

Track parameter pull from KalmanFitter in ACTS

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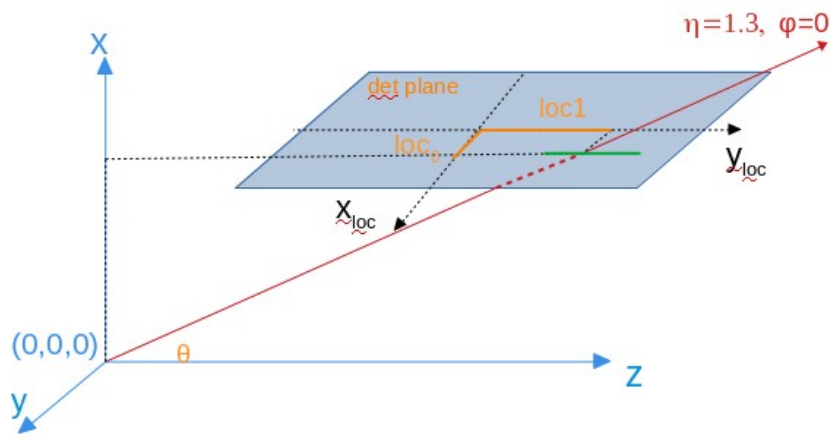
Test setup

	10000 single muon generated with parameter: $d_0=0$, $z_0=0$, $p_T=10\text{GeV}$	Truth smeared to make inputs for KalmanFitter
Test 1	$\eta=1.3$, $\phi = 0$	Only smear parameter θ
Test 2	$\eta=0$, $\phi = 0$	Only smear parameter ϕ
Test 3	$\eta=1.3$, $\phi=0$	Smear both truth parameter and truth hits
Test 4	$\eta=0$, $\phi = 0$	Smear both truth parameter and truth hits

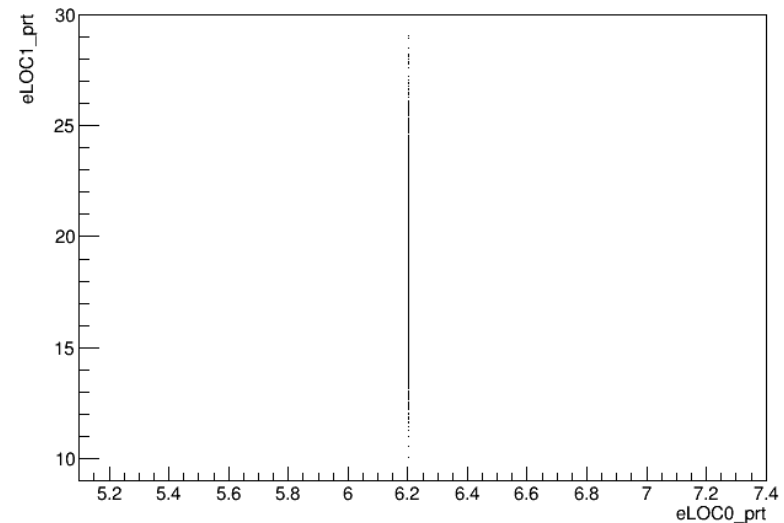
Test 1: only θ smeared

10000 Truth particle generated with:

- $d0 = 0, z0 = 0$
- $\eta = 1.3, \phi = 0$
- $pT = 10 \text{ GeV}/c$



loc0: loc1 at first measurement surface

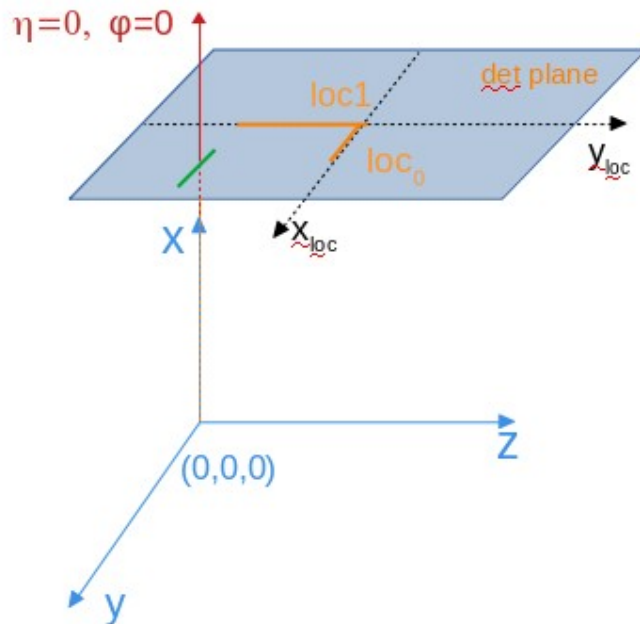


- Only smearing of truth track parameter θ
 - The predicted $loc0, \phi, q/p$ on the first measurement surface will be fixed
 - Only variance of $loc1$ and θ is present
- No smearing of truth hit
 - Measurement is the same with truth hit & measurement covariance = 0

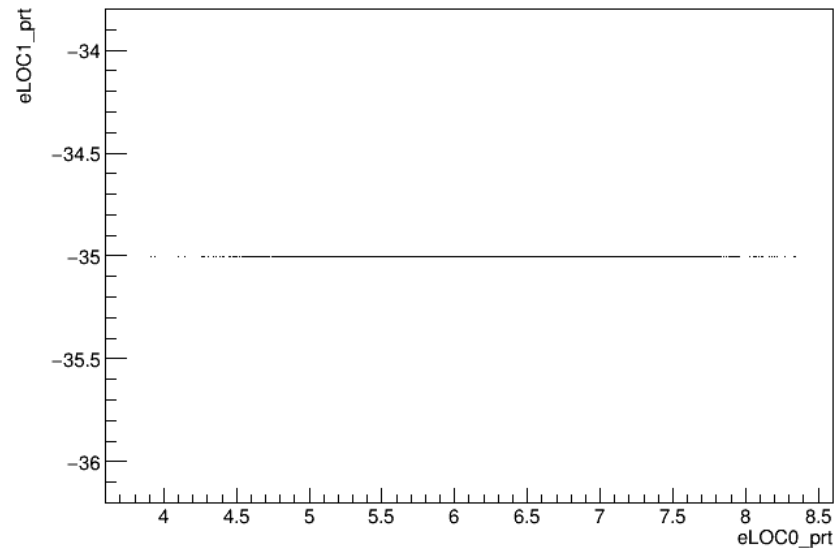
Test 2: only φ smeared

10000 Truth particle generated with:

- $d0 = 0, z0 = 0$
- $\eta = 0, \phi = 0$
- $pT = 10 \text{ GeV}/c$



loc0: loc1 at first measurement surface

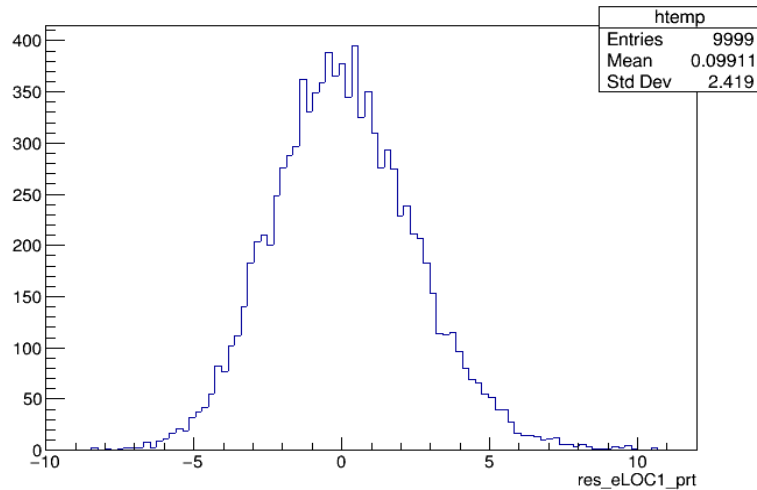


- Only smearing of truth track parameter φ
- The predicted loc1, θ , q/p on the first measurement surface will be fixed
- Only variance of loc0 and φ is present
- No smearing of truth hit
 - Measurement is the same with truth hit & measurement covariance = 0

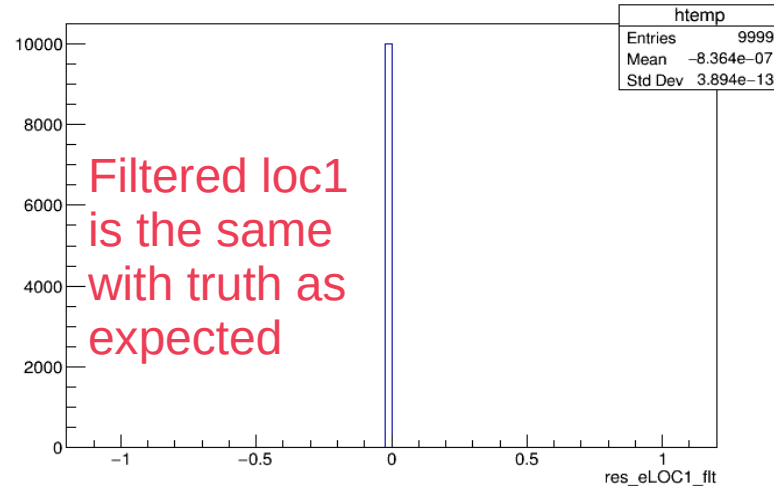
Scenario 1:
no material effects

Test 1: residual of $\text{loc1}/\theta$ (on first measurement surface)

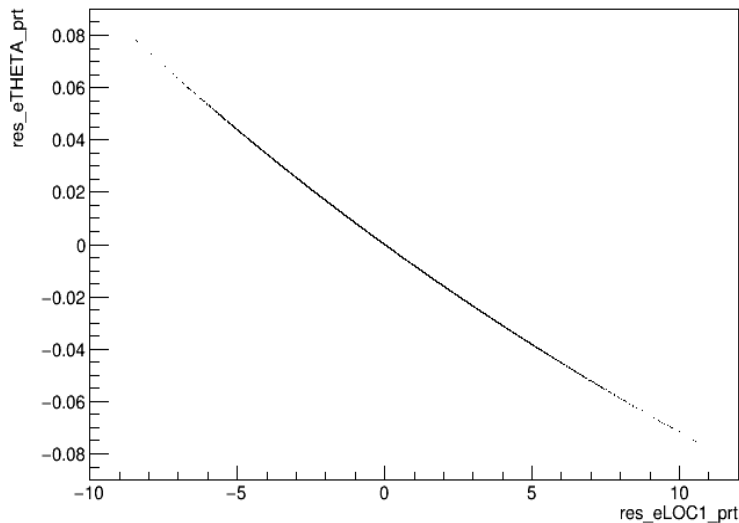
resid_loc1_pred



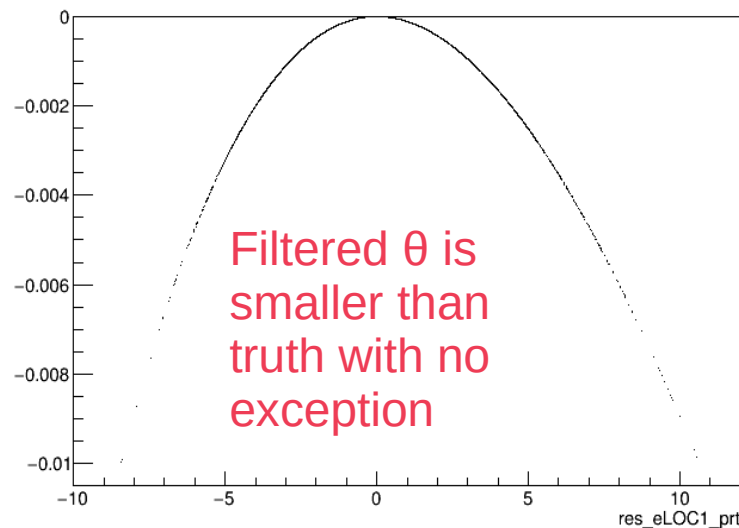
resid_loc1_filtering



resid_θ_pred vs. resid_loc1_pred



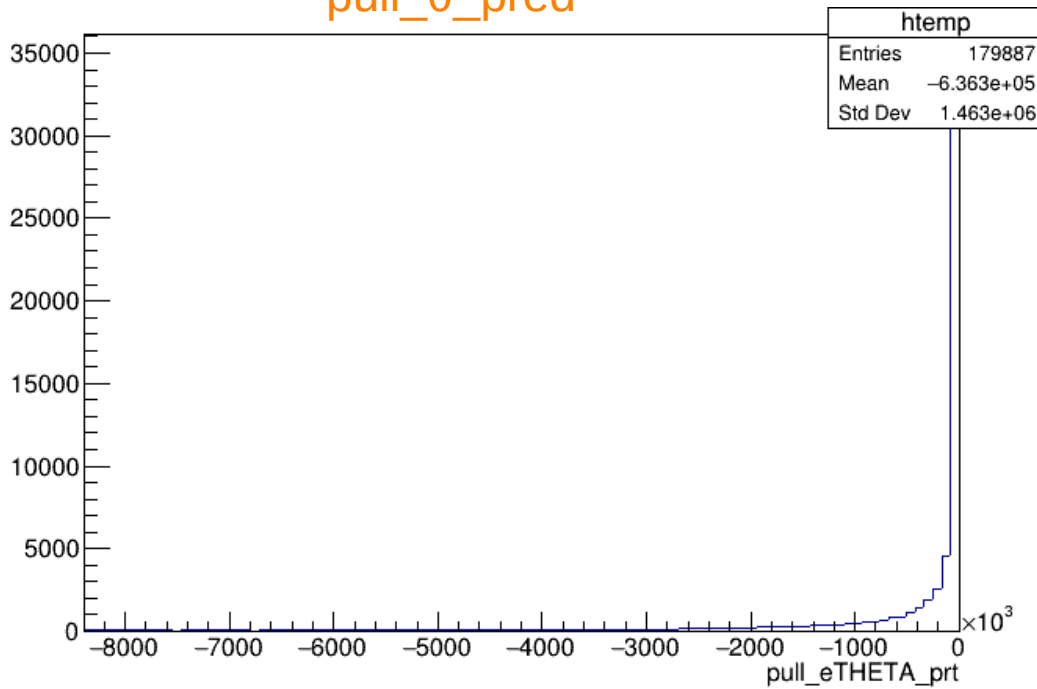
resid_θ_filtering vs. resid_loc1_pred



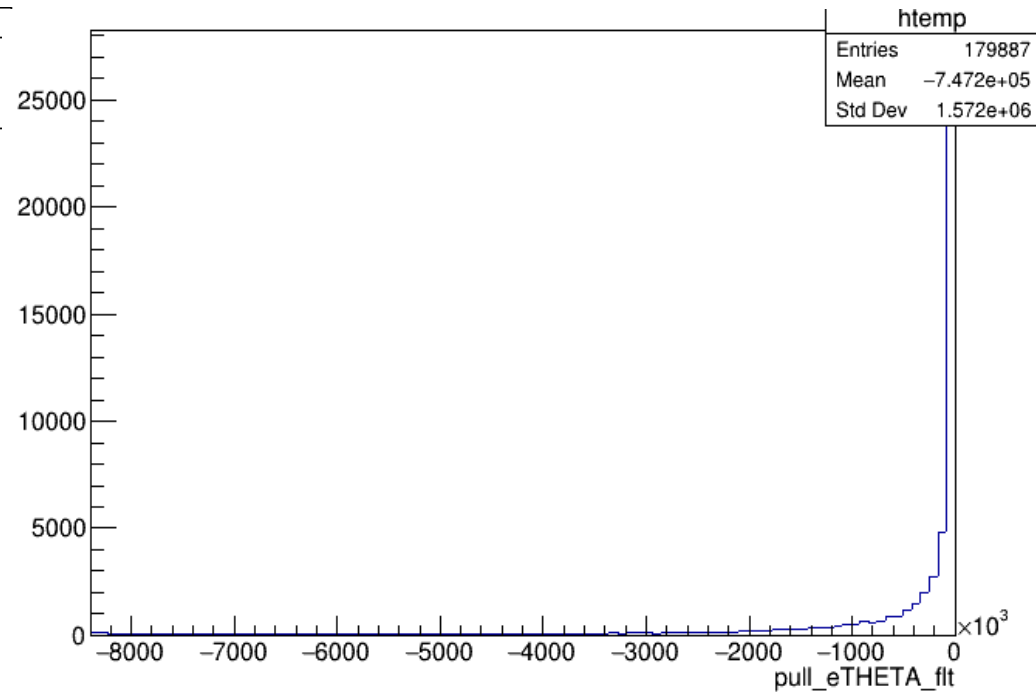
Test 1: pull of θ

(on all measurement surface)

