$B_c \to J/\psi$ Form Factors and $R(J/\psi)$ using Lattice QCD

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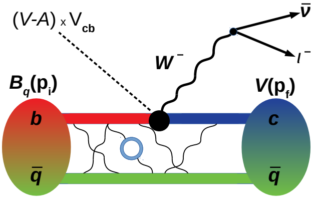
Outline

- SM background, overview of experimental status and need for precise theoretical predictions;
- Present a brief overview of methodology of lattice QCD;
- Heavy-HISQ;
- Preliminary Results for $B_c \to J/\psi$ case;
- Preliminary results for $B_s \to D_s^*$;
- Outlook;

$B_c \to J/\psi$ Semileptonic decays

Consider the heavy-light semileptonic pseudoscalar to vector decay in which the b decays to a c



The differential rate wrt $q^2 = (p_i - p_f)^2$ and angular variables is given by

$$\frac{d\Gamma}{dq^2 d\ldots} = \frac{G^2}{(2\pi)^3}|V_{cb}|^2 \frac{(q^2 - M_\ell^2)^2 p'}{12 M_{B_c}^2 q^2} \times \mathcal{F} \qquad (1)$$

where $\mathcal{F}$ is a function of kinematic variables, helicity amplitudes H_i and the lepton mass M_ℓ.

- The helicity amplitudes H_i defined in terms of form factors.
- form factors defined in terms of matrix elements of vector and axial vector currents.

Schematically:

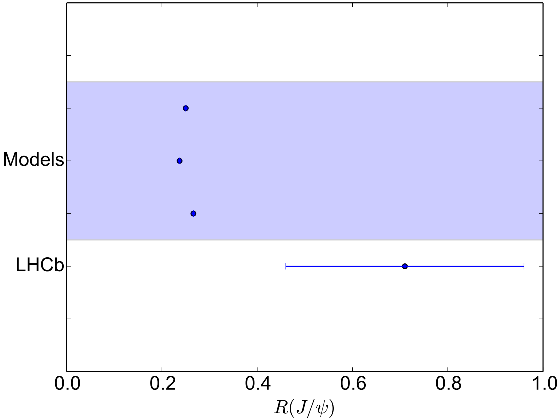
$$\begin{aligned}
\langle V(p',\epsilon)|\bar{c}\gamma^\mu b|P(p)\rangle =& V(q^2) \times \text{Kinematic}^V(p,p',\epsilon)^\mu \\
\langle V(p',\epsilon)|\bar{c}\gamma^\mu\gamma^5 b|P(p)\rangle =& A_0(q^2) \times \text{Kinematic}^{A_0}(p,p',\epsilon)^\mu \\
&+ A_1(q^2) \times \text{Kinematic}^{A_1}(p,p',\epsilon)^\mu \\
&+ A_2(q^2) \times \text{Kinematic}^{A_2}(p,p',\epsilon)^\mu
\end{aligned}$$

$\rightarrow$ Matrix elements between meson states are nonperturbative - difficult to compute!

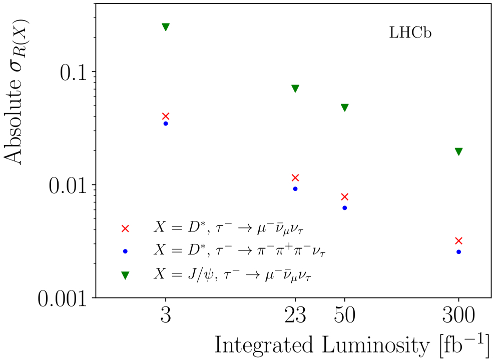
Experimental Status - $R(J/\psi)$

Useful to define dimensionless ratio of total decay rates to approximately massless μ/e lepton final state and massive τ final state.

$$R(J/\psi) = \frac{\Gamma(B_c \to J/\psi \tau^- \overline{\nu}_\tau)}{\Gamma(B_c \to J/\psi \mu^- \overline{\nu}_\mu)}$$



Experimental Status

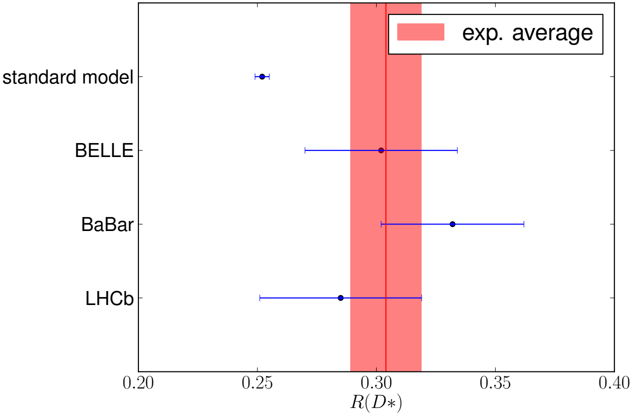


Projected uncertainties in $R(D^*)$ and $R(J/\psi)$ reproduced from arXiv:1808.08865v4

Aside - $B \to D^*$

For $B \to D^*$ case:

$$R(D^*) = \frac{\Gamma(B \to D^* \tau^- \overline{\nu}_\tau)}{\Gamma(B \to D^* \mu^- \overline{\nu}_\mu)}$$



Aside - V_{cb}

Historically determinations of V_{cb} use $B \to D^* \ell^- \overline{\nu}_\ell$

- Extrapolate experimental data to $q^2_{\rm max}$ using some parameterisation.
- Extract V_{cb} using lattice evaluation of form factor at $q^2_{\rm max}$.

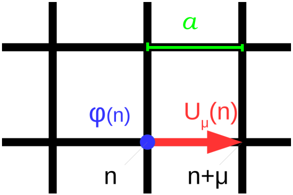
$\to$ Dependent on choice of parameterisation scheme, moves value more than quoted errors.
$\to$ Would be ideal to compute form factors across full momentum range and compare directly.

Lattice QCD

We want to extract matrix elements, amplitudes and energies from Euclidean correlation functions computed in the path integral formalism,

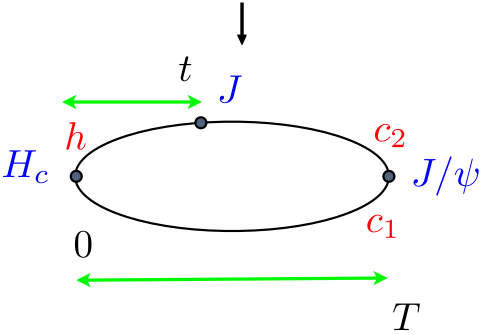
$$\int \mathcal{D}[\psi, \overline{\psi}, A] \mathcal{O}^1(t) \mathcal{O}^2(0) e^{-S^E[\psi,\overline{\psi},A]} = \sum_n \langle 0|\hat{\mathcal{O}}^1|n\rangle\langle n|\hat{\mathcal{O}}^2|0\rangle e^{-E_n t},$$

- discretise QCD onto a lattice
- Fermion integrals exact $\rightarrow$ need to invert dirac operator
- Monte-carlo integral over gauge fields U



Extracting Matrix Elements

