Charm rescattering effects in B decays

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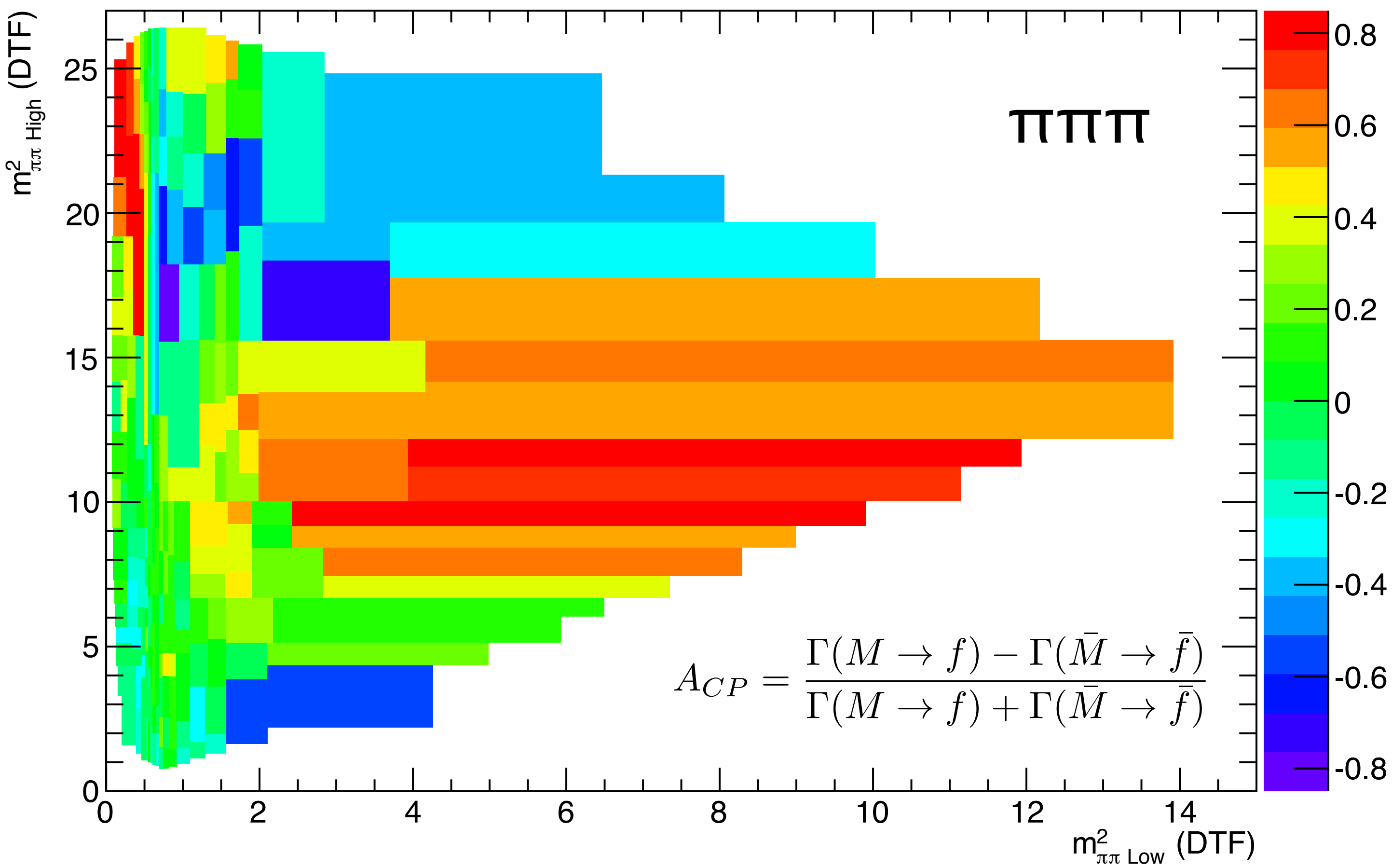
- CP-Violation
  - $B^{\pm} \to h^{\pm} h^{-} h^{+}$ massive localized Acp



dynamic effect !!

at low and high mass

LHCb PRD90 (2014) 112004 new one coming soon

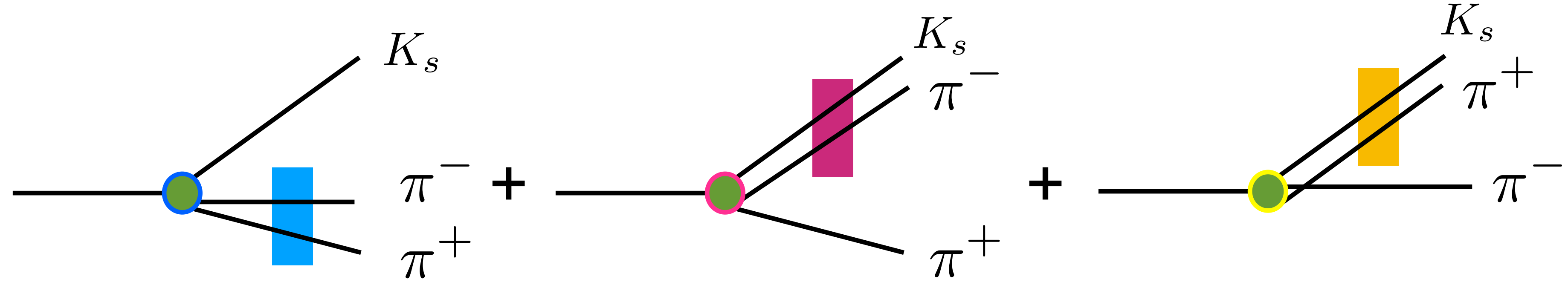
- 1st observation in charm $D^0(\bar{D}^0) \to \pi^+\pi^-, K^+K^-$

→ can lead to new physics

↳ CPV on three-body?

- common cartoon to described 3-body decay

$D^0 \to K_s \pi^- \pi^+$



- If true, one expect 2-body resonances → But in reality……. not all of them are clearly present

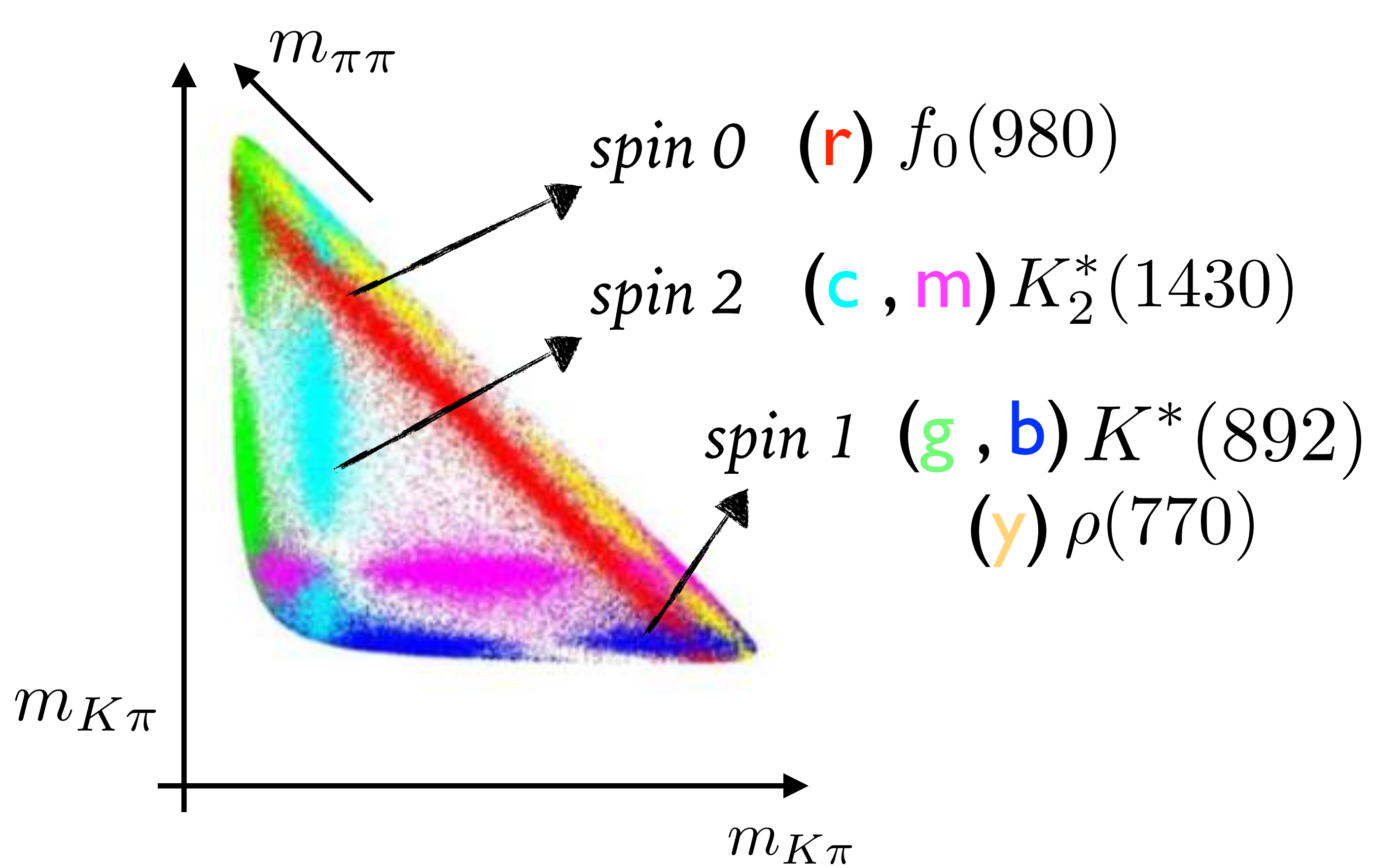
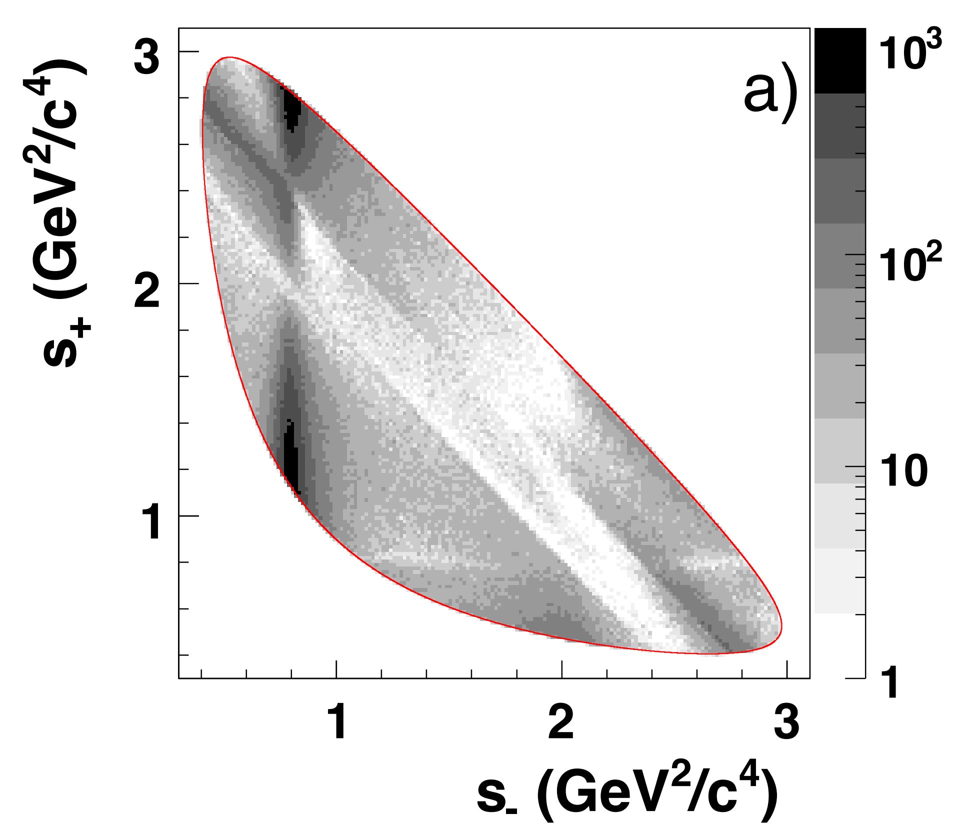


