Measuring the CKM angle γ with $B^{\pm} \to D(\to K_S^0 h^+ h^-)h'^{\pm}$ decays

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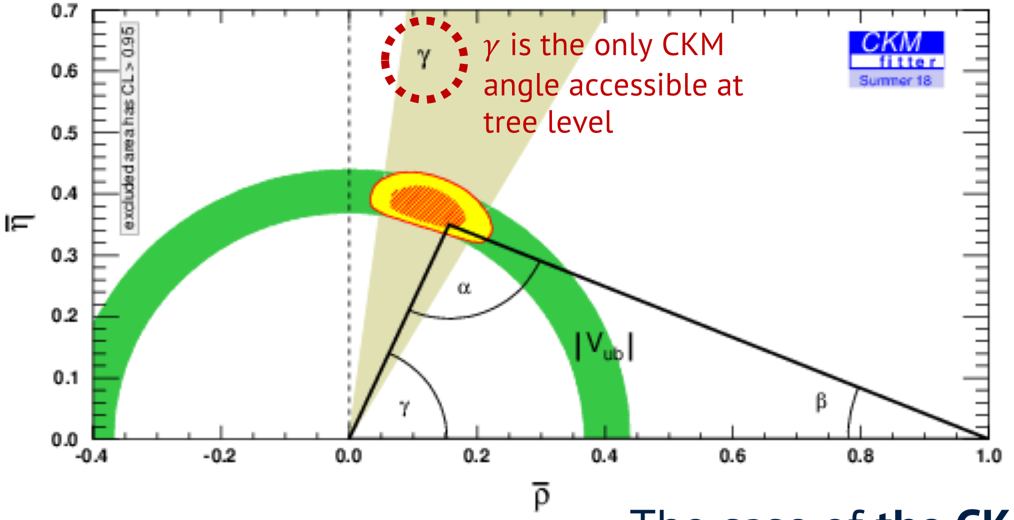
LHCb UK ● 6 January, 2020

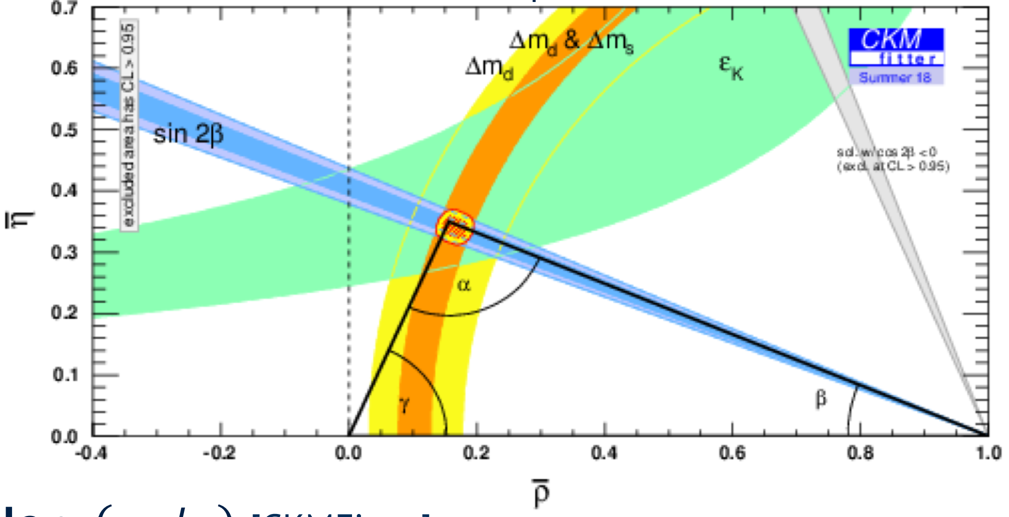
Is the Unitarity Triangle a triangle?

Crucial flavour physics goal: experimentally test consistency of the CKM mechanism by over-constraining parameters

Constraints from “tree-level” observables



Constraints from “loop-level” observables



The case of **the CKM angle γ** $(= \phi_3)$ [CKMFitter]

$$\gamma_{\text{direct}} = 72.1^{+5.4}_{-5.7}{}^{\circ}$$

= ?

$$\gamma_{\text{indirect}} = 65.64^{+0.97}_{-3.42}{}^{\circ}$$

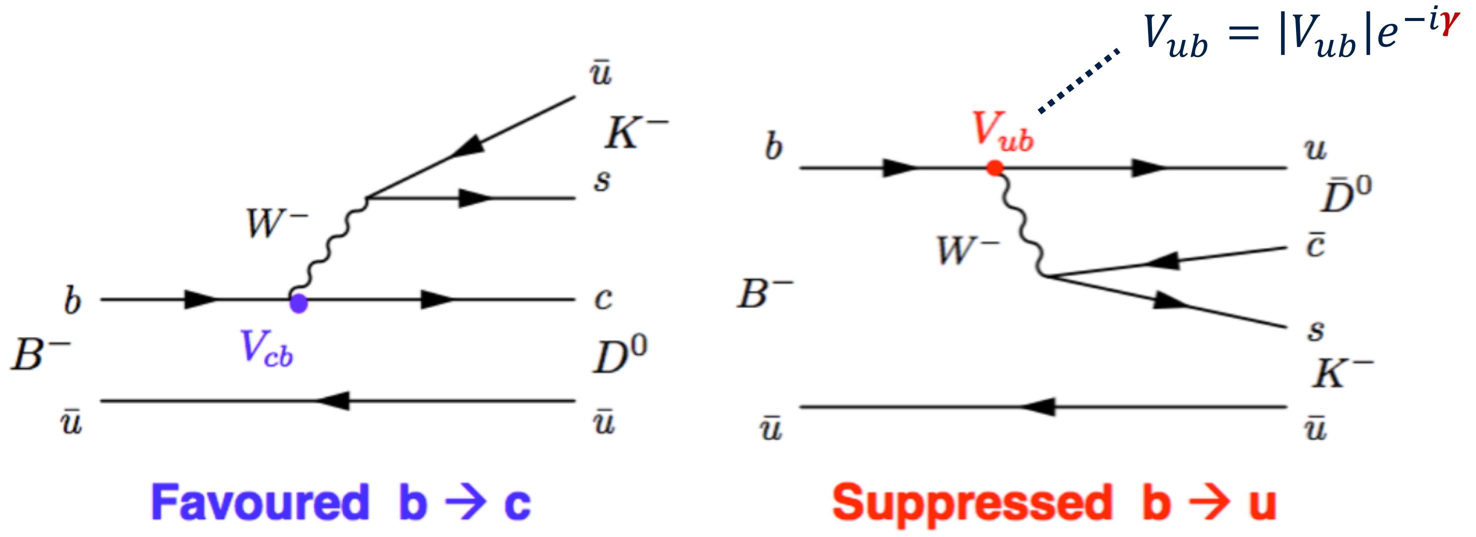
Measured in $B \to DK$ decays and friends

- Theoretically **clean**: $\delta\gamma_{\text{theory}}/\gamma \simeq 10^{-7}$ [JHEP.01(14)51]
- **SM benchmark**: NP contributions to tree level process expected to be small (although not excluded by data: [PRD.92(15)033002])

Indirect determination from other CKM parameters

- $\sin 2\beta$ from $B^0 \to J/\psi K^0_S$
- Δm_d and Δm_s from $B^0_{(s)}$ mixing
- ϵ_k from neutral kaon CPV measurements
- Uncertainties from LQCD

How to measure γ in $B^{\pm} \to DK^{\pm}$: "the text book example"



Access to γ via **interference between $b \to c$ and $b \to u$** transitions

- γ is the EW (CP-violating) phase between the $B^{\pm} \to D^0 K^{\pm}$ and $B^{\pm} \to \bar{D}^0 K^{\pm}$ amplitudes *(up to relative corrections of $O(\lambda^4) \simeq 2\times10^{-3}$)*

$$\frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} = r_B \exp[i(\delta_B - \gamma)]$$

$$\frac{A(B^+ \to D^0 K^+)}{A(B^+ \to \bar{D}^0 K^+)} = r_B \exp[i(\delta_B + \gamma)]$$

With $\boldsymbol{D \to K_S^0 h^+ h^-}$ **final state**: interference $\propto \cos[\delta_B \pm \gamma + \Delta\delta_D(m^2_{K_S^0\pi^-}, m^2_{K_S^0\pi^+})]$

Phase-space dependent $\overline{D^0} - D^0$ decay amplitude phase difference

The model-independent GGSZ method

Divide phase space of $D \to K_S^0 h^+ h^-$ decay into bins and measure yields in each

