2 hrs lectures on inflation

- Definition
- What happens after inflation? See references
- Are there alternatives? Yes
  - Bouncing cosmologies 0902.4731
  - String-gas cosmology
  - Galilean Genesis 1007.0027
  - ⋮
- Predictions of inflation See references
- Problems? Perceived (or not)
  - initial conditions
  - multiverse

REFERENCES

- COSMOLOGY by Weinberg 2008
- MODERN COSMOLOGY by Dodelson

Books

- THE PHYSICS OF INFLATION by Baumann

online lecture notes self-consistent

.. We will address the first

.. How do photons (with CMB photons in mind) propagate in a universe made up of standard stuff: radiation and matter.

Basic assumptions / starting points

(1) General relativity as our theory of gravity valid at inflationary scales. OK if $\frac{H}{M_p} \ll 1$

(2) Universe homogeneous and isotropic at large scales $\lambda \gtrsim 100\,\mathrm{Mpc}$

Spacetime described by solution to GR equations w/ homogeneous + isotropic metric field

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu}$$] rules of the game

FLRW: $$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$] solution we seek given observations of current interest: 1911.02087

(0,0) component: $3M_P^2H^2 = \rho - 3M_P^2 \cancel{\frac{K}{a^2}}$ → 0 for simplicity

where $H \equiv \frac{\dot{a}}{a}$

• What makes up "ρ"?

One typically writes $T_{\mu\nu}$ as a perfect fluid, i.e. one that can be put in the form

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu} \quad \underbrace{+ \Pi_{\mu\nu}}_{0905.4352}$$

density   pressure   4-velocity

where terms such as those due to viscosity are missing

A perfect fluid, for example, emerges quickly from a Lagrangian formulation of the type $\mathcal{L} = \mathcal{L}(X, \phi)$ w/ $X \equiv -g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$ and ϕ is a scalar field.

quick exercise

show that for $\mathcal{L} = P(X, \phi)$

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu} \quad \text{where} \quad \begin{cases} p = P(X, \phi) \\ \rho = 2X P_X - P \\ u_\mu = -\dfrac{\partial_\mu \phi}{\sqrt{X}} \end{cases}$$

Let us go back to EE:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}}_{G_{\mu\nu}} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu}$$

Bianchi identity $\nabla_\mu G^{\mu\nu} = 0 \Rightarrow \nabla_\mu T^{\mu\nu} = 0$

take $\nu = 0$ and verify that $\boxed{\dot{\rho} + 3H(\rho + p) = 0}$

where $\rho = \rho(t)$, $\gamma \simeq 1$ $v \ll 1$

Newtonian limit

$$\partial_t \rho + \nabla \cdot [(\rho + p)\, \bar{v}] = 0.$$

Curved space adds the $\propto 3H(\rho + p)$ term define $w \equiv \frac{p}{\rho}$, typically it is $w = \frac{\bar{p}}{\bar{\rho}}$ $\Rightarrow$ background values

and assume, for simplicity, that $\dot{w} = 0$

$$\dot{\rho} + 3H(\rho + p) = \dot{\rho} + 3H\rho(1+w) = 0$$

$$\frac{d\rho}{dt} = -3H\rho(1+w) \Rightarrow \frac{d\rho}{\rho} = -3\frac{\cancel{dt}}{a}\frac{da}{\cancel{dt}}(1+w)$$

$$\Rightarrow \ln\rho \sim -3(1+w) \Rightarrow \boxed{\rho \sim a^{-3(1+w)}}$$

standard content: $\begin{cases} \text{matter, } w = 0 \\ \text{radiation, } w = 1/3 \end{cases}$

