Semileptonic $B_{(s)}$ decays on the lattice

Ryan Hill

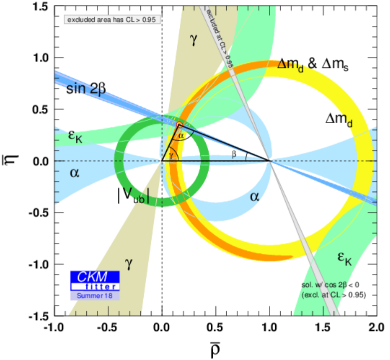
J. Flynn, A. Jüttner, A. Soni, J. Tsang, O. Witzel
with the RBC-UKQCD collaborations
University of Southampton

6^{th} January 2020

arXiv:1912.09946

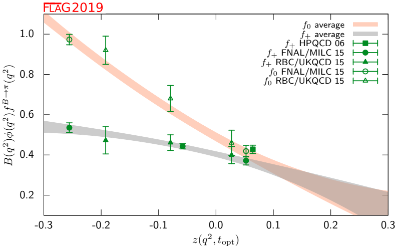
Motivation

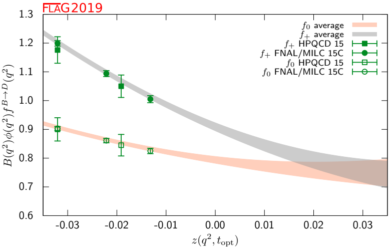
- Test unitarity of CKM matrix
- $B \to \pi\ell\nu$, $B_s \to K\ell\nu$: constrains $|V_{ub}|$
- $B \to D\ell\nu$, $B_s \to D_s\ell\nu$: constrains $|V_{cb}|$
- 2-3σ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$)
- Lepton universality ratio predictions.

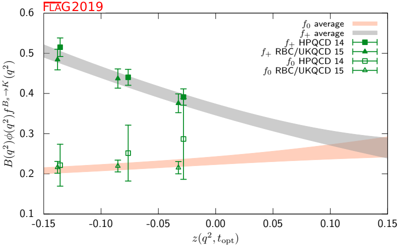


CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Motivation







- Relatively little data for heavy-light decays in 2019 FLAG averages

HPQCD 06	[PRD 73 (2006) 074502]
FNAL/MILC 15	[PRD 92 (2015) 014024]
RBC/UKQCD 15	[PRD 91 (2015) 074510]
HPQCD 14	[PRD 90 (2014) 054506]
HPQCD 15	[PRD 92 (2015) 054510]
FNAL/MILC 15C	[PRD 92 (2015) 034506]

Goal

- Differential $B_{(s)} \to P\ell\nu$ decay rate:

$$\underbrace{\frac{d\Gamma(B_{(s)} \to P\ell\nu)}{dq^2}}_{\text{Experiment}} = \underbrace{|V_{xb}|^2}_{Target} \times \left(\frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - M_P^2}}{q^4 M_{B_{(s)}}^2} \right.$$

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_{(s)}}^2 (E_P^2 - M_P^2) \underbrace{|f_+(q^2)|^2}_{non-pert.} + \frac{3m_\ell^2}{8q^2} (M_{B_{(s)}}^2 - M_P^2)^2 \underbrace{|f_0(q^2)|^2}_{non-pert.} \right]$$

$$P = \pi, K, D, D_s$$

$$x = u, c$$

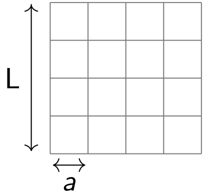
q^2 — momentum transfer to $\ell\nu$

- So seek to compute form factors theoretically

Lattice QCD

- Lattice QCD is a first-principles, systematically improvable non-perturbative formulation of QCD valid at weak and strong coupling.

- Lattice length L acts as an IR regulator
- Lattice spacing a acts as a UV regulator



Lattice QCD

- Based on path integral formulation of QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\ldots] \mathcal{O} e^{-S[\ldots]}$$

- In continuum, infinite number of integration variables
- On the lattice, there are instead a finite (but very large!) number of integration variables
- QCD becomes a (computationally expensive) problem in numerical integration!