- Analysis is **independent of modelling** of D decay
- Sensitivity from **phase-space distribution**, not overall asymmetries
  → overall production/detection/K_S^0-CPV asymmetries have no impact

$$r_B \exp[i(\delta_B \pm \gamma)] = x_\pm + i y_\pm$$

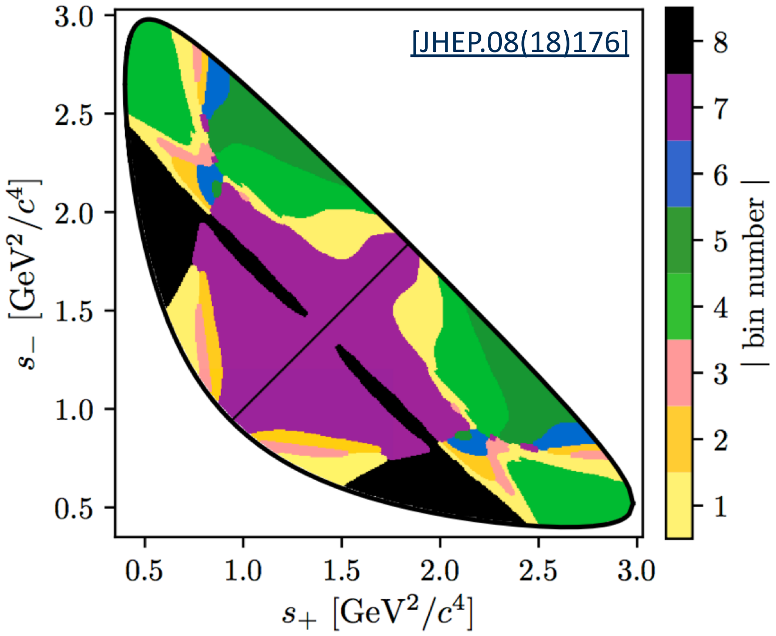
$$N_{\pm i}^{-} \propto F_{\pm i} + \left(x_\pm^2 + y_\pm^2\right) F_{\mp i} + 2\sqrt{F_i F_{-i}}\left(x_\pm c_{\pm i} \mp y_\pm s_{\pm i}\right)$$

F_i : Fractional yield of flavour tagged D^0 into bin i

Earlier analyses: measured in control channel: $\bar{B}^0 \to D^{*+}\mu^-\nu_\mu X$

c_i / s_i : Strong phase difference of $D^0 - \bar{D}^0$ decays

External input from CLEO-c/BESIII measurement



CP asymmetries

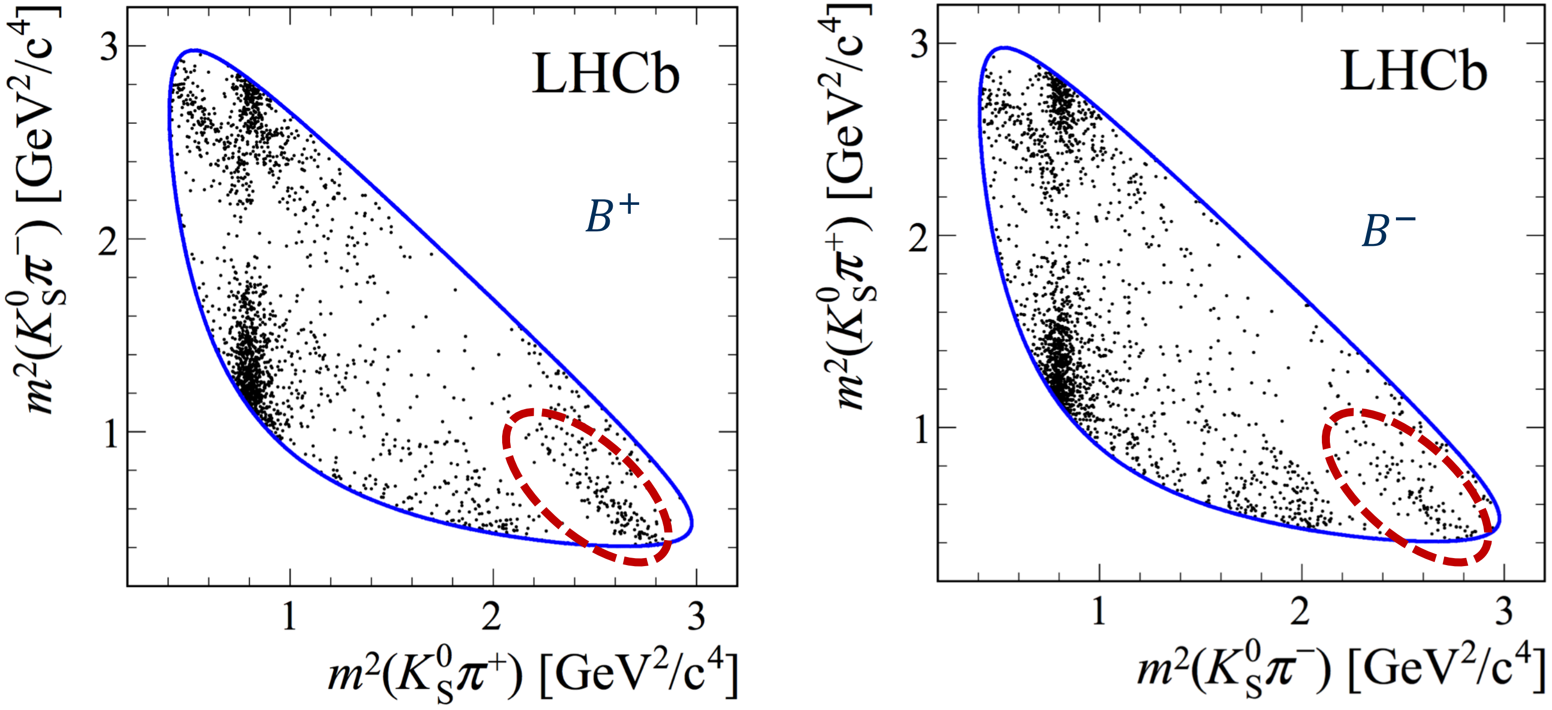
Overall CP asymmetry
small ≃ 1%

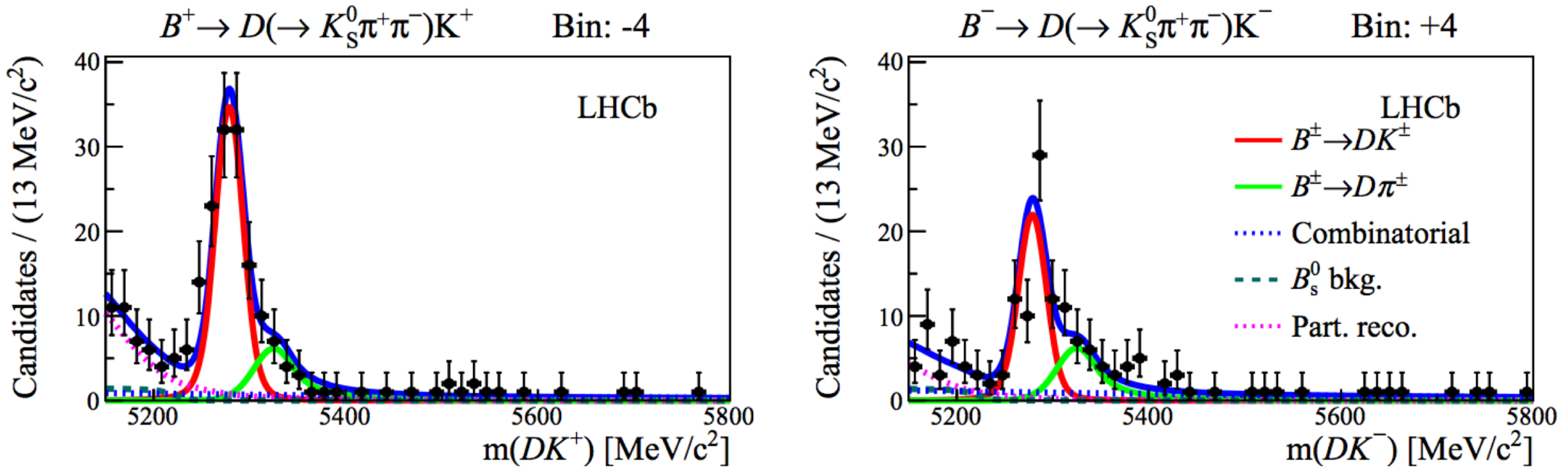
But large in certain regions of phase-space

LHCb has published two model-independent GGSZ measurements with $B^{\pm} \to DK^{\pm}$

- Run 1 data: [JHEP.08(18)176]
- 2015+16 data: [JHEP.08(18)176]

[JHEP.08(18)176]





[JHEP.08(18)176]