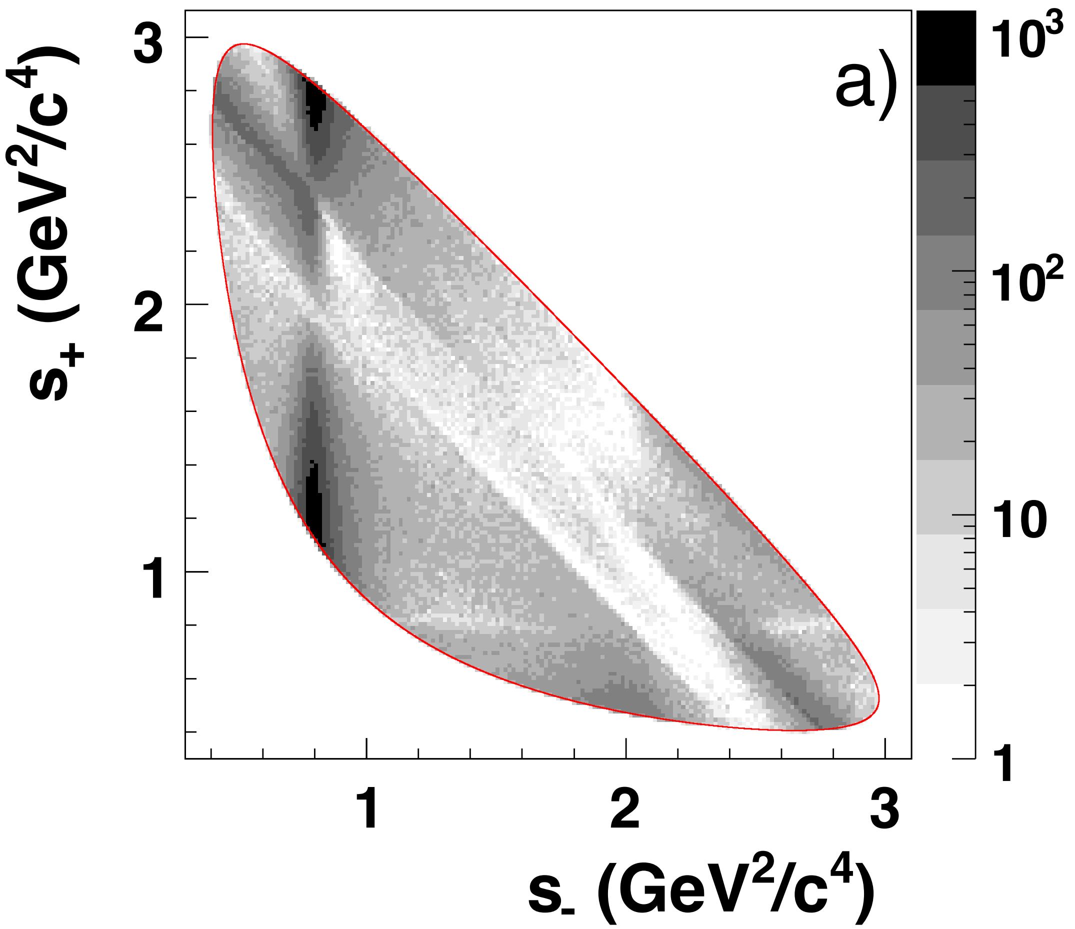
image credit:Tom Latham



BABAR Phys.Rev. Lett. 105 (2010) 081803

Three-body kinematics : DALITZ plot

- $D^0 \to K_s \pi^- \pi^+$



- $D^0 \to K^- \pi^+ \pi^0$

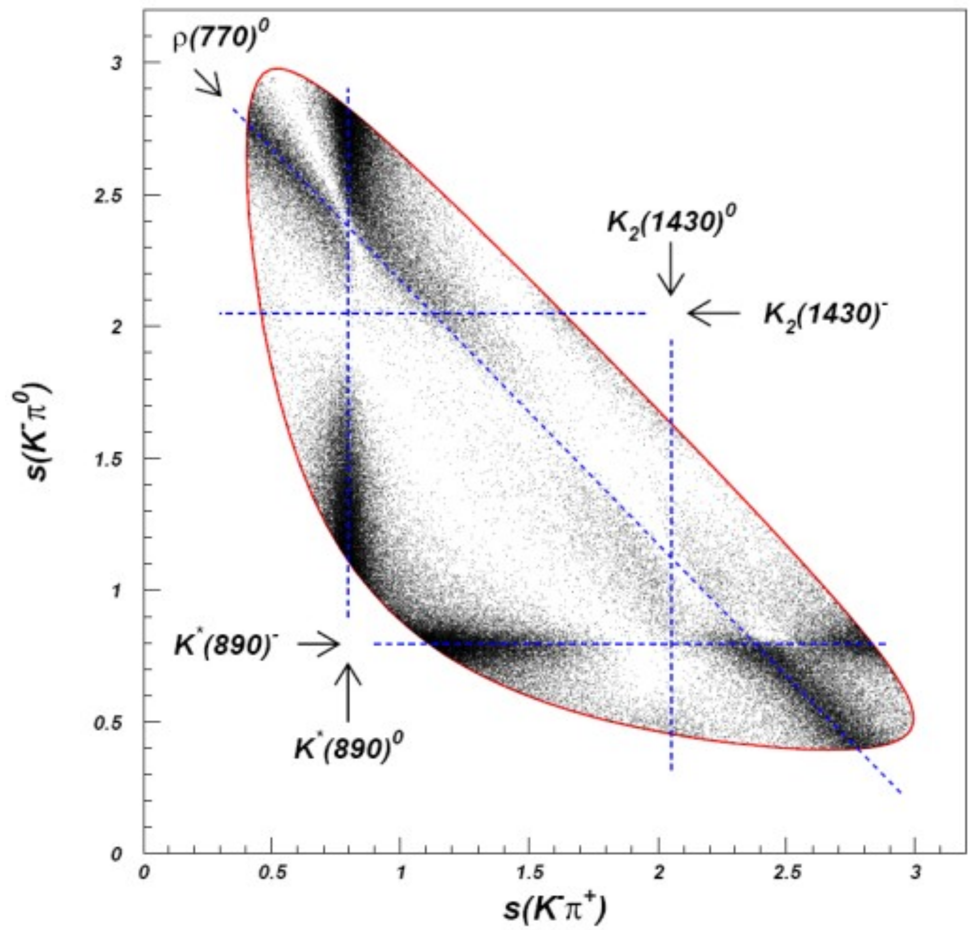


image credit:Brian Meadows

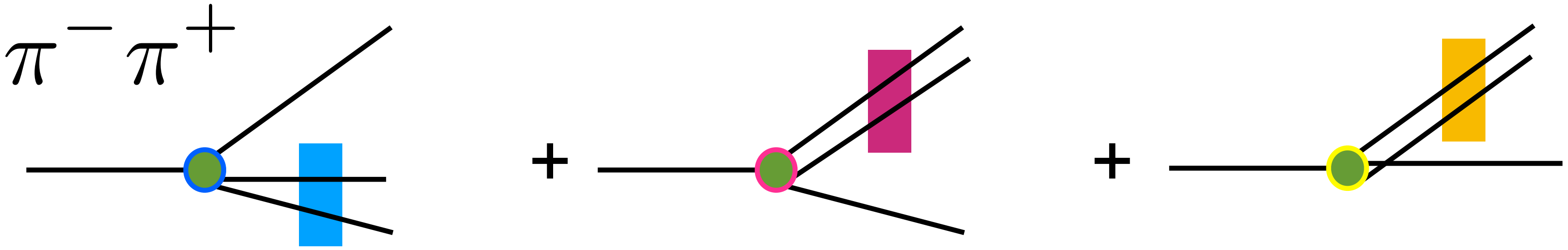
⟶ Similar final state but different interference pattern

↳ different dynamics to be understood

⟶ to disentangle the interference we need amplitude analysis

- new hight sample data cannot be described only by adding resonances!

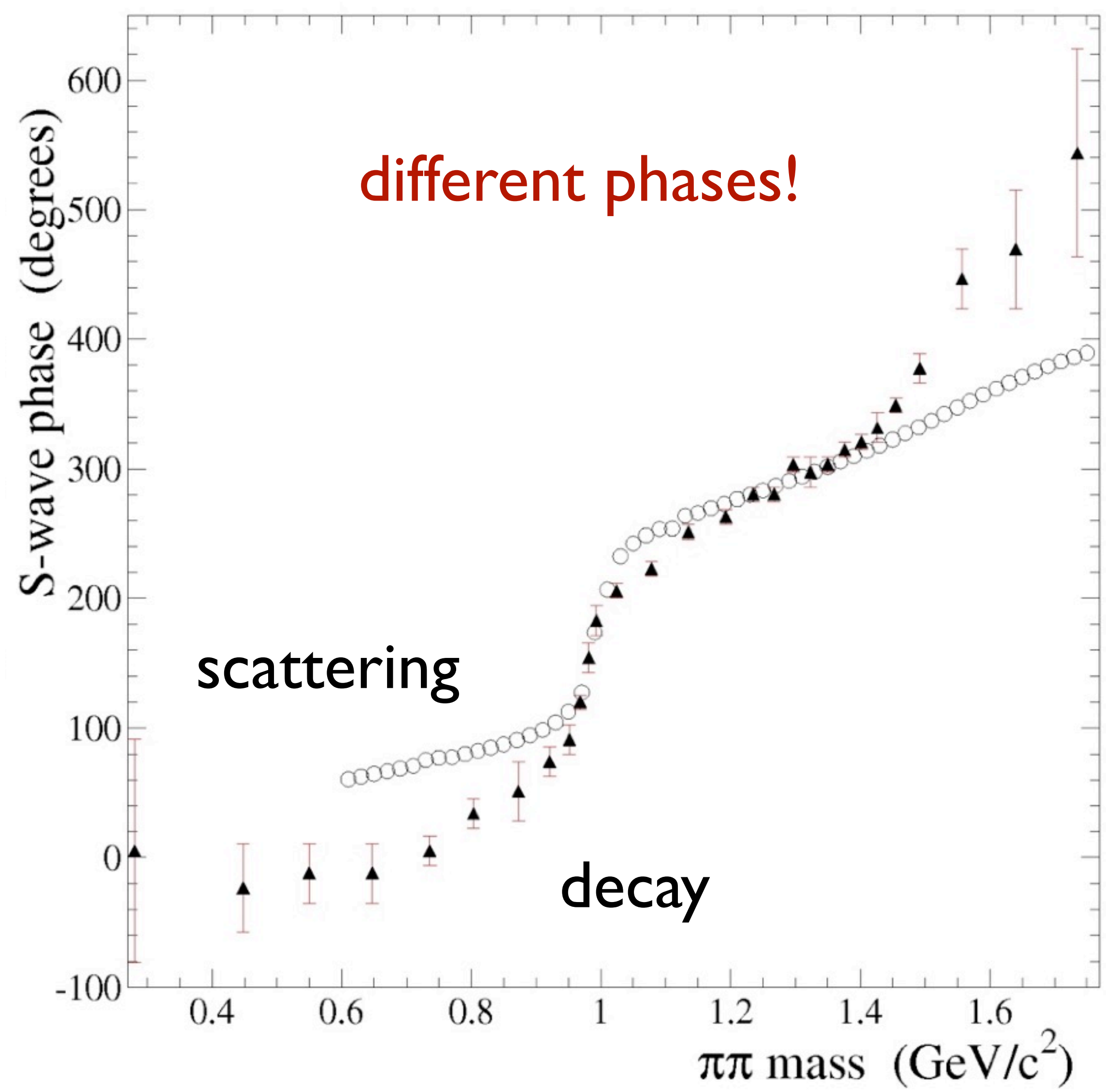
- $D_s^+ \to \pi^+\pi^-\pi^+$



- If this picture is the reality:

It should only contain 2-body informations! $\longrightarrow$ phase from decay should be the same as scattering

$\rightarrow$ Is not as simple as it look like!

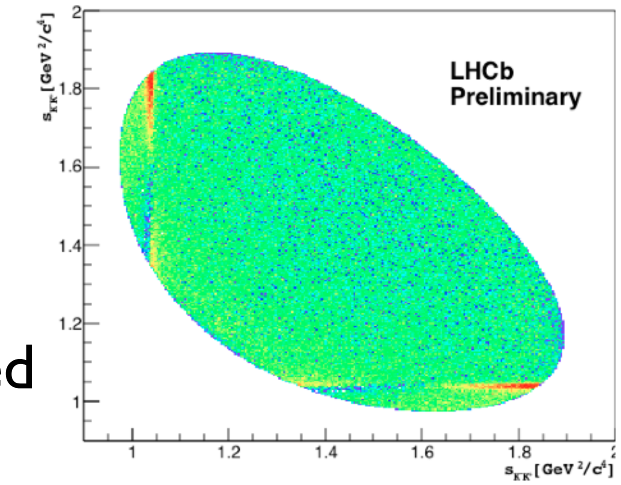


Phys.Rev. D 79 (2009) 032003

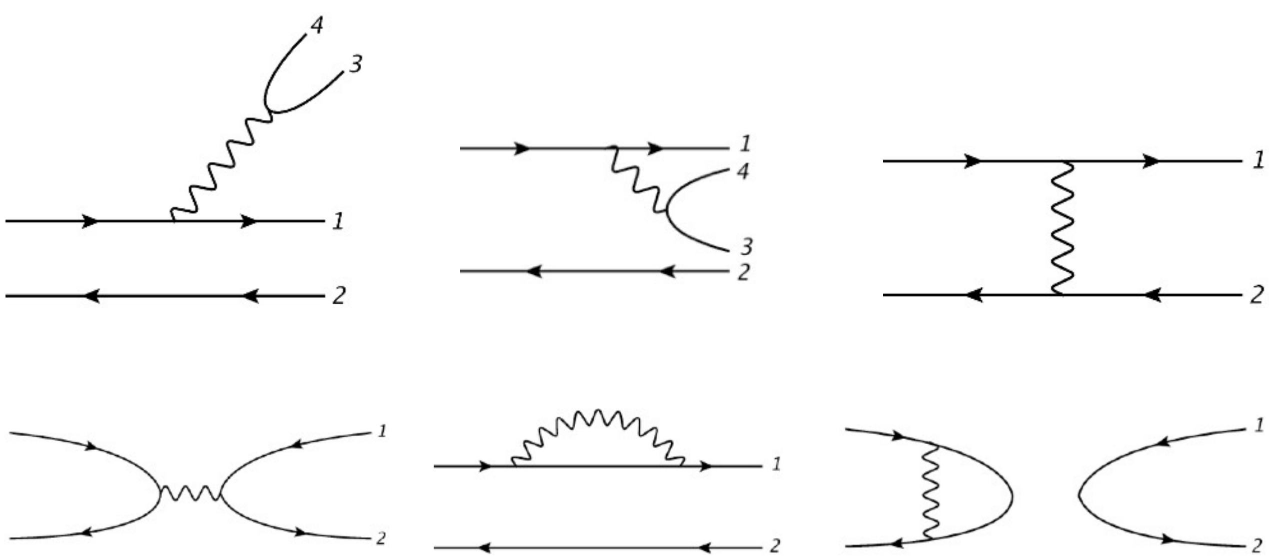
- Quantum numbers:
  - 2-body amplitude: spin and isospin well defined!
  - 3-body data: only spin! and $\neq$ dynamics

Three-body heavy meson decay

- dynamics $D^+ \to K^- K^+ K^-$

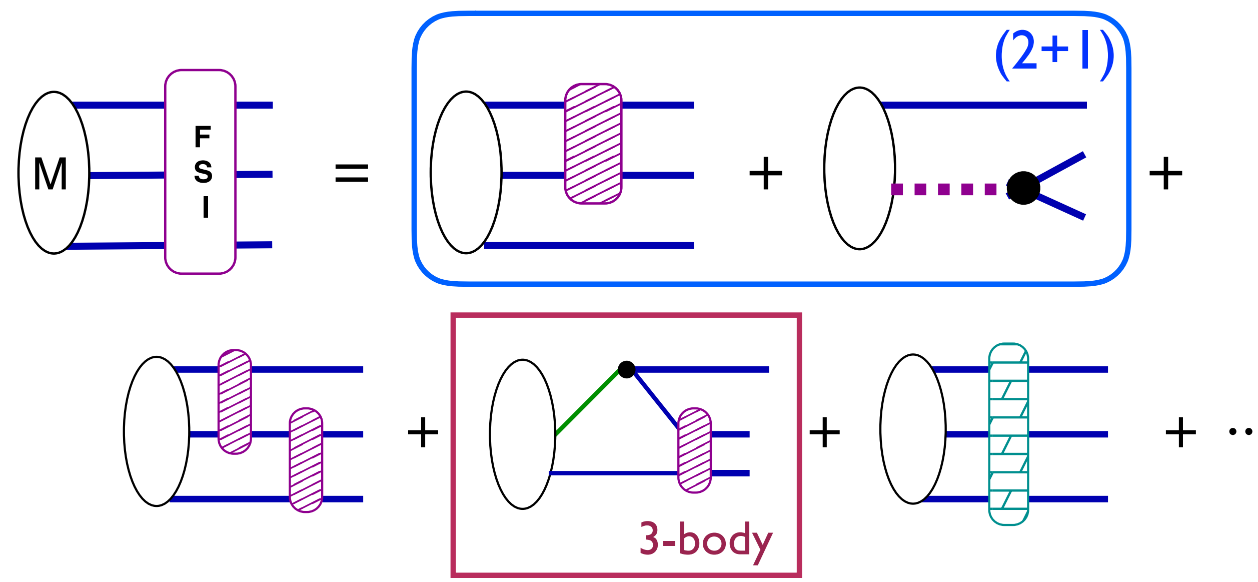


primary vertex - weak -



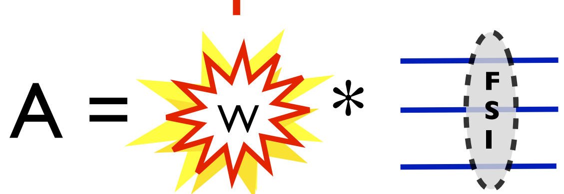
QCD, CKM coupling and phase

Final State Interactions - strong -



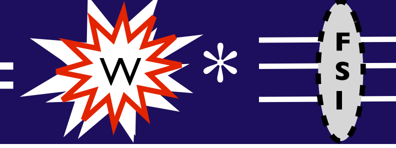
2-body is crucial!!!!

To extract information from data we need an amplitude MODEL

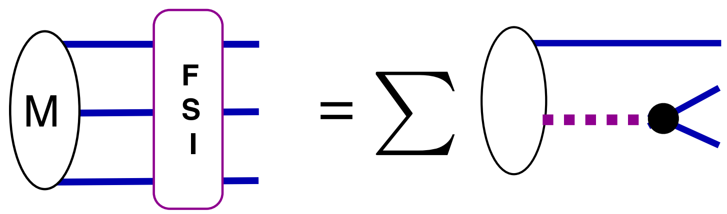


$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12}, s_{23})|^2$$

dynamics

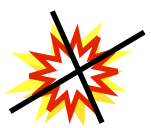
A = 

- isobar model: widely used by experimentalists
  - (2+1) approximation:

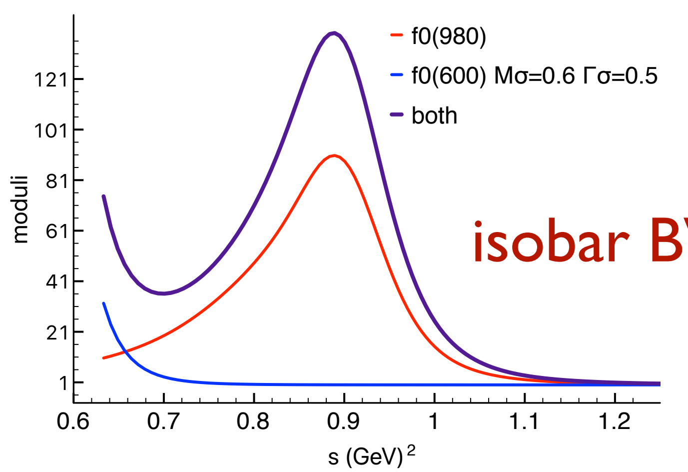


→ ignore the 3rd particle (bachelor)

$A = \sum c_k A_{k;}$ + NR { non-resonant as constant or exponential!
each resonance as Breit-Wigner $\mathrm{BW}(s_{12}) = \frac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})},$

 weak vertex is not considered explicitly

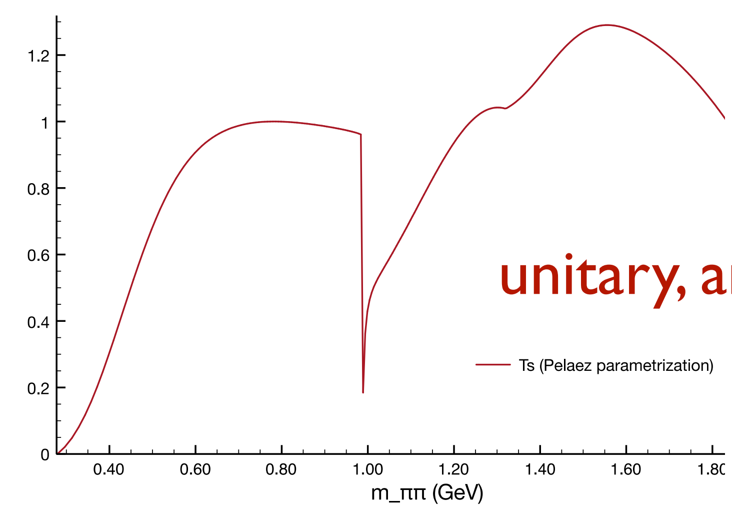
- worst problems: ππ S-wave



isobar BW

→ fit could change this interference

more than 2 scalars ←



unitary, analytic,…

Pelaez, Yndurain PRD71(2005) 074016

- sum of BW violates two-body unitarity (2 res in the same channel);
- do NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !

!

Models available A = W * FSI

- movement to use better 2-body (unitarity) inputs in data analysis

  - “K-matrix" : ππ S-wave 5 coupled-channel modulated by a production amplitude

    ↳ used by Babar, LHCb, BES III — Anisovich PLB653(2007)

  - rescattering $\pi\pi \to KK$ contribution in LHCb $\begin{cases} B^\pm \to \pi^+\pi^-\pi^\pm & \text{[arXiv:1909.05212; 1909.05211]} \\ B^\pm \to K^-K^+\pi^\pm & \text{[arXiv:1905.09244]} \end{cases}$

    Pelaez, Yndurain PRD71(2005) 074016

    ↳ new parametrization Pelaez, and Rodas EPJ. C78 (2018) 11, 897

⟶ other scalar and vector form factors available

Limited to low E (2 GeV)!

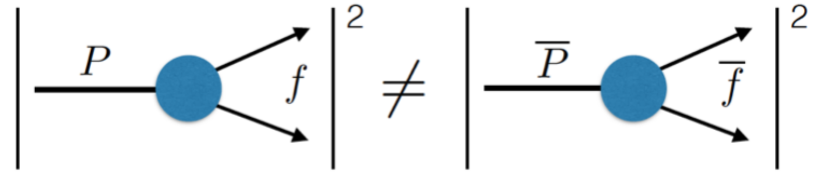
$<\pi\pi\|0>$	scalar	Moussallam EPJ C 14, 111 (2000); Daub, Hanhart, and B. Kubis JHEP 02 (2016) 009.
	vector	Hanhart, PL B715, 170 (2012); Dumm and Roig EPJ C 73, 2528 (2013).
$<K\pi\|0>$	scalar	Moussallam EPJ C 53, 401 (2008); Jamin, Oller and Pich, PRD 74, 074009 (2006)
	vector	Boito, Escribano, and Jamin EPJ C 59, 821 (2009).
$<KK\|0>$ (no data)	Fit from 3-body data	PCM, Robilotta + LHCb JHEP 1904 (2019) 063
	extrapolate from unitarity model	Albaladejo and Moussallam EPJ C 75, 488 (2015).
	quark model with isospin symmetry	Bruch, Khodjamirian, and Kühn, EPJ C 39, 41 (2005)

Final State Interaction in B decays as a source of CP violation

CPV on data: Puzzle!

- Charge Parity Violation

$$\Gamma(M \to f) \neq \Gamma(\bar{M} \to \bar{f})$$



  - condition to CPV

    → $2 \neq$ amplitudes, SAME final state with strong (δ_i) and weak (ϕ_i) phase

$$\langle f \,|\, T \,|\, M \rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

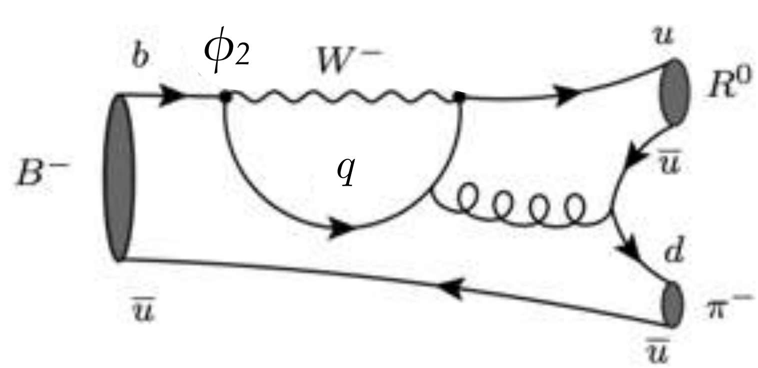
$$\Big\downarrow \text{CP}$$

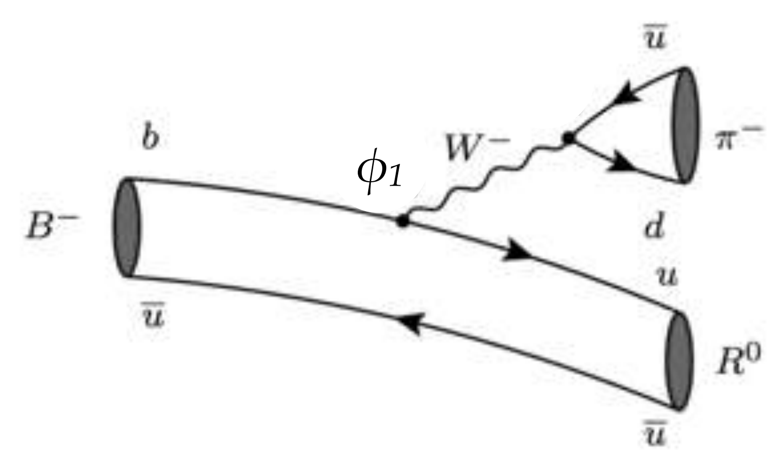
$$\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

$$\therefore\ \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f \,|\, T \,|\, M \rangle|^2 - |\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle|^2 = -4A_1A_2 \sin(\delta_1-\delta_2)\, \sin(\phi_1-\phi_2)$$

- BSS model  + 

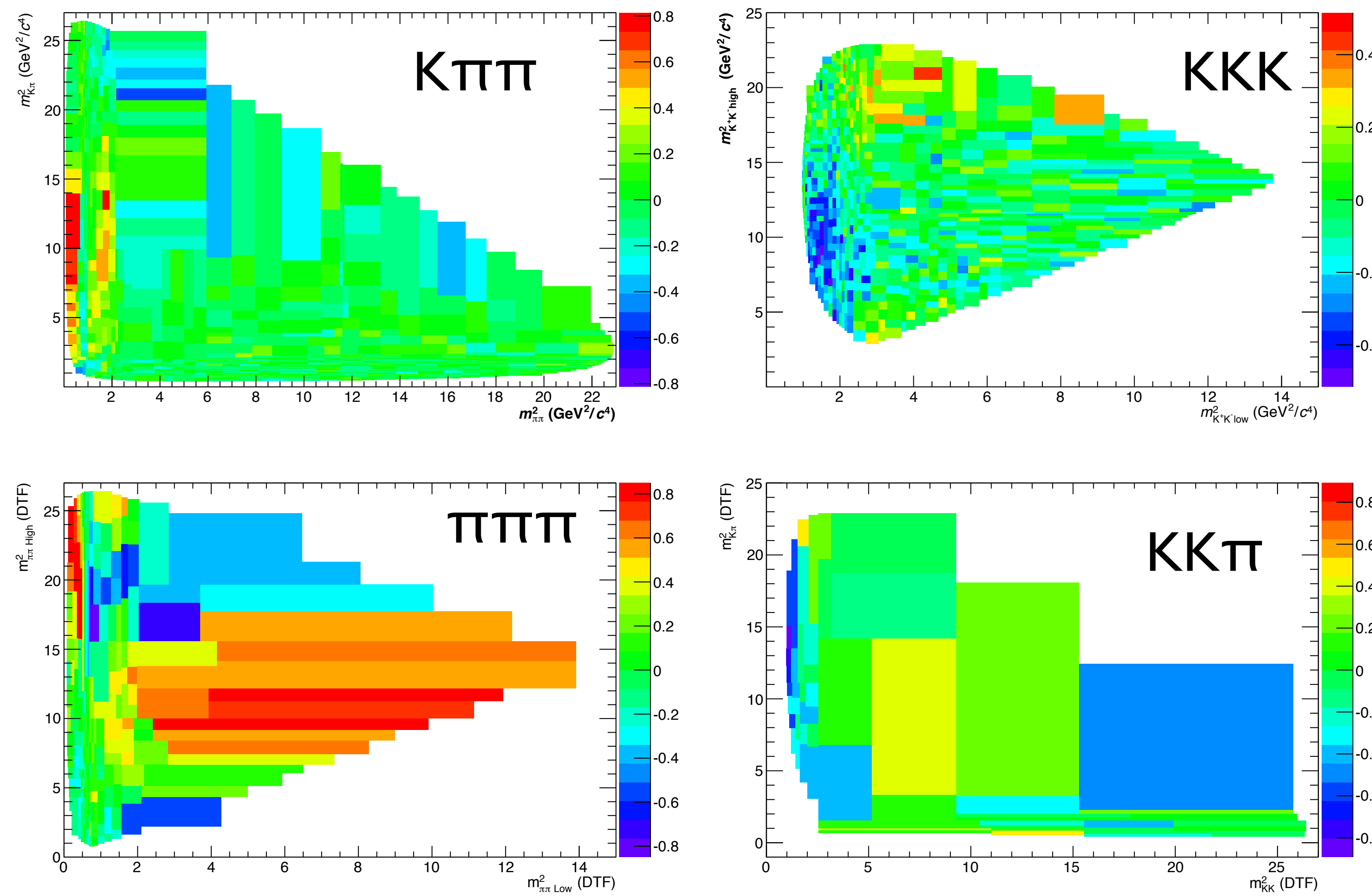
Bander Silverman & Soni PRL 43 (1979) 242





- $B^{\pm} \to h^{\pm}h^{-}h^{+}$ LHCb massive localized Acp
  - suggest dynamic effect
  - middle looks "empty" $\to$ CPV

$$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$



- BSS model  + 

not enough!!

- hadronic interactions $\to$ strong phase

- $B^{\pm} \to h^{\pm}\pi^{-}\pi^{+}$ and $B^{\pm} \to h^{\pm}K^{-}K^{+}$ low-energy CPV with opposite signs

$\boxed{\pi\pi \to KK}$

Frederico, Bediaga, Lourenço PRD89(2014)094013

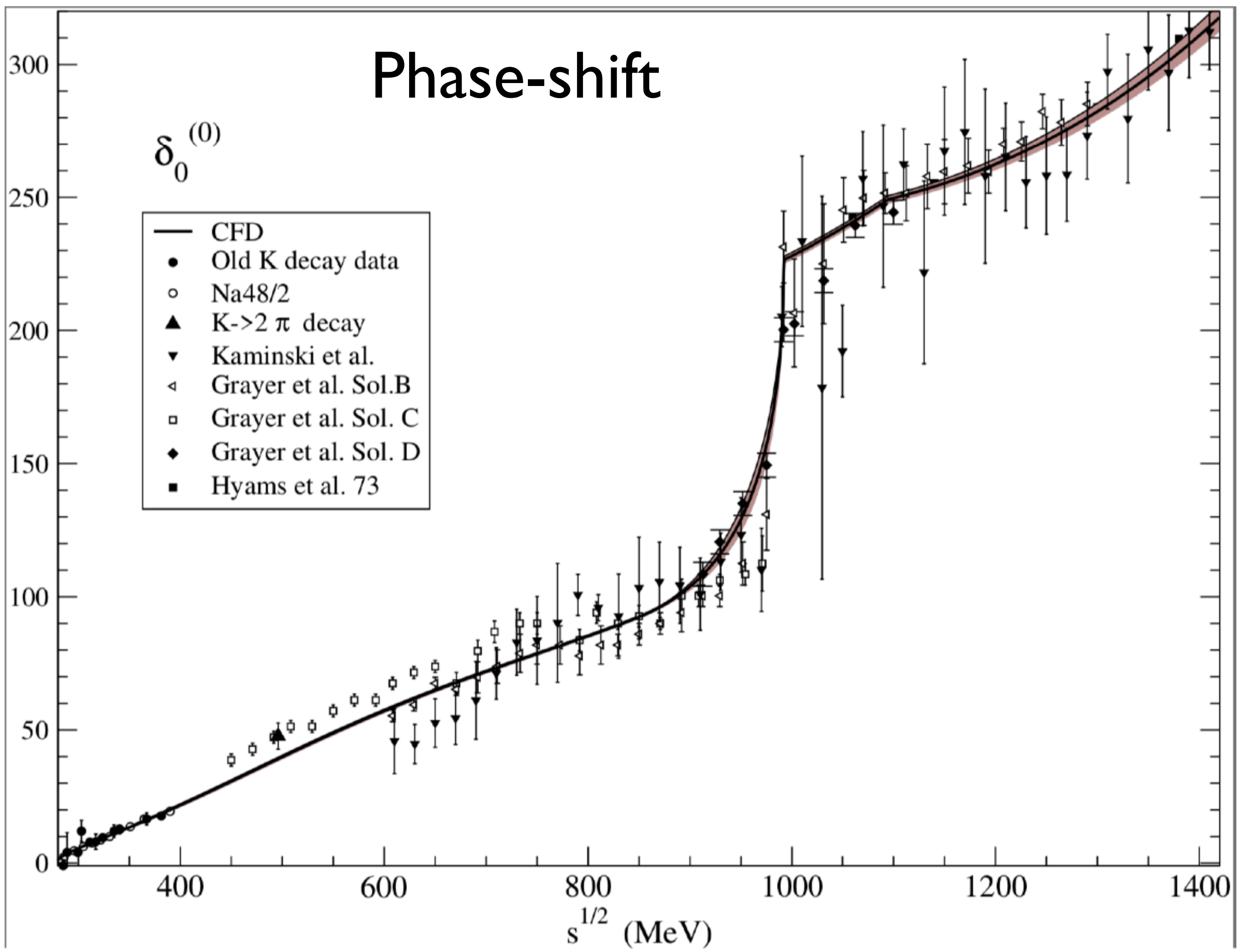
Parentheses…..

- $\pi\pi$ scattering data S-Wave

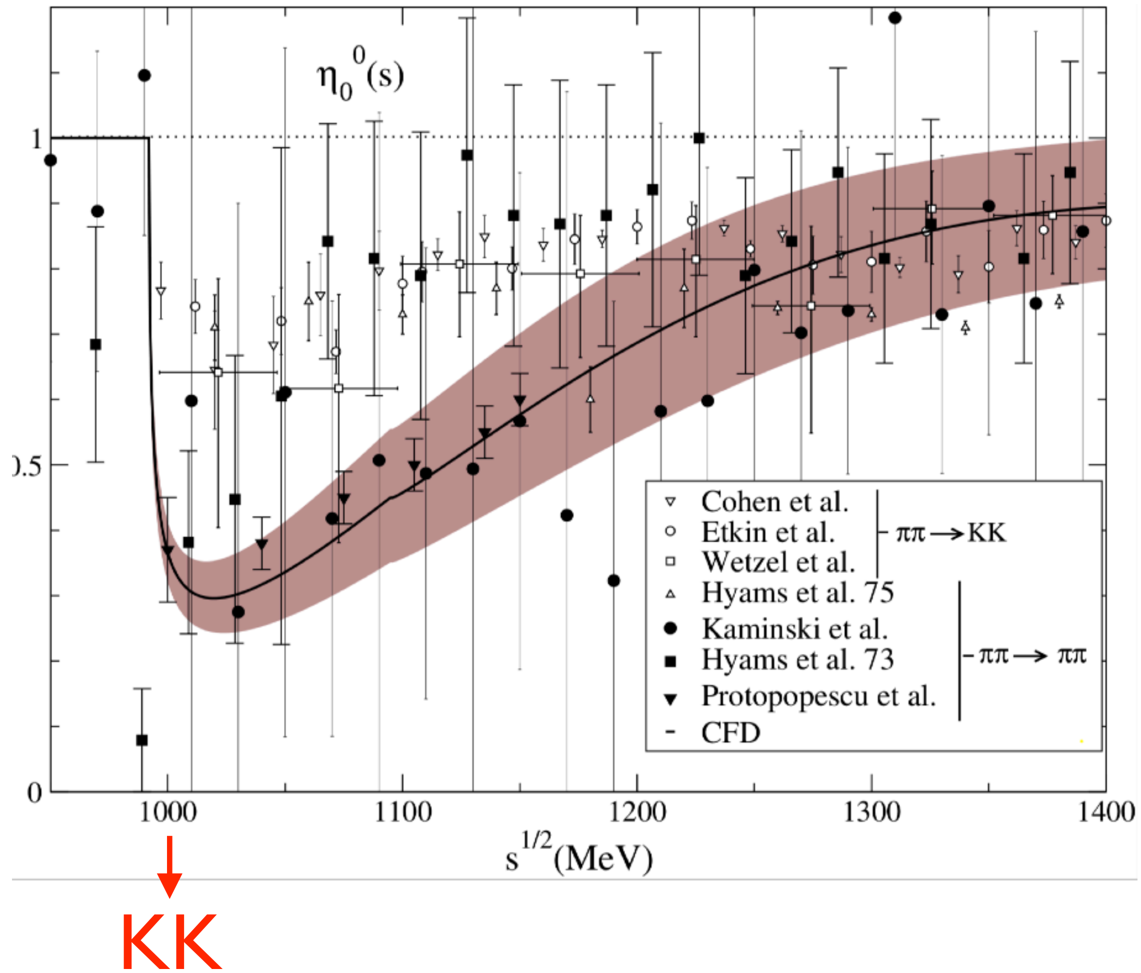
Pelaez, Yndurain PRD71(2011) 074016

- amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$

Phase-shift



Inelasticity



$$\sigma_l^{\rm el} = \frac{1}{2}\left\{\frac{1+\eta_l^2}{2} - \eta\cos 2\delta_l\right\},$$