Lattice QCD

- Chiral symmetry is not necessarily respected on the lattice.
- Bare quark masses am_q are input parameters given in "lattice units".
- Mass as a free parameter is useful: small masses are expensive to compute, large masses can run into uncontrolled discretisation errors.
- Results must be:
  - Inter/extrapolated in physical mass values
  - Taken to the continuum limit (remove discretisation effects)
  - Taken to infinite volume (remove finite volume effects)
- Choice of fermion **action** can have a significant impact on discretisation errors and the mass range available to simulate.

Regulation requirements: Quick calculation

- To control finite size effects, need $m_\pi L \gtrsim 4$
  For $m_\pi = m_\pi^{\text{phys}} \approx 140$ MeV, $\Rightarrow L \approx 5.6 fm$
- To control discretisation effects, need $am_q \ll 1$
  For $m_b \approx 4.2$ GeV, $\Rightarrow a \approx 0.05 fm$
- $N = L/a \gtrsim 120$, $\Rightarrow N^3 \times (2N) \gtrsim 4 \times 10^8$ lattice sites
- This is a very expensive constraint to satisfy.

Quark Actions

- We choose Domain wall fermions which is a discretisation that is exponentially close to chiral symmetry
- We use a different action to simulate the b-quark because we don't have access to fine enough lattices (yet) to do this directly at the *b*-mass.

Strategy

Outline of strategy:

1. Simulate form factors at various lattice spacings, masses, daughter energies
2. Extrapolate the results to the continuum
3. Assess all sources of systematic errors and build a complete error budget
4. Extrapolate continuum result over full q^2 range.

Strategy

- In order to find f_+, f_0, seek to compute the hadronic matrix element for the flavour-changing vector currents $\langle P|\mathcal{V}^\mu|B_{(s)}\rangle$
- Standard parameterisation in terms of the scalar and vector form factors f_+ and f_0:

$$\langle P|\mathcal{V}^\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{M^2_{B_{(s)}} - M^2_P}{q^2} q^\mu\right) + f_0(q^2)\left(\frac{M^2_{B_{(s)}} - M^2_P}{q^2} q^\mu\right)$$

Strategy

- Parallel and perpendicular form factors $f_{\|}$ and $f_{\perp}$ are simpler to relate to lattice data in the rest frame of the $B_{(s)}$-meson:

$$\langle P|\mathcal{V}^{\mu}|B_{(s)}\rangle = \sqrt{2M_{B_{(s)}}}\left[v^{\mu} f_{\|}(E_P) + p_{\perp}^{\mu} f_{\perp}(E_P)\right]$$

with
v^{μ} — $B_{(s)}$-meson 4-velocity
$p_{\perp}^{\mu}$ — $p_P^{\mu} - (p_P \cdot v)v^{\mu}$
p_P^{μ} — momentum of pseudoscalar particle

$$f_{\|} = \frac{\langle P|\mathcal{V}^{0}|B_{(s)}\rangle}{\sqrt{2M_{B_{(s)}}}} \qquad f_{\perp} = \frac{\langle P|\mathcal{V}^{i}|B_{(s)}\rangle}{\sqrt{2M_{B_{(s)}}}}\frac{1}{p^{i}}$$

- Neatly separates into spatial and temporal components.

Strategy

- The matrix elements can be extracted from three-point correlation functions by cancelling off the other contributions.
- These other contributions involve two-point correlation functions of the involved mesons and their energies.
- Energies can be extracted from the two-point correlation functions.
- So the data used in this analysis is a set of three- and two-point correlation functions evaluated using lattice QCD.

Ensembles

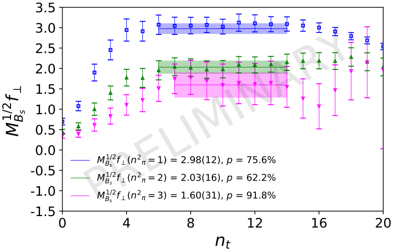
	$L^3 \times T \ / \ a^4$	a^{-1} / GeV	m_π / MeV
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1	$48^3 \times 96$	2.77	230

- 2+1f ensembles: Degenerate light quark
- Sea quarks: Domain-wall fermions
- **F1** ensemble: New for this update of RBC-UKQCD 2015 analyses.
- Future plans to include physical pion mass ensemble C0 to the analysis.