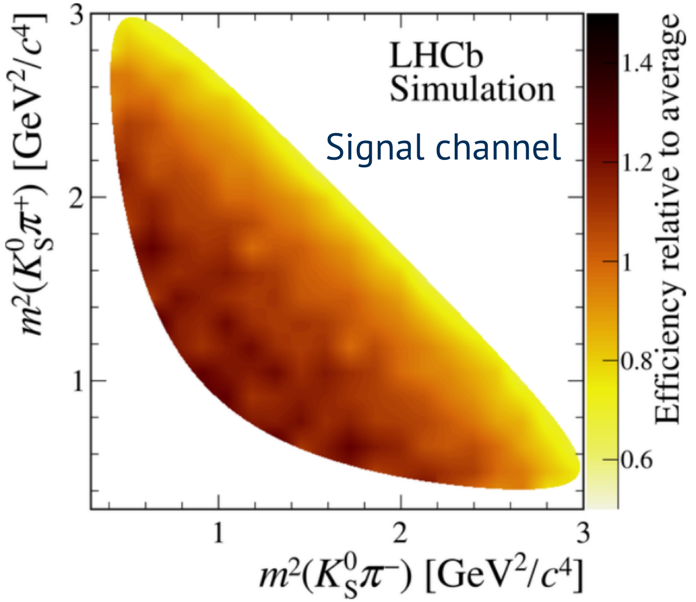
Phase-space dependent efficiency

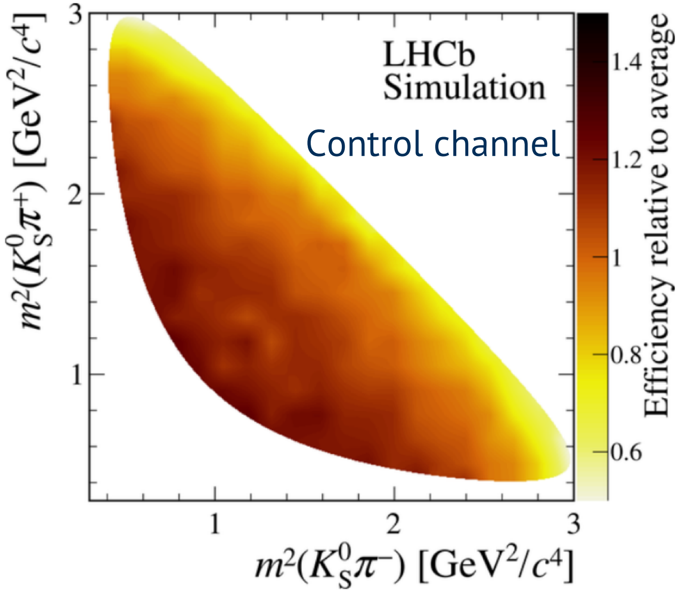
Significant variation in reconstruction efficiency over phase-space

Handled in data-driven manner: Measure F_i in control channel: $\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\mu^-\nu_\mu X$

- Flavour tagged D^0 from B decay: similar efficiency profile → still correct for signal-control efficiency difference
- Signal-control channel efficiency **ratio** from simulation → effects of simulation imperfections cancel to first order
- Naturally incorporates effects of bin migration and D-mixing [PRD.82(10)034033]

Efficiency in simulation





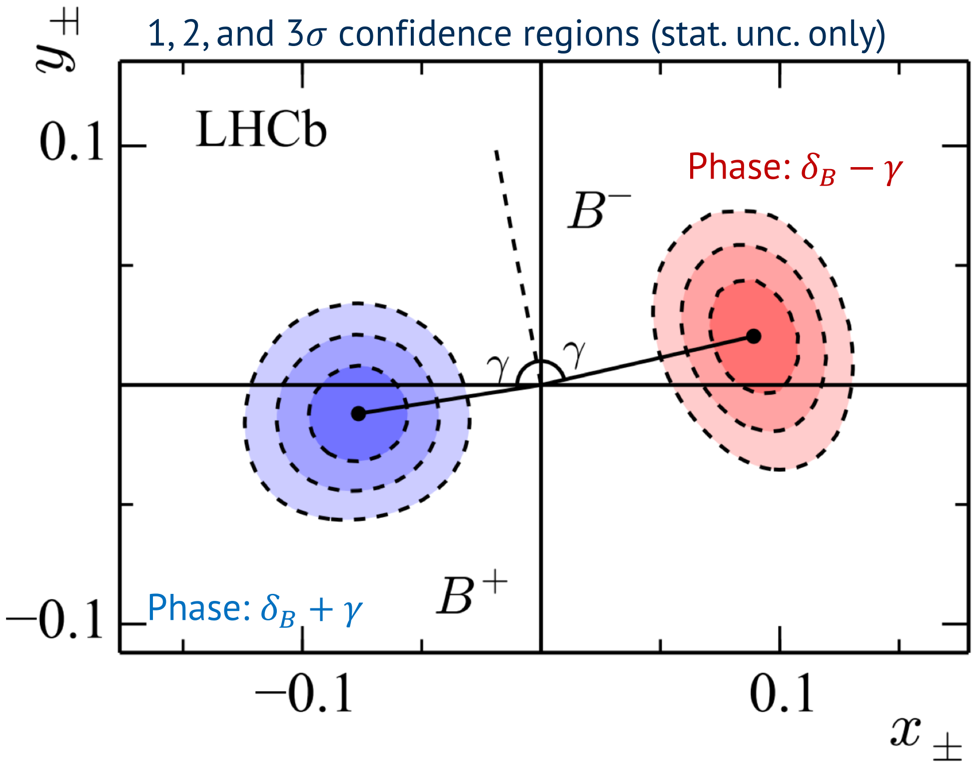
[JHEP.08(18)176]

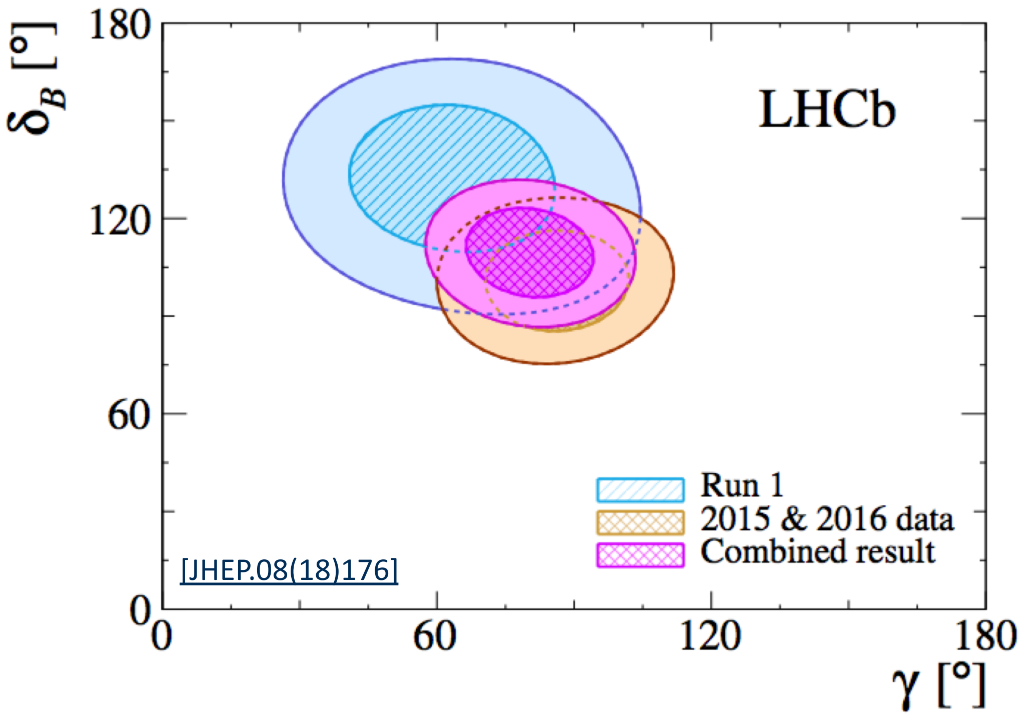
Interpreted and combined with Run 1 GGSZ measurement [JHEP.10(14)97] using gammacombo [JHEP.12(16)87]

$\gamma = 87^{\circ}{}^{+11^{\circ}}_{-12^{\circ}}$

Combined Run 1 and 15/16: $\gamma = 80^{\circ}{}^{+10^{\circ}}_{-9^{\circ}}$

Most precise stand-alone determinations of γ to date (at 2σ level)





Statistical uncertainty dominates ($\simeq 8^\circ$ on γ)

Leading systematic uncertainties

- **Strong-phase** measurements from CLEO ($\simeq 4^\circ$ on γ)
  → Inclusion of forthcoming BESIII result expected to reduce strong-phase-related systematic to → 1.5°
- **Efficiency correction** between signal and semi-leptonic control channels ($\simeq 1.5^\circ$ on γ)