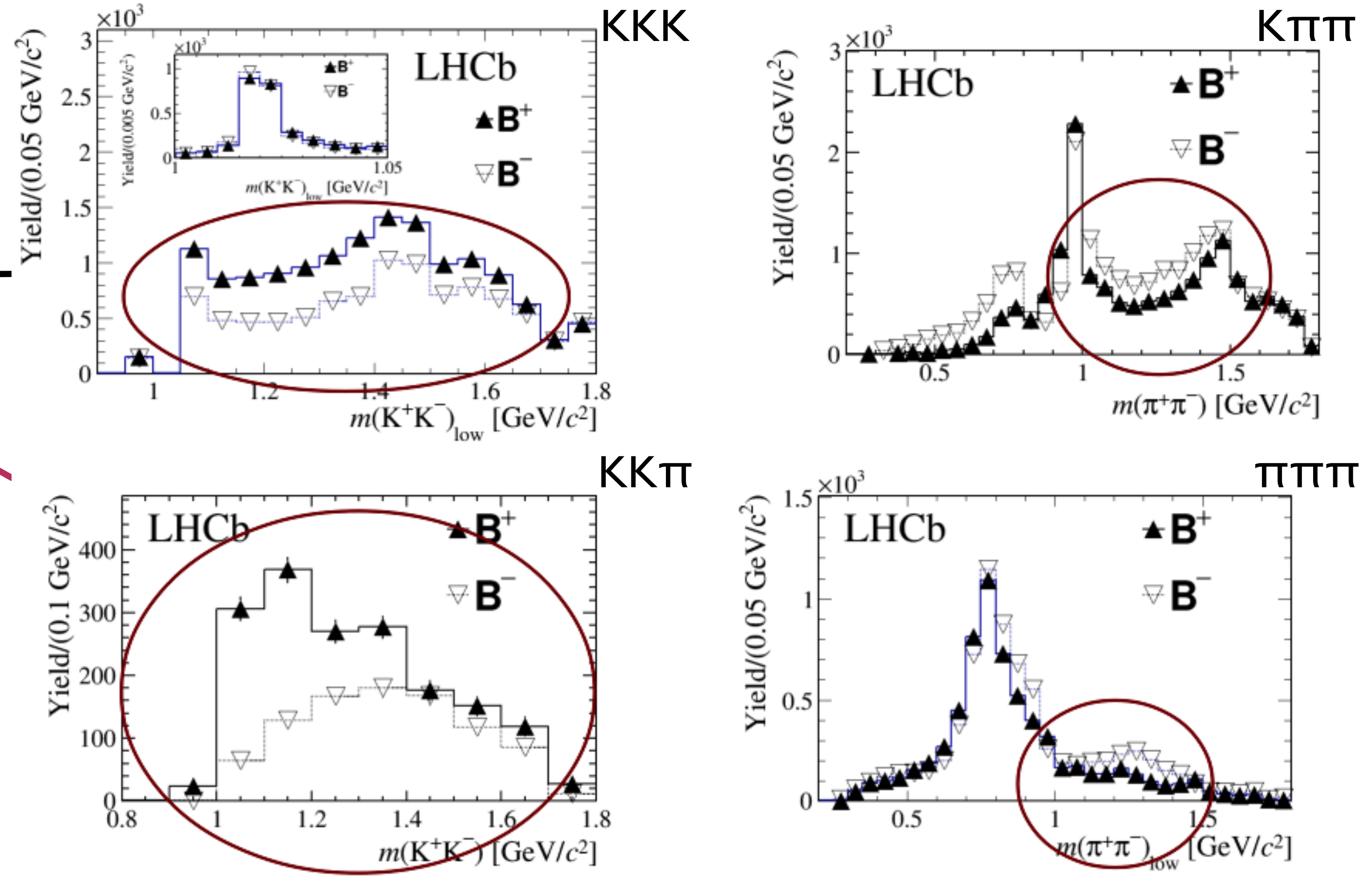
Inelasticity: one minus the probability of losing signal (1==elastic)

CPV on data

- low-energy CPV [1 - 2] GeV

$$\boxed{\pi\pi \rightarrow KK}$$

Frederico, Bediaga & Lourenço
PRD89(2014)094013



- FSI → strong phase

Wolfenstein PRD43 (1991) 151

- CPT:

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \bar{\Gamma}_{total}$

$$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots$$

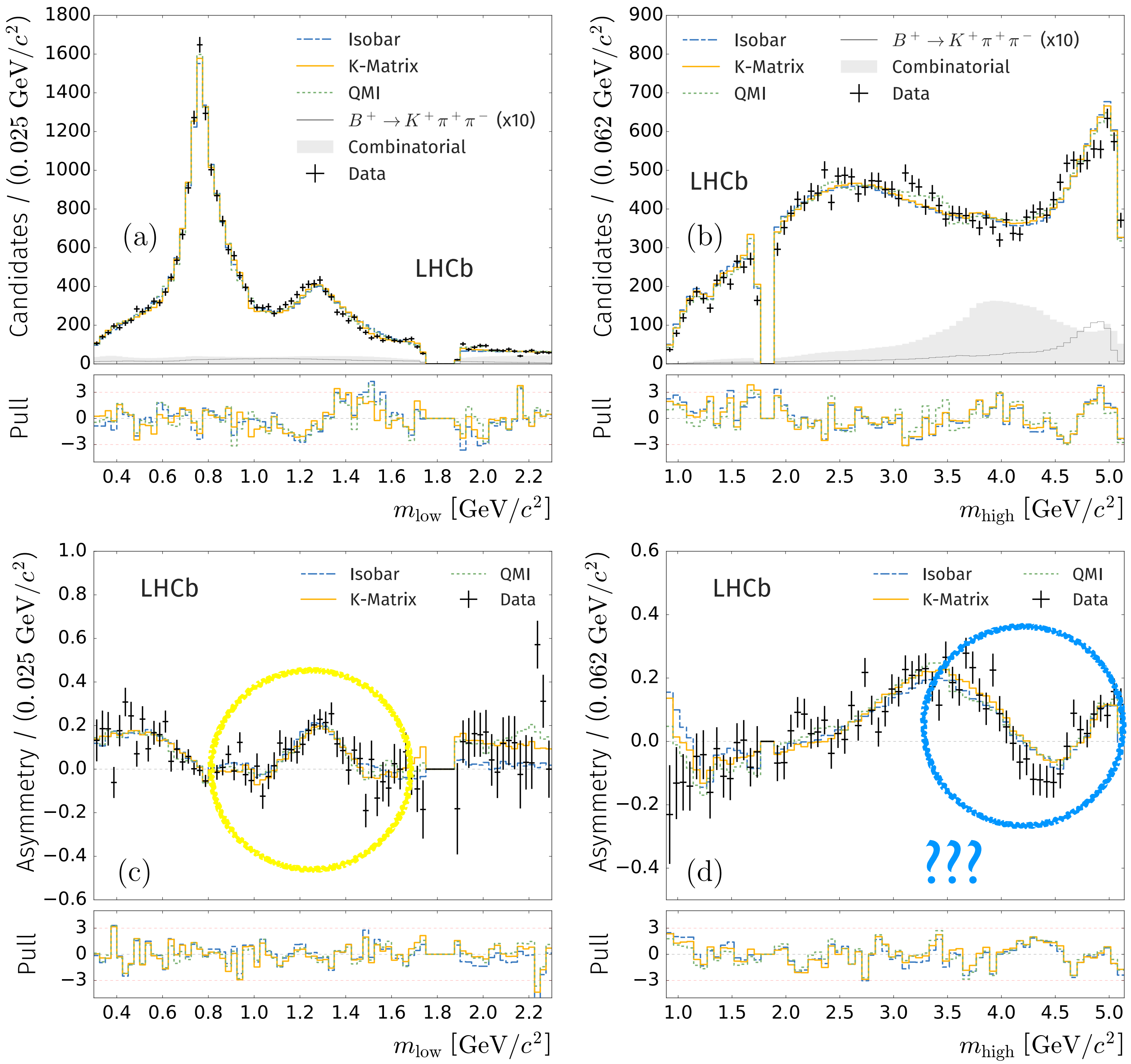
$$\bar{\Gamma}_{total} = \bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \dots$$

→ CPV in one channel should be compensated by another one with opposite sign

CPV: amplitude analysis $B^{\pm} \to \pi^- \pi^+ \pi^{\pm}$

- LHCb recent Amplitude analysis $B^{\pm} \to \pi^- \pi^+ \pi^{\pm}$ [arXiv:1909.05212(PRD); 1909.05211(PRL)]

  - $(\pi^- \pi^+)_{S-Wave}$ 3 different model:
    - ↳ σ as BW (!) + rescattering;
    - ↳ P-vector K-Matrix;
    - ↳ binned freed lineshape (QMI);

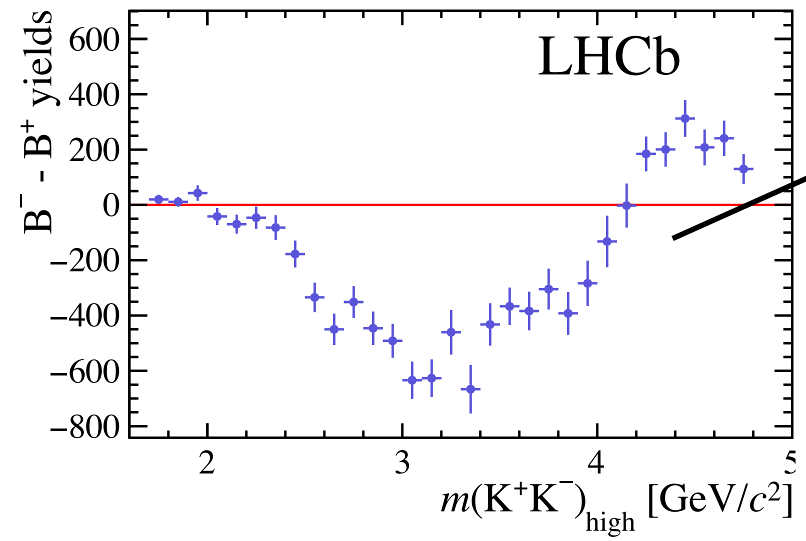


Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

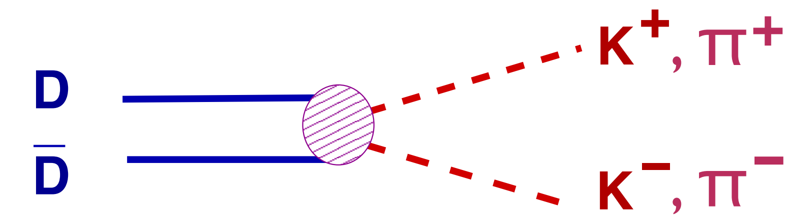
- ANA for $B^{\pm} \to \pi^{\pm} K^- K^+$ [arXiv:1905.09244]

Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[^{o}] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$	0 (fixed)
			$1.06 \pm 0.04 \pm 0.02$	0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$	$-176 \pm 10 \pm 16$
			$0.82 \pm 0.09 \pm 0.10$	$136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$	$-138 \pm 7 \pm 5$
			$1.97 \pm 0.12 \pm 0.20$	$166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$	$-175 \pm 10 \pm 15$
			$1.92 \pm 0.10 \pm 0.07$	$140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$	$-106 \pm 11 \pm 10$
			$1.13 \pm 0.08 \pm 0.05$	$-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$	$-56 \pm 12 \pm 18$
			$0.86 \pm 0.07 \pm 0.04$	$-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$	$-52 \pm 23 \pm 32$
			$0.22 \pm 0.06 \pm 0.04$	$107 \pm 33 \pm 41$

- CPV high mass?



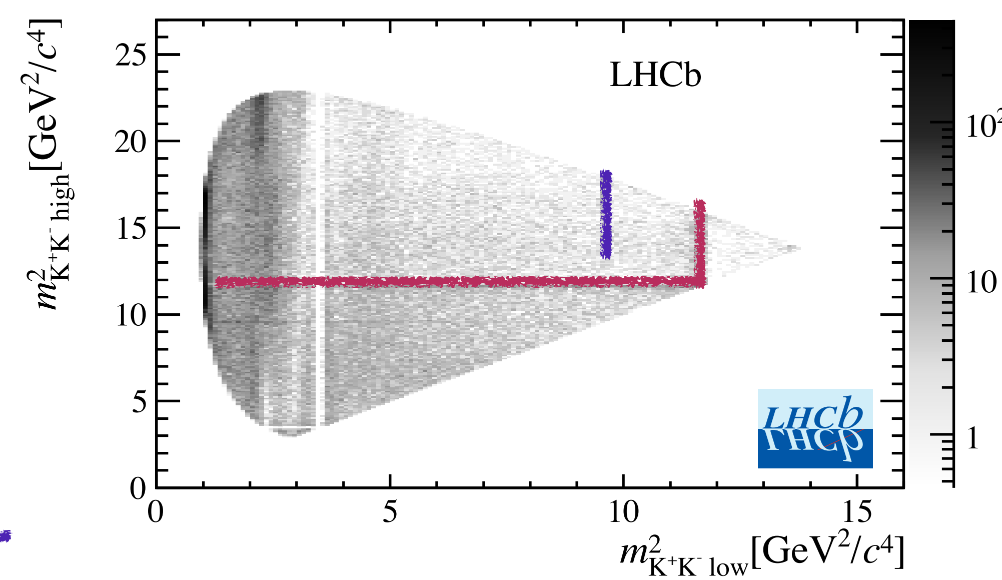
~ $D\bar{D}$ open channel →

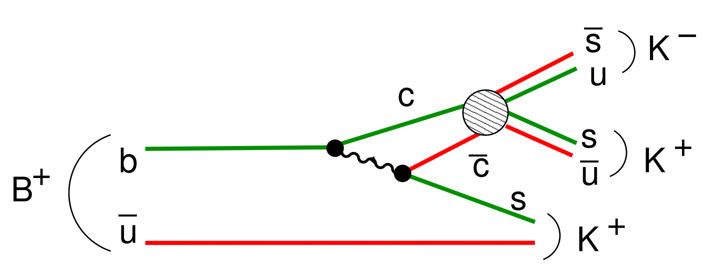


same observed in coupled-channels

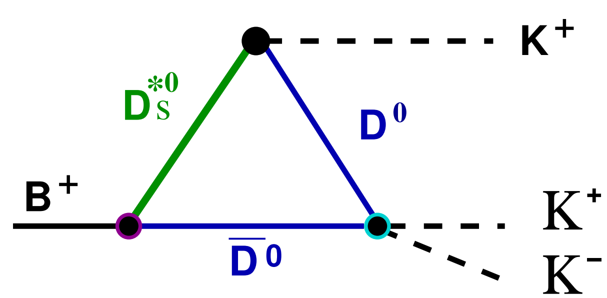
→ **charm intermediate processes as source of strong phase**

- $B^+ \to K^- K^+ K^+$
  - high statistic 109k
  - nonresonant → all phase-space
  - presence of charm resonances: χ_{c0} J/ψ
  - dominated by penguin





→

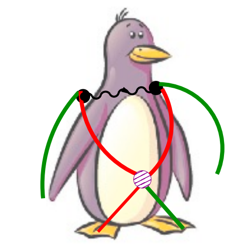


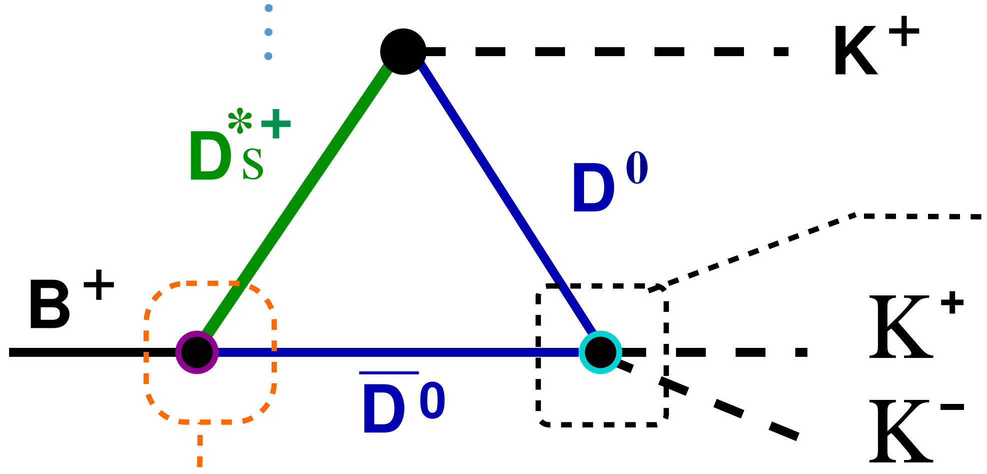
charm rescattering!