pull_θ_pred

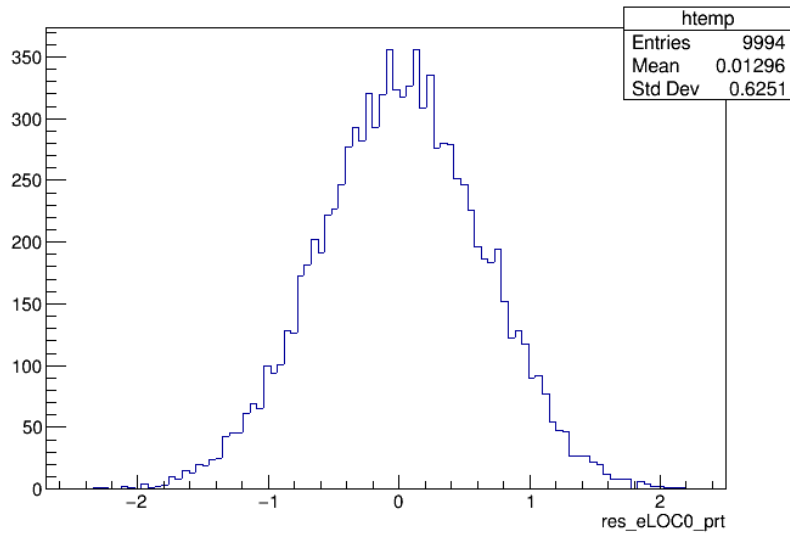


pull_θ_filtering

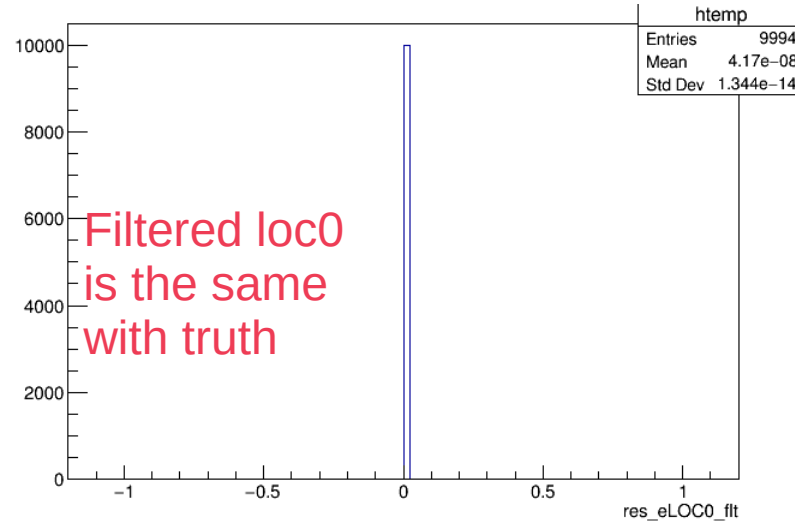


Test 2: residual of $\text{loc0}/\varphi$ (on first measurement surface)

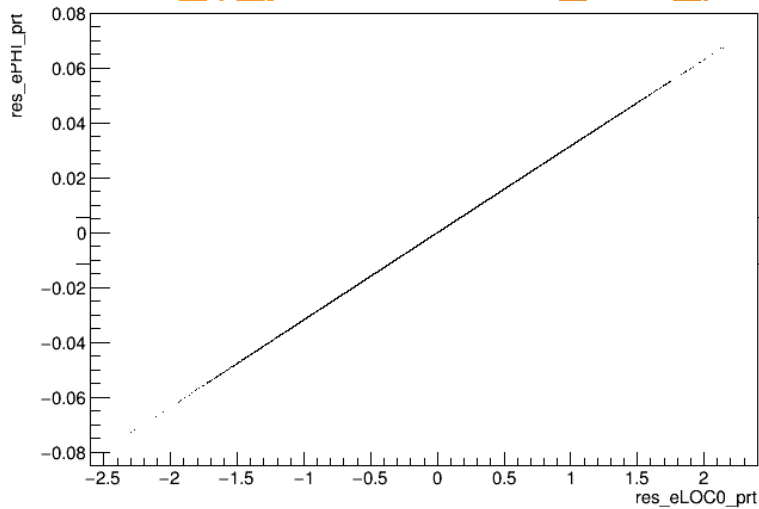
resid_loc0_pred



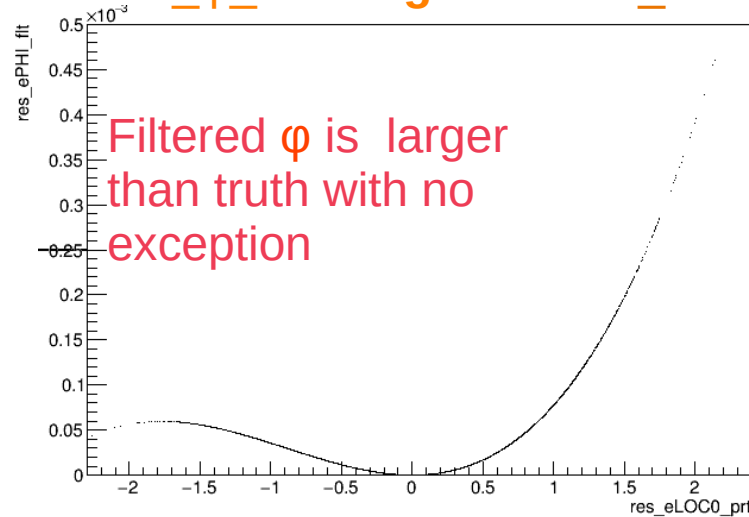
resid_loc0_filtering



resid_φ_pred vs. resid_loc0_pred



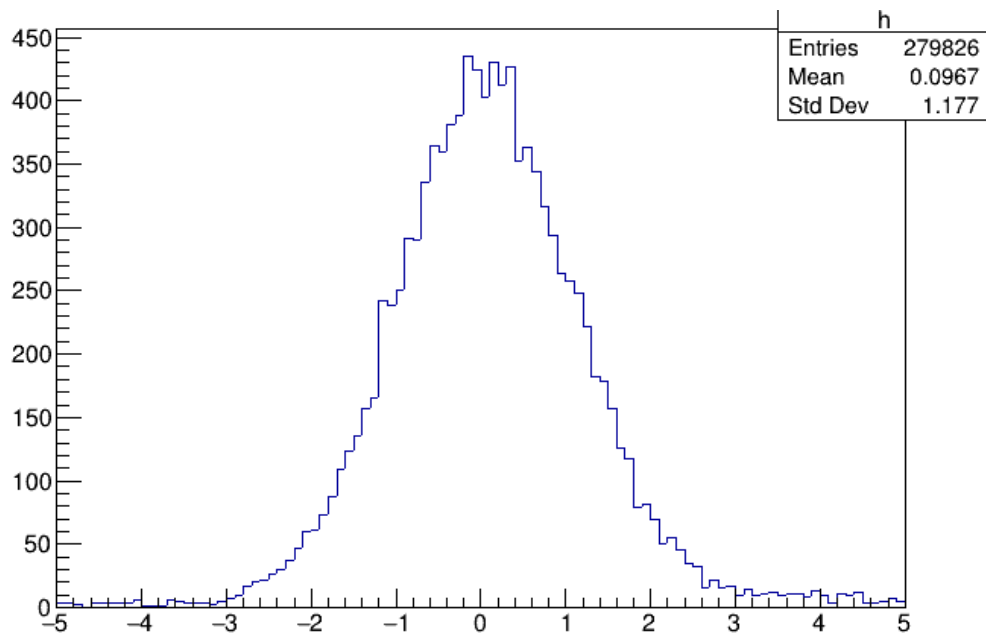
resid_φ_filtering vs. resid_loc0_pred



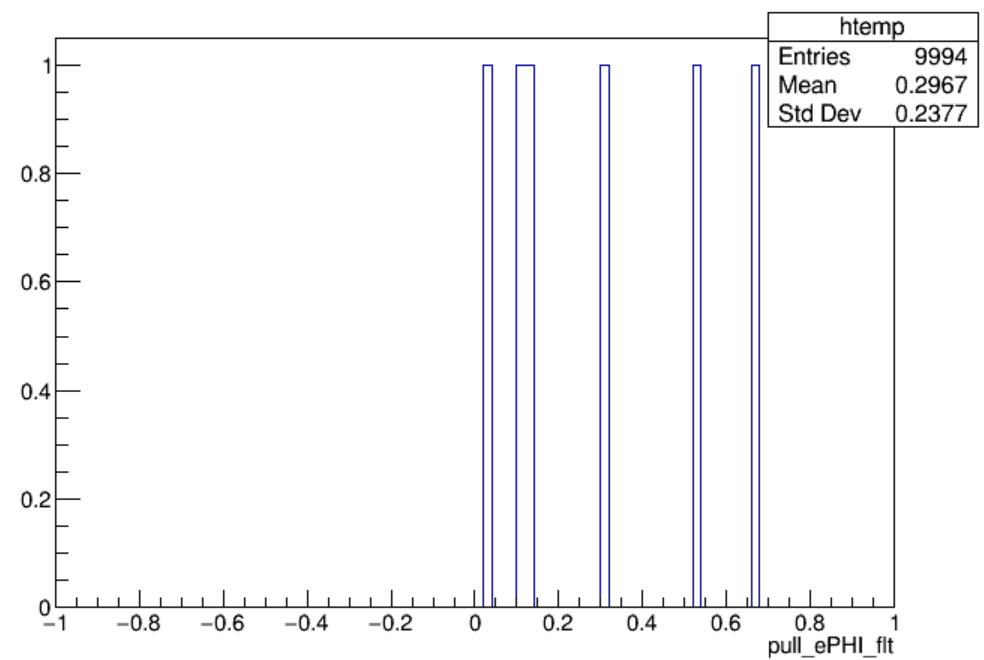
Test 2: pull of φ

(on all measurement surface)

pull_φ_pred



pull_φ_filtering



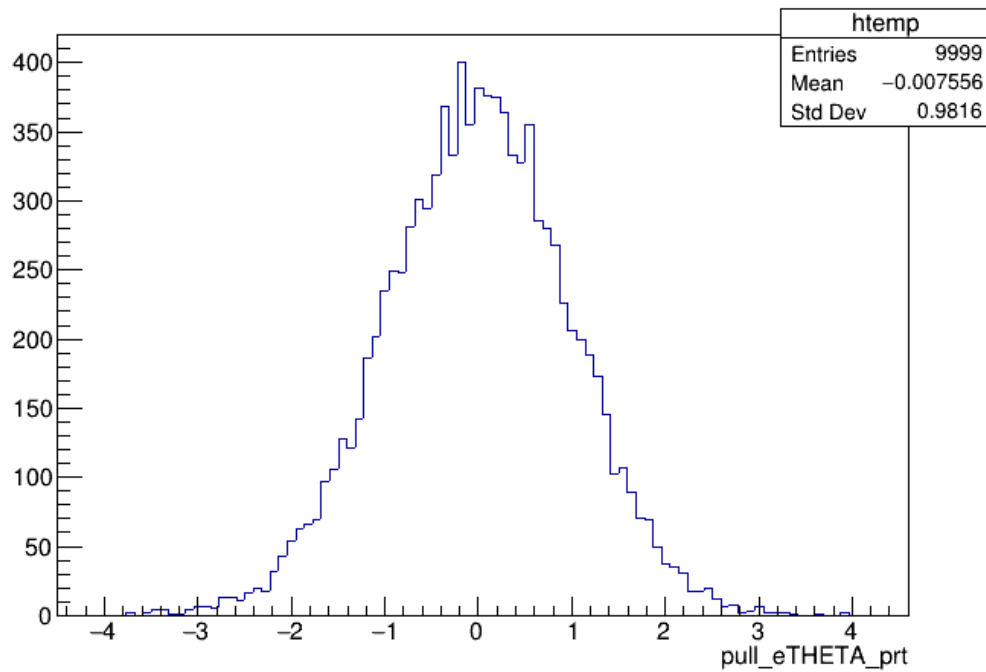
Scenario 2: with material effects

- In simulation:
 - theta/phi/E is updated with a predefined Gaussian value on each detector layer:
 - $\sigma_{\text{Theta}} = 2.0e-05$
 - $\sigma_{\text{Phi}} = 1.5e-05$
 - $\sigma_{\text{EnergyLoss}} = 4.5e-06$
- In Reconstruction:
 - The predefined Gaussian sigma in simulation is added to track parameter covariance

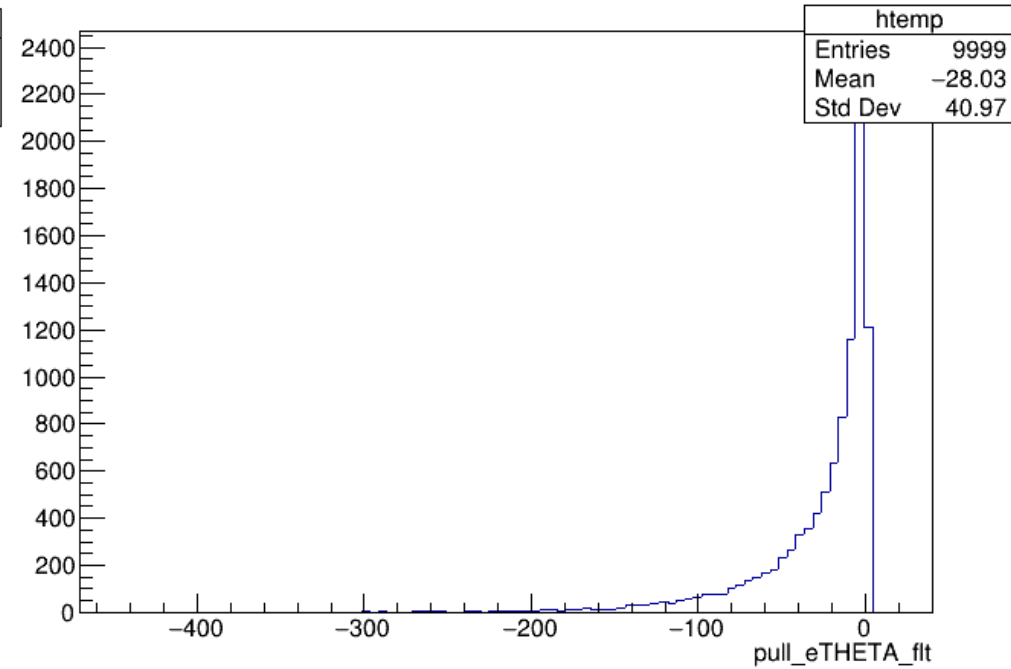
Test 1: pull of θ

(on first measurement surface)

pull_θ_pred



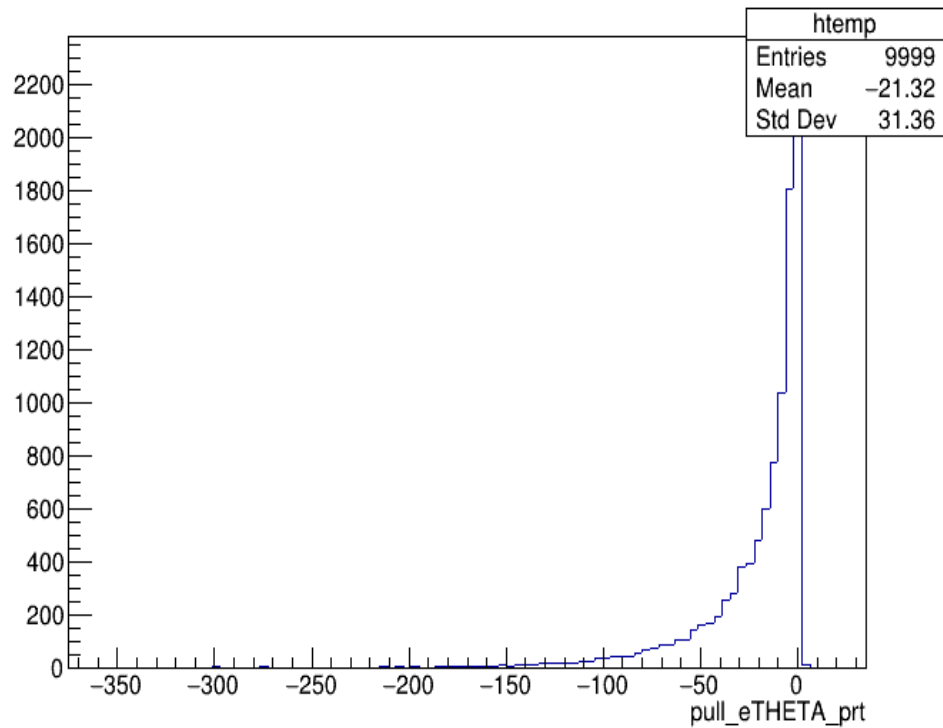
pull_θ_filtering



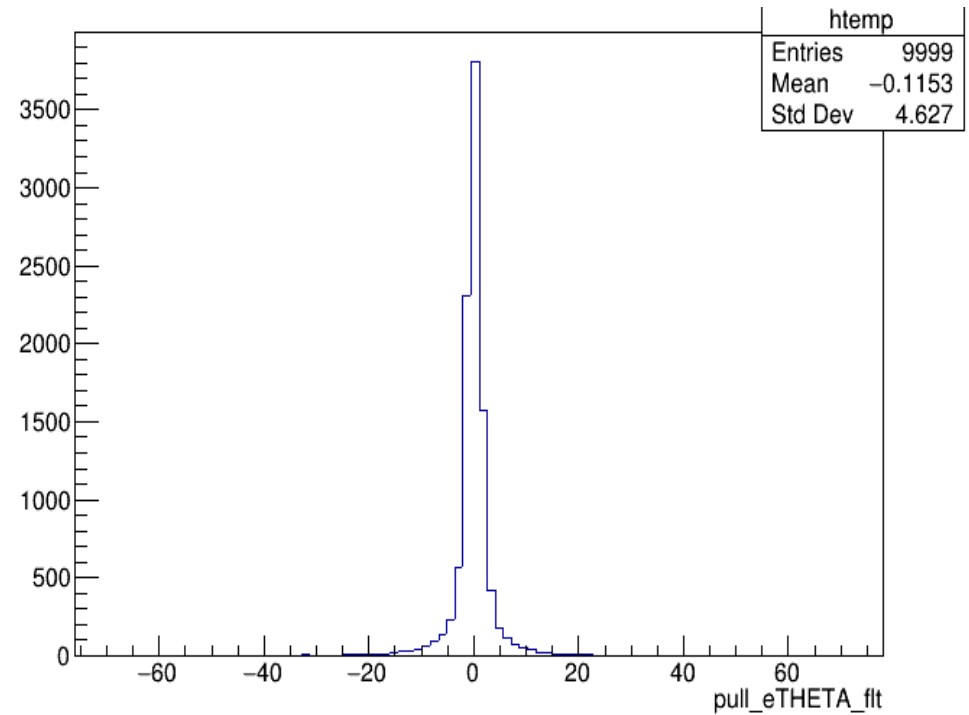
Test 1: pull of θ

(on second measurement surface)

