet function
* Deep Inelastic Scattering involving large off-shell Higgs boson can give Gluon Jet function, we now have three loops result which is new