$B \to \gamma$ Form Factors @ NLO

LHCb UK Annual Meeting Huddersfield '20

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Motivation

Reasons to look at $B \to \gamma$ FF's...

- $B_s \to \ell^+\ell^-\gamma$ as a test of Lepton Flavour Universality.
  - Sensitive to the same set of operators as $b \to s\, \ell^+\ell^-$ due to identical effective Hamiltonian.
  - Shifts in the Wilson coefficient of these operators could explain anomalies in flavour data (e.g. $R_{K(*)}$).
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- Determination of the first inverse moment of the leading B-meson distribution amplitude, λ_B.
  - Currently a large source of uncertainty in B-meson DA calculations.
  - Lots of indirect determinations with results ranging from $\lambda_B = 200$ - 600 MeV ...Descotes-G. Sachrajda '02, Rohrwild,Beneke '11, Braun,Khodjamirian '12, Wang '16, Braun, Beneke, Ji, Wei '18 and more.
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  - Should be able to give competitive results by extending the work of Ball & Kou '03 to NLO.
- An input for flavoured axion searches. Albrecht, Stamou, Ziegler, Zwicky '19.

Definition of the Form Factors

Form factors of interest parameterise the hadronic matrix elements of operators from the effective weak Hamiltonian.

$$\langle\gamma(k,\epsilon)|\,\mathcal{O}_\mu^V\,|\bar{B}_q(p_B)\rangle = P_\mu^\perp V_\perp(q^2) - P_\mu^\parallel V_\parallel(q^2) \qquad \mathcal{O}_\mu^V = -\frac{m_B}{e}\bar{q}\gamma_\mu(1-\gamma_5)b,$$

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Two independent structures

$$P_\mu^\perp = \varepsilon_{\mu\alpha\beta\gamma}\,\epsilon^{*\alpha}p_B^\beta k^\gamma, \qquad P_\mu^\parallel = i(p_B\cdot k\,\epsilon_\mu^* - p_B\cdot\epsilon^*\,k_\mu).$$

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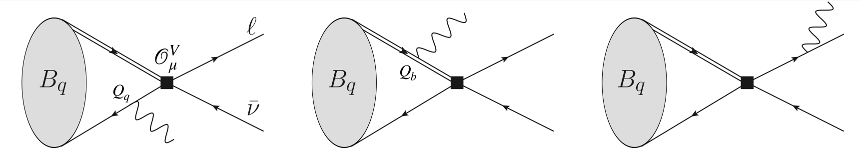

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- Photon DA: Photon is not exactly point-like (hard) but also has a (soft) hadronic contribution.
  - Hadronic contribution related to probability for photon to dissociate into partons .
  - Reminiscent of massless vector mesons $\rightarrow$ description in terms of DA.

The Computation

Information on the FF is contained in the correlation function:

$$\Pi_\mu^\Gamma(p_B^2, q^2) = i \int_x e^{-ip_B \cdot x} \langle \gamma(k, \epsilon)| \, T\{J_B(x) \mathcal{O}_\mu^\Gamma(0)\} \, |0\rangle \,,$$
$$= P_\mu^\perp \Pi_\perp^\Gamma(p_B^2, q^2) - P_\mu^\parallel \Pi_\parallel^\Gamma(p_B^2, q^2)$$

Interpolating operator for the B-meson $J_B = m_b \bar{b} \gamma_5 q$, $\Gamma = T, V$.

For the perturbative photon contribution:
$\langle \gamma(k, \epsilon)| \to -ie \int_y e^{ik \cdot y} \langle 0| \sum_{\psi=b,q} Q_\psi \bar{\psi} \gamma_\rho \psi(y)$

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Step-by-step:

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- Apply Borel transform - reduces contributions from higher resonances.

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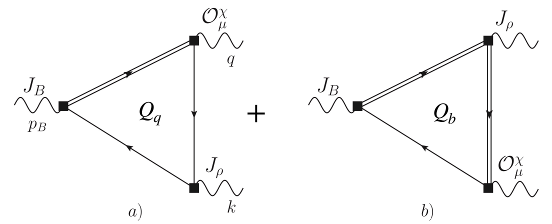
Sum rule:

$$\Gamma_{\perp,\parallel}(q^2) = \frac{1}{m_B f_B} \int_{m_b^2}^{s_0} \mathrm{d}s \, \frac{1}{\pi} \mathrm{Im}[\Pi^{\Gamma}_{\perp,\parallel}(s, q^2)] e^{(m_B^2 - s)/M^2}.$$