Systematic uncertainties in 2015+16 $B^+ \to DK^+$ measurement

All uncertainties x10^{-2}:

Source	$\sigma(x_+)$	$\sigma(x_-)$	$\sigma(y_+)$	$\sigma(y_-)$
Statistical	1.9	1.7	1.9	2.2
Strong phase measurements	0.4	0.4	0.9	1.1
Efficiency corrections	0.6	0.6	0.1	0.2
Mass fit PDFs	0.2	0.2	0.3	0.3
Different misID shape over Dalitz plane	0.1	0.2	0.1	0.1
Different low mass shape over Dalitz plane	0.1	0.1	0.1	0.2
Uncertainty on $B_s^0 \to \overline{D}^0[\pi^+]K^-$ yield	0.1	0.1	0.1	0.1
Bias correction	0.1	0.1	0.1	0.1
Migration	0.1	0.1	0.1	0.1
K^0-CPV and material interaction	0.1	0.1	0.1	0.2
Total experimental source	0.7	0.7	0.4	0.5

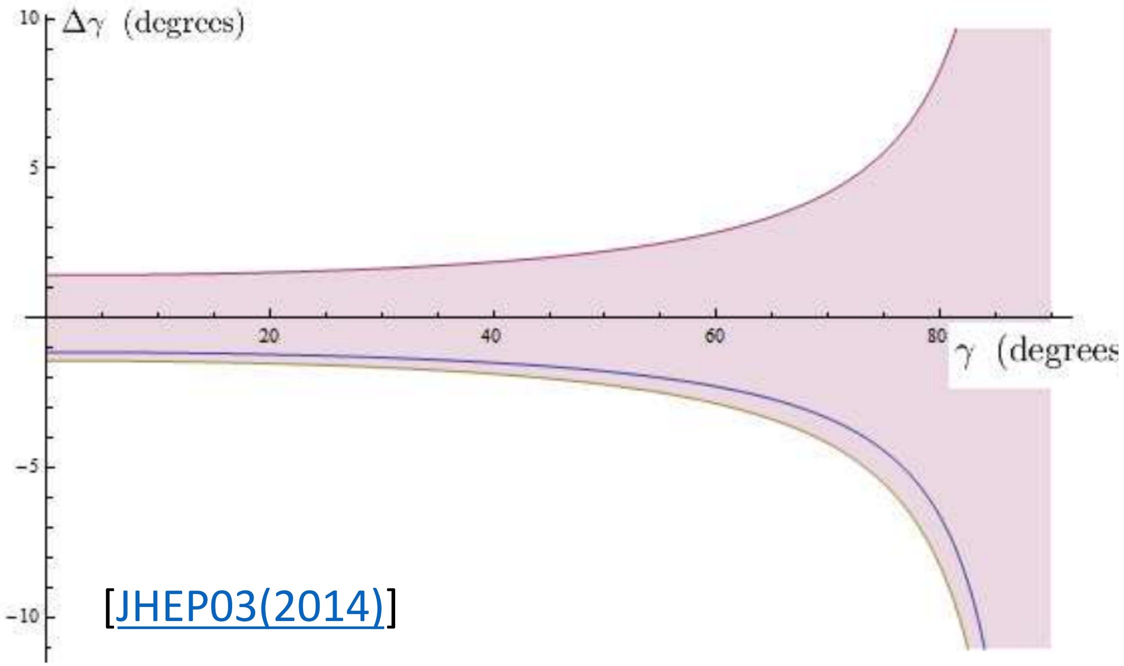
Dalitz-plot efficiency profiles expected to be very similar in $B \to DK$ and $B \to D\pi$

- $B \to D\pi$ already included as control channel for cross feed/shape fits

Why was $B \to D\pi$ **not** used as efficiency control mode?

a) Bias from non-zero CPV in $B \to D\pi$

b) Worries about effect of $\boldsymbol{K_S^0}$**-CPV in** $\boldsymbol{D\pi}$ if promoting to signal channel
→ Effect cf. [JHEP03(2014)]: $\Delta\gamma/\gamma = O(|\epsilon|/r_B)$



[JHEP03(2014)]

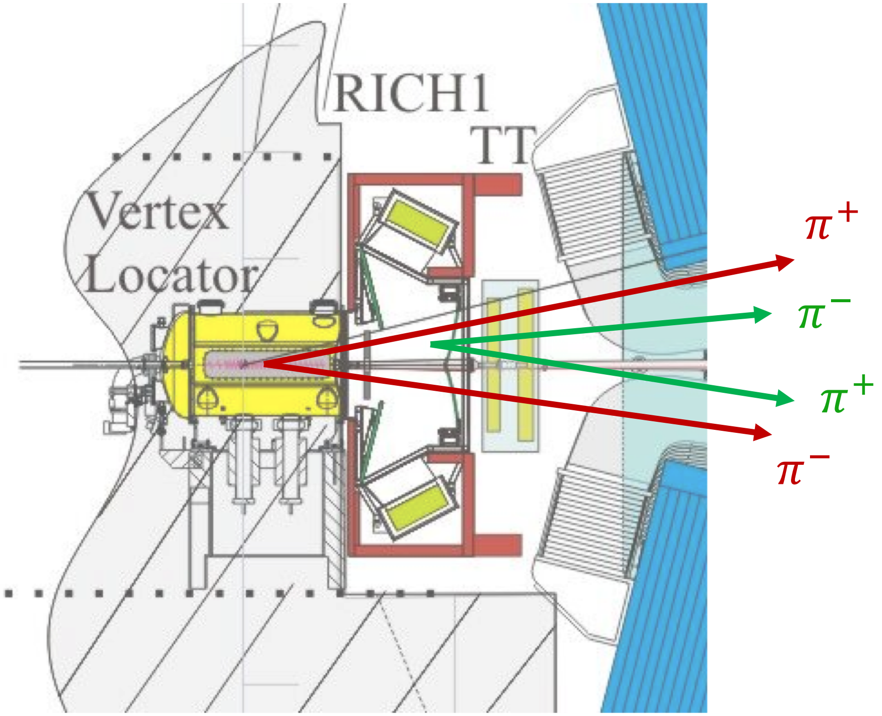
$|\epsilon|/r_B^{DK} \simeq 2\ \%$	$|\epsilon|/r_B^{D\pi} \simeq 40\ \%$

Detailed studies of phase-space dependent effects

- CPV: $A_{D^0}\left(m^2_{K^0_S\pi^+}, m^2_{K^0_S\pi^-}\right) \neq A_{\bar{D}^0}(m^2_{K^0_S\pi^-}, m^2_{K^0_S\pi^+})$
- Material interaction leads to dependence on $D \to K^0_L\pi^+\pi^-$ amplitude

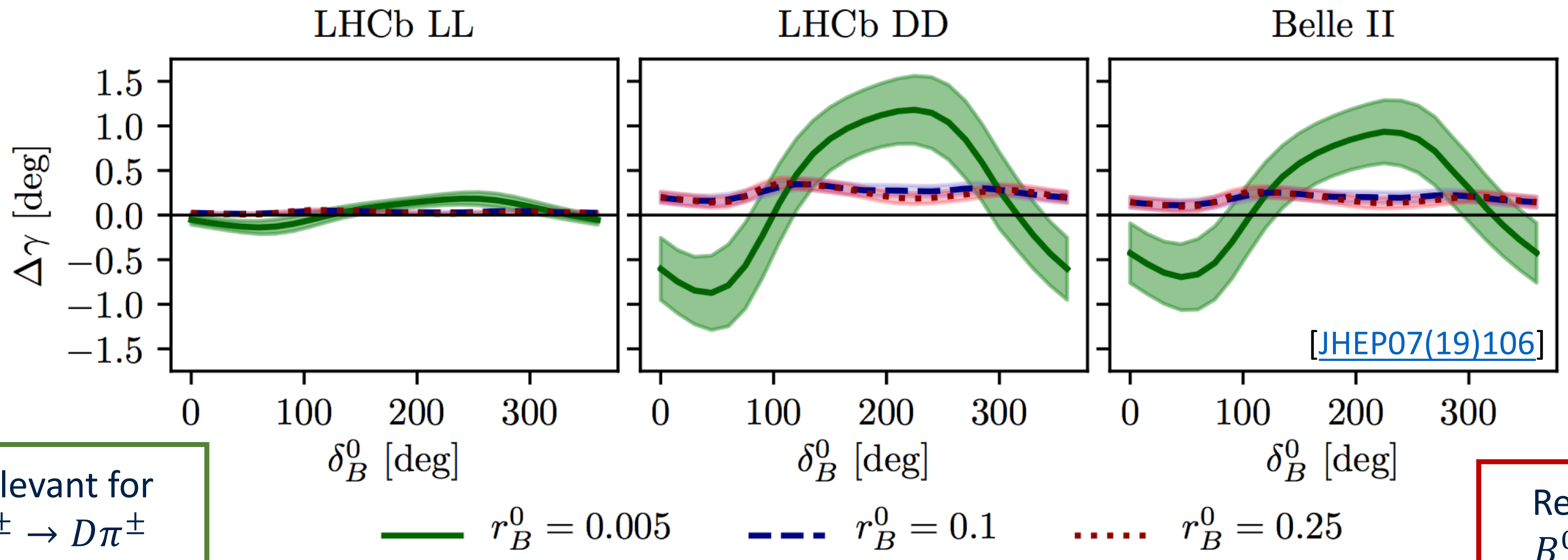
Effects studied using latest BaBar amplitude model [PRD98(2018)112012] & simplified detector description of LHCb (and Belle II)