pull_θ_pred

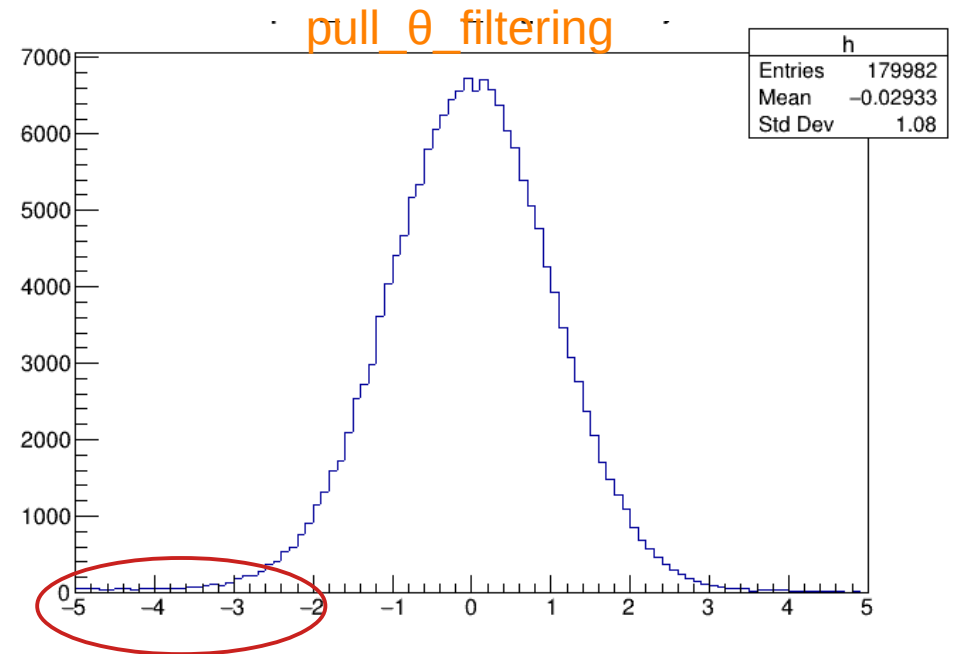
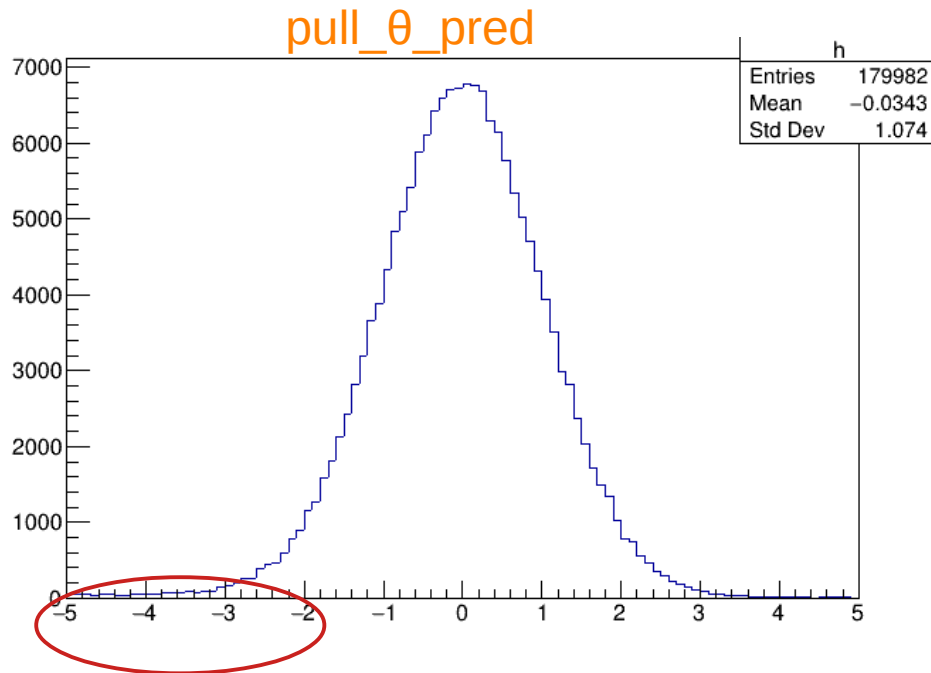


pull_θ_filtering



Test 1: pull of θ

(on all measurement surfaces)

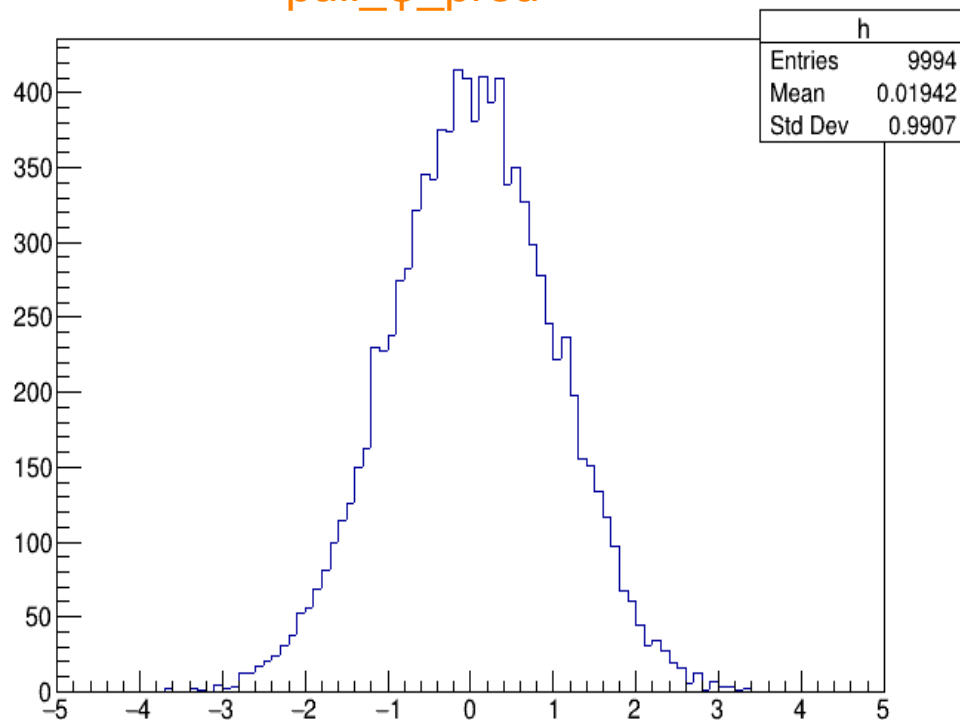


Bias still there, but not obvious

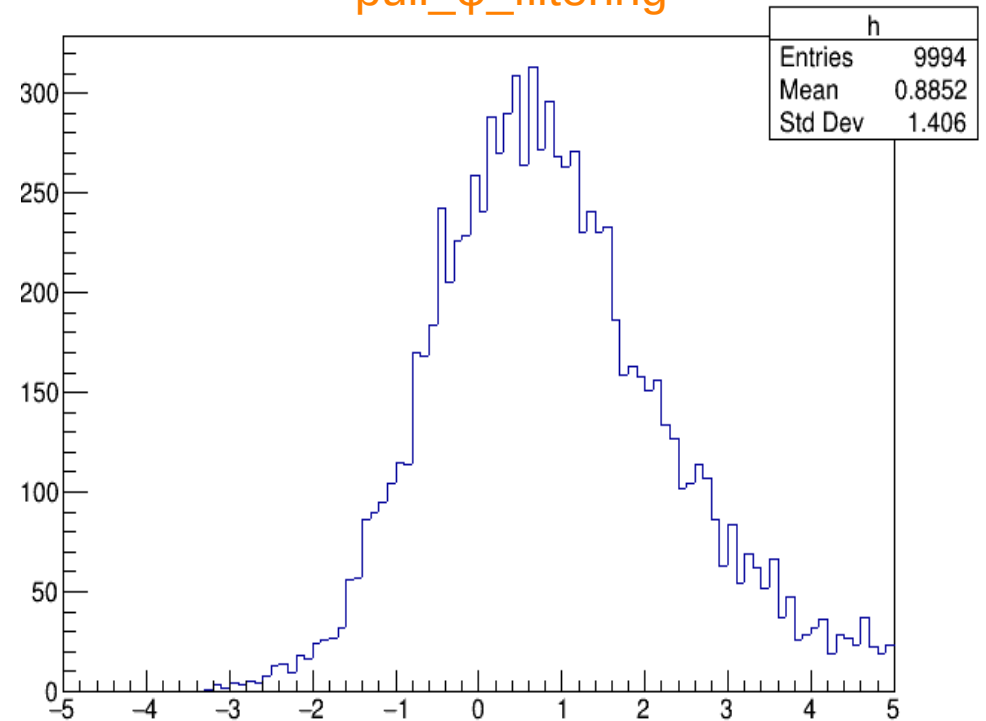
Test 2: pull of φ

(on first measurement surface)

pull_φ_pred



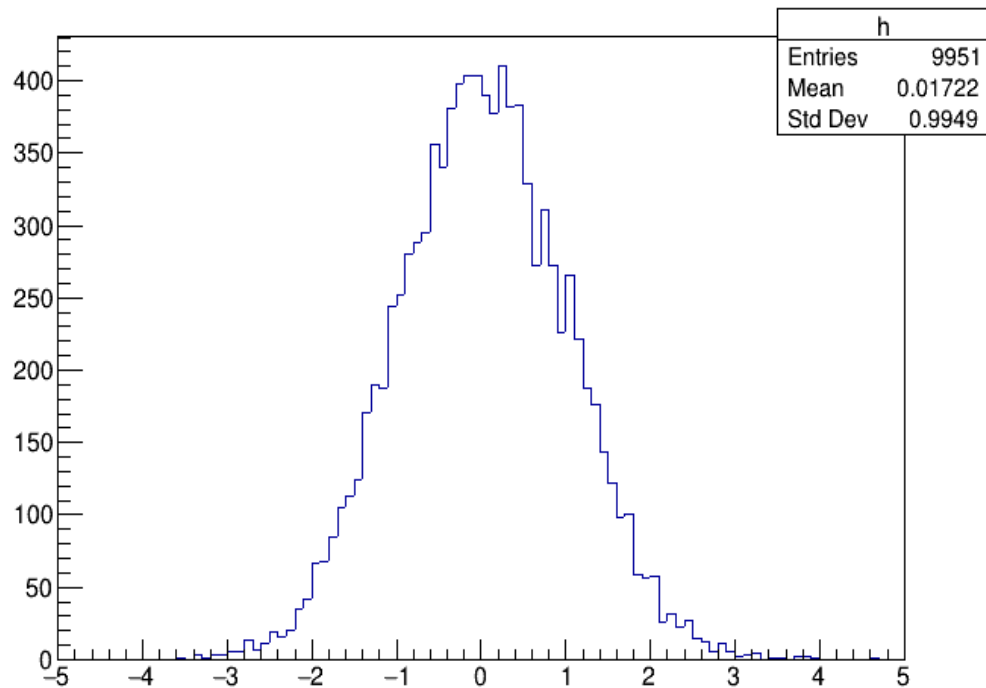
pull_φ_filtering



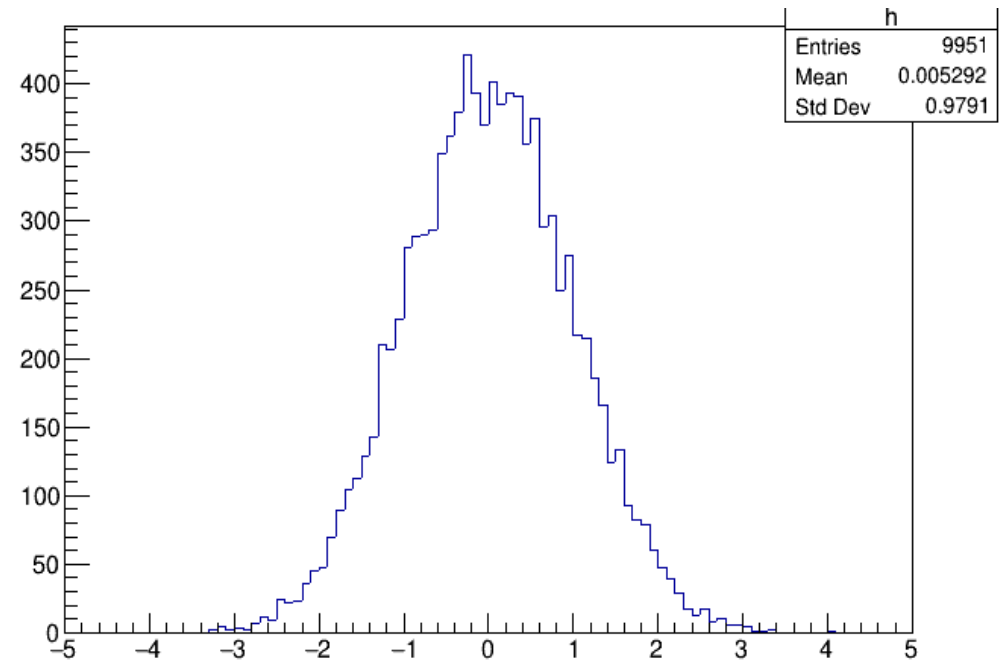
Test 2: pull of φ

(on second measurement surface)