I. Bediaga, PCM, T Frederico
PLB 780 (2018) 357

hadronic loop

I. Bediaga, PCM, T Frederico
PLB 780 (2018) 357





form factor for $W^+ \to D^0 K^+$

$D^0 \bar{D}^0 \to K^+ K^-$ scattering amplitude

phenomenological:
S- matrix unitarity + Regge theory

weak transition $B^+ \to W^+ \bar{D}^0$ → $C_0 \times$ form factor to regulate

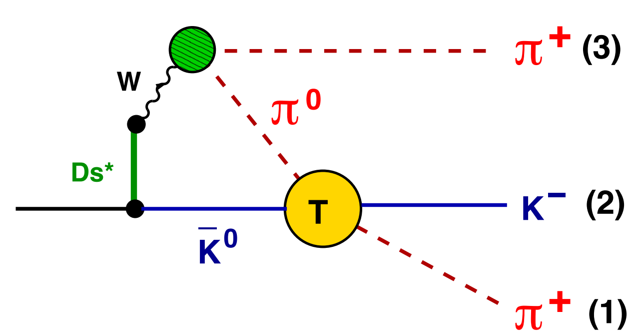
- $Br\,[B \to D D_s^*]$ ~1% → 1000 x $Br\,[B \to KKK]$

- hadronic loop → three-body FSI - introduce new complex structures

- $B^+ \to \pi^+ \pi^- \pi^+$

PCM & I Bediaga
arXiv:1512.09284

- $D^+ \to \pi^+ K^- \pi^+$

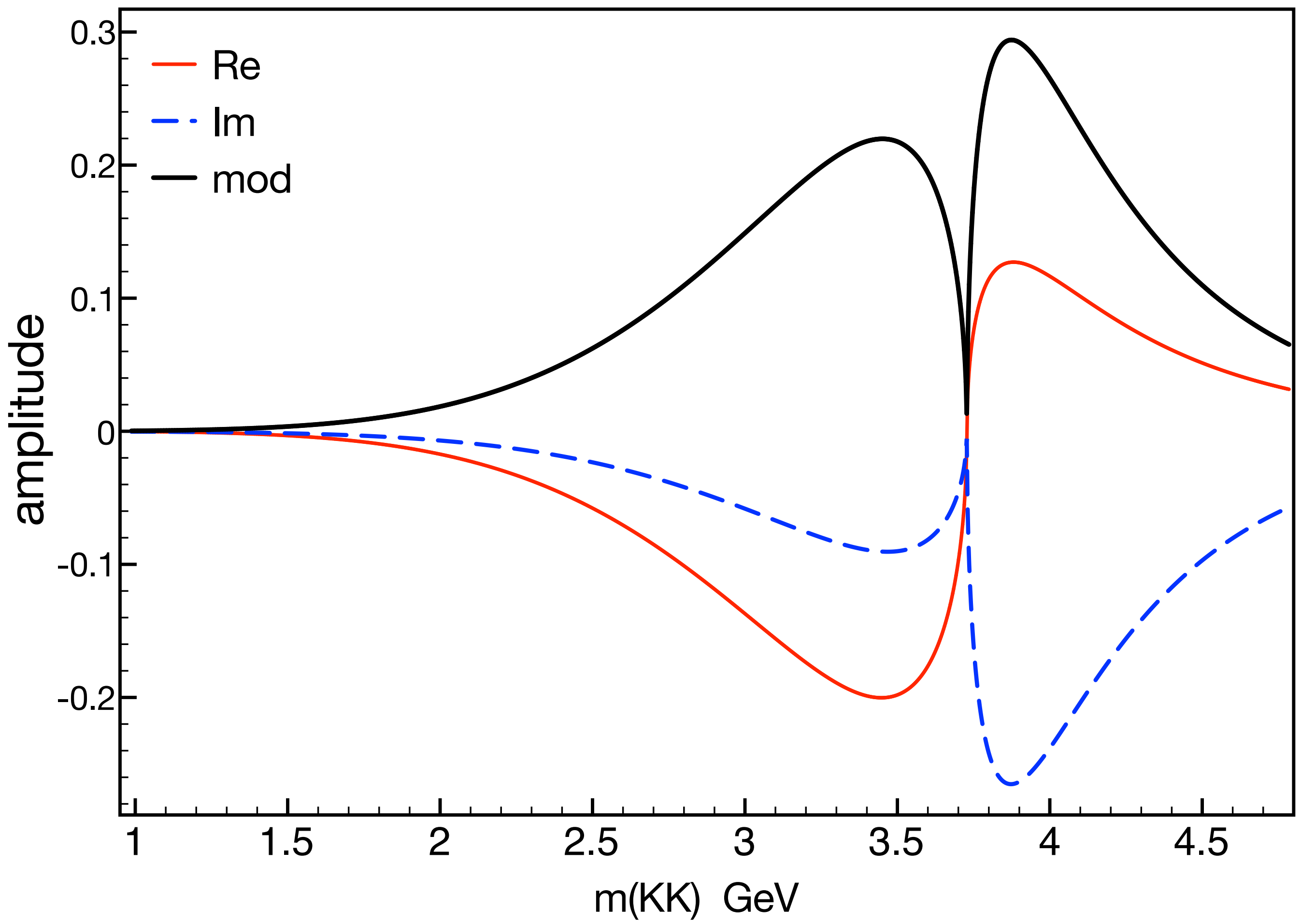


PCM & M Robilotta
PRD 92 094005 (2015) [arXiv:1504.06346]
PCM et al
PRD 84 094001 (2011) [arXiv:1105.5120]

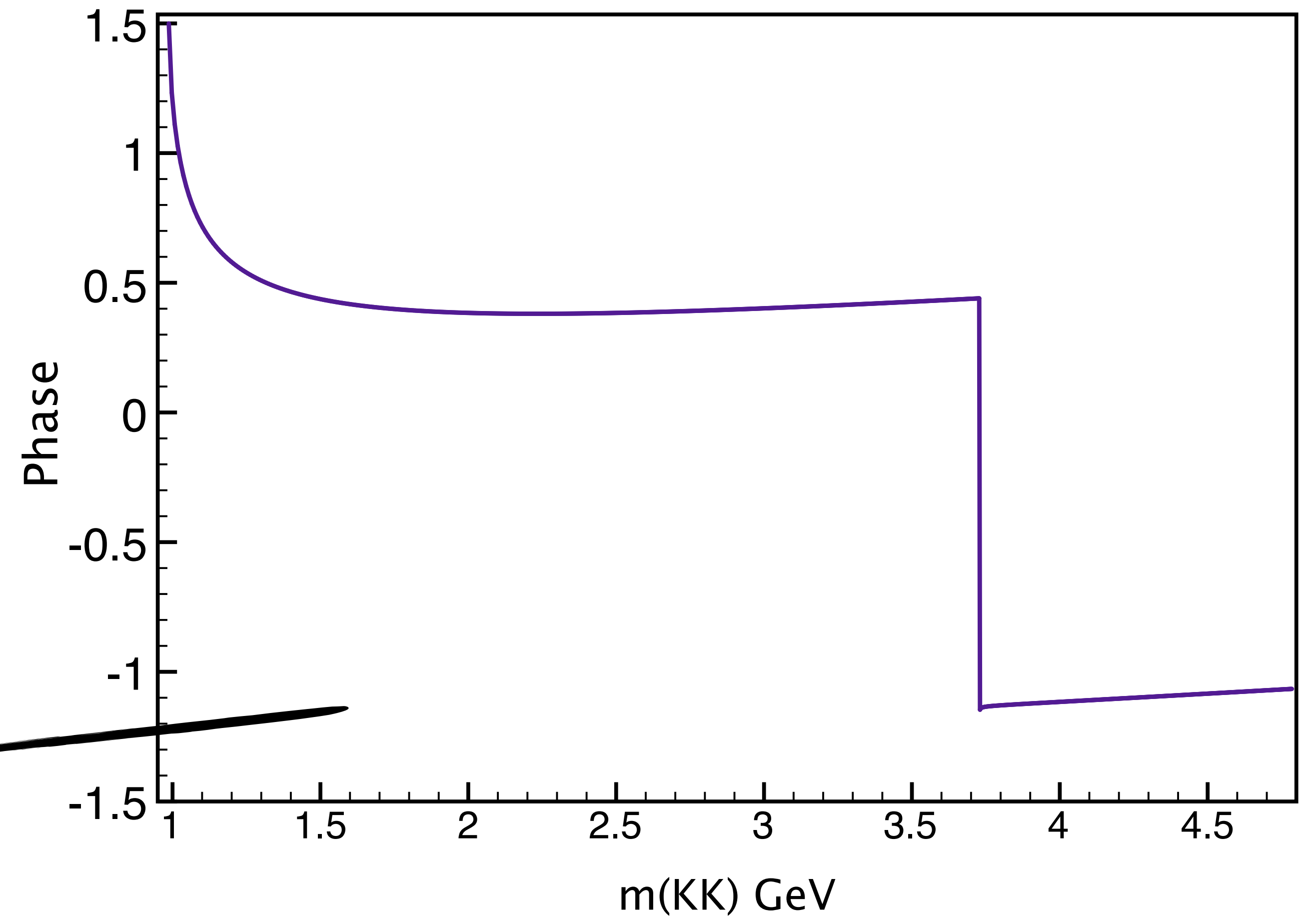
$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^\alpha}{s^\alpha_{th\,D\bar{D}}} \frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}}, \quad s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi} \left(\frac{m_0}{s-m_0}\right)^{\beta} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}}, \quad s \geq s_{th\,D\bar{D}}$$

→ parameters fix by data!



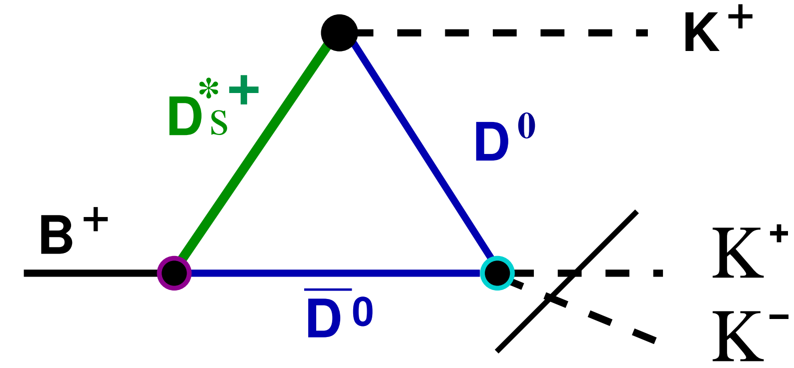
→ zero at threshold

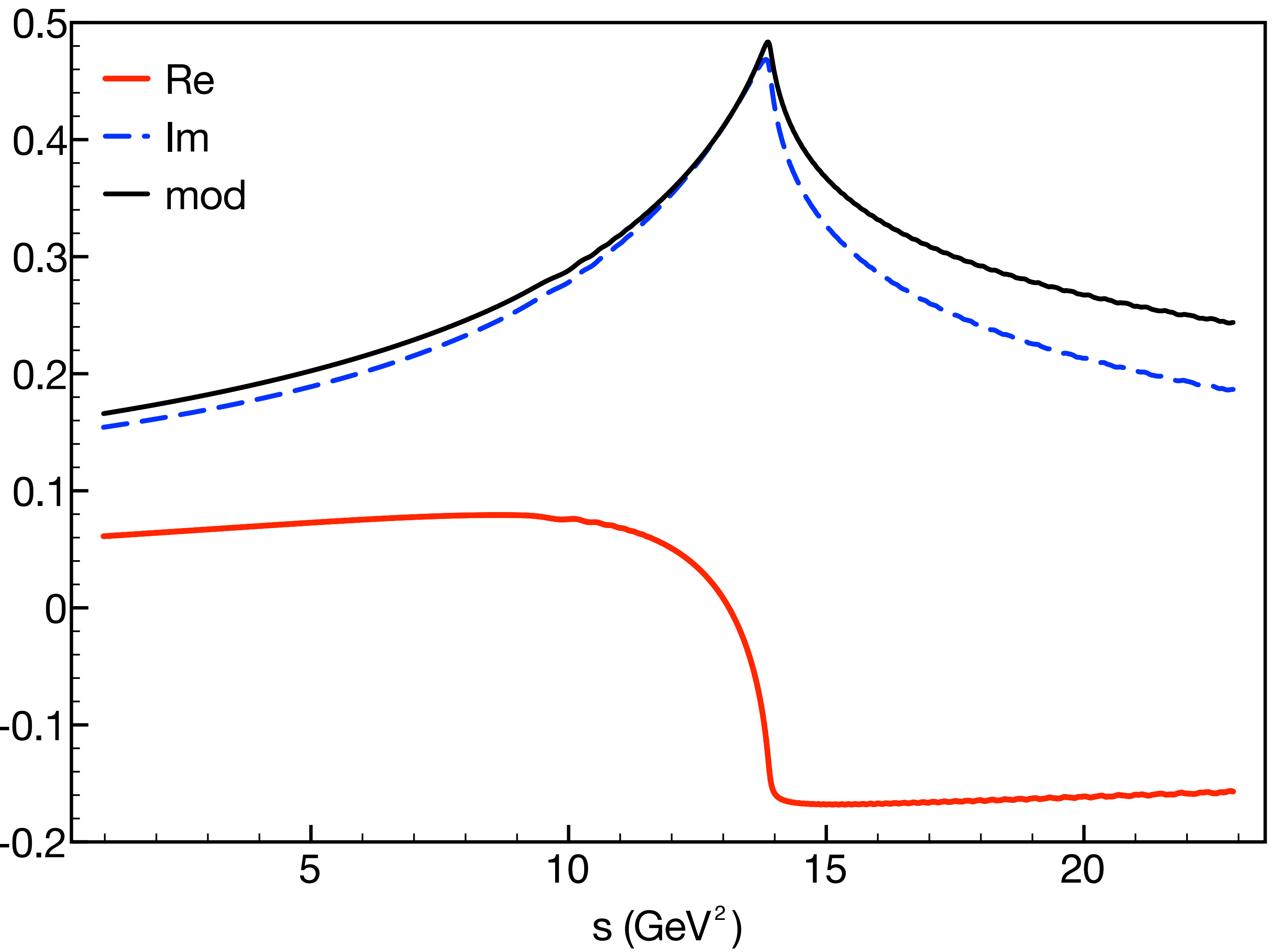


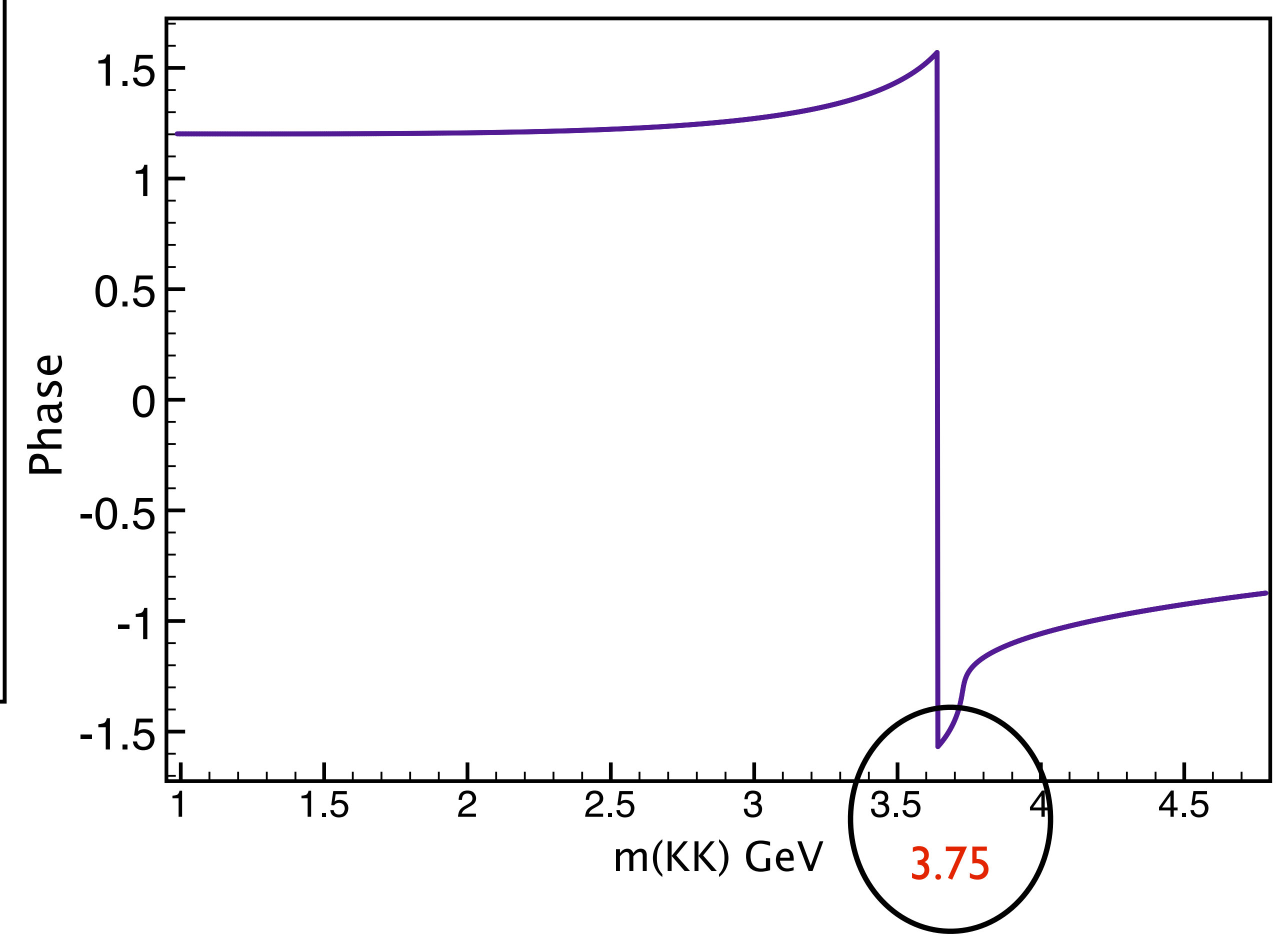
discontinuity at threshold ←

hadronic loop

- $$Loop = i \int \frac{d^4\ell}{(2\pi)^4} \, \frac{\Delta_{D^0} + 2\,\Delta_{\bar{D}^0} - \, 2\, s_{23} + 3\, M_K^2 + M_B^2 - l^2}{\Delta_{D^0}\, \Delta_{\bar{D}^0}\, \Delta_{D*}\, [l^2 - m_{B^*}]}$$