Time-dependent effects: different impact on **long track** and **downstream** K^0_S-decay categories of LHCb



Conclusions: leading order effects have no phase-space dependence:

- Effect on GGSZ measurements $\simeq O(|\epsilon|/r_B \times 0.05)$
  → negligible compared to statistical uncertainties for both DK and $D\pi$ observables
  → feasible to promote $D\pi$ to a signal channel



Relevant for $B^\pm \to D\pi^\pm$

Relevant for $B^0 \to DK^{*0}$

$B \to DK$ and $B \to D\pi$ Dalitz-plot efficiency profiles identical

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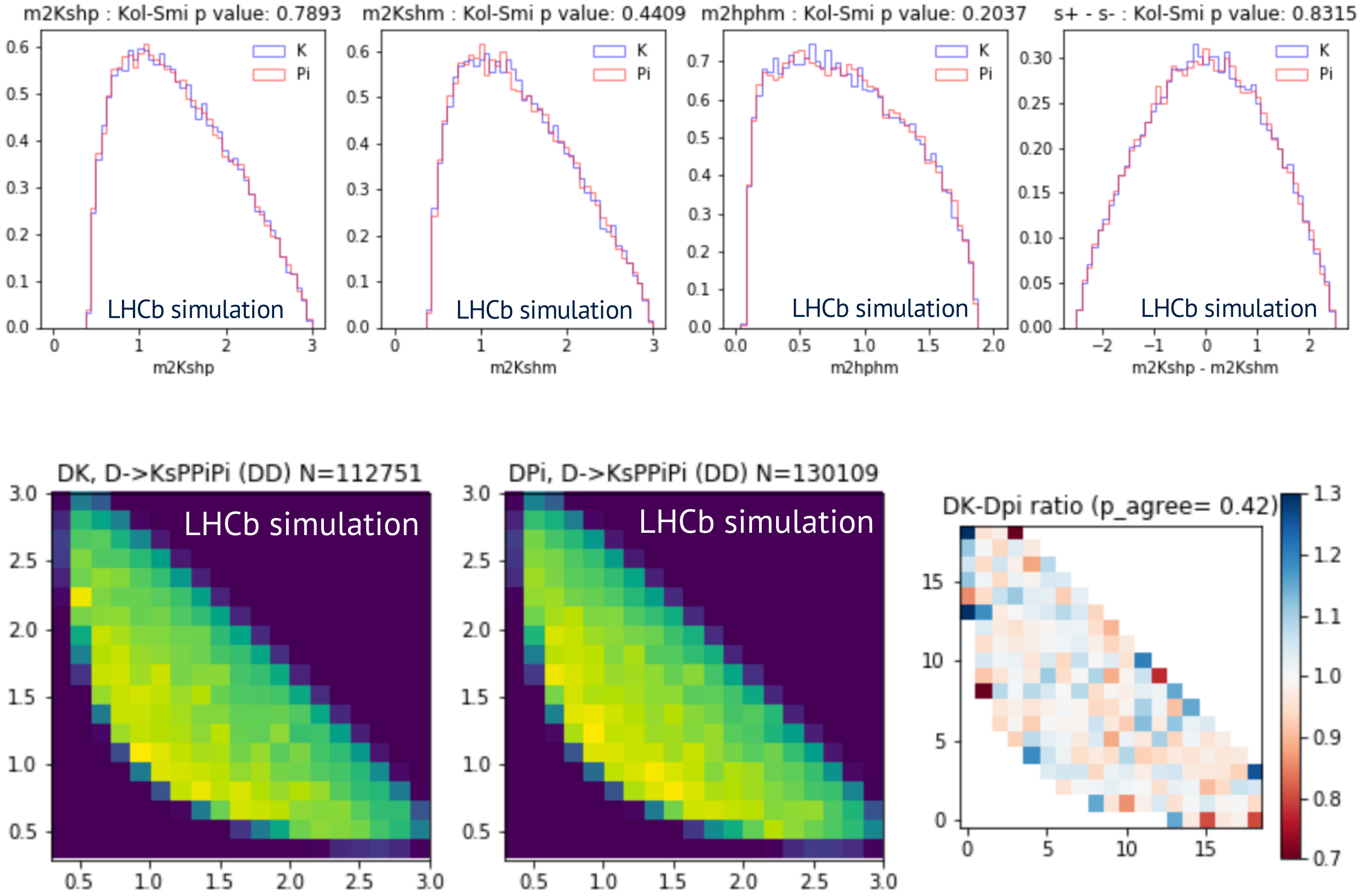
Efficiency profiles identical (given MC statistics)

Promoting $D\pi$ to signal channel

- Fit F_i and CPV parameters simultaneously

Extra bonus:

- Should be possible to resolve multiple solutions for $r_B^{D\pi}/\delta_B^{D\pi}$ in LHCb gamma combination



Efficiency profiles in LHCb simulation
PHSP MC: distribution ∝ efficiency profile

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$$N^{-}_{\pm i} \propto F_{\pm i} + \left(x_-^{Dh^2} + y_-^{Dh^2}\right) F_{\mp i} + 2\sqrt{F_i F_{-i}}\left(x_\pm^{Dh} c_{\pm i} - y_\pm^{Dh} s_{\pm i}\right)$$

Bin-yield parameterisation used for both $DK^\pm$ and $D\pi^\pm$

Adding **four** extra free parameters $(x_\pm^{D\pi}, y_\pm^{D\pi})$

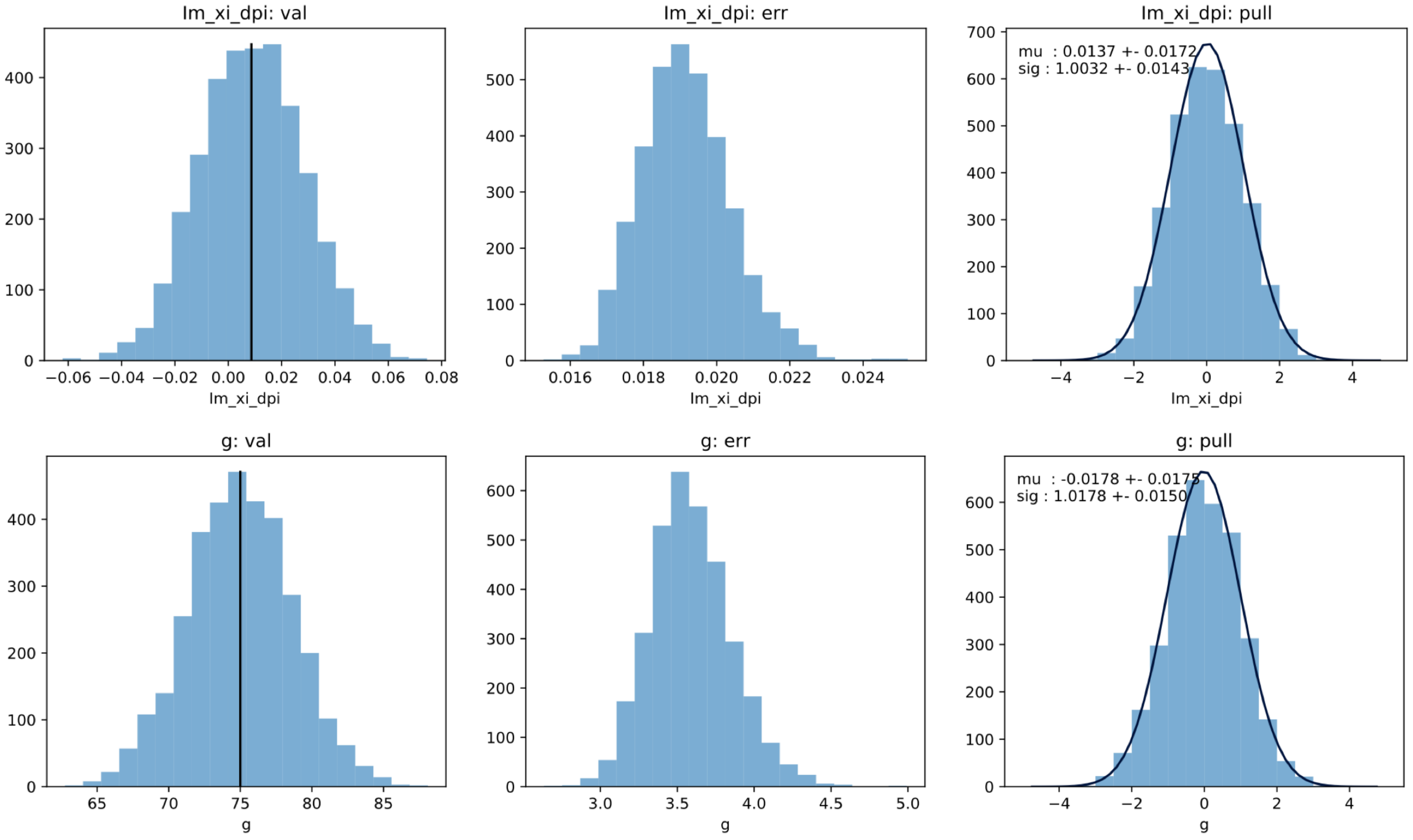
→ large correlations/unstable fits with free F_i