pull_φ_pred



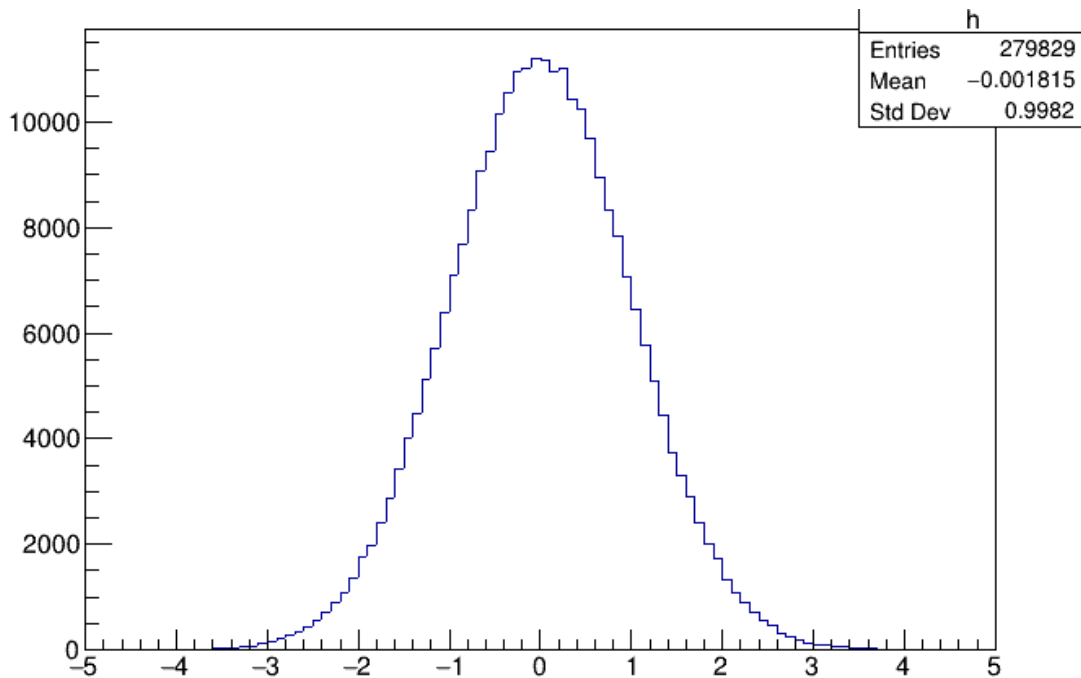
pull_φ_filtering



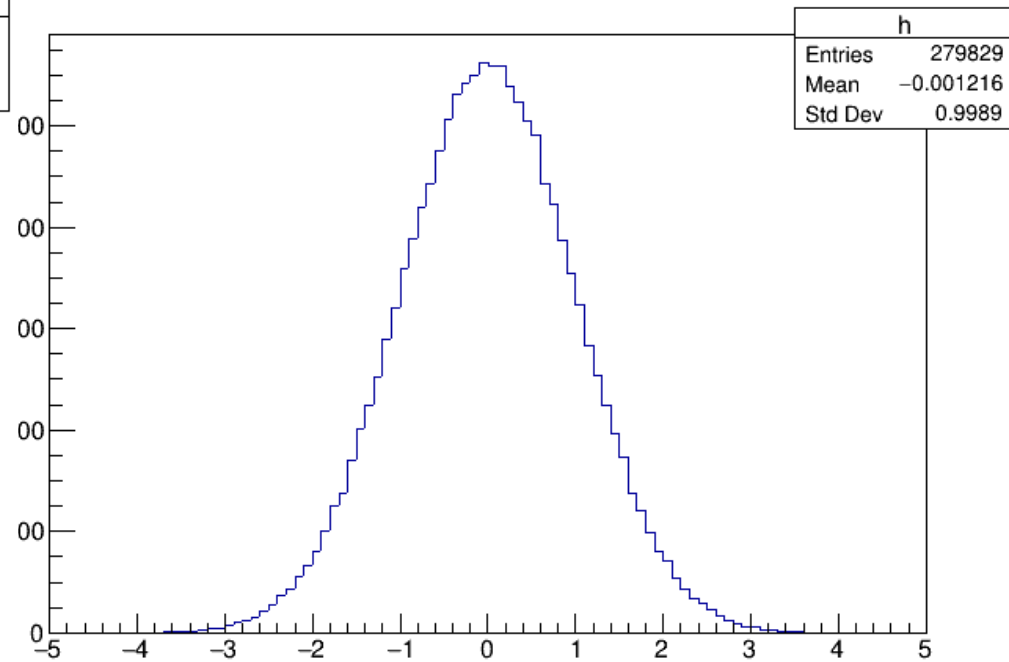
Test 2: pull of φ

(on all measurement surface)

pull_φ_pred

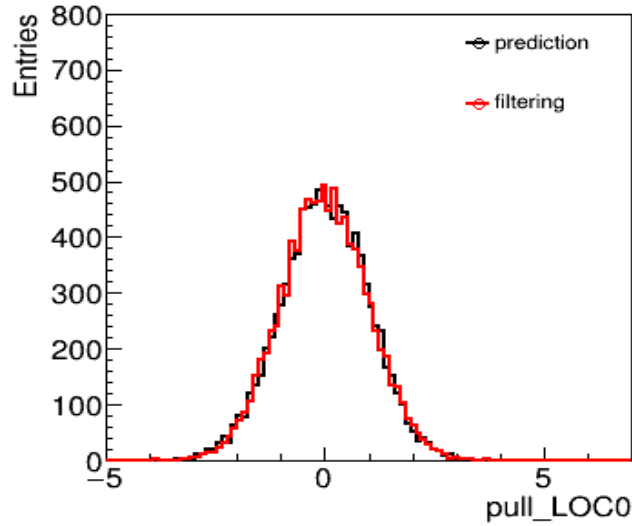


pull_φ_filtering

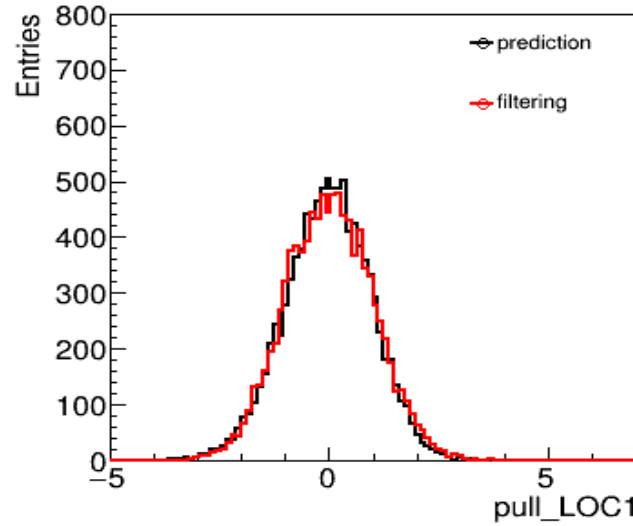


Test 3: pull of track parameter (on first measurement surface)

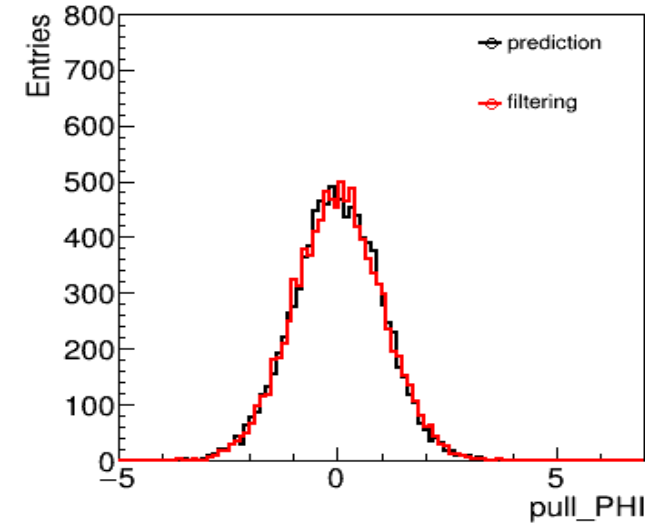
pull of LOC0



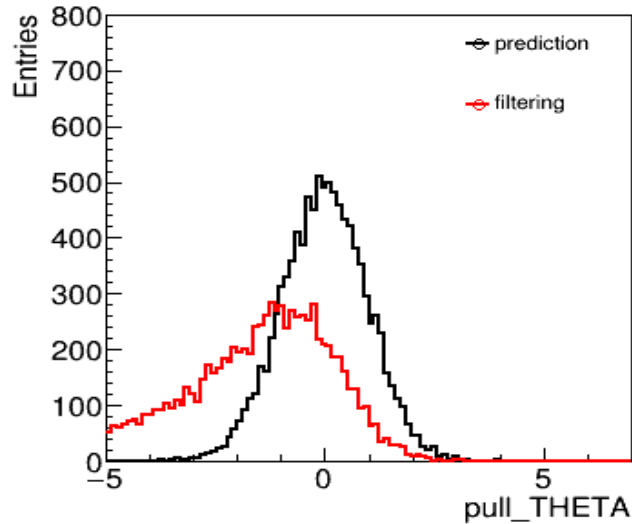
pull of LOC1



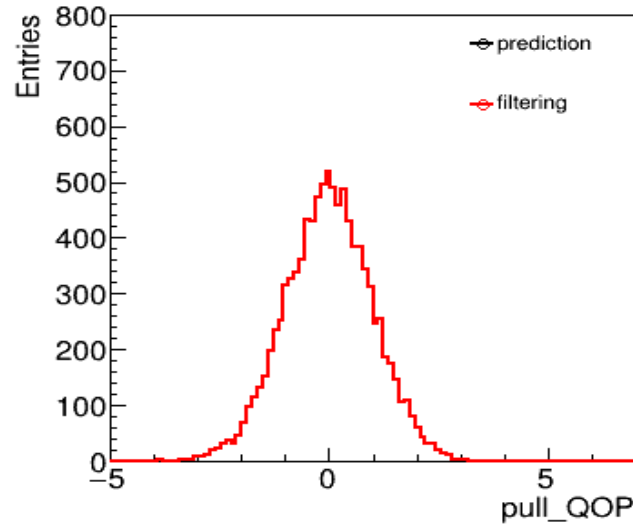
pull of PHI



pull of THETA

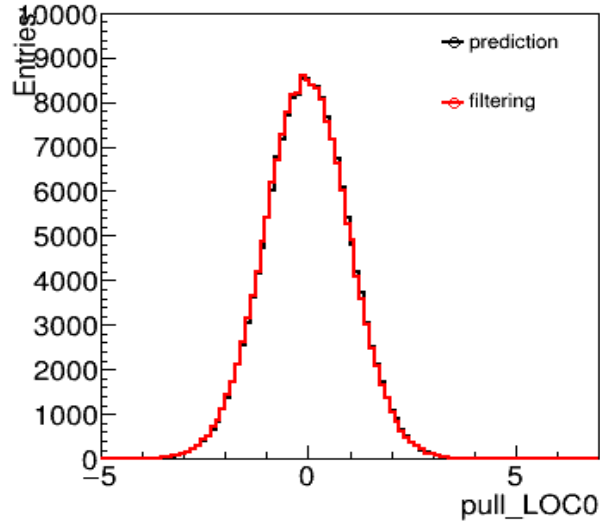


pull of QOP

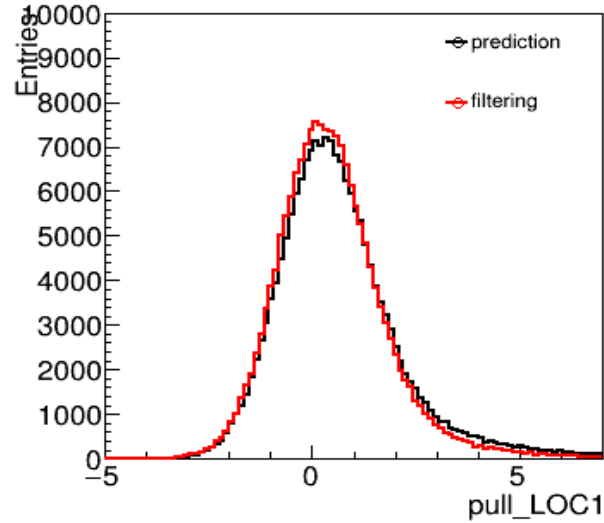


Test 3: pull of track parameter (on all measurement surface)

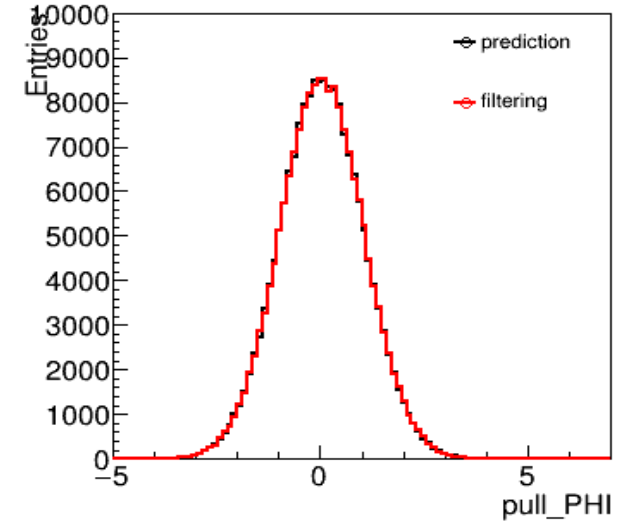
pull of LOC0



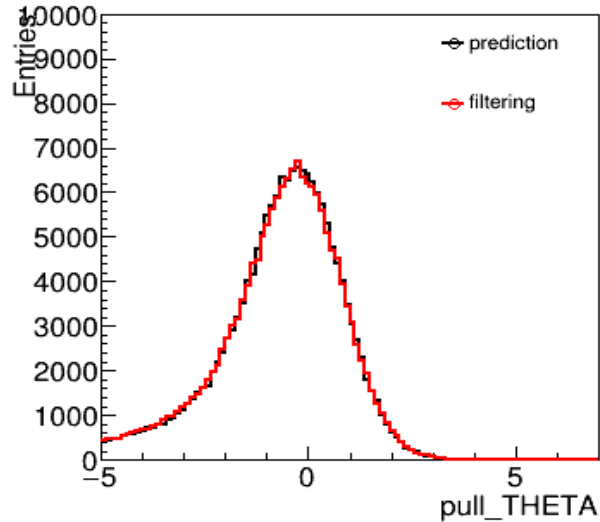
pull of LOC1



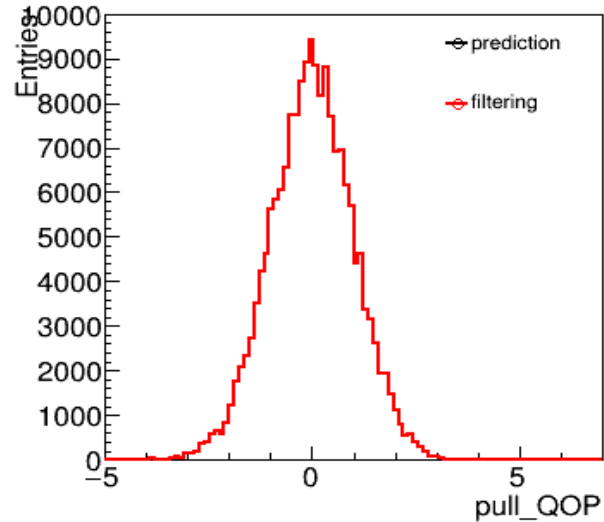
pull of PHI



pull of THETA

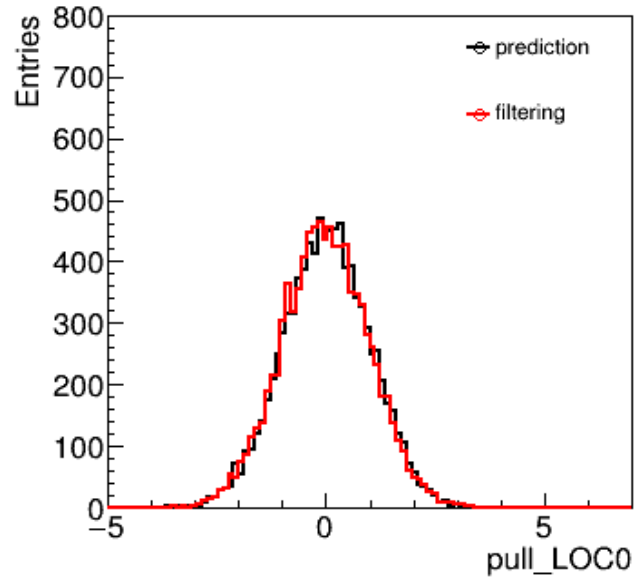


pull of QOP

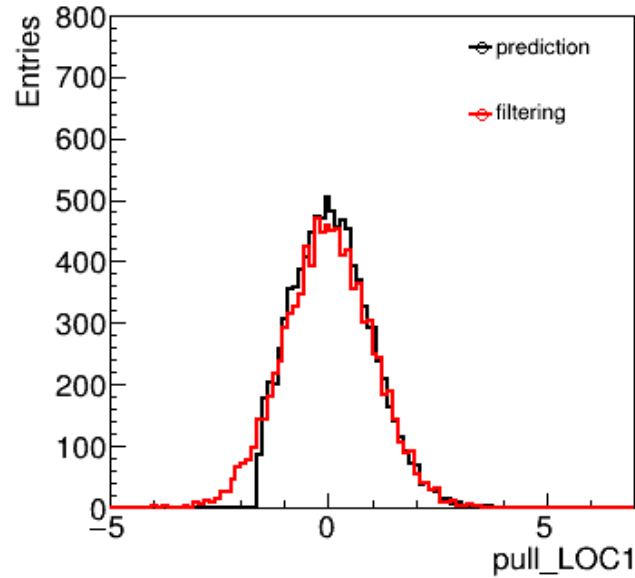


Test 4: pull of track parameter (on first measurement surface)