$B \to \pi$ Analysis

$B \to \pi$ Form Factors

- Extract energies from two-point functions.
- Calculate $f_\parallel$ and $f_\perp$ from lattice data.
- Use this to find f_0 and f_+:



$f_\perp$ on M1 ensemble

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M^2_{B_{(s)}} + E_P^2}\left[(M_{B_{(s)}} - E_P)f_\parallel(q^2) + (E_P^2 - M_P^2)f_\perp(q^2)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}}\left[f_\parallel(q^2) + (M_{B_{()}} - E_P)f_\perp(q^2)\right]$$

$B \to \pi$ Chiral Continuum Fits

- Extrapolate to physical pion mass and zero lattice spacing simultaneously
- Use NLO hard-pion SU(2) HMχPT [PRD 67 (2003) 054010]

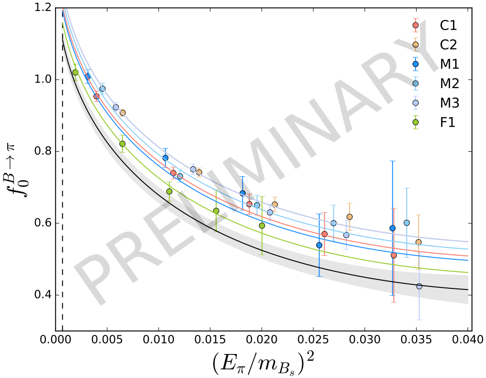
$$f(M_\pi, E_\pi, a) = \frac{c_1}{\Delta + E_\pi}\left(1 + \frac{\delta f}{(4\pi f_\pi)^2} + c_2 \frac{M_\pi^2}{\Lambda^2} + c_3 \frac{E_\pi}{\Lambda} + c_4 \left(\frac{E_\pi}{\Lambda}\right)^2 + c_5 (a\Lambda)^2\right)$$

- $\Lambda = 1$ GeV
- $\Delta_0 = 0.263$ GeV
- $\delta f^{B\to\pi} = -\frac{3}{4}(3g_b^2 + 1)M_\pi^2 \log\left(\frac{M_\pi^2}{\Lambda^2}\right)$

- $f_\pi = 0.1304$ GeV
- $\Delta_+ = -0.0416$ GeV
- $g_b = 0.57$

$B \to \pi$ Chiral Continuum Fits

- Three values of a
- Three/four values of E_π per ensemble
- Six ensembles/pion masses
- Simultaneously fit coefficients c_{1-5} to all data
- Continuum form factor given by $f(M_\pi^{\text{phys}}, E_\pi, a = 0)$



Systematic Error Analysis

- In preparation for an extrapolation of the continuum results to the full q^2 range, we construct **synthetic data points**.
- Systematic error analysis required for this construction: discretisation error, lattice scale uncertainty, variations in chiral continuum fit ansatz...
- Error budget not yet mature for $B \to \pi$: coming in the future.
- The synthetic data points are constructed at **reference q^2 values** and account for both systematic and statistical errors.
- The extrapolation coefficients are lattice-independent as a result.

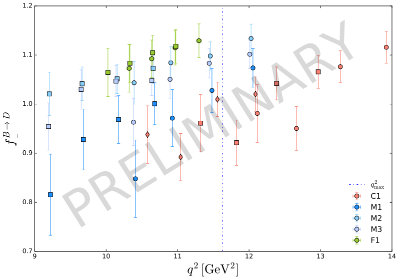
$B \to D$ Analysis

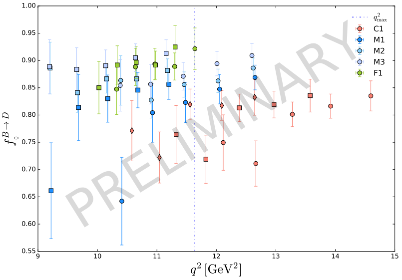
$B \to D$ Form Factors

- Charm quarks simulated with Möbius Domain-Wall Fermions for heavy quarks

  [JHEP 1604 (2016) 037] [JHEP 1712 (2017) 008]

- Simulate charm quarks at 2/3 different masses in order to inter/extrapolate to the physical value of the charm quark mass.