The $D\pi$ CPV observables are parameterised using **two** nuisance parameters [1804.05597]:

$$\xi^{D\pi} = x_\xi^{D\pi} + i\, y_\xi^{D\pi} = \frac{r^{D\pi} e^{i\delta^{D\pi}}}{r^{DK} e^{i\delta^{DK}}}$$

$$x_\pm^{D\pi} = x_\xi^{D\pi} x_\pm^{DK} - y_\xi^{D\pi} y_\pm^{DK}$$
$$y_\pm^{D\pi} = x_\xi^{D\pi} y_\pm^{DK} + y_\xi^{D\pi} x_\pm^{DK}$$

Fit stability has been verified in a series of toy studies



Signal only toys: expected uncertainty in final measurement is larger

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Total yields in $D \to K_S^0\pi^+\pi^-$ channels:

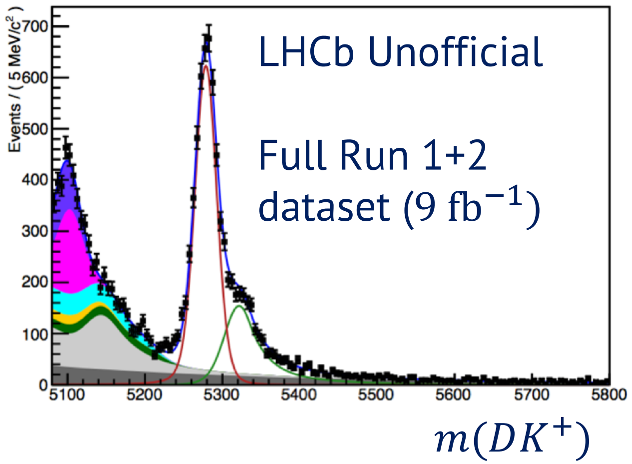
- $D\pi \simeq 220k$
- $DK \simeq 15k$

Toy studies suggest expected statistical uncertainty $\boldsymbol{\sigma(\gamma) \simeq 4.5 - 6^\circ}$

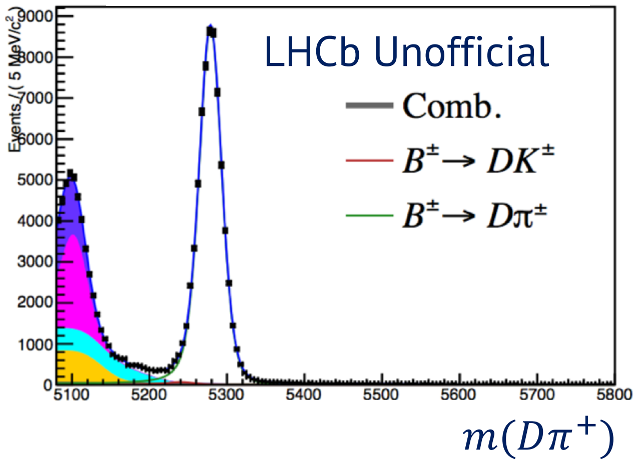
- Depends on central value of r_B^{DK}

Inclusion of forthcoming BESIII result expected to reduce strong-phase-related systematic from $4^\circ \to 1.5^\circ$

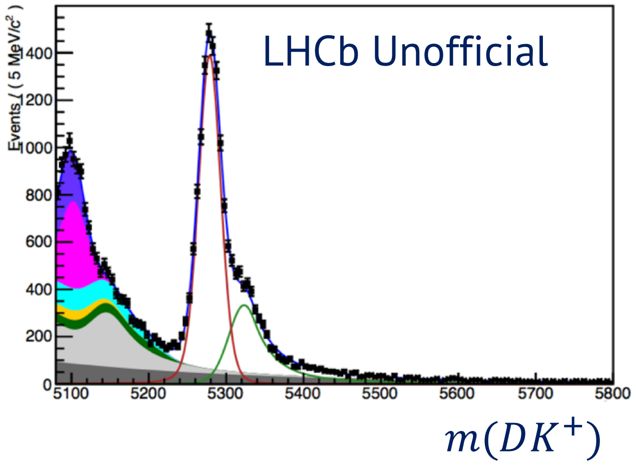
$B^\pm \to DK^\pm$ (long tracks)



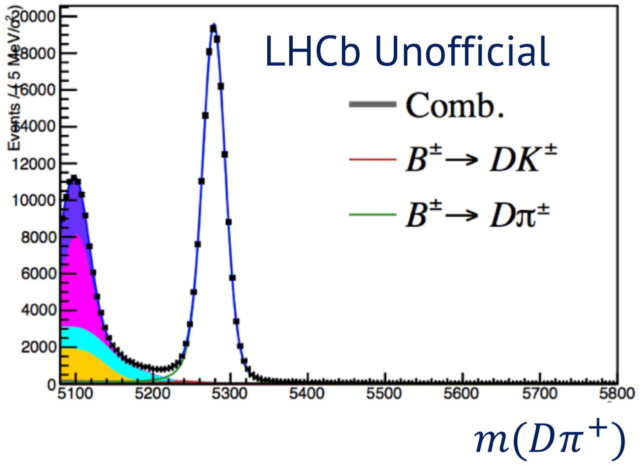
$B^\pm \to D\pi^\pm$ (long tracks)



$B^\pm \to DK^\pm$ (downstream)



$B^\pm \to D\pi^\pm$ (downstream)



Summary

Combined model-independent GGSZ measurement with $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$ decays and the the full Run 1+2 LHCb dataset well under way

Avoids relying on semi-leptonic control channel for efficiency profile

- removes a leading systematic uncertainty and the need for large MC samples

Expected single-measurement precision on γ comparable to current world averages

Backup

Neutral Kaon CP Violation

CP violation in the kaon sector changes GGSZ equations → $\boldsymbol{A^D(s_-,s_+) \neq A^{\bar{D}}(s_+,s_-)}$!

$$d\Gamma \propto |\psi_S(t,s_-,s_+) + \epsilon \cdot \psi_L(t,s_-,s_+)|^2$$

K_S^0 is not an exact CP eigenstate

$$A_S^D(s_-,s_+) \propto A_S^{\bar{D}}(s_+,s_-) + 2\epsilon A_2^{\bar{D}}(s_+,s_-)$$

Contribution from $K_S^0 - K_L^0$ interference

$$A_L^D(s_-,s_+) \propto -A_L^{\bar{D}}(s_+,s_-) - 2\epsilon A_1^{\bar{D}}(s_+,s_-)$$

Corrected yields simple to calculate in terms of $A_{1/2}^D = A(D^0 \to K_{1/2}\pi^+\pi^-)$

$$A_1^D(s_-,s_+) = A_1^{\bar{D}}(s_+,s_-)$$

$$A_2^D(s_-,s_+) = -A_2^{\bar{D}}(s_+,s_-)$$

$$A_S^D \propto A_1^D - \epsilon A_2^D$$

$$A_L^D \propto A_2^D - \epsilon A_1^D$$

$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \epsilon,$$

$$\epsilon \simeq (2.2\times10^{-3})e^{0.24\pi i}$$

$$\widehat{CP}\ K_1 = K_1 \qquad \widehat{CP}\ K_2 = -K_2$$

Kaon regeneration in matter

$$i\partial_t\psi = \mathcal{H}_{vac}\begin{pmatrix}K^0\\ \overline{K}^0\end{pmatrix} + \begin{pmatrix}\chi & 0\\ 0 & \bar{\chi}\end{pmatrix}_{matter}\begin{pmatrix}K^0\\ \overline{K}^0\end{pmatrix}$$