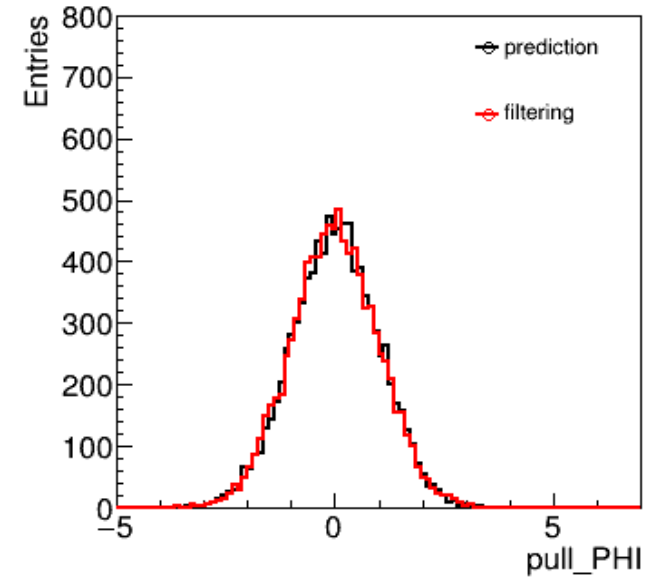
pull of LOC0



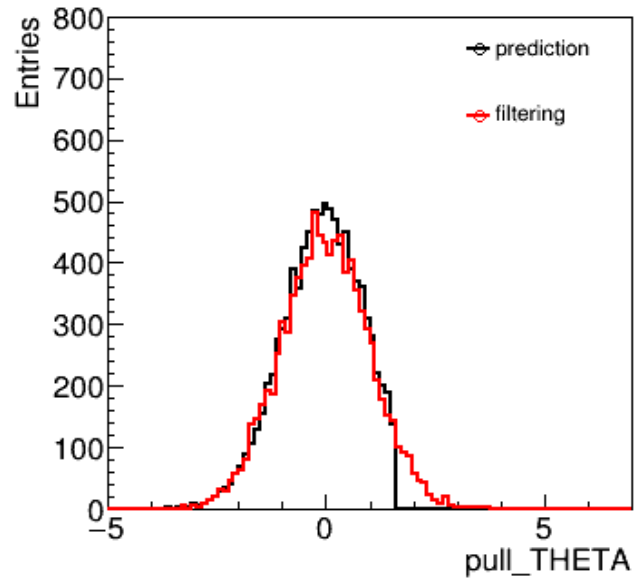
pull of LOC1



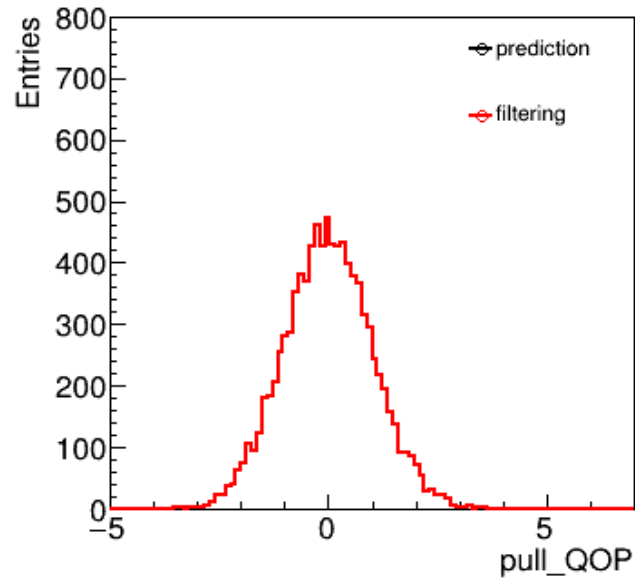
pull of PHI



pull of THETA

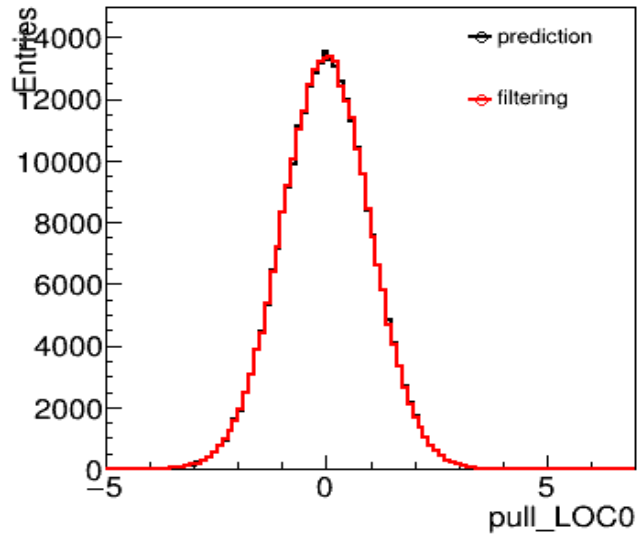


pull of QOP

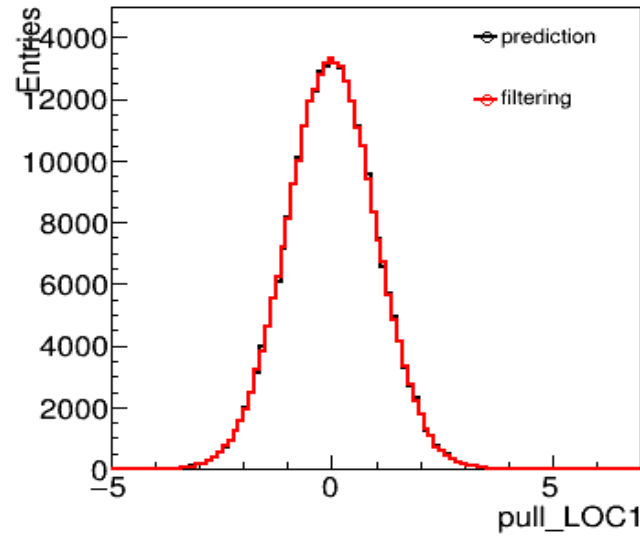


Test 4: pull of track parameter (on all measurement surface)

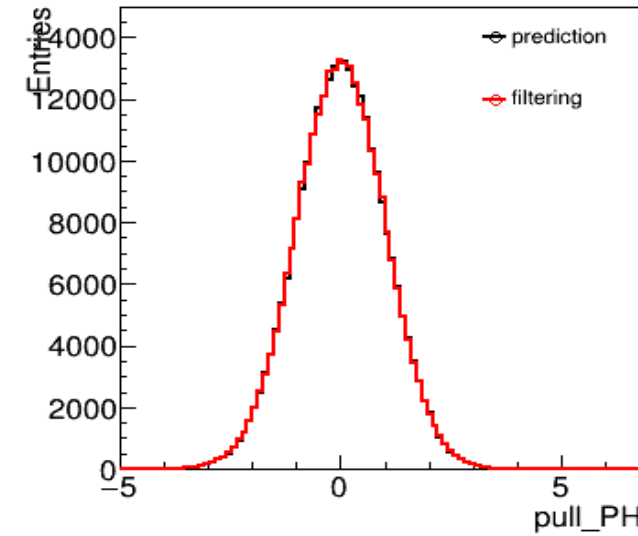
pull of LOC0



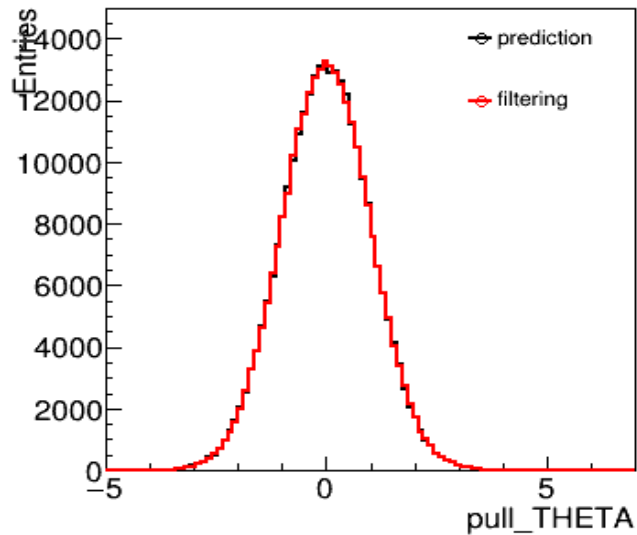
pull of LOC1



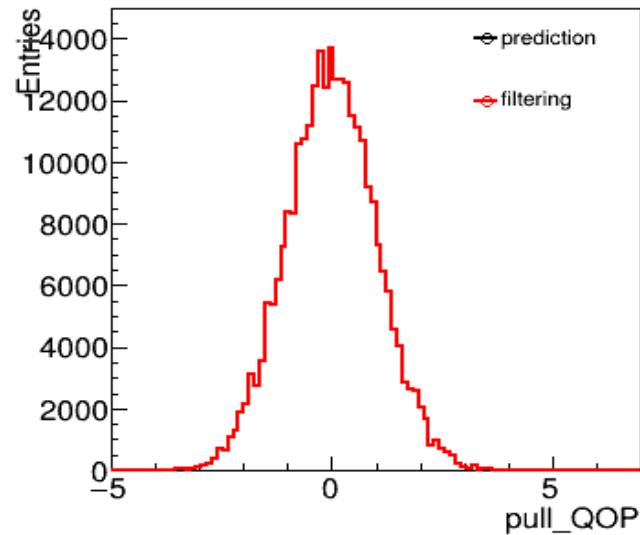
pull of PHI



pull of THETA



pull of QOP



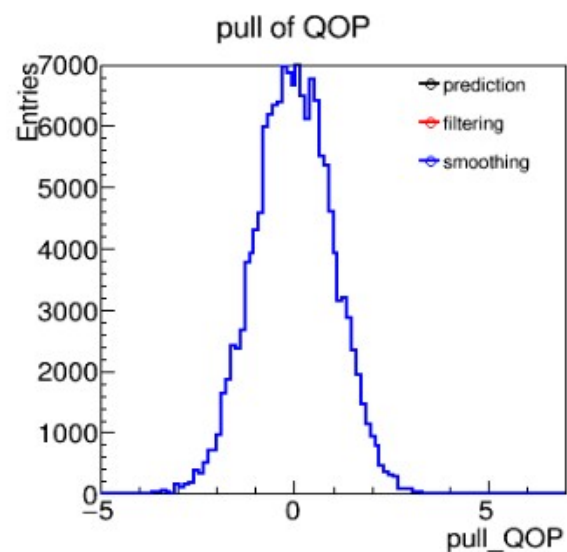
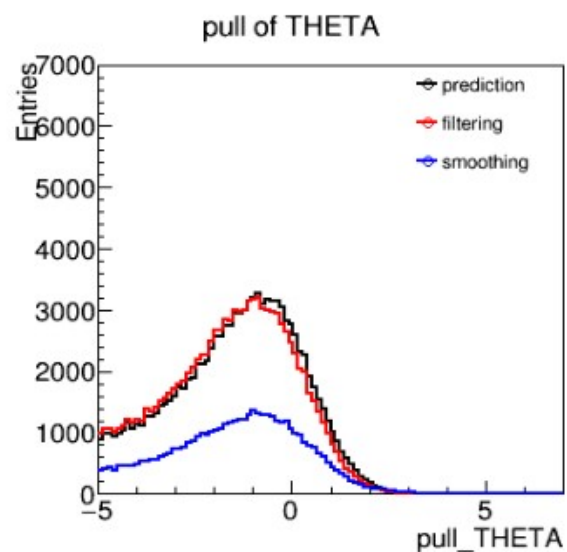
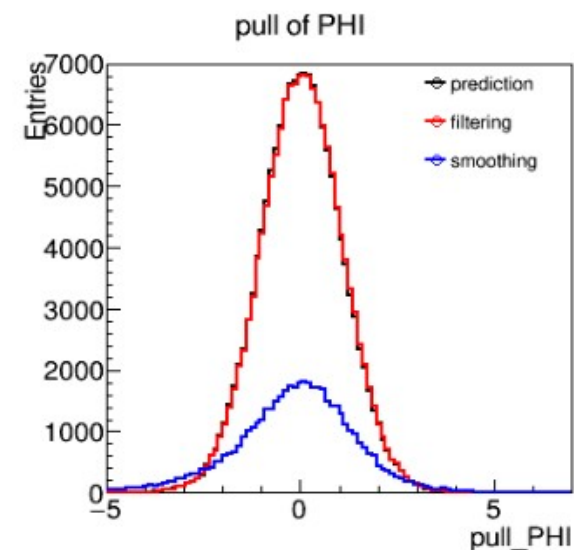
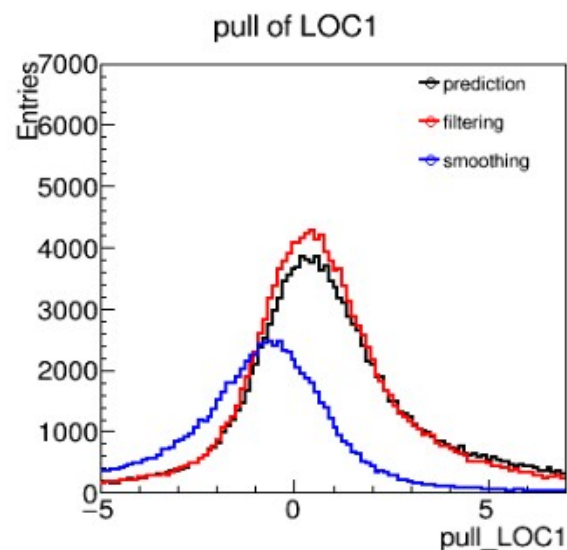
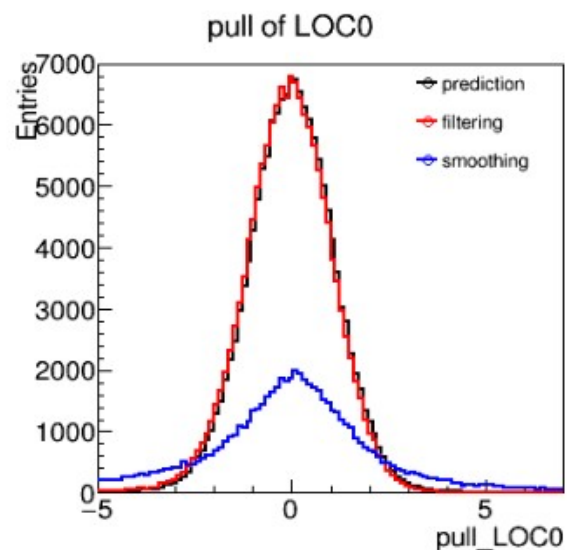
Summary

- KalmanFitter filtering has a limited accuracy for a non-linear system
 - pull of track parameter could be biased
- However, the limitation of KalmanFitter is not obvious with material effects
 - pull of track parameter will be affected mainly by material effects
- Remaining issue: Kalman Fitter smooth gives negative covariance matrix

```
G =  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
smoothedCov =  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan  
-nan -nan -nan -nan -nan
```