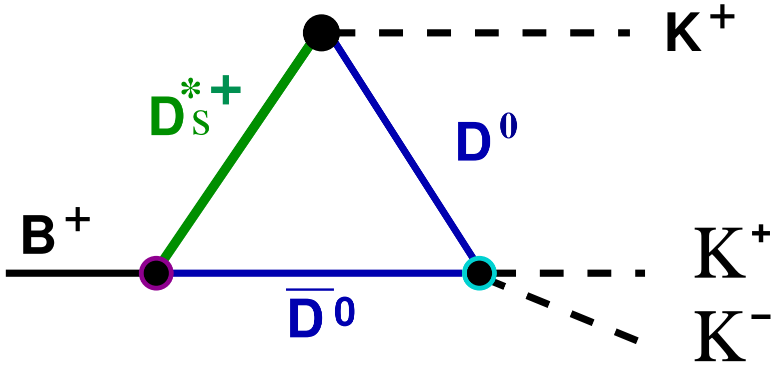


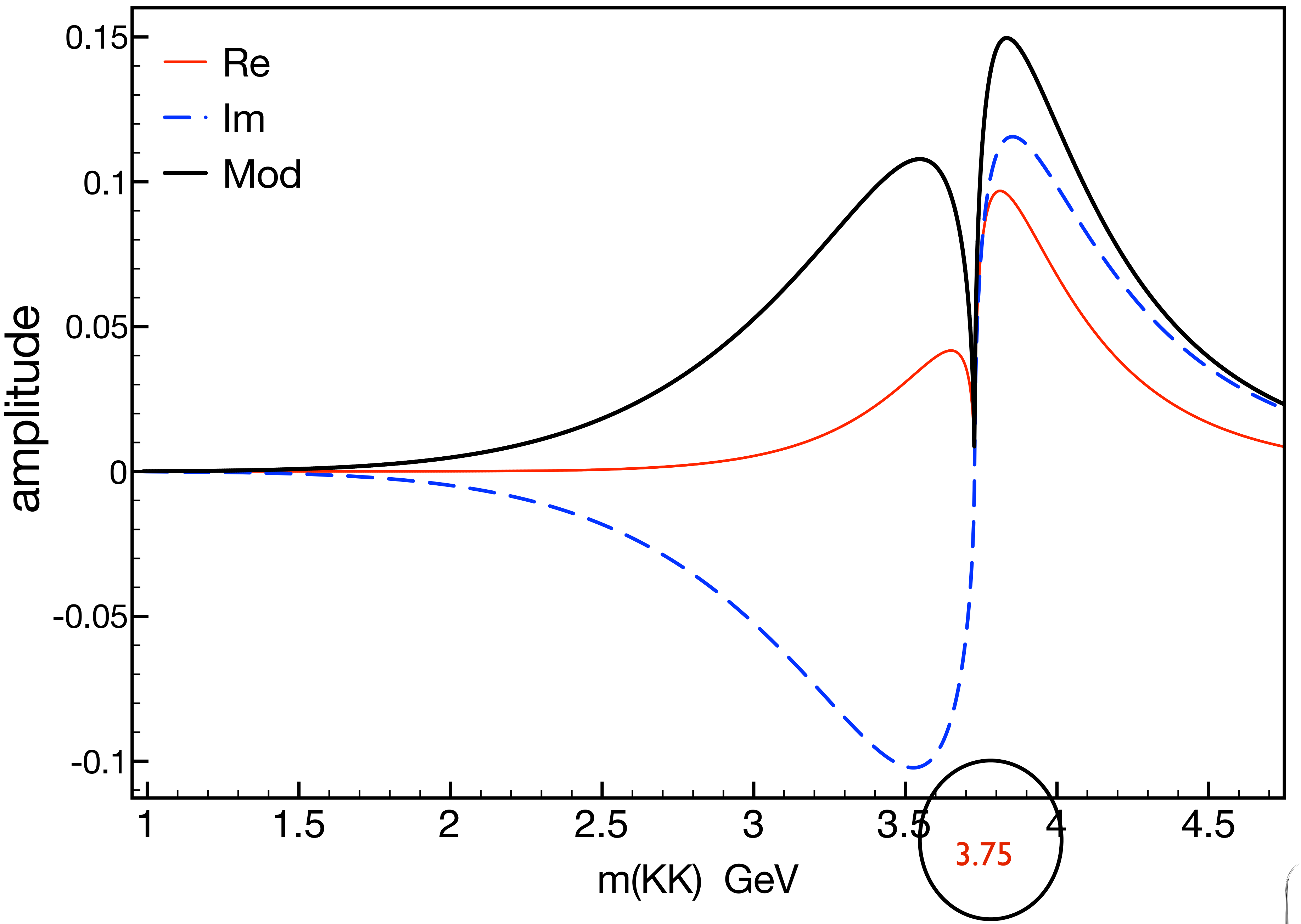
discontinuity at threshold

→ change sign at threshold

Final Amplitude

- $$A = iC \ m_a^2 \int \frac{d^4\ell}{(2\pi)^4} \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23}) \left[-2\, p_3' \cdot (p_2' - p_1)\right]}{\Delta_{D^{+*}} \Delta_{D^0} \Delta_{\bar{D}^0} \Delta_a},$$



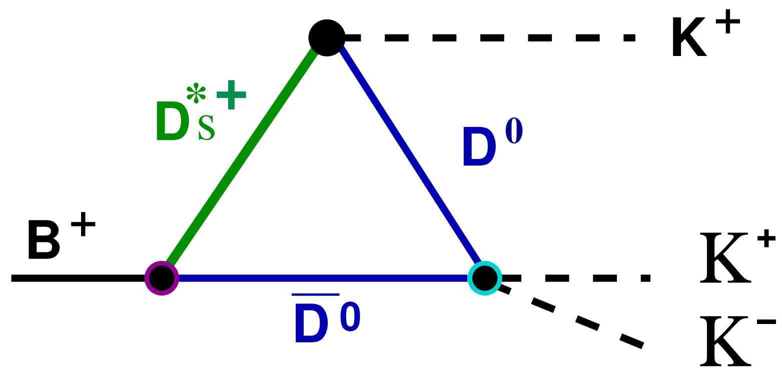


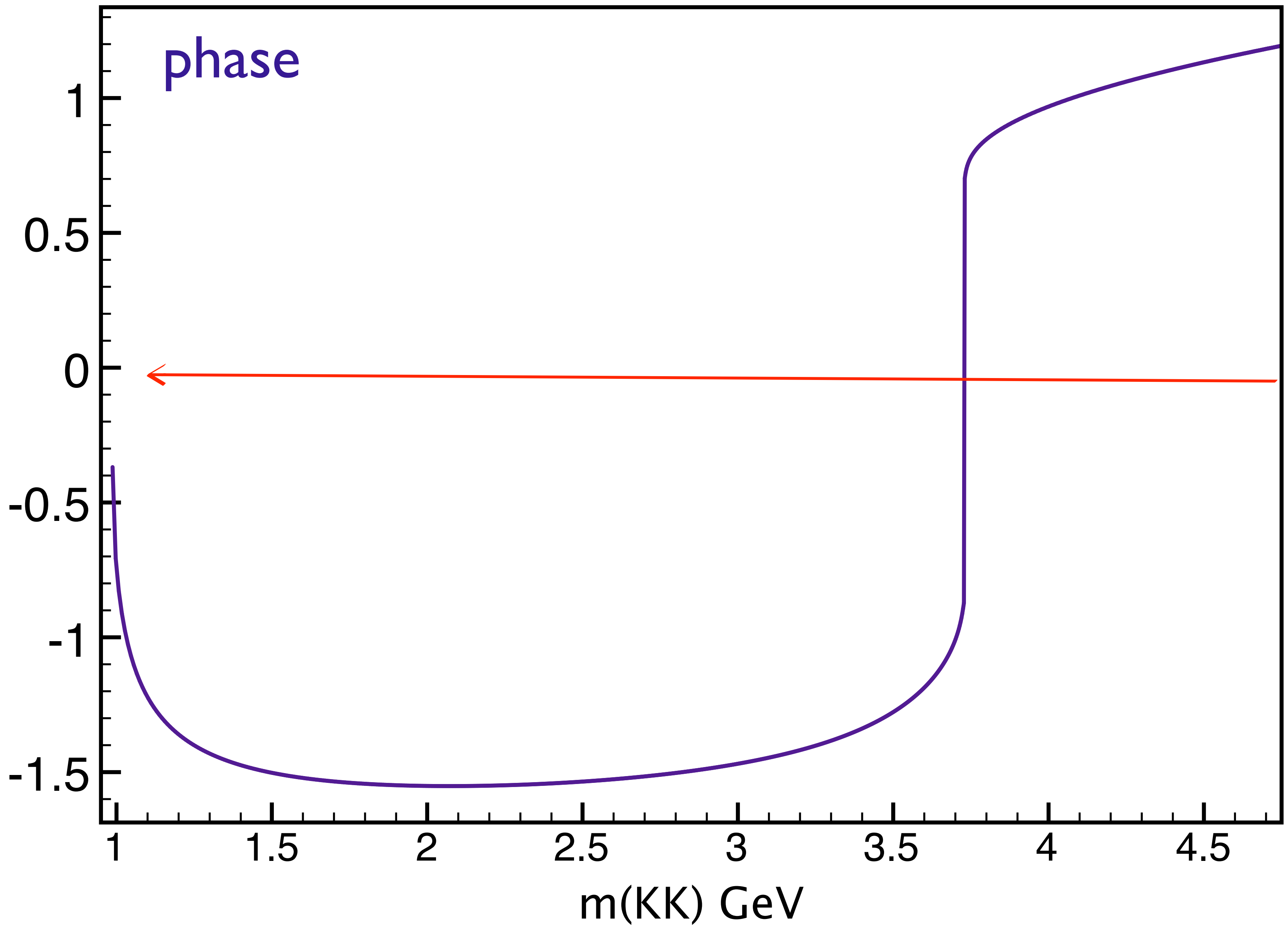
zero in between two bumps

rescattering $D^0\bar{D}^0 \to K^+K^-$ play a major role

superposition of triangles

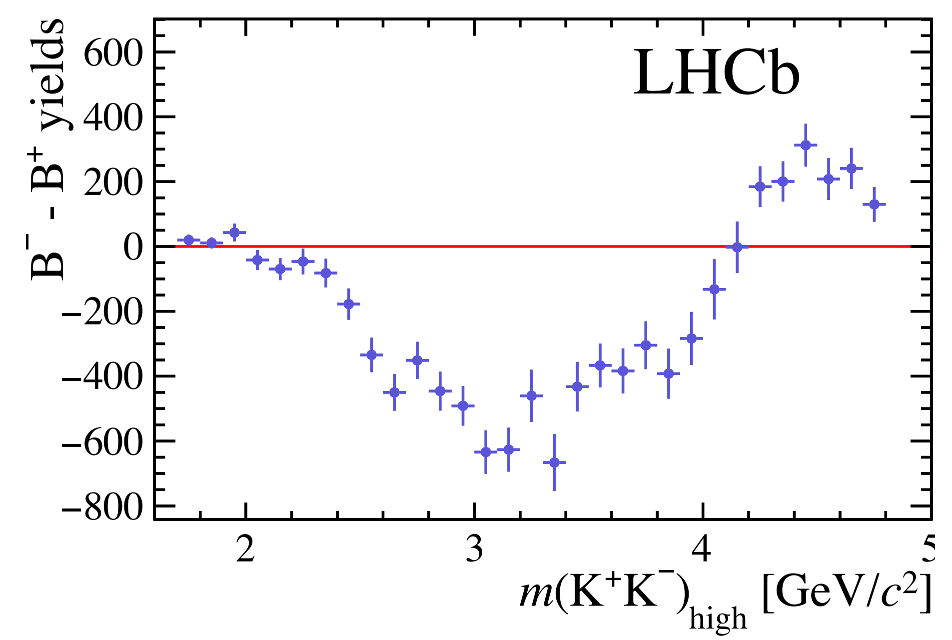
$$A = iC \; m_a^2 \int \frac{d^4\ell}{(2\pi)^4} \; \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23})\,[-2\,p_3' \cdot (p_2' - p_1)]}{\Delta_{D^{+*}}\,\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_a} \,,$$





→ Phase change signal in the same region as Acp data

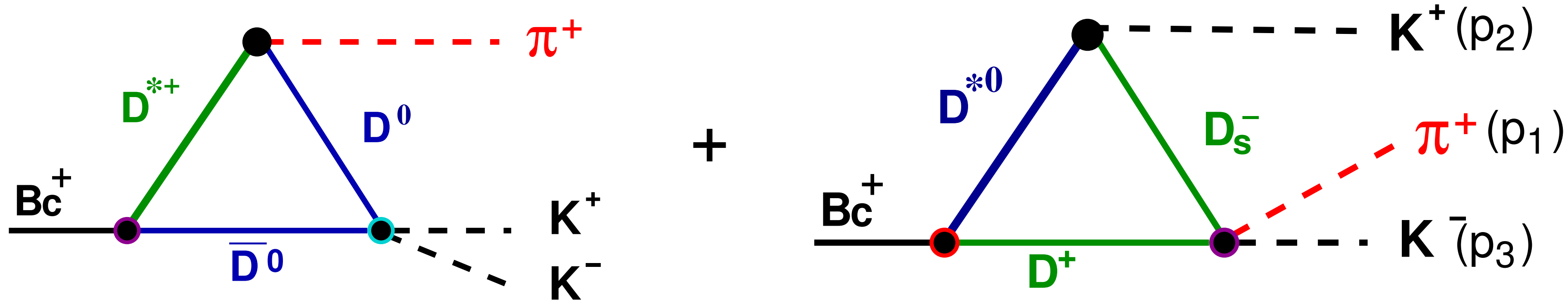
Promising mechanism !



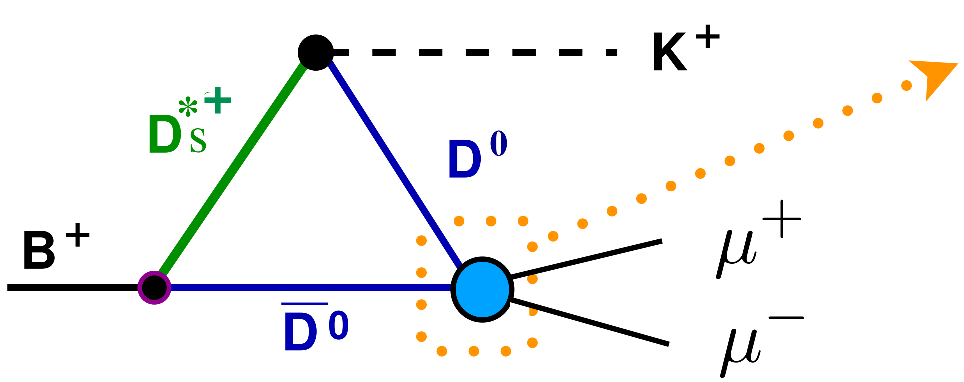
→ can explain change CPV signal in DP!!!

charm rescattering to $B_c^+ \to K^- K^+ \pi^+$

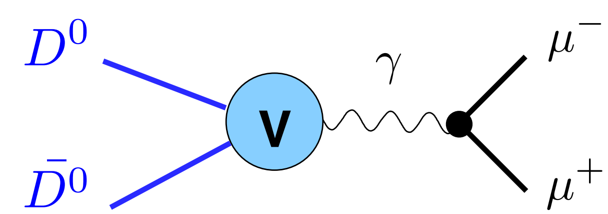
I. Bediaga, PCM,T Frederico
PLB 785 (2018) 581



Next: investigating Hadronic effect in $B \to K\mu\mu$



$\bar{D}^0 D^0 \to \gamma \to \mu^+\mu^-$



vector meson dominance

V= all vector family psi, …

How much of the anomalies can be understood as hadronic effects?