χ ($\bar{\chi}$) is proportional to K^0 ($\overline{K}^0$) forward scattering amplitude in material

Kaon regeneration in matter introduces further dependence on $A(D \to K_L^0\pi^+\pi^-)$:

Non-zero $K_L^0 \leftrightarrow K_S^0$ transition amplitudes in matter

To lowest order in $r_\chi = \frac{1}{2}\frac{\chi-\bar{\chi}}{(m_L-m_S)+i/2(\Gamma_L-\Gamma_S)}$ [Z.Phys.C.72.543]

$$\psi_S(t) = e^{-i/2(\chi+\overline{\chi})t}\left((\psi_S^0 - r_\chi\psi_L^0)e^{i\lambda_S t} + r_\chi\psi_L^0 e^{i\lambda_L t}\right)$$

For LHCb and Belle II:

$$< r_\chi > \simeq 10^{-3}$$

The A1 and A2 amplitude relations

The amplitudes $A_1 = A(D^0 \to K_1\pi^+\pi^-)$ and $A_2 = A(D^0 \to K_2\pi^+\pi^-)$ are related under the assumption that they are well described by isobar-like models [1010.2817]

$$A_2^D(s_-, s_+) = -A_1^D(s_-, s_+) + \boldsymbol{r_A \Delta A(s_-, s_+)}, \qquad \boldsymbol{r_A \simeq \tan^2 \theta_C}$$

$$A_1^D = \alpha^0 + \sum_{CF} \alpha_i A(K^{*-}\pi^+) + \sum_{DCS} \alpha_j A(K^{*+}\pi^-) + \sum_{\pi\pi} \alpha_k A(K_1\pi^+\pi^-)$$

$\to \bar{K}^0$: relative "$-$"

$\to K^0$: relative "$+$"

$\to \bar{K}^0$ & K^0: but K^0 coupling is DCS

$$A_2^D = \alpha^{0\prime} - \sum_{CF} \alpha_i A(K^{*-}\pi^+) + \sum_{DCS} \alpha_j A(K^{*+}\pi^-) + \sum_{\pi\pi} -(1 - 2r_k e^{i\delta_k})\alpha_k A(K_1\pi^+\pi^-)$$

$$K^0 \propto K_1 + K_2$$
$$\bar{K}^0 \propto K_1 - K_2$$

Where the DCS amplitudes satisfy : $\alpha_{DCS}/\alpha_{CF} \simeq r_k \simeq \tan^2 \theta_C \simeq 0.05$

Modified Dalitz bin yields

$$N_{\pm i}^{B^-} = h^-\left(1 + \Gamma_S/\Gamma_L\left|\epsilon + r_\chi\right|^2 + \boldsymbol{\Delta h}\right)\left\{\hat{K}_{\pm i} + (x_-^2 + y_-^2)\hat{K}_{\mp i} + 2\sqrt{\hat{K}_i\hat{K}_{-i}}(x_-\hat{c}_{\pm i} + y_-\hat{s}_{\pm i}) + \boldsymbol{O(r\epsilon)}\right\}$$

$$N_{\pm i}^{B^+} = h^+\left(1 + \Gamma_S/\Gamma_L\left|\epsilon + r_\chi\right|^2 - \boldsymbol{\Delta h}\right)\left\{\hat{K}_{\pm i} + (x_-^2 + y_-^2)\hat{K}_{\mp i} + 2\sqrt{\hat{K}_i\hat{K}_{-i}}(x_-\hat{c}_{\pm i} + y_-\hat{s}_{\pm i}) + \boldsymbol{O(r\epsilon)}\right\}$$

Dalitz-plot distribution is only changed at $\boldsymbol{O(r\epsilon)}$

→ **expected impact on GGSZ measurements is $\boldsymbol{O(r\epsilon/r_B)}$ = permille level**

→ need to look at higher order terms to assess bias

$O(r\epsilon) =$

$O(r_A\epsilon) + O(r_A r_\chi)$

$+(r_B\epsilon) + O(r_B r_\chi)$

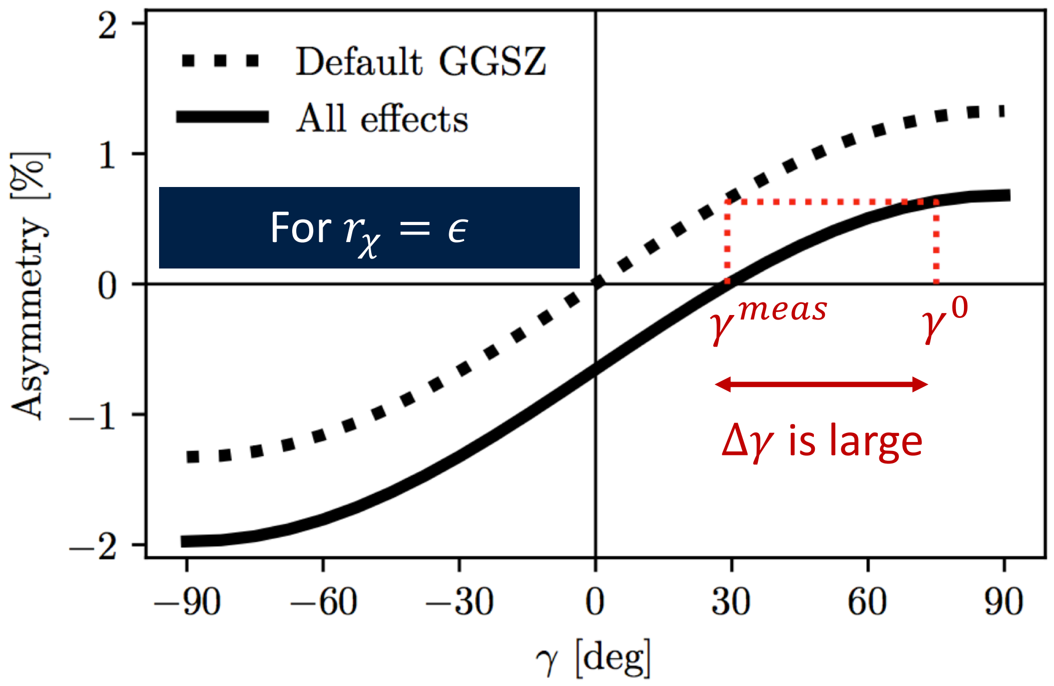
Well known overall yield asymmetry [eg. 1110.3790]

- Here shown for infinite time-acceptance
- And including contribution from material interaction

$$\Delta h(\epsilon, r_\chi) = 2\mathrm{Re}[\epsilon + r_\chi]\left(1 - 2\frac{\Gamma_S}{\Gamma_S + \Gamma_L}\frac{1 + \mu \cdot \mathrm{Im}[\epsilon + r_\chi]/\mathrm{Re}[\epsilon + r_\chi]}{1 + \mu^2}\right) + O(r\epsilon), \qquad \mu = \frac{2\Delta m}{\Gamma_S + \Gamma_L}$$

Bias on γ measurements from the integrated-yield asymmetry

$$A = \frac{N^- - N^+}{N^- + N^+} = \frac{2\sum c_i\sqrt{K_i K_{-i}}\, r_B \sin\delta_B \sin\gamma + \Delta h(\epsilon, r_\chi)}{1 + r_B^2 + r_B \cos\delta_B \cos\gamma}$$



Effect of K_S^0 CPV and material effect: Δh

Non-trivial phase $\Delta\delta_D(s_-, s_+)$

→ $\kappa = \sum c_i \sqrt{K_i K_{-i}} \simeq \mathbf{0.1} \ll \mathbf{1}$ (small coherence fact

→ $\Delta\gamma = O(|\epsilon|/(\mathbf{0.1} \times r_B))$ = **10's of degrees**

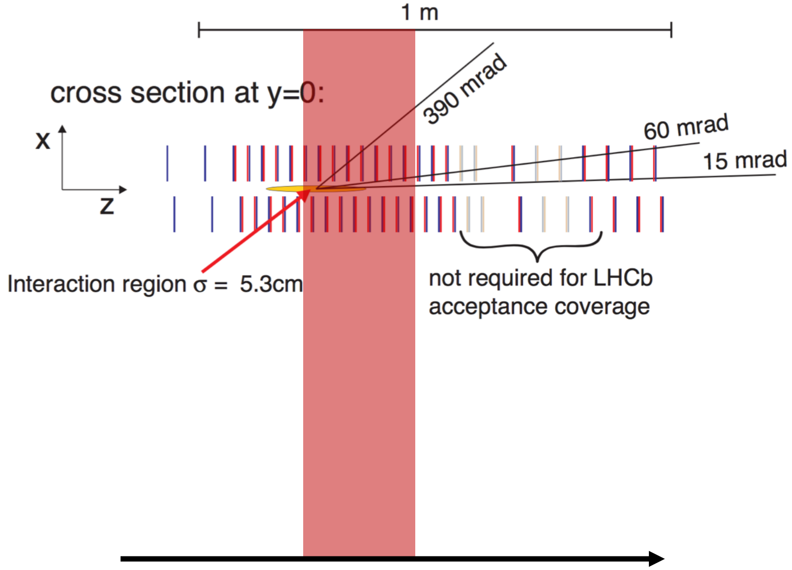
Corrections are possible, but

- $\Delta h(\epsilon, r_\chi)$ depends significantly on experimental time acceptance [1110.3790]
- $\Delta h(\epsilon, r_\chi)$ depends on experiment material budget and geometry
- Terms of $O(r_A r_\chi)$ and $O(r_A \epsilon)$ lead to % level biases and must be included