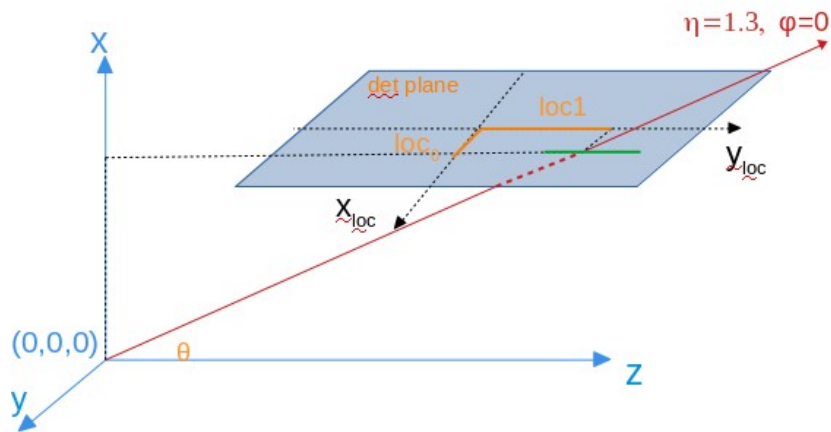
backup

Prediction/filtering/smoothing at all possible measurement surfaces

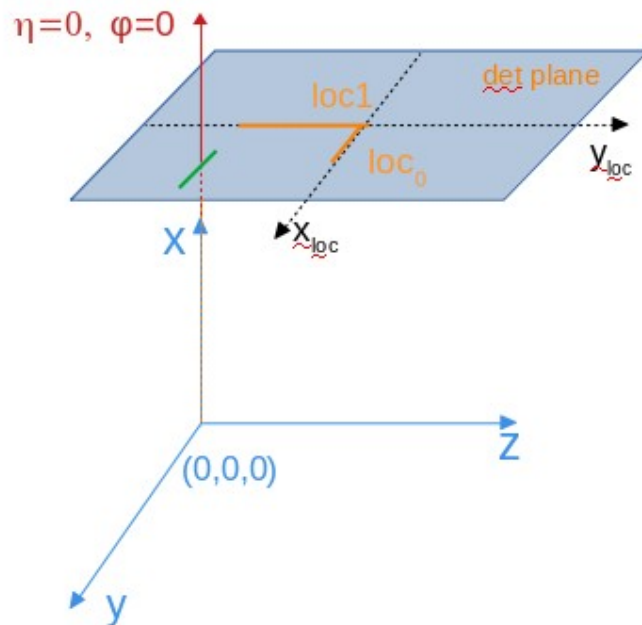


No material

$d_0 = 0$, $z_0 = 0$
 $\eta = 1.6$
 $\phi = 0$
 $p_T = 10 \text{ GeV}/c$

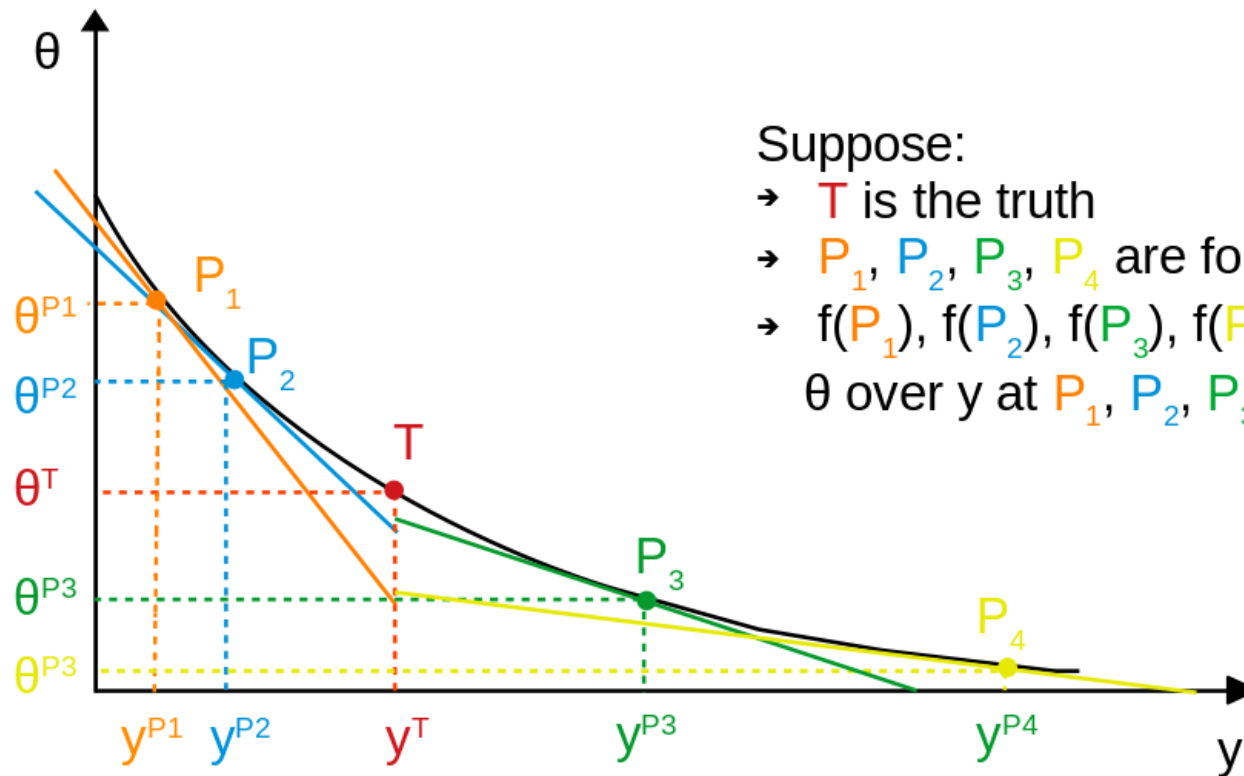


(x_s, y_s, z_s) is the global coordinate of the surface, $loc_1 = r \cot \theta - Z_s$
 $\Rightarrow d\theta/dloc1 = -\sin^2 \theta / r$



$$Loc_0 = r \sin \varphi - c_0$$

What's happening in the filtering



Suppose:

- T is the truth
- P_1, P_2, P_3, P_4 are four possible prediction
- $f(P_1), f(P_2), f(P_3), f(P_4)$ are the derivative of θ over y at P_1, P_2, P_3, P_4 , respectively.

When we use information at prediction to estimate the value of θ at truth, we will get:

- $\theta^{P1} + f(P_1) * (y^{P1} - y^T) < \theta^T$
- $\theta^{P2} + f(P_2) * (y^{P2} - y^T) < \theta^T$
- $\theta^{P3} + f(P_3) * (y^{P3} - y^T) < \theta^T$
- $\theta^{P4} + f(P_4) * (y^{P4} - y^T) < \theta^T$

$$K_k = C_k^{k-1} H_k^T (V_k + H_k C_k^{k-1} H_k^T)^{-1}$$

$$x_k = x_k^{k-1} + K_k (m_k - h_k(x_k^{k-1}))$$

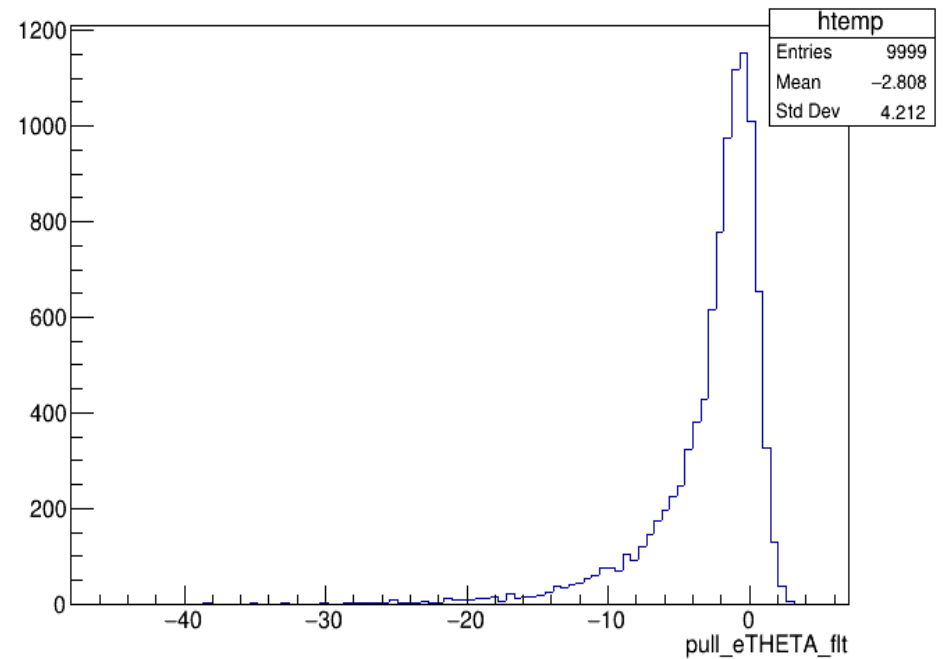
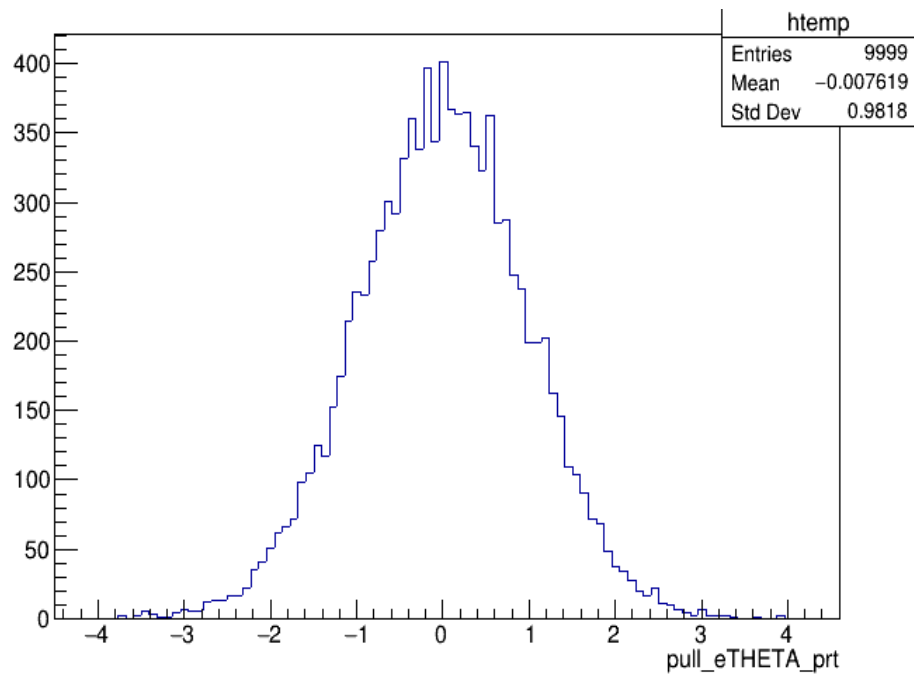
$$C_k = (1 - K_k H_k) C_k^{k-1}$$

i.e. the estimated value of θ will always be smaller than θ^T !

Scenario 3: more material effects

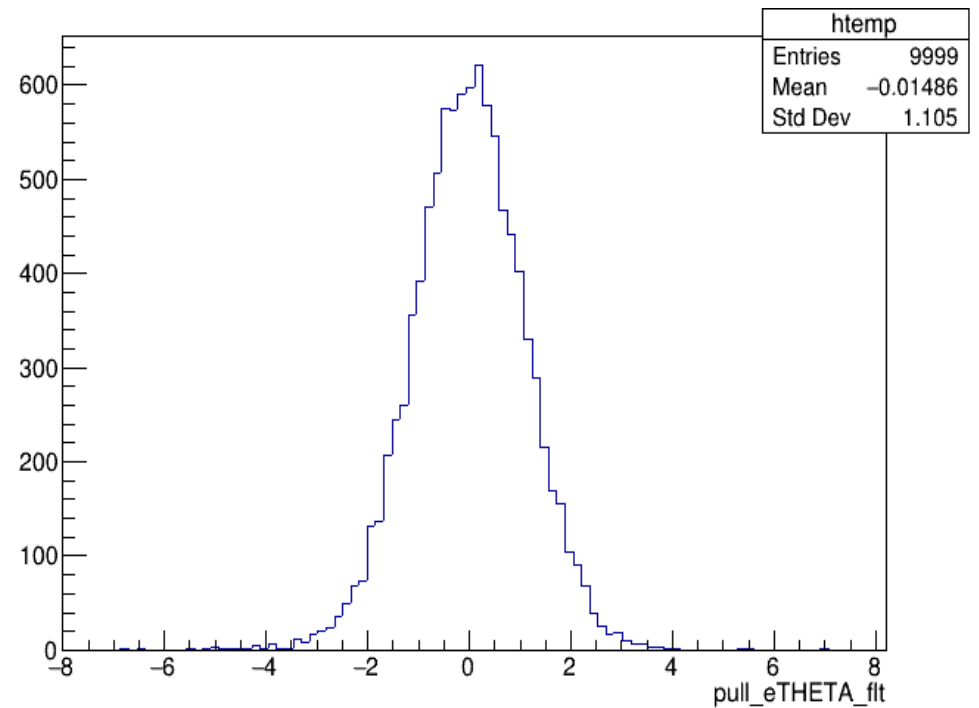
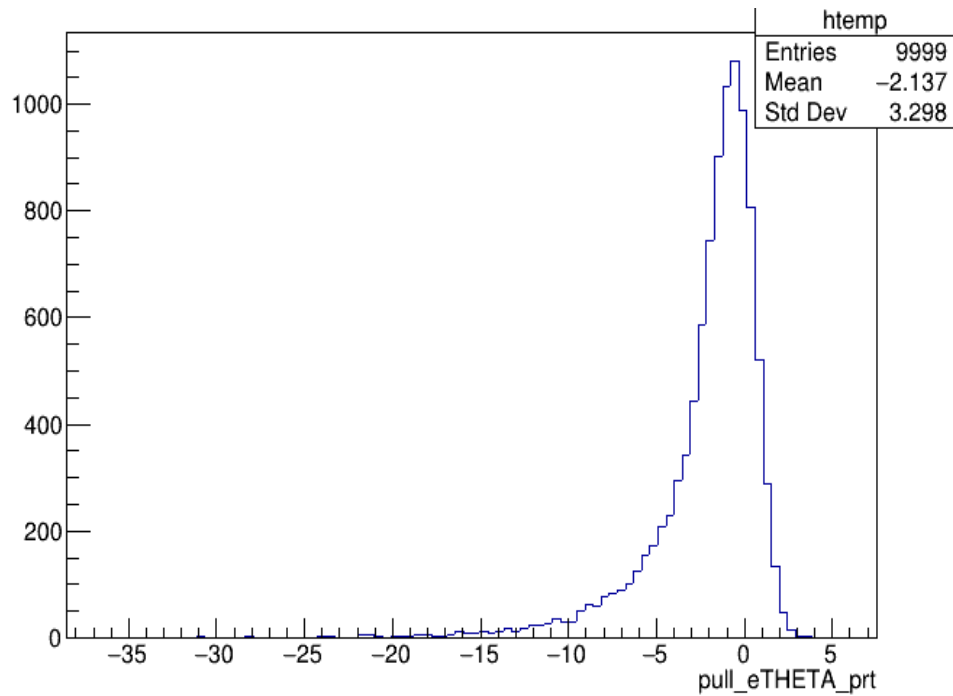
Test 1: pull of θ

(on first measurement surface)



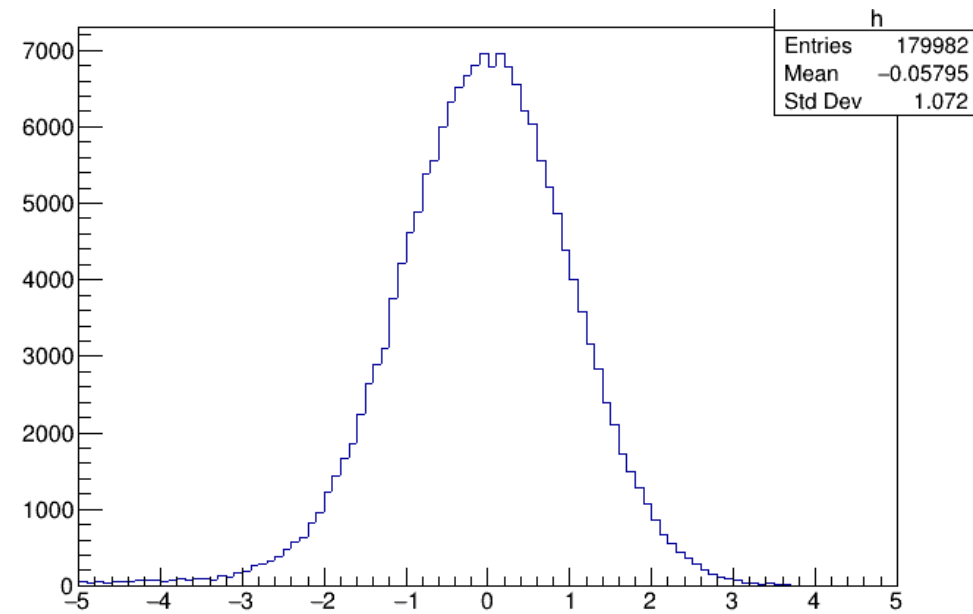
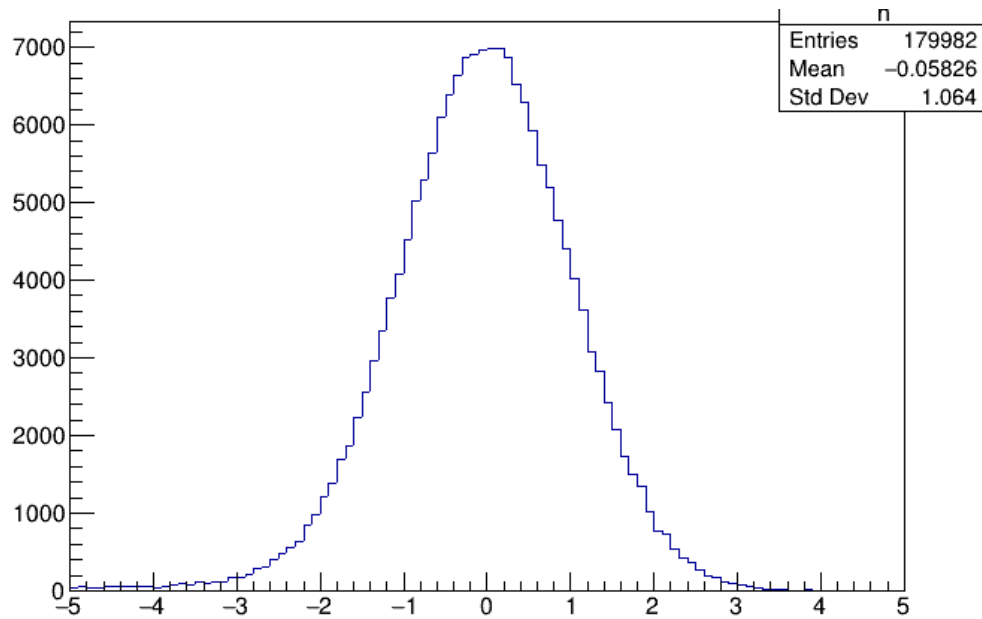
Test 1: pull of θ