Charm rescattering

- $B_c^+ \to K^- K^+ \pi^+$

I. Bediaga, PCM, T Frederico
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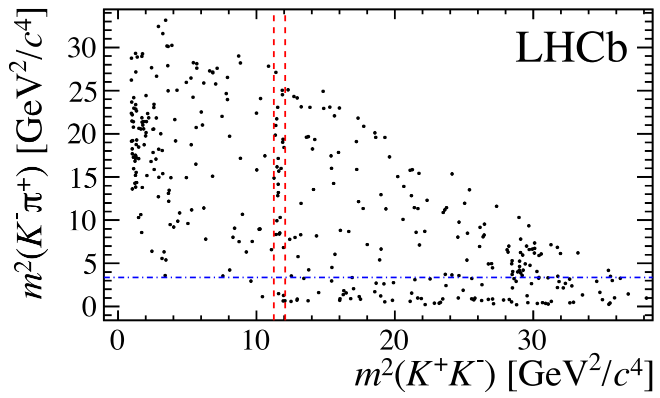
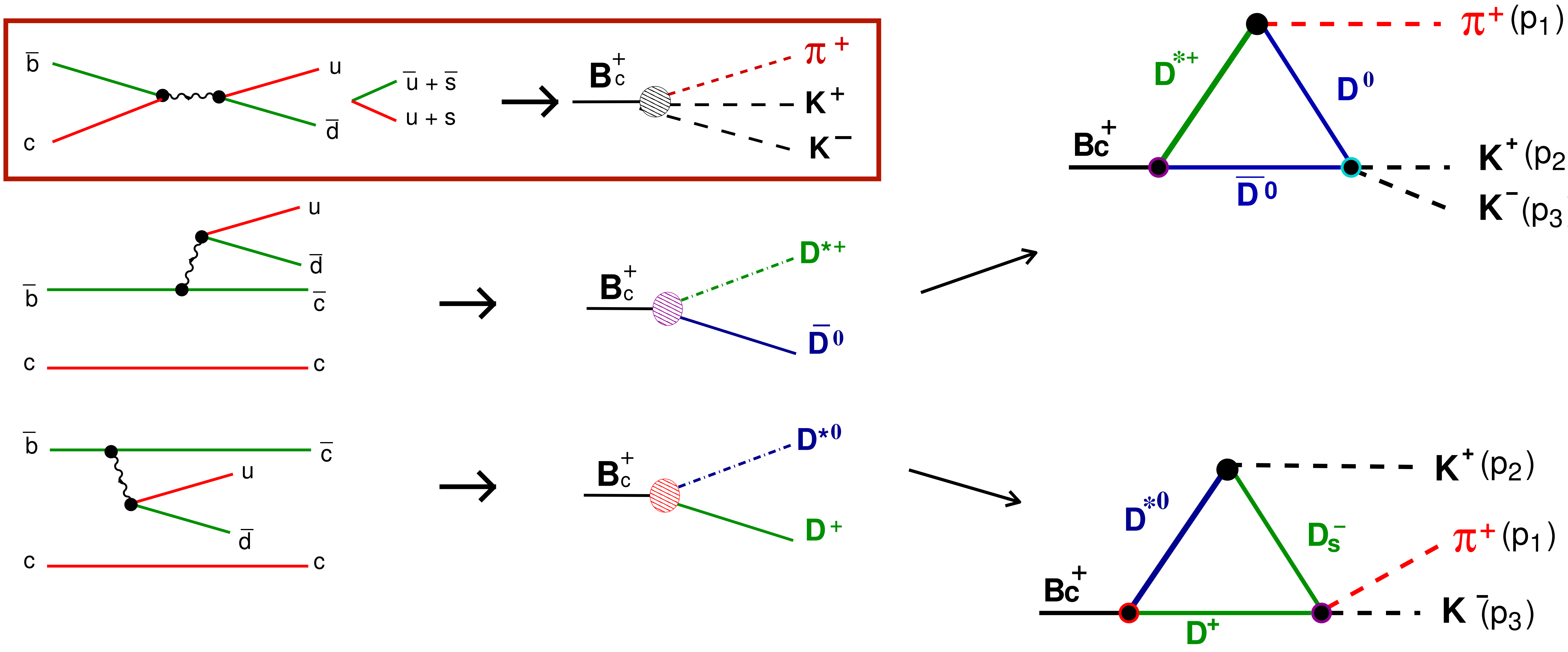
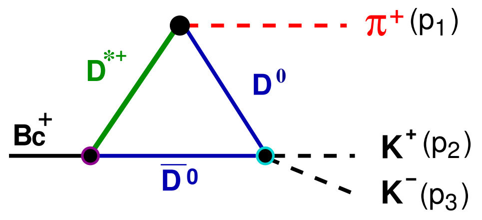


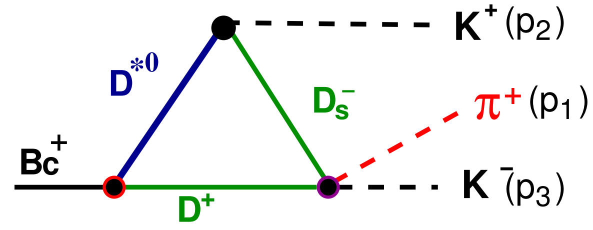
- very suppressed direct production (annihilation)
- charm rescattering can be the dominant mechanism

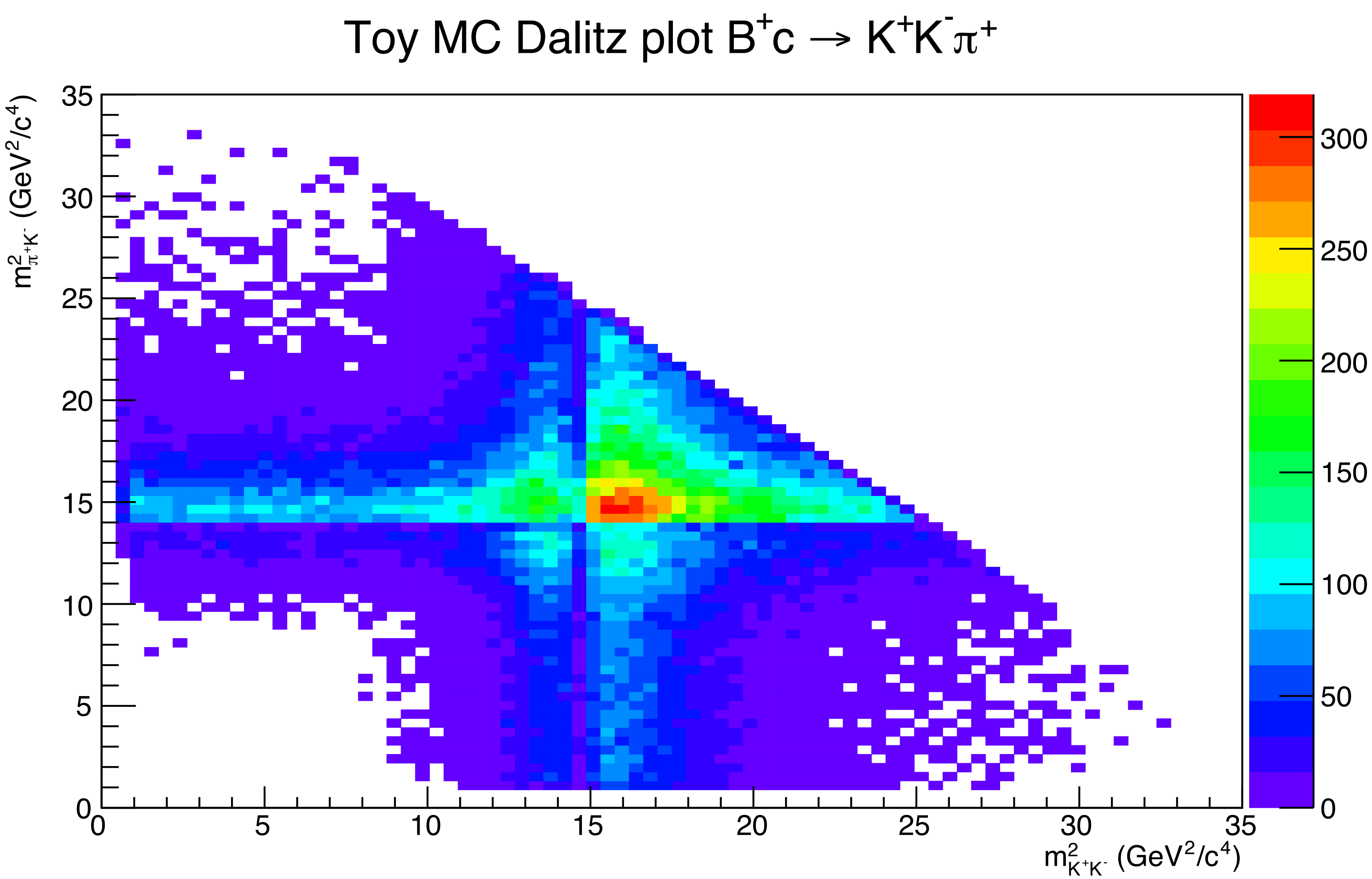


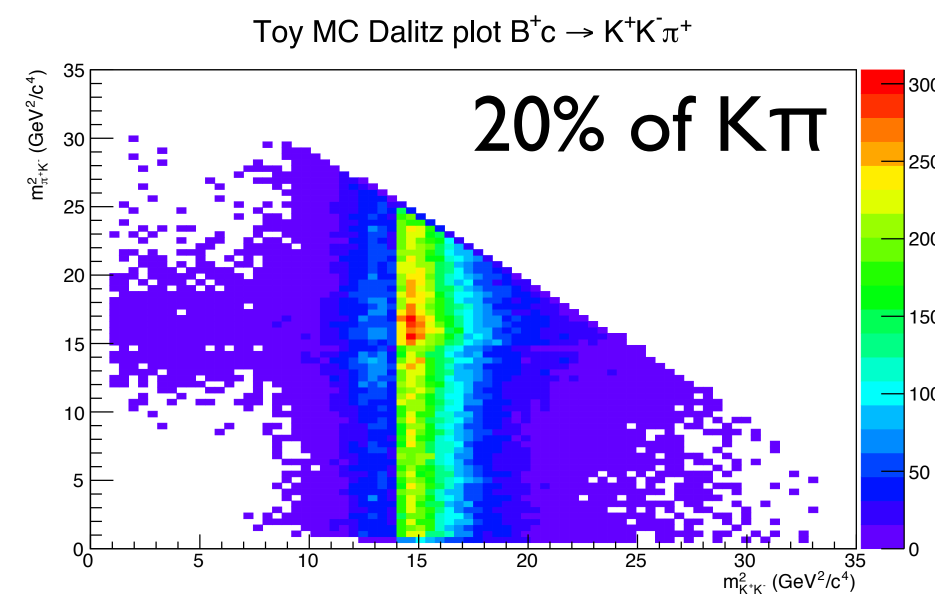
- toy Monte Carlo generator



&



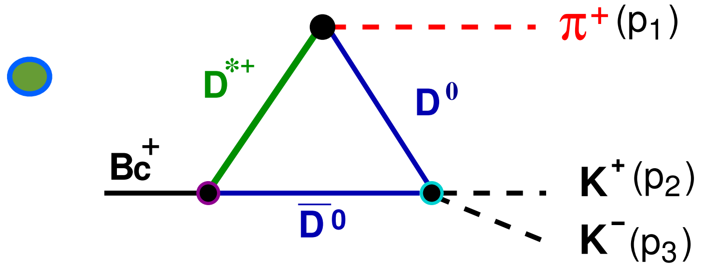


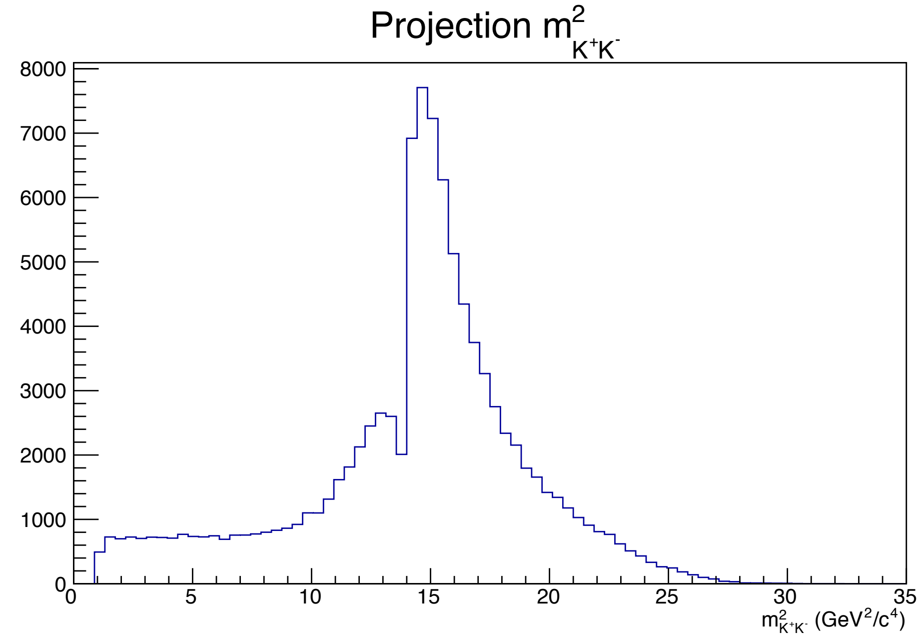


change side bands but not the minimum position (thresholds)

→ leave a signature in the middle of the Dalitz plot

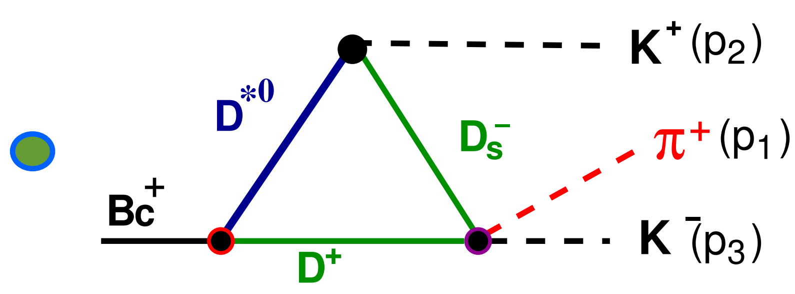
- Amplitudes projections

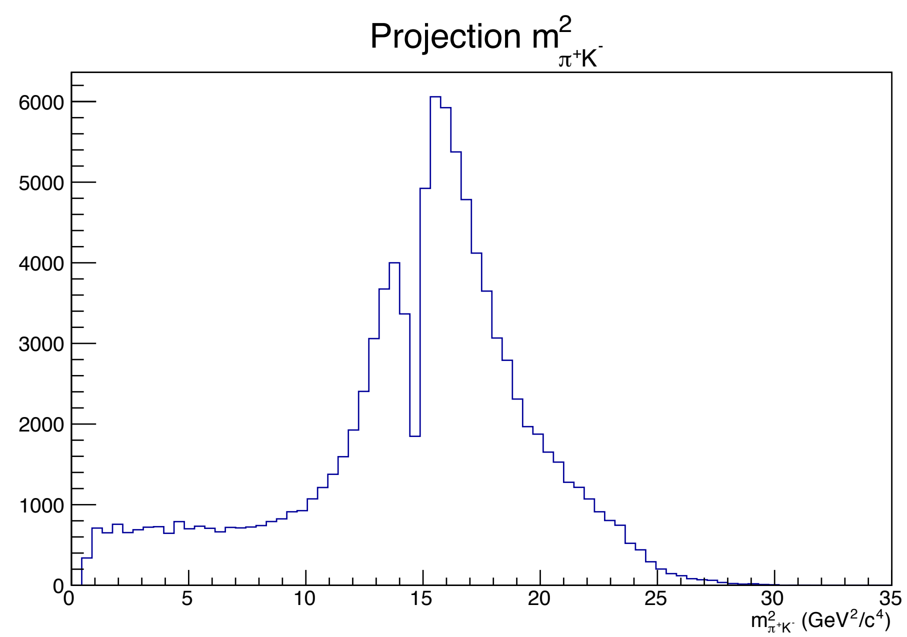




⟶ minima in different positions (≠ thresholds)

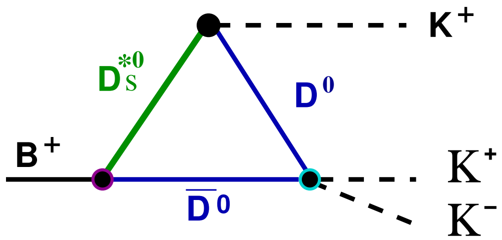
⟶ ≠ mass parameters inside triangle and rescattering amplitudes are relevant





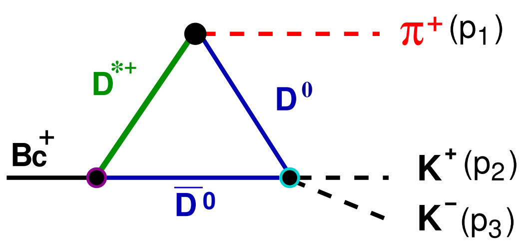
Triangle hadronic loop with charm FSI play an important role!