$\exists$ dust particles that only interact gravitationally

quick route is $T_\mu{}^\mu = 0$ for EM so $\rho - 3p = 0$ $\Rightarrow \frac{p}{\rho} = \frac{1}{3}$

$$\Rightarrow \rho \sim \begin{cases} \frac{1}{a^3} & \text{matter} \\ \frac{1}{a^4} & \text{radiation} \end{cases}$$

$$3M_p^2 H^2 \propto a^{-3(1+w)}$$

$$\Rightarrow \frac{1}{a}\frac{da}{dt} \propto a^{-\frac{3}{2}(1+w)}$$

$$da\; a^{\frac{1}{2}(1+3w)} \propto dt$$

$$\Rightarrow a^{\frac{3}{2}(1+w)} \propto t$$

$$\Rightarrow t \propto \begin{cases} a^{3/2} & \text{matter} \\ a^{2} & \text{radiation} \end{cases} \longleftrightarrow a(t) \sim \begin{cases} t^{2/3} & \text{matter} \\ t^{1/2} & \text{radiation} \end{cases}$$

Photons propagate along null geodesics, so in FLRW

$$ds^2 = 0 = -dt^2 + a^2(t)\, d\vec{x}^2 = a^2(\tau)\left(-d\tau^2 + d\vec{x}^2\right)$$

where τ is the conformal time.

For $d\vec{x}^2$ or $d\vec{r}^2$ we have $dr^2 = d\tau^2 \Rightarrow r(\tau) = \pm\tau + \text{const}$

$\Rightarrow$ photons travel at $\pm 45°$ in the τ-r plane.

The maximal distance a photon can travel (from t_i to $t > t_i$)

is $\Delta r = \Delta\tau = \tau - \tau_i = \int_{t_i}^{t} \frac{d\tilde{t}}{a(\tilde{t})}$

w/o loss of generality $t_i = 0$

$$\Delta x = \Delta r = \int_0^t \frac{d\tilde{t}}{a(\tilde{t})}$$

$\equiv$ comoving particle horizon, which depend on the make up of the universe because, as we have seen, $a(t)$ does.

$$\Delta r = \int \frac{dt}{da}\,\frac{da}{a}\cdot\frac{a}{a} = \int \frac{1}{H}\frac{da}{a^2} \sim \int da\,\frac{1}{a^2}\,a^{\frac{3}{2}(1+w)}$$

$$\Rightarrow \Delta r = \Delta\tau \propto \frac{2}{(1+3w)}\,a^{\frac{1}{2}(1+3w)}$$

now, conventionally $\tau_i = 0$ so $\tau \sim \frac{2}{1+3w}\,a^{\frac{1}{2}(1+3w)}$

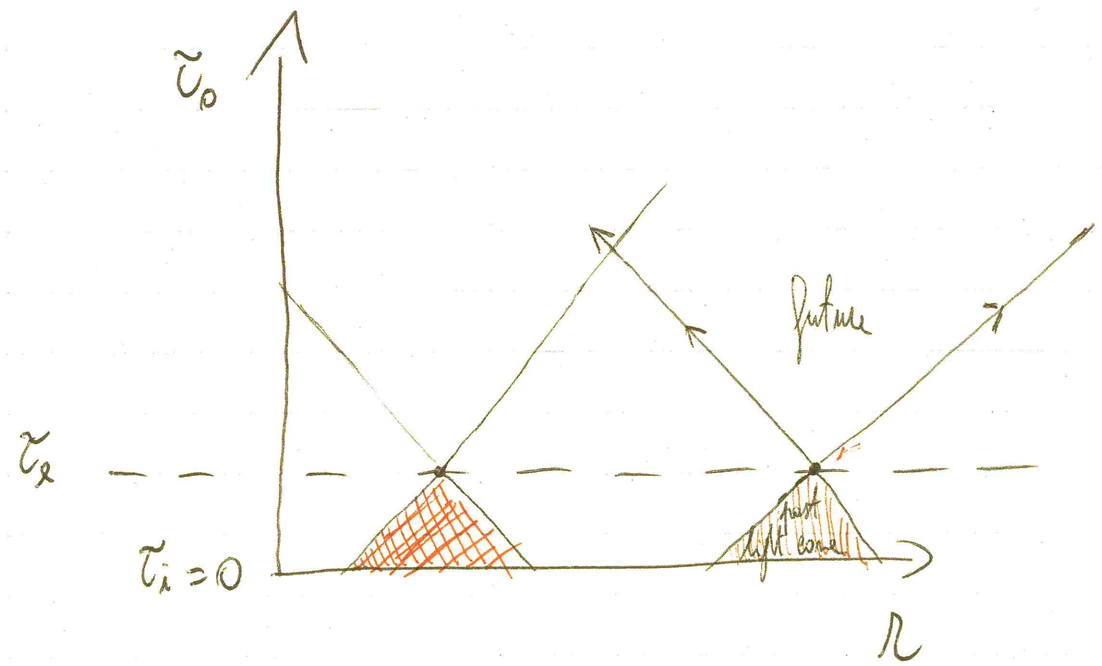
the important bit is that $a(t)$ for conventional sources grows with time and so long as ~~$\frac{2}{1+3w}$~~ ~~$a^{\frac{1}{2}(1+3w)}$~~ $\underbrace{\frac{2}{1+3w}}_{>0}\,a^{\overbrace{\frac{1}{2}(1+3w)}^{>0}}$