OPE Expansion @ LO

Leading contributions to the correlation function:

- Pertubative



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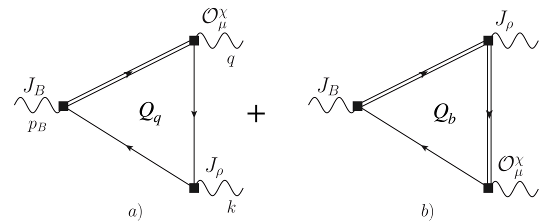
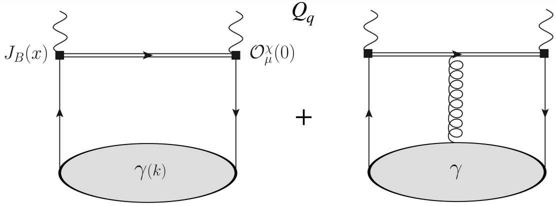


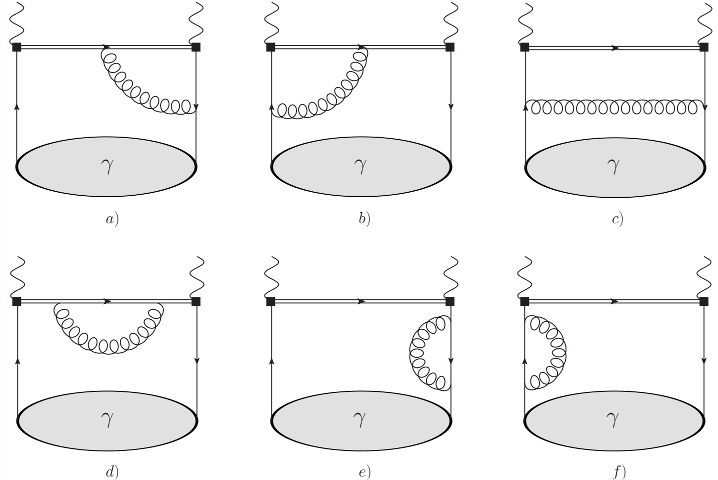
- Soft*



*soft contribution proportional to Q_b is treated using local OPE

OPE Expansion @ NLO

$O(\alpha_s)$ corrections to twist-2 have been computed:



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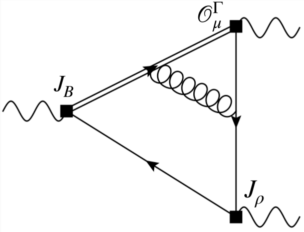

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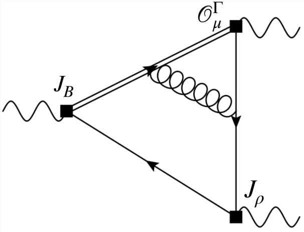

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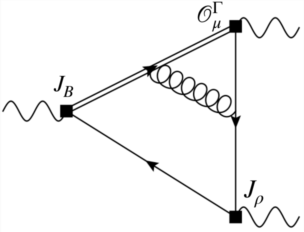

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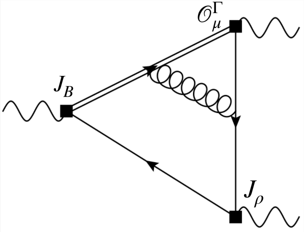

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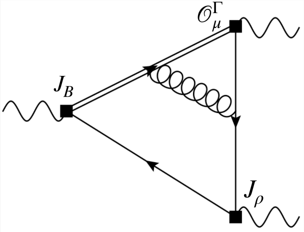

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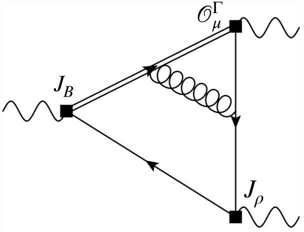

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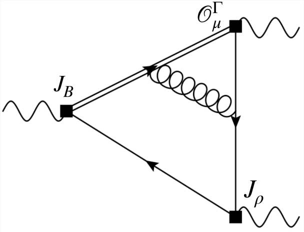

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- Goncharov functions $\to$ Classical polylogs (PolyLogTools)
- Numerical tests (SecDec, PolyLogTools)

Sanity Checks

1) Any FF determination has to obey the EOM:

$$\bar{q}i(D^\nu i\sigma_{\mu\nu} + \overleftrightarrow{D}_\mu)[\gamma_5]b + (m_q \mp m_b)\bar{q}\gamma_\mu[\gamma_5]b = 0$$