Not an issue in current experiments: overall asymmetry is not used to determine γ

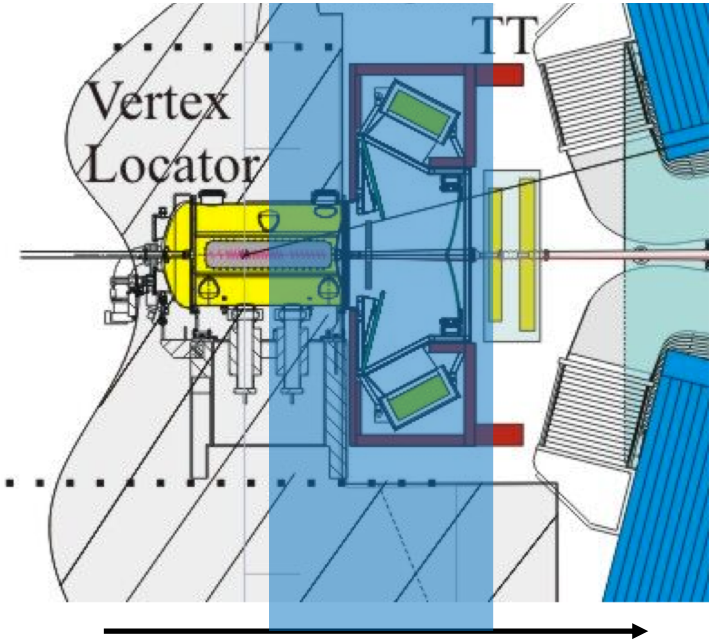
Time-acceptance ($\tau_{\text{decay}} \in [\tau_1, \tau_2]$) for kaon with $p = (p_z, p_T)$ determined by geometry

- **LL LHCb**/**Belle II**: require kaon decay products to cross at least 3 vertex detector segments
- **DD LHCb**: require kaon decay before the TT tracking station



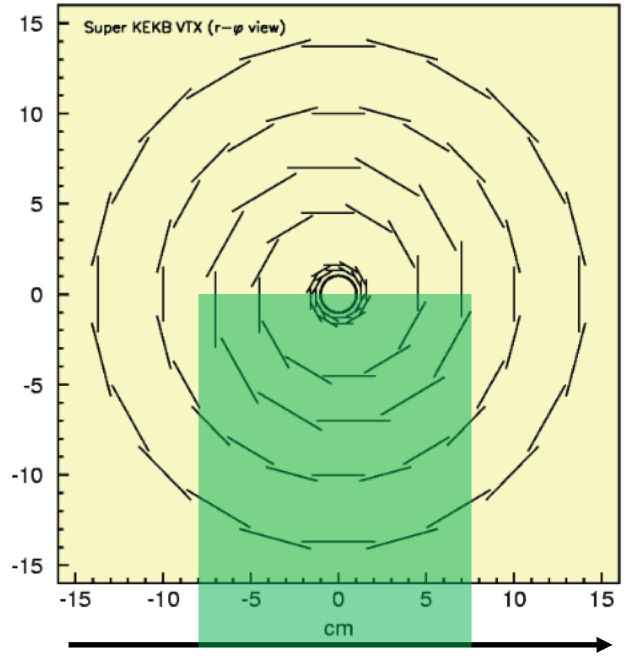
$z_{LL} \in [0, 280]$ mm

$< \tau_{LL} > \simeq 0.1\tau_{K_S^0}$



$z_{DD} \in [280, 2350]$ mm

$< \tau_{DD} > \simeq 0.8\tau_{K_S^0}$



$r_{Belle\ II} \in [0, 80]$ mm

$< \tau_{Belle\ II} > \simeq 0.7\tau_{K_S^0}$

Calculation procedure and input requirements

1. Model for $A_1(s_-, s_+)$ used to get $A_2(s_-, s_+)$ and then $A_{S(L)}^{D(\overline{D})}(s_-, s_+)$
2. $\psi_{S/L}^{\pm}(t, s_-, s_+)$ calculated keeping all orders of $r_{A,B,\chi}$ and ϵ
   - For inputs $(\gamma^0, r_B^0, \delta_B^0)$
3. $N_i^{\pm}$ from integrating $\mathrm{d}\Gamma^{\pm}$ over
   - experimental time-acceptance
   - Dalitz-plot bins
4. Fit $(x_\pm, y_\pm)$ with default GGSZ eq.
5. Interpret $(x_\pm, y_\pm)$ in terms of (γ, r_B, δ_B)
   $\rightarrow \quad \Delta\gamma = \gamma - \gamma^0$

Belle 2018 model [1804.06153] used to represent A_1

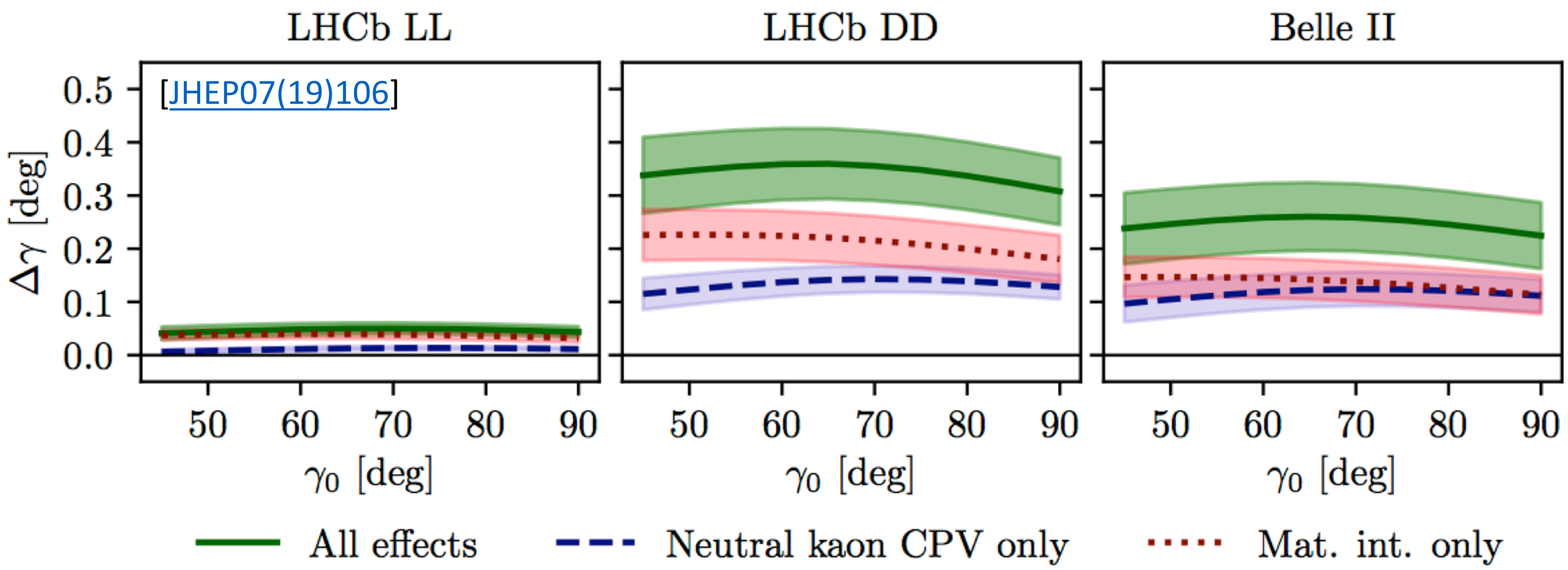
Material parameter $\Delta\chi$ from average material budget cf. technical design reports

$\Delta\chi$ and (t_1, t_2) depend on momentum distribution, estimated using `RapidSim` [1612.07489]

Time acceptance(t_1, t_2) from detector geometry and p_K

K_S^0 CPV study results for DK channel

[JHEP07(19)106]



Negligible bias on γ from neutral kaon CPV and material interactions in analyses of $B^{\pm} \to DK^{\pm}$ decays based on $D \to K_S^0\pi^+\pi^-$ phase-space **distribution**