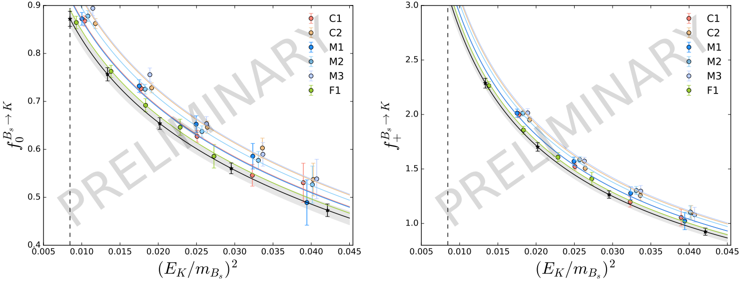
$B \to D$ Further Analysis

- Analysis still in progress
- Extrapolate to physical masses and continuum
- Proceed hereafter as with $B \to \pi$
- Alter fit ansatz etc. in order to estimate systematics

$B_s \to K$ Analysis

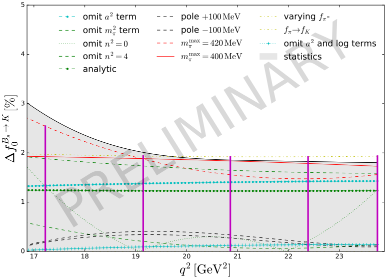
$B_s \to K$ Chiral Continuum Fit

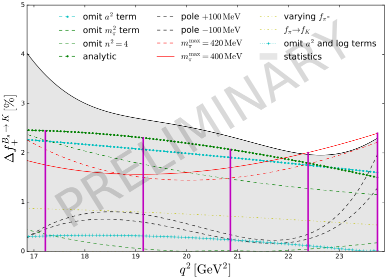
- Similar chiral continuum fit as $B \to \pi$
- Only structural difference is the non-analytic term $\delta f^{B_s \to K} = -\frac{3}{4} M_\pi^2 \log\left(\frac{M_K^2}{\Lambda^2}\right)$



$B_s \to K$ Error Budget

- Largest systematic due to a fit ansatz variation at each reference q^2 point is taken as the size of the chiral continuum fit systematic error for that reference q^2 value.
- Added to other systematics in quadrature
- These systematics + statistics are then used to construct the **synthetic data** at the reference q^2 values.





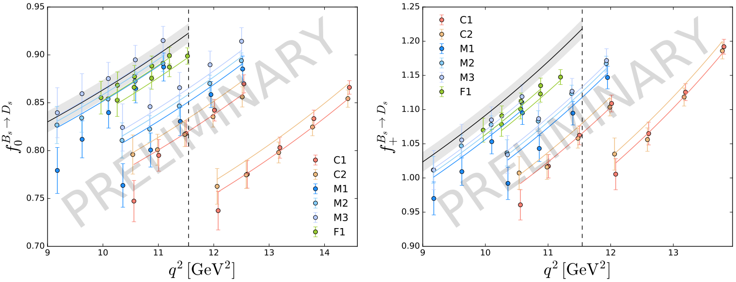
$B_s \to D_s$ Analysis

$B_s \to D_s$ Global Fit

- Use a chiral-continuum-charm fit over lattice spacing and charm quark mass to obtain physical result.