(on second measurement surface)



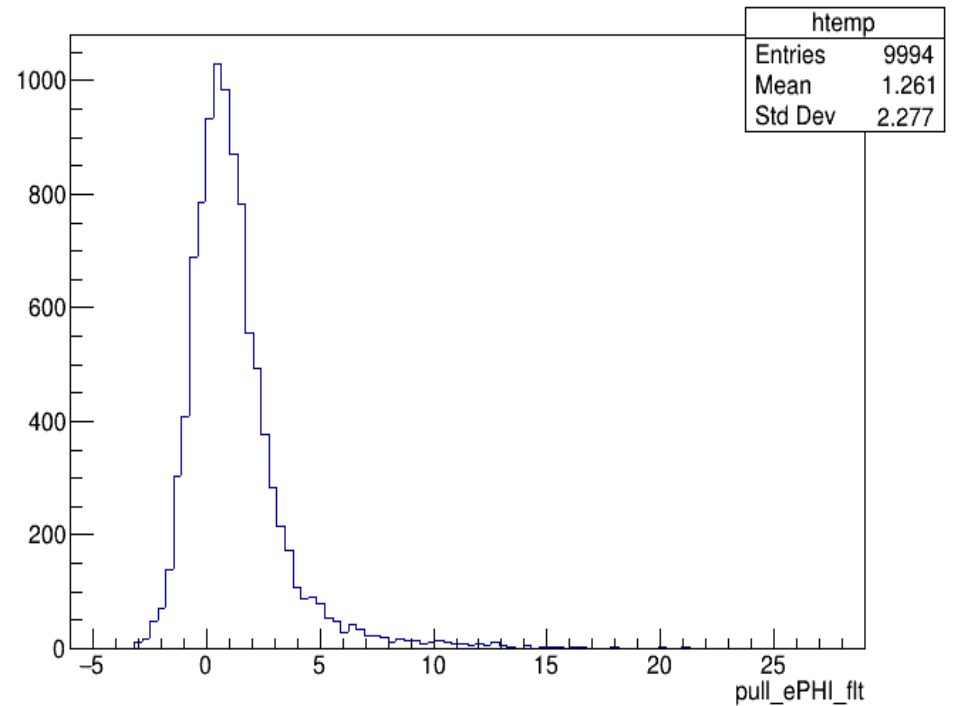
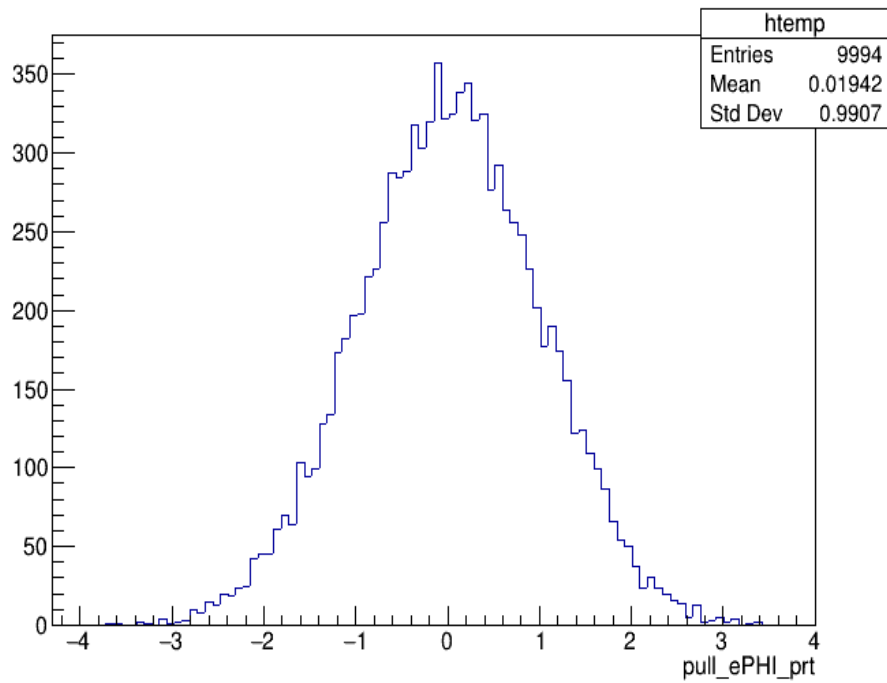
Test 1: pull of θ

(on all measurement surface)



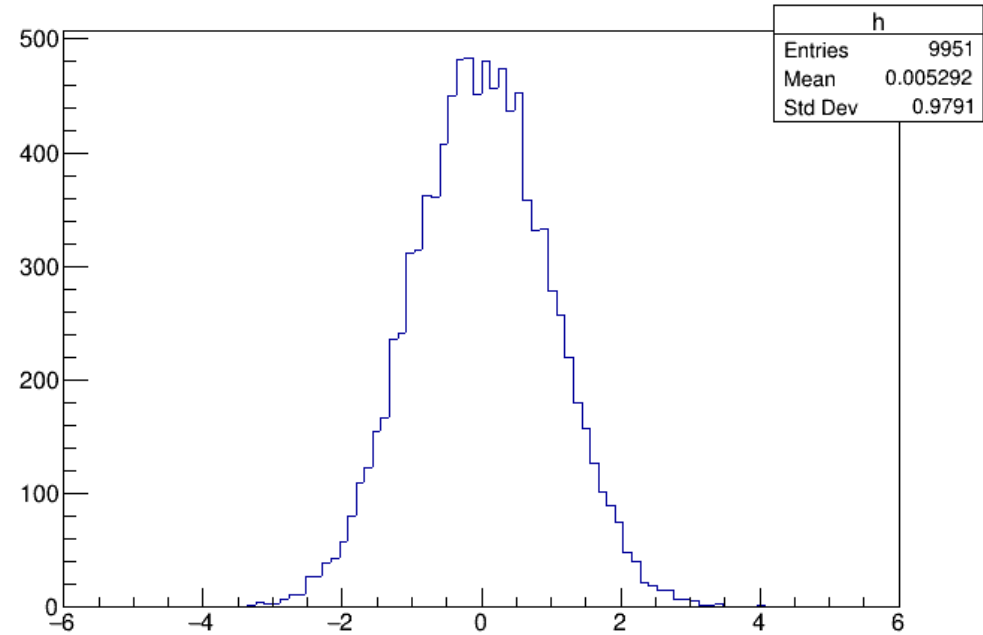
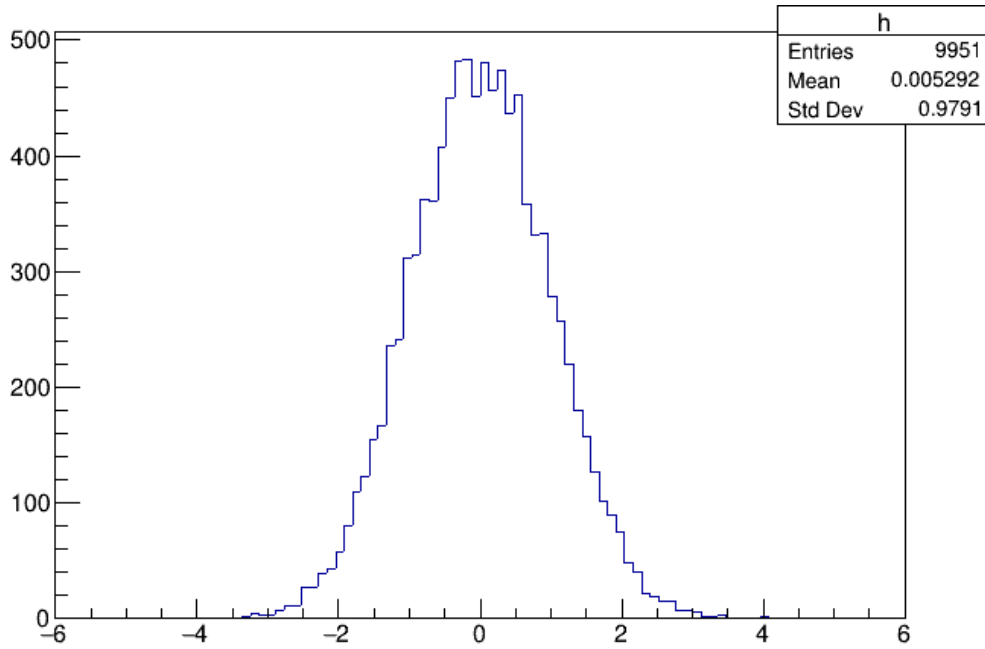
Test 2: pull of φ

(on first measurement surface)



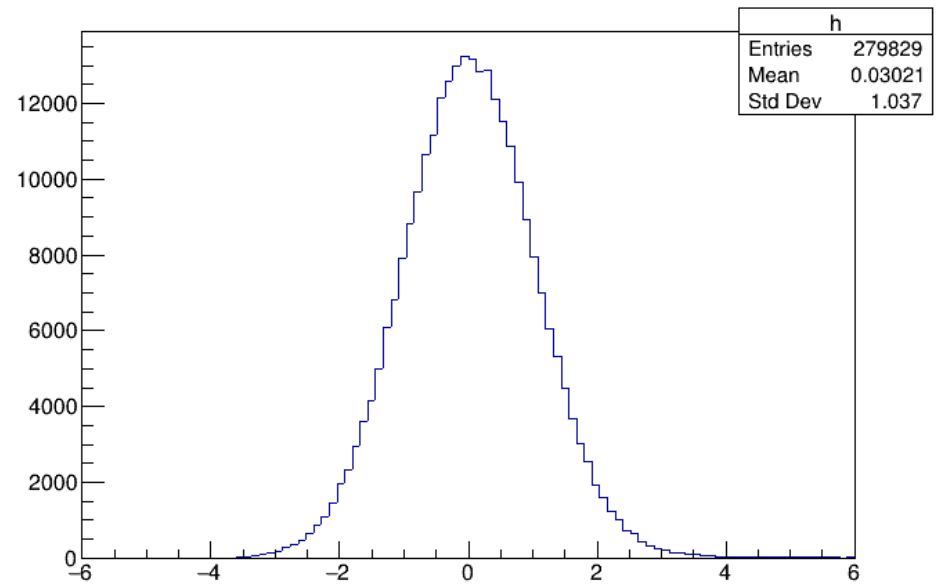
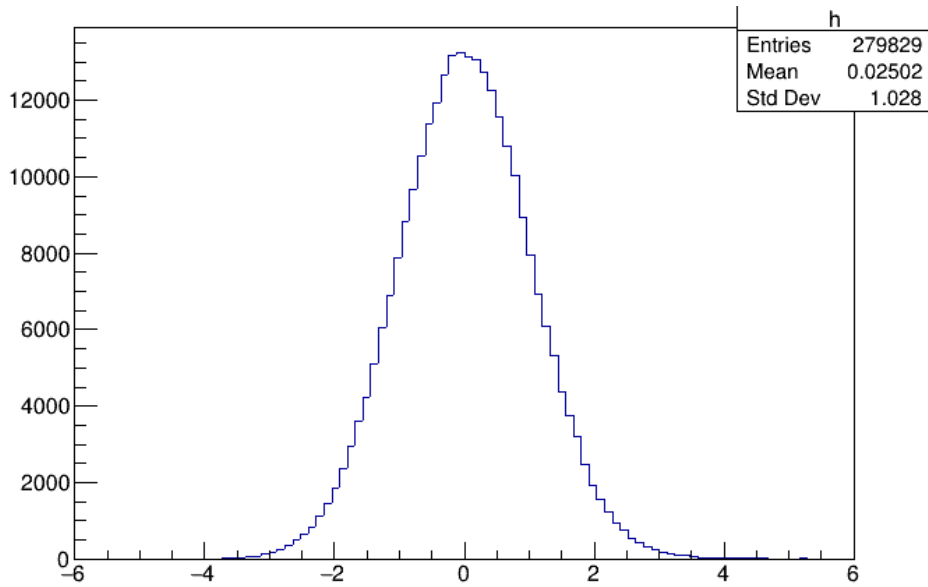
Test 2: pull of φ

(on second measurement surface)

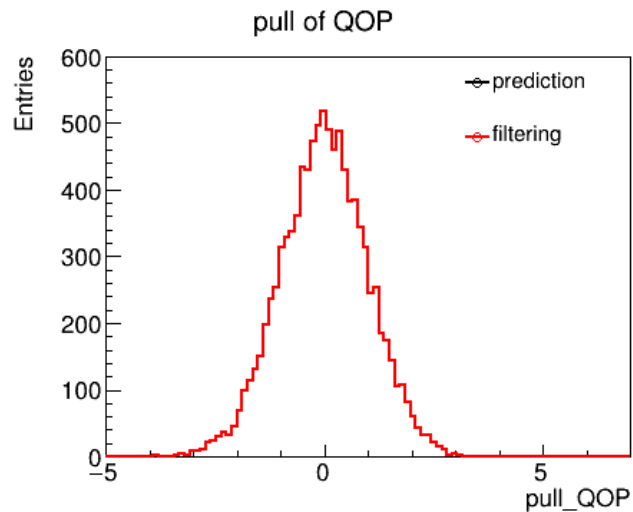
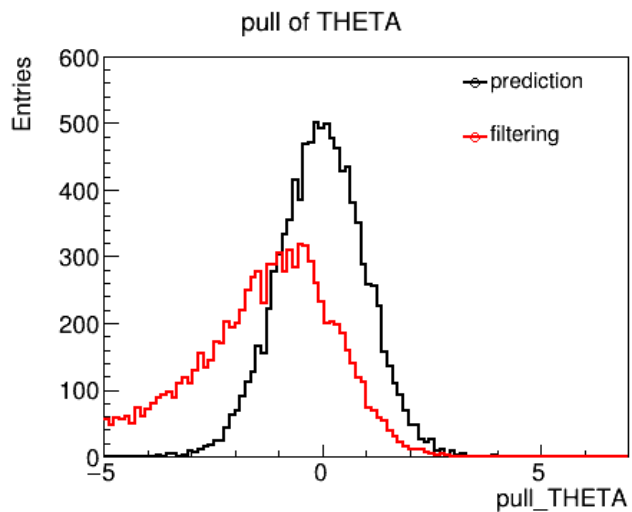
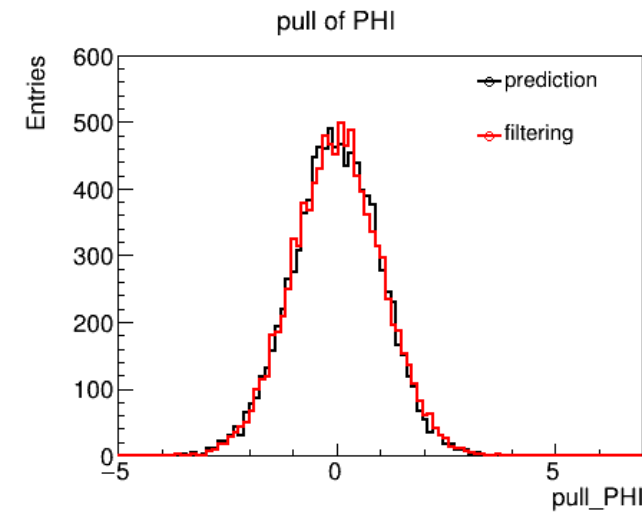
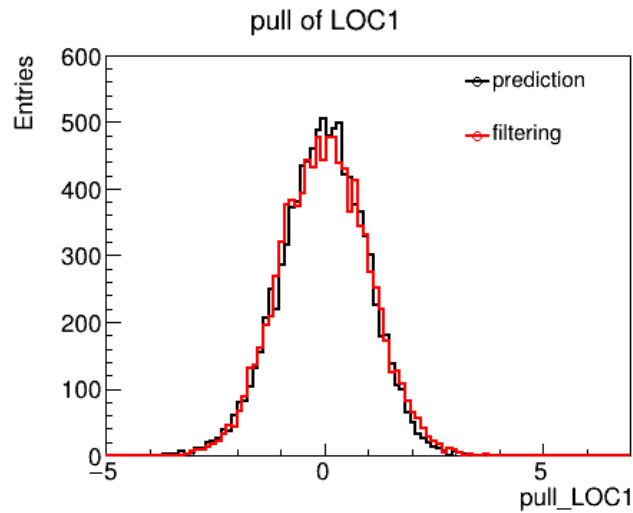
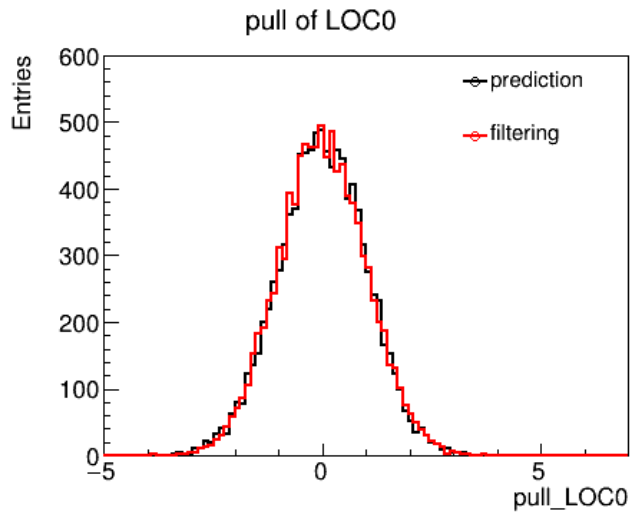