$\Delta r = \Delta \tau$ is dominated by a_{Final} or, analogally

$\Delta r = \int_{t_i=0}^{t_F=t} \frac{d\tilde{t}}{a(\tilde{t})}$ is dominated by integral at t_F

so we have established that $\tau \propto \frac{2}{1+3w} a^{\frac{1}{2}(1+3w)} > 0$

take $\tau_i = 0$ and draw conformal diagram (i.e. diagram in the (τ, r) plane) ~~[illegible]~~



this is what photons do, and their $\tau_i = 0$. Let us talk about CMB photons and what we know about them

CMB RADIATION

- thermal blackbody spectrum at 2.7 °K
- photons that propagated to us approximately unimpeded, from time of "photon decoupling" in the early universe

(a) hot and dense universe filled with plasma of photons, electrons protons ... (b) The universe got increasingly cooler as a result of the expansion. (c) Protons and electrons combined to form neutral hydrogen (recombination) $\Rightarrow$ no more scattering of the thermal radiation by Thomson scattering $\Rightarrow$ (d) photons start to travel freely (i.e. photon decoupling)

when? $\simeq$ 380.000 y after the big-bang

or $a(t) = \frac{1}{1+z}$; remember $a(t=0)=0$ and $a(t)$ acts as a clock

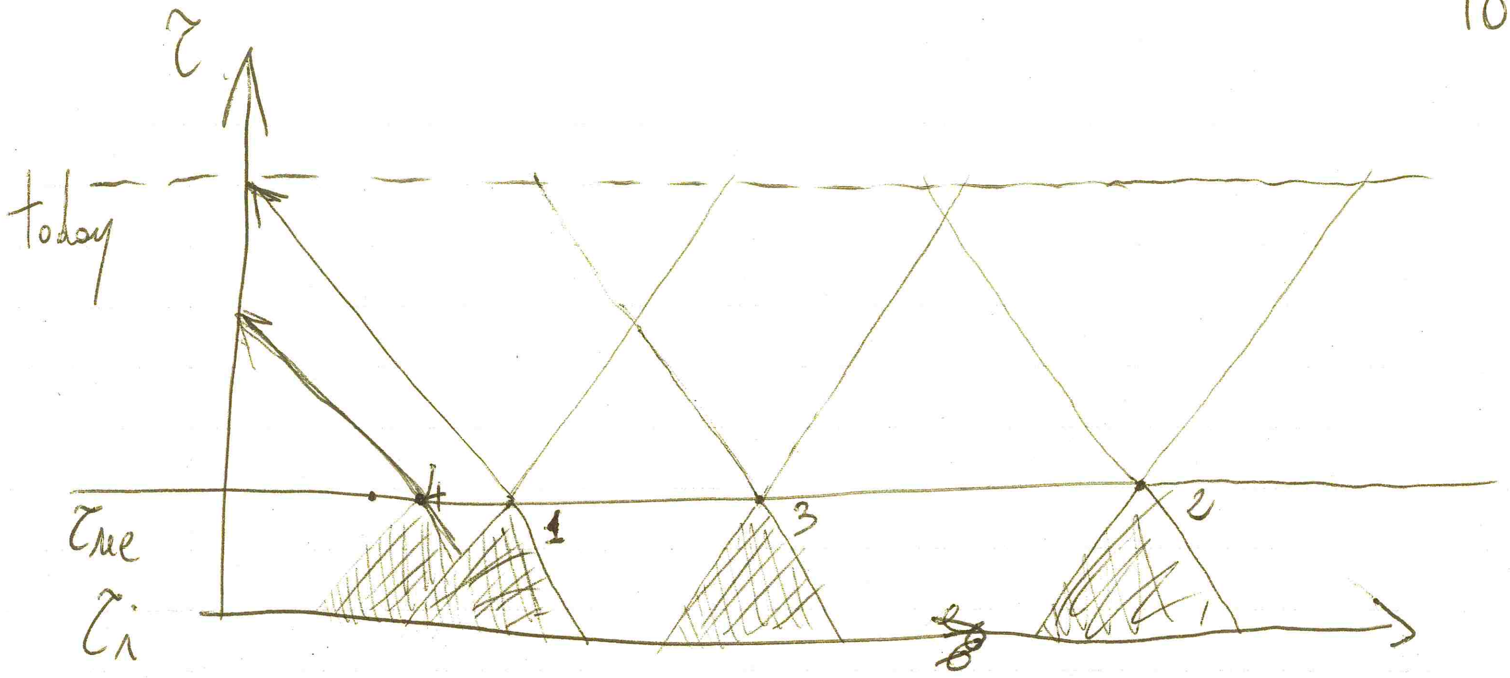
$z \simeq 1100$

the point: $\exists$ specific time in the past about which CMB photons carry information, ~~this~~ we call this finite time $t_{rec} \Leftrightarrow \tau_{rec}$