Tensor "New" Vector

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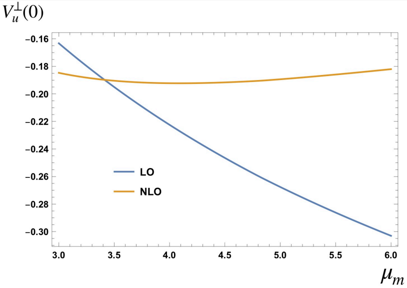
3) Renormalisation:

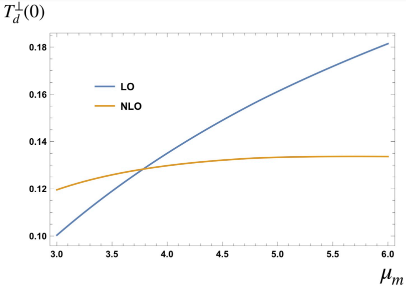
$\rightarrow$ Non-trivial mixing of operators

$\rightarrow$ Cancellation of thousands of Goncharov functions

$\rightarrow$ Cancellation of explicit scale dependence at NLO

Cancellation of explicit scale dependence at NLO





Similar results for the other FF

Breakdown and Borel stability

Breakdown

	$V_u^{\perp}(0)$	$V_d^{\perp}(0)$
Tw=1, LO	82%	85%
Tw=1, NLO	-11%	-10 %
Tw=2, LO	49%	41%
Tw=2, NLO	-11%	-9%
Tw=3, LO	-1%	<1%
Tw=4, LO	-8%	-6%

- Both charged and neutral FF has similar breakdown with reasonable convergence
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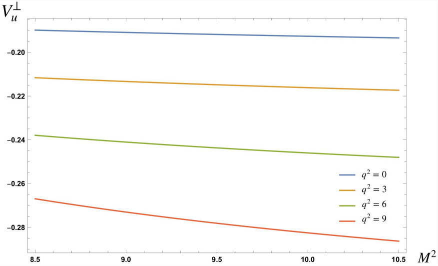
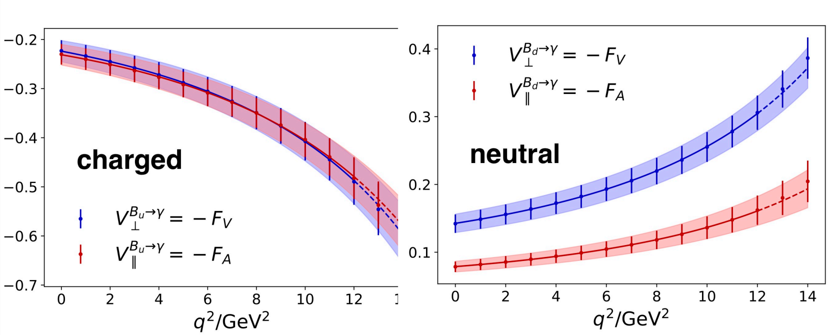


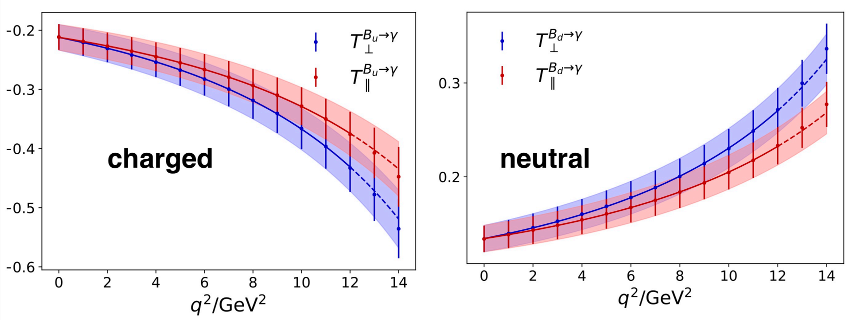
- Stability under variation of Borel parameter signals "good" sum rule.
- Variation increases with q^2 but stays under control ($< 10\%$).

Preliminary Plots - Vector



- Q_q Dominates $\rightarrow$ difference between charged and neutral

Preliminary Plots - Tensor



- Recall $T_\perp(0) = T_\parallel(0)$

Summary

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- $D_{uds} \to \gamma$ FF can also be extracted but only for a very small kinematic range.
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Thanks for you attention!