Test 2: pull of φ

(on all measurement surface)

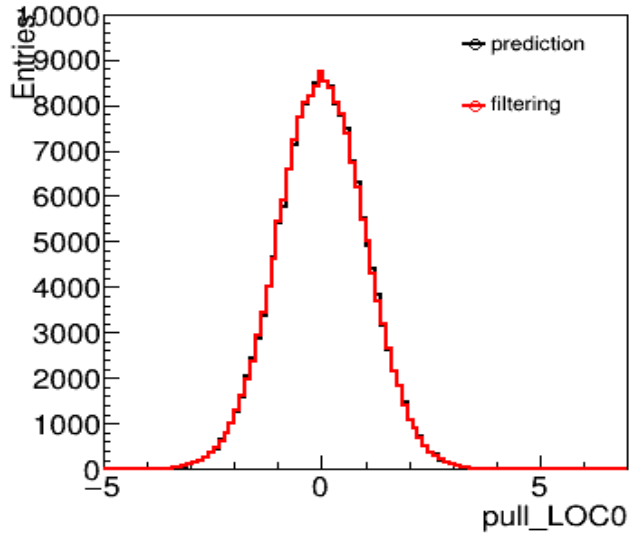


Test 3: pull of track parameter (on first measurement surface)

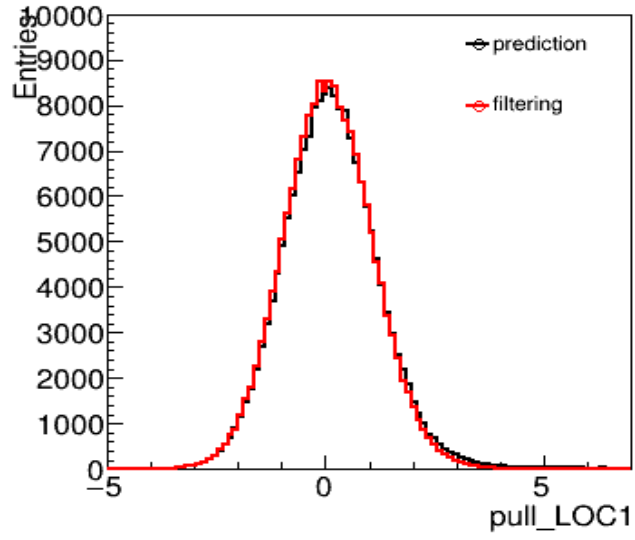


Test 3: pull of track parameter (on first measurement surface)

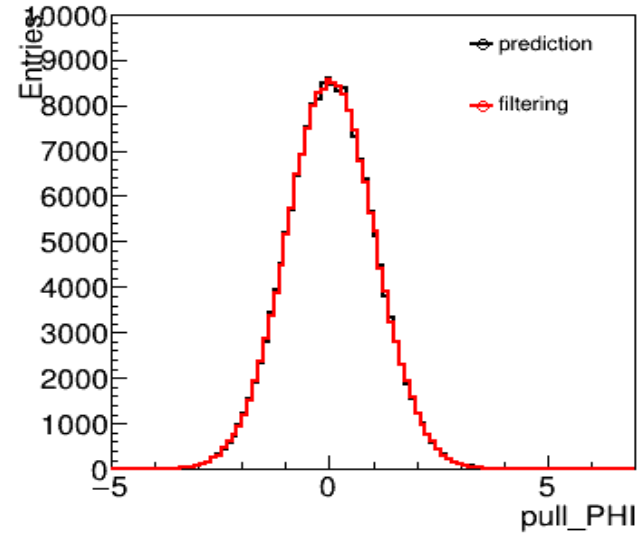
pull of LOC0



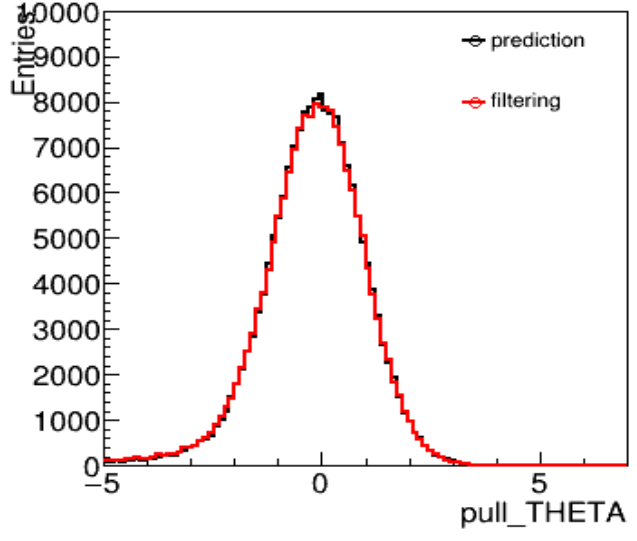
pull of LOC1



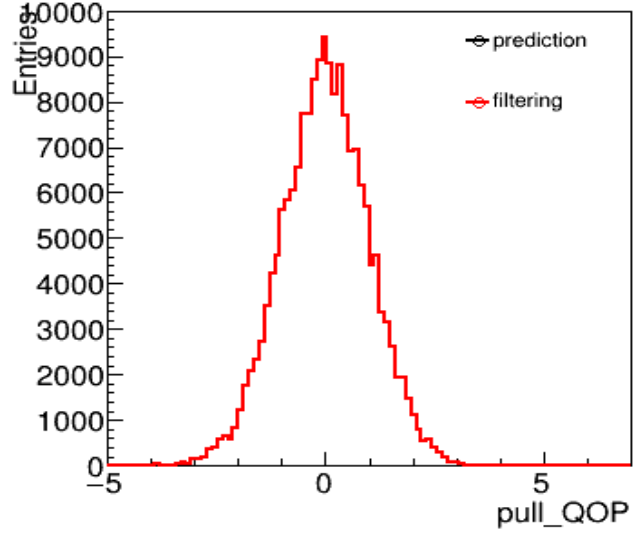
pull of PHI



pull of THETA

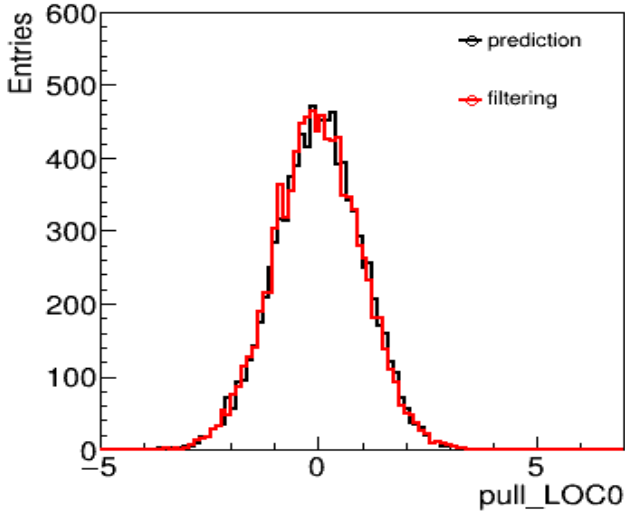


pull of QOP

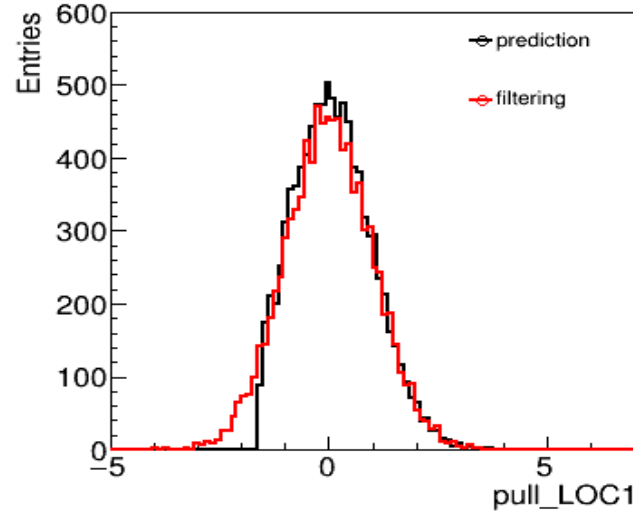


Test 4: pull of track parameter (on first measurement surface)

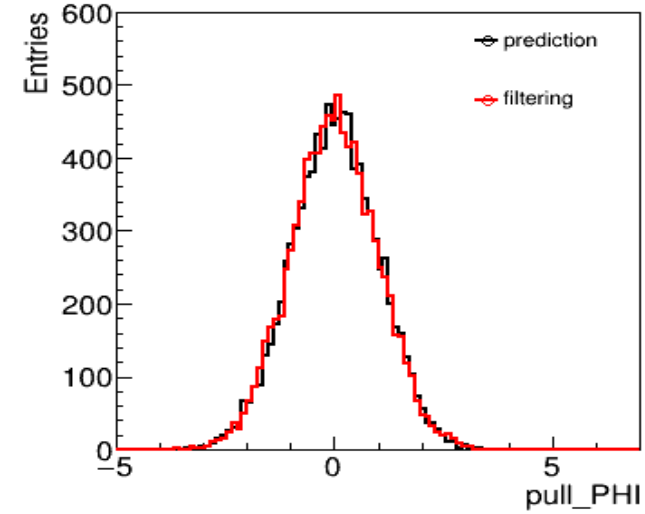
pull of LOC0



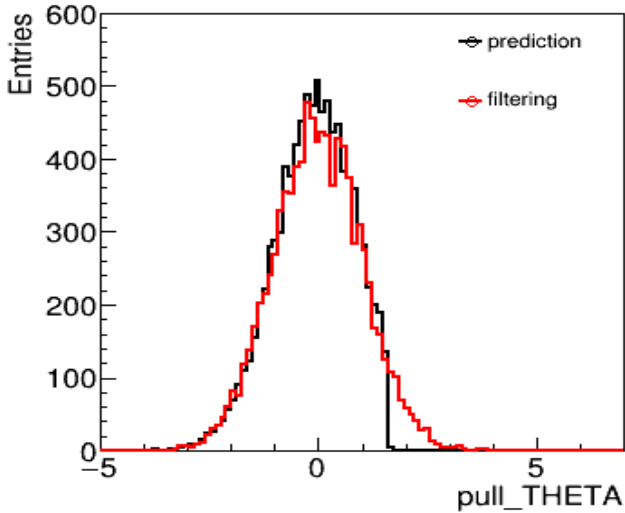
pull of LOC1



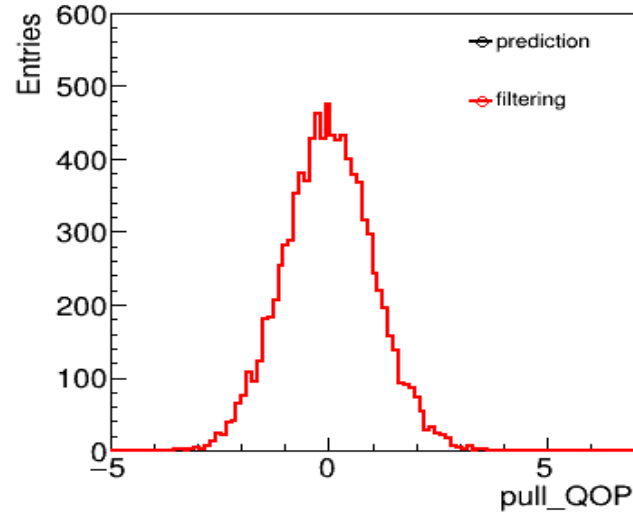
pull of PHI



pull of THETA

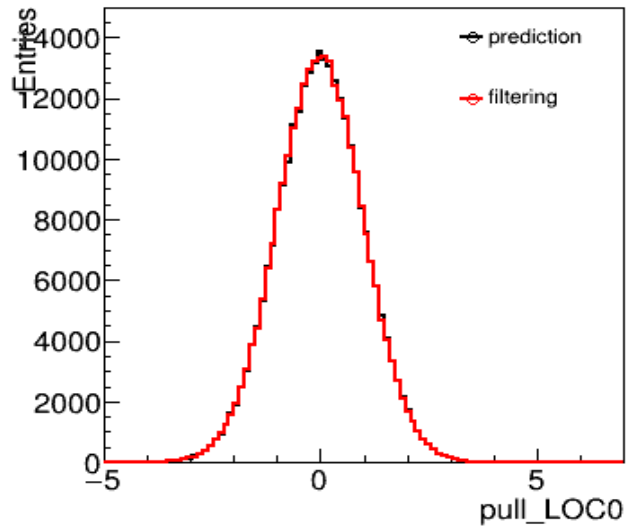


pull of QOP

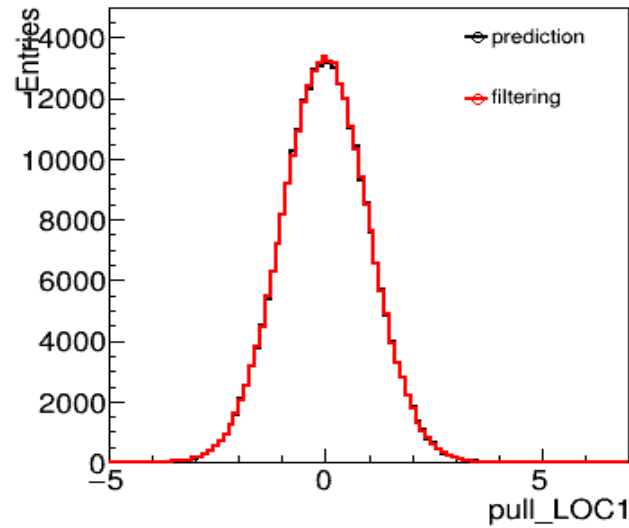


Test 4: pull of track parameter (on all measurement surface)

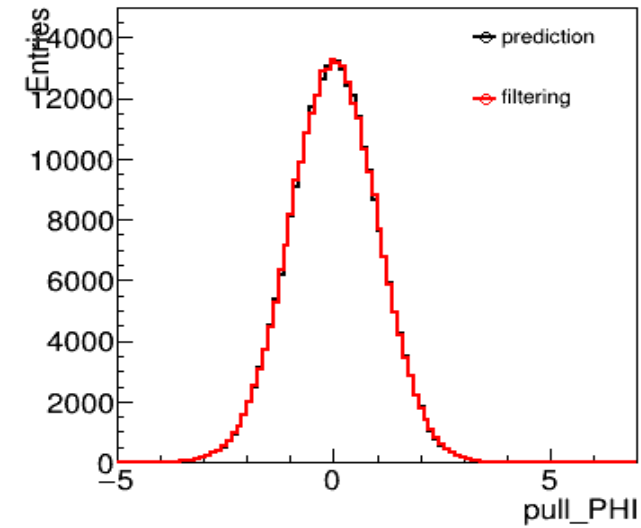
pull of LOC0



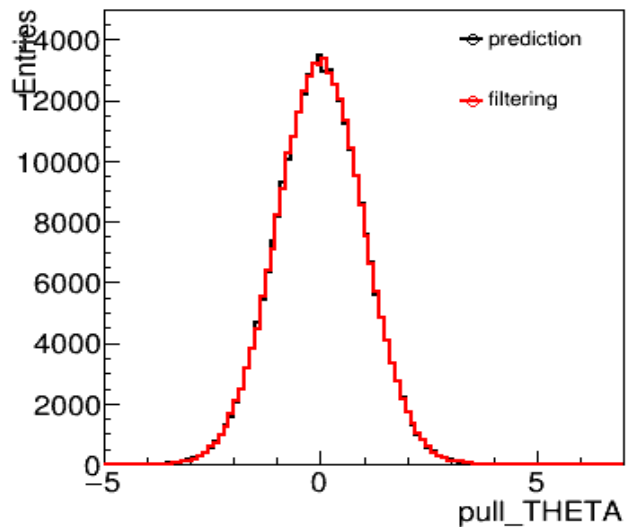
pull of LOC1



pull of PHI



pull of THETA



pull of QOP