One piece of information is the temperature*. Consider the quantity $\frac{\Delta T(\hat{n})}{T}$. Photons we observe from different directions may (not) have different temperature.

IF PHOTONS HAVE NOT BEEN IS CAUSAL CONTACT from τ_i to τ_{rec}, we do not expect them to have the same temperature, i.e. $\left.\frac{\Delta T}{T}\right|_{0} \simeq \mathcal{O}(1)$

* $T(z) \simeq 2.7\,(1+z)\ K^{\circ}$



1 has never "talked" to 2 or to 3 before recombination but it was in causal contact with 4

the angle in the sky today corresponding to photons that have been in contact before τ_{re} is intuitively proportional to $\propto \dfrac{\tau_{re}-\tau_i}{\tau_0-\tau_{re}}$

use $a(z) = \dfrac{1}{1+z}$ to write

$$\tau_2-\tau_1 = \int_{z_1}^{z_2} \frac{dz}{H(z)}$$

and the Friedman equation

$$3M_P^2 H^2 = \frac{\rho_{m,0}}{a^3} + \frac{\rho_{r,0}}{a^4} + \Lambda_{OBS}$$

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda}$$

$\Omega_m \simeq 0.3$; $\Omega_r \simeq \cancel{\frac{0.3}{3400}}$ $\quad \Omega_\Lambda \simeq 0.7$

this gives $\dfrac{\tau_{rec}-\tau_i}{\tau_0-\tau_{rec}} \simeq 1.16^\circ$

angle in the sky is actually $2\times$

$$\boxed{\theta = 2.3^\circ}$$

Comparing points at angular separation larger than 2.3° we should see a $\boxed{\dfrac{\Delta T}{T} \sim 1}$

What do we actually see?

$$\frac{\Delta T}{T} \sim 10^{-5} \text{ at } \theta \text{ scales}$$

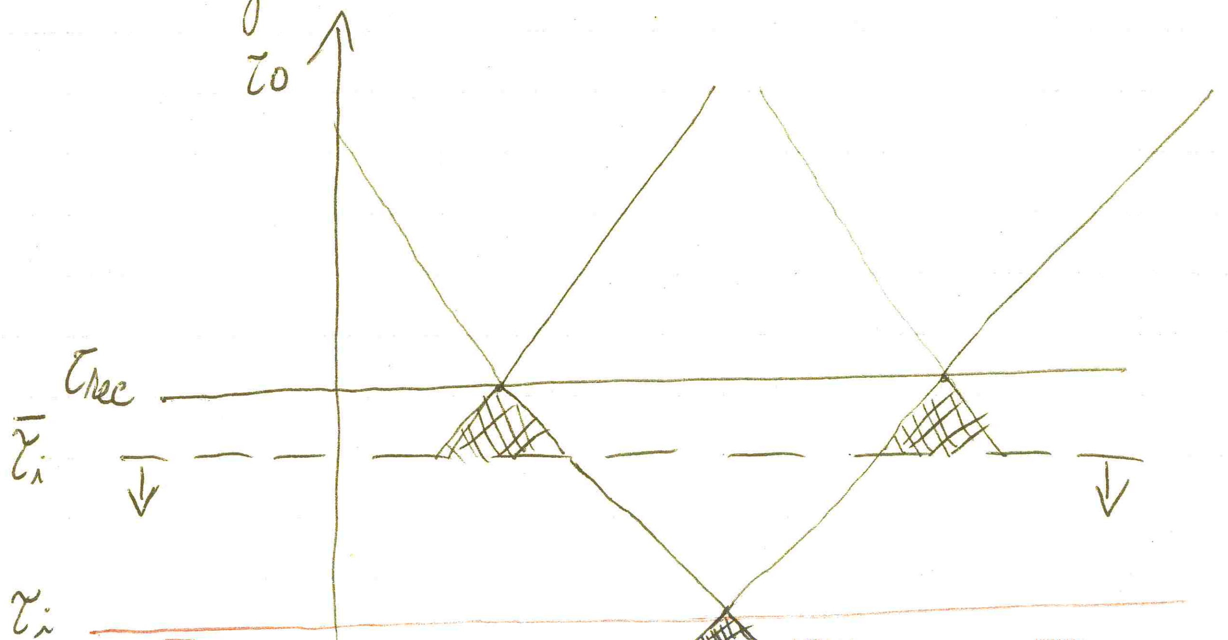
some how photons were in contact before recombination, otherwise hard to accept $\frac{\Delta T}{T} \ll 1$

want $\frac{\boxed{\tau_{rec} - \tau_i}}{\tau_0 - \tau_{rec}}$ larger

remember: $\tau \propto \frac{2}{1+3w} a^{\frac{1}{2}(1+3w)}$

standard constituents $\Rightarrow w \geq 0$; $w \begin{cases} \to 0 & \text{matter} \\ \to 1/3 & \text{rad} \end{cases}$

but if $\tau < 0$ ~~were~~ allowed then $\tau_{rec} - \tau_i$ could be much larger



an earlier set τ_i would do the job. For that to occur, we need

$$\tau_i \propto \frac{2}{1+3w}\, a^{\frac{1}{2}(1+3w)}$$

<0

require $1+3w<0$

$\Rightarrow w<-\frac{1}{3}$

$\frac{p}{\rho}<-\frac{1}{3}$, constituent with negative pressure.

We call the period in the early universe when expansion was driven by a component such that $w<-\frac{1}{3}$ "INFLATION".

Is there a clear intuitive notion of what $w<-\frac{1}{3}$ means for the dynamic? YES

go back to EE: the (0,0) component gave us $3M_p^2H^2 \sim \rho$

(0,0) component + trace of EE $\quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$

gives: $-6M_p^2 \frac{\ddot{a}}{a} = \rho + 3p$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p) = \frac{-1}{6M_p^2}\rho(1+3w)$$

so $w < -\frac{1}{3} \Rightarrow \ddot{a} > 0$

$$\begin{cases} a > 0 \\ \dot{a} > 0 \rightarrow \text{expansion} \\ \ddot{a} > 0 \quad \text{accelerated expansion} \end{cases}$$

<u>inflation</u>: period of accelerated expansion in the early universe.

1 reason to postulate it is make sense of $\frac{\Delta T}{T} = 10^{-5}$ ~~at~~ in all directions for the CMB.

re-phrase as: inflation solves the <u>horizon problem</u>

$\exists$ at least two more "problems" inflation solves:

- flatness problem — $|\Omega_k - 1| < 10^{-2}$ today and much much smaller in the past; why?
- monopole problem — From GUT (grand unification theories)

expect: $r \equiv \frac{n_{\text{monopoles}}}{n_{\text{photons}}} \sim 10^{-9}$, observe $r < 10^{-39}$ $\Rightarrow$ need inflation before photons are created.

these are automatically solved if inflation lasts long enough to solve horizon problem. Look them up in Weinberg's book, it's all in just a couple of paragraphs.

<u>Back to inflation</u>: need $w < -\frac{1}{3}$ to make sense of

$\frac{\Delta T}{T} \sim 10^{-5}$ at all scales in CMB

How to implement a phase when $w < -\frac{1}{3}$?

$3M_p^2 H^2 = \rho \sim a^{-3(1+w)}$; need $w < -\frac{1}{3}$, e.g. $w = -1$

$\Rightarrow \rho = \text{const}$ ~~[illegible]~~. Does that work?

- It does provide $\ddot{a} > 0$
- We might have seen it before, call $\rho_{const} \equiv \Lambda$

$$3M_p^2 H^2 \sim \Lambda + \underbrace{\frac{\rho_{m,0}}{a^3} + \frac{\rho_{r,0}}{a^4}}_{\text{sub-leading}}$$

but, if $\Lambda > \left\{\frac{\rho_{m,0}}{a^3}, \frac{\rho_{r,0}}{a^4}\right\}$ in the early universe, so

it will be later on because decrease with time

$\Rightarrow$ this mechanism for inflation will never end! We need radiation and matter to eventually dominate! Not a constant then. What's the next simplest thing? A scalar field perhaps...

Classical field treatment for scalar field ϕ

$$\mathcal{L}\phi \sim -(\partial_\mu \phi)^2 - V(\phi)$$

$$S \sim \int d^4x \sqrt{-g}\, \mathcal{L}(\phi)$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

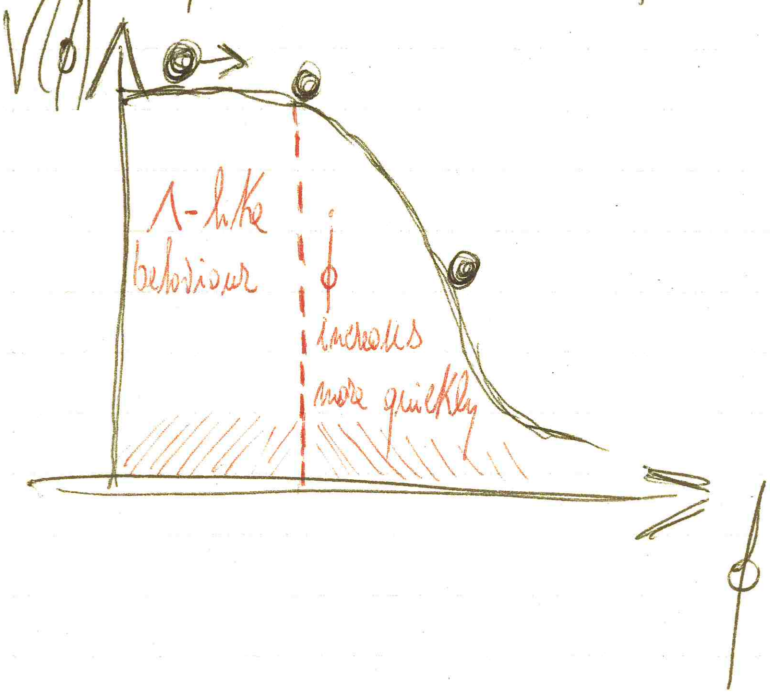
$$\rho \sim T_{00} \sim \dot{\phi}^2 + V(\phi)$$

$$p \sim T_{ii} \sim \dot{\phi}^2 - V(\phi)$$

- easy way to implement $w < -\frac{1}{3}$ is $V(\phi) \gg \dot{\phi}^2 \Rightarrow p \simeq -\rho \Leftrightarrow w \simeq -1$
- this will <u>end</u> when kinetic energy of ϕ becomes large enough

inflation $\exists$ when $\dot{\phi}^2 \ll V(\phi)$

can dynamically end when $\dot{\phi}^2(t) = V(\phi)$



A little more details

ϕ e.o.m.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

FLRW background, only time dependent

Further assume $\ddot{\phi} \ll V'(\phi)$

$$\Rightarrow \dot{\phi} \simeq \frac{-V'(\phi)}{3H}$$

$\dot{\phi}^2 \ll V(\phi)$

time derivative

$2\cancel{\dot{\phi}}\ddot{\phi} \ll V'(\phi)\cancel{\dot{\phi}}$

$\ddot{\phi} \ll V'(\phi)$

i.e. $\dot{\phi}^2 \ll V(\phi)$ lasts longer when $\ddot{\phi} \ll V'(\phi)$

more illuminating (or not) form for $\dfrac{\dot\phi^2}{V(\phi)} \ll 1$

is $\dfrac{V'^2}{9H^2V} \ll 1 \Longrightarrow \underbrace{\dfrac{1}{3} M_p^2 \left(\dfrac{V'}{V}\right)^2}$ $\ll 1$

we have used
$3M_p^2H^2 \sim \rho = \dot\phi^2 + V(\phi)$
but $\dot\phi^2 \ll V(\phi)$ so
$3M_p^2H^2 = V(\phi)$

i.e. slope of the potential very flat, V almost constant when inflation taking place.

the conditions $\dot\phi^2/V(\phi) \ll 1$ and $\ddot\phi \ll \begin{cases} H\dot\phi \\ V'(\phi) \end{cases}$

are called slow-roll conditions.

More often than not, these are expressed in different terms

$$\boxed{\begin{aligned} \varepsilon &= -\frac{\dot H}{H^2} \ll 1 \\ \eta &= \frac{\dot\varepsilon}{H\varepsilon} \ll 1 \end{aligned}}$$

Exercise: show these two are equivalent to our previous requirements

start from $3M_p^2 H^2 \sim \rho \sim \dot\phi^2 + V(\phi) \simeq V(\phi)$

time derivative $6M_p^2 H \dot H \sim V'(\phi)\dot\phi$

remember $\dot\phi \simeq -\frac{V'(\phi)}{3H} \Rightarrow 6M_p^2 H\dot H \sim -\frac{V'^2}{3H} \Rightarrow -\frac{\dot H}{H^2} \sim \frac{V'^2}{18M_p^2 H^4}$

$$\Rightarrow \varepsilon \sim \frac{V'^2}{\underbrace{3M_p^2 H^2}\, 3H^2 \cdot 2} \simeq \boxed{\frac{M_p^2}{2}\frac{V'^2}{V^2}} \ll 1$$

[we showed $\dot\phi^2/V(\phi) \ll 1$ $\Rightarrow \frac{M_p^2}{3}\left(\frac{V'}{V}\right)^2 \ll 1$]

$$\eta \equiv \frac{\dot\varepsilon}{H\varepsilon} \propto \frac{\frac{d}{dt}\left(\dot\phi^2/V\right)}{H\cdot \dot\phi^2/V} \sim \frac{\cancel{V}}{H\dot\phi^{\cancel{2}}}\left[\frac{2\cancel{\dot\phi}\ddot\phi}{\cancel{V}} - \frac{\dot\phi^{\cancel{2}} V'\cancel{\dot\phi}}{V^{\cancel{2}1}}\right]$$

$$\sim 2\frac{\ddot\phi}{H\dot\phi} - \frac{V'}{V}\frac{\dot\phi}{H}$$

$\ll 1$?

$$-\frac{V'}{V}\frac{\dot\phi}{H} \sim \frac{V'}{V}\frac{V'}{3H^2} \sim \frac{V'^2}{V^2}M_p^2 \sim \varepsilon \ll 1 \checkmark$$

How long does inflation need to last?

for CMB observations to make we need ~60 e-folds

$$60 \sim \ln \frac{a_F}{a_i} = \int_{a_i}^{a_F} \frac{da}{a} = \int \frac{da}{dt}\, dt \, \frac{1}{a} = \int H\, dt$$

we $\dot{H}$ small $\qquad \simeq H \int dt = H \int \frac{dt}{d\phi}\, d\phi \simeq H \int \frac{d\phi}{\dot{\phi}}$

$$\simeq 3H^2 \int_{\phi_i}^{\phi_F} \frac{d\phi}{V'(\phi)} \longleftarrow \text{flat potential helps!}$$

Inflation conventionally ends when $\varepsilon = 1$.

We can implement the inflationary mechanism via a slowly rolling scalar field. That will explain

$\frac{\Delta T}{T} \ll 1$ across widely different angles the same, simple setup also explains how

$\frac{\Delta T}{T} \sim 10^{-5}$ (as opposed to zero).

What we have done so far is taking the background part of

$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}g_{\mu\nu} = \overline{T}_{\mu\nu}$

, that is $\overline{R_{\mu\nu}} - \frac{1}{2}\overline{Rg_{\mu\nu}} = \overline{T_{\mu\nu}}$. Quantum fluctuations of the inflaton field should be studied, these account for temperature fluctuations in the CMB.

We need to study

$$\delta R_{\mu\nu} - \frac{1}{2}\delta[Rg_{\mu\nu}] = \delta T_{\mu\nu}$$

when working w/ a theory with e.g. metric field $g_{\mu\nu}$ and scalar ϕ one has to first identify the number of propagating degrees of freedom

---- big gap here ---- see e.g. chapter 5 of Weinberg

we can work with

$$\delta g_{ij} = a^2(1-2\underbrace{\mathcal{R}}_{\text{scalar perturbation}})\delta_{ij} + a^2 \underbrace{h_{ij}}_{\text{tensor perturbation}}$$

$$\mathcal{R} \sim \Psi + H \frac{\delta\phi}{\dot{\phi}}$$

- $$\Theta(\hat{n}) = \frac{\Delta T(\vec{n})}{T_0} = \sum_{lm} a_{lm} Y_{lm}(\vec{n})$$

- $$a_{lm} = \int d\Omega \, Y^*_{lm}(\vec{n}) \, \Theta(\vec{n})$$

- $$a_{l,m} = 4\pi (-i)^l \int \frac{d^3k}{(2\pi)^3} \Delta_{Tl}(k) \sum_k Y_{lm}(\hat{k})$$

$$\left\{ \Delta_{Tlk} = \int_0^{\tau_0} \underbrace{S_T(k,\tau)}_{\text{SOURCE}} \underbrace{P_{Tl}(k|\tau_0-\tau)}_{\text{PROJECTION}} \right.$$

$$C_l^{TT} = \frac{1}{2l+1} \sum_m \langle a_{lm} a^*_{lm} \rangle \quad \text{or} \left\{ \langle a^*_{lm} a_{l'm'} \rangle = C_l^{TT} \delta_{ll'} \delta_{mm'} \right.$$

$$C_l^{TT} = \frac{2}{\pi} \int k \, dk \, \underline{\underline{P_2(k)}} \, \Delta_{Tl}(k) \, \Delta_{Tl}(k)$$

essentially (22) tells us about $\left(\frac{\Delta T}{T}\frac{\Delta T}{T}\right)$ and its statistics.

$$P_R \sim \frac{H^2}{M_p^2 \varepsilon}\left(\frac{k}{k_0}\right)^{-2\varepsilon-\eta}$$

$$P_\gamma \sim \frac{H^2}{M_p^2}\left(\frac{k}{k_0}\right)^{-2\varepsilon}$$

$r \sim 16\varepsilon$