- $B^{\pm} \to K^+ K^- K^+$



→ mechanism to produce CP asymmetry at high mass

- $B_c^+ \to K^- K^+ \pi^+$



→ main mechanism to produce this final state

- If direct production (annihilation) → expect resonances in KK and Kπ channels

  → not observed

- real data: interference between ≠ triangles & NR sources & resonances

  → canNOT change the signature of a minima between the bumps and phases!

  → LHCb run 2 to confirm

FSI are important and play a major role in hadronic 3-body decays!

→ superposition of resonant and non-resonant at low and high energy

→ Charm rescattering is under intense investigation : CPV on B, exotics, anomalies, ……

→ Will be tested in data ANA!

- Successful examples of cooperation between theory and experiment !!!

→ Important tool !

Thank you very much!

Backup slides

Backup slides

$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- not well understand on literature

- important as FSI in B two-body decays

Donoghue *et al.*, *PRL* 77(1996)2178;
Suzuki,Wolfenstein, PRD 60 (1999)074019;
Falk et al. PRD 57,4290(1998);
Blok, Gronau, Rosner, PRL 78, 3999 (1997).

- phenomenological amplitude

Antunes, Bediaga, Frederico, PCM
ICHEP2016 - proceedings

  - unitarity of the S-matrix $S = \begin{pmatrix} \eta\, e^{2i\alpha} & \sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} \\ -\sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} & \eta\, e^{2i\beta} \end{pmatrix}$

  - inspired in the damping factor of the S matrix i.e. $\pi\pi \to KK$

$$\eta = \mathcal{N}\sqrt{s/s_{th}-1}/(s/s_{th})^{2.5}$$

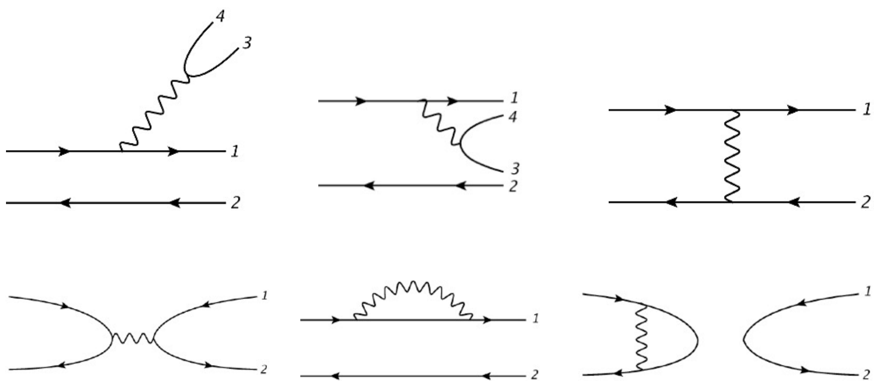
KK: $e^{2i\alpha} = 1 - \dfrac{2ik_1}{\frac{c}{1-k_1/k_0} + ik_1}$, DD: $e^{2i\beta} = 1 - \dfrac{2ik}{\frac{1}{a} + ik}$

$k = \sqrt{\frac{s-s_{th}}{4}}$, $k_1 = \sqrt{\frac{s-s_{th1}}{4}}$ and $k_0 = \sqrt{\frac{s_0-s_{th}}{4}}$

$$S_{\beta,\alpha} = \delta_{\beta,\alpha} + it_{\beta,\alpha}$$

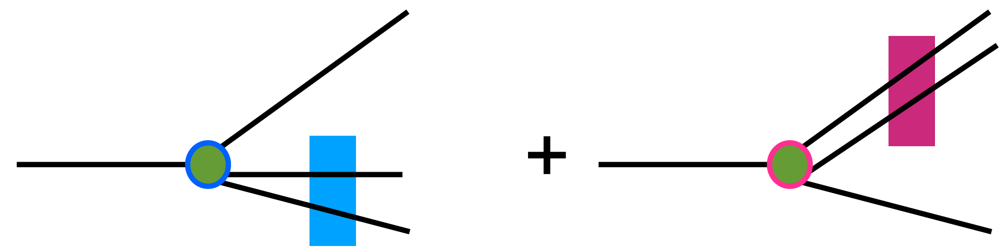
$$\boxed{t_{\beta,\alpha} = \sqrt{1-\eta^2}e^{i(\alpha+\beta)}}$$

- QCD factorization approach → factorize the quark currents



Chau [Phys. Rep. 95,1(1983)]

- challenging for 3-body
- not all FSI and 3-body NR
- scale issue with charm !



$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \left[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right] + \text{h.c.} \,,$$

→ ex: $B^+ \to \pi^+\pi^-\pi^+$

how to describe it?

$$A \sim \underbrace{\langle [\pi^+(p_2)\pi^-(p_3)]}_{R} \; |(\bar{u}b)_{V-A}|B^-\rangle \, \langle \pi^-(p_1)|(\bar{d}u)_{V-A}|0\rangle + \langle \pi^-(p_1)|(\bar{d}b)_{sc-ps}|B^-\rangle \, \underbrace{\langle [\pi^+(p_2)\pi^-(p_3)] \; |(\bar{d}d)_{sc+ps}|0\rangle}_{FF}$$

- naive factorization
  - intermediate by a resonance R;
  - FSI with scalar and vector form factors FF

↳ parametrizations for B and D→3h Boito et al. PRD96 113003 (2017)

- modern QDC factorization: improvement to include "long distance" still developing

Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)

FSI in three-body decay :

references

I. Bediaga, I., T. Frederico, T. and O. Louren   Phys. Rev. D89, 094013(2014),[arXiv:1307.8164]

J. H. Alvarenga Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico and O. Louren¸ Phys. Rev. D92, 054010 (2015) [ArXiv:1506.08332].

PC Magalhães and I Bediaga   arXiv:1512.09284;

P. C Magalhães and R.Robilotta, Phys. Rev. D92 094005 (2015) [arXiv:1504.06346] ; P.C.Magalhães et. al. Phys. Rev. D84 094001 (2011) [arXiv:1105.5120]; P.C. Magalhães and Michael C. Birse, PoS QNP2012, 144 (2012).

I. Caprini, Phys. Lett. B 638 468 (2006).

Bochao Liu, M. Buescher, Feng-Kun Guo, C. Hanhart, and Ulf-G. Meissner, Eur. Phys. J. C 63 93 (2009).

F Niecknig and B Kubis - JHEP 10 142 (2015) ArXiv:1509.03188

H. Kamano, S.X. Nakamura, T.-S.H. Lee and T. Sato, Phys. Rev. D 84, 114019 (2011).

S. X. Nakamura, arXiv:1504.02557 (2015).

J. -P. Dedonder, A. Furman, R. Kaminski, L. Lesniak, L. and B. Loiseau, Acta Phys. Polon. B42, 2013 (2011), [Arxiv: 1011.0960]

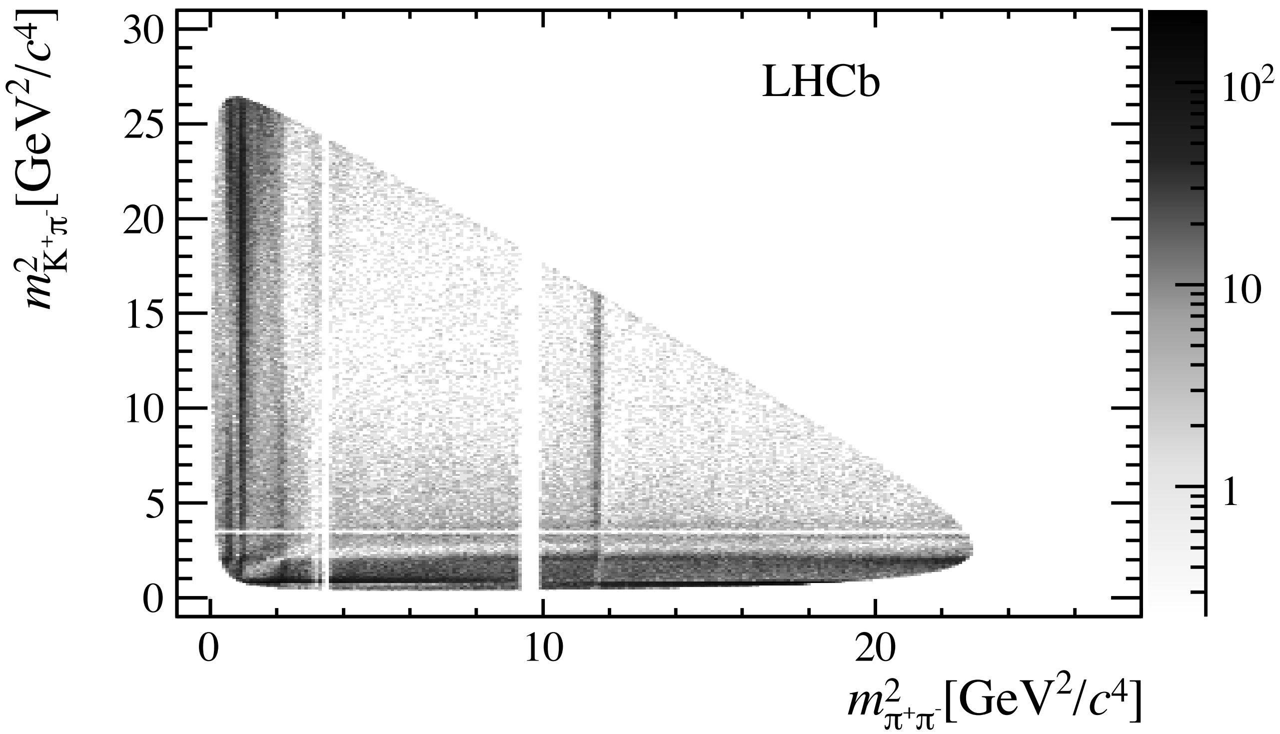
J.-P. Dedonder, R. Kaminski, L. Lesniak, and B. Loiseau, , Phys. Rev.D89, 094018 (2014).

Donoghue *et al.*, *Phys. Rev Letters* 77(1996) 2178;

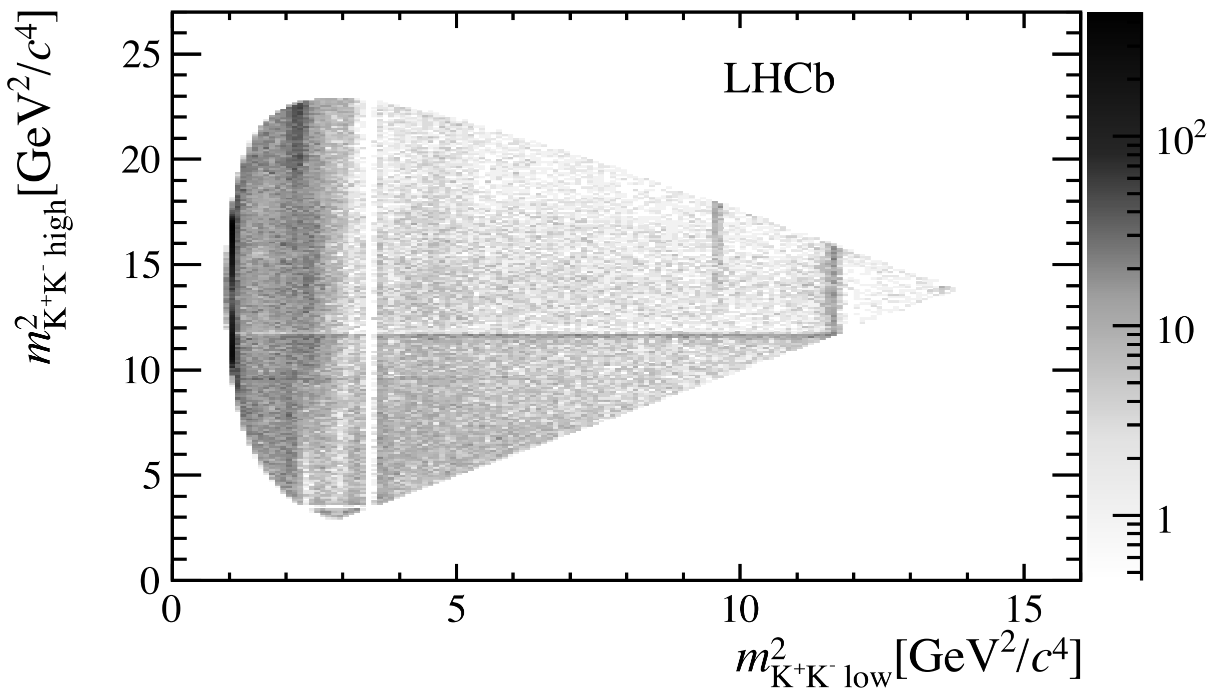
Suzuki,Wolfenstein, Phys. Rev. D 60 (1999)074019;
Falk et al. Phys. Rev. D 57,4290(1998);
Blok, Gronau, Rosner, *Phys. Rev Letters* 78, 3999 (1997).

many more ...

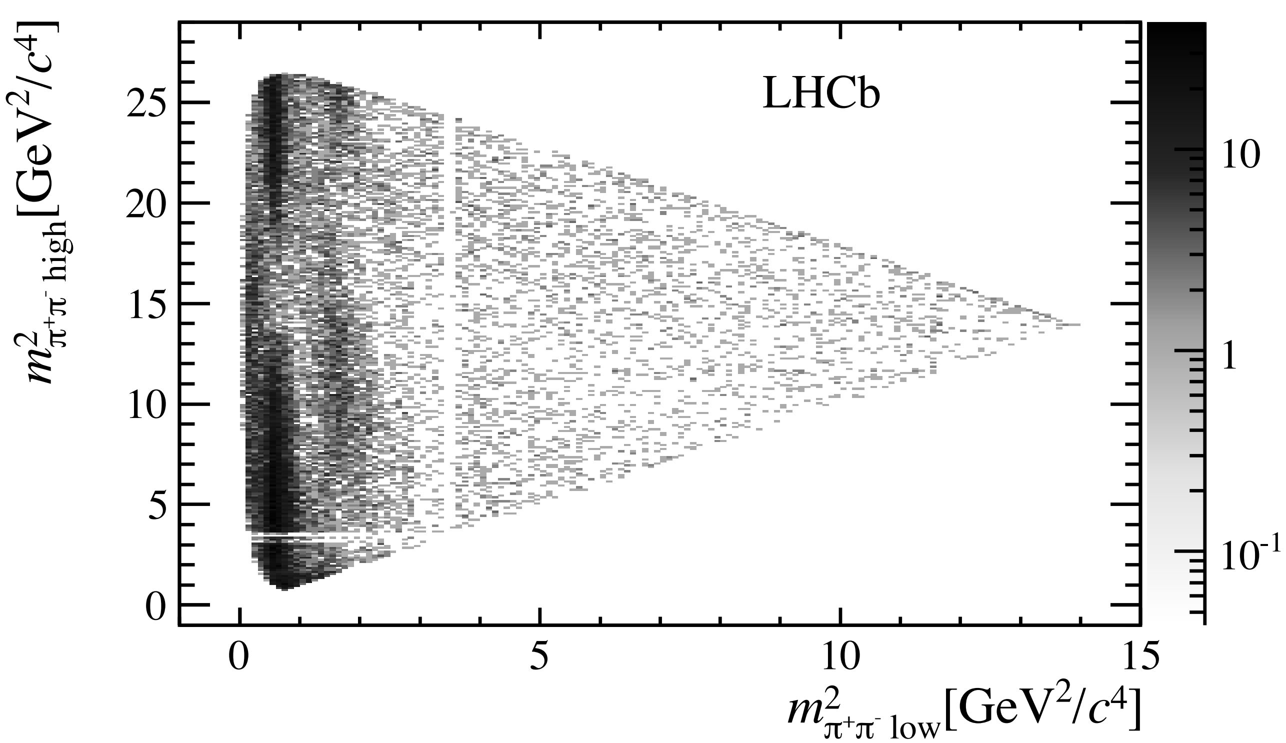
Kpp



KKK



PPP



KKp