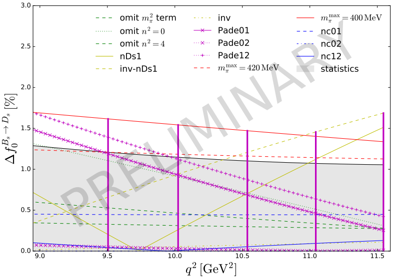
$$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + c_1 \frac{M_\pi^2}{\Lambda^2} + \sum_{j=1}^{n_{D_s}} c_{2,j} \left[\Lambda \cdot \Delta M_{D_s}^{-1}\right]^j + c_3 (a\Lambda)^2 \right] P_{\alpha,\beta}(q^2/M_{B_s}^2)$$

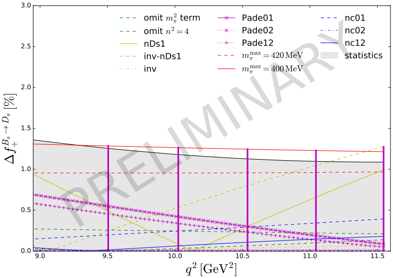
$$\Delta M_{D_s}^{-1} \equiv \left(\frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\text{phys}}} \right) \qquad P_{\alpha,\beta}(x) = \frac{1+\sum_{i=1}^{N_\alpha} \alpha_i x^i}{1+\sum_{i=1}^{N_\beta} \beta_i x^i}$$



$B_s \to D_s$ Error Budget

- Same strategy as other three decays: make fit ansatz variations to estimate fit ansatz systematic error by taking largest deviation from the preferred fit at reference q^2 points.
- As previously, other systematics are added in quadrature.





Full q^2 range extrapolation

z-expansion

- Use z-expansions: a model-independent q^2 extrapolation scheme
- Change variables from q^2 to z with

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

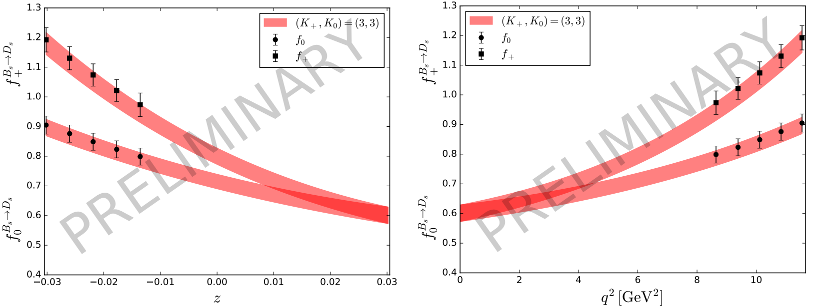
$$t_+ = (M_{B_{(s)}} + M_P)^2$$

$$t_0 = (M_{B_{(s)}} + M_P)(\sqrt{M_{B_{(s)}}} - \sqrt{M_P})^2$$

- Allows the form factors to be expanded as a power series in z

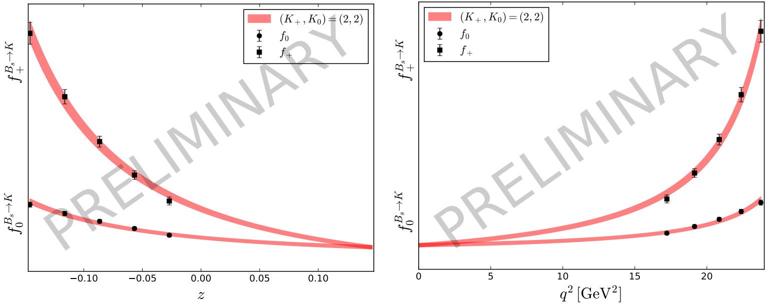
z-expansions

- Specifics determined by choice of form and parameters
- Two commonly chosen fits: Boyd-Grinstein-Lebed (BGL) and Bourelly-Caprini-Lellouch (BCL)
- z-expansions shown are BCL fits



z-expansions

- z-expansions usually given the additional physical constraint that $f_0 = f_+$ at $q^2 = 0$.
- Can be combined with experimental data to determine $|V_{xb}|$.
- By integrating over q^2, predictions of the lepton-universality ratios can be obtained.



Summary

- Updates to RBC-UKQCD 2015 results in the pipeline
- More precisely determined lattice spacing and inclusion of third lattice spacing via F1 ensemble reduces errors
- More details available in conference proceedings: arXiv:1912.09946
- Currently **still preliminary**
- $B_s \to K\ell\nu$, $B_s \to D_s\ell\nu$ being finalised.
- $B \to \pi\ell\nu$, $B \to D\ell\nu$ will follow at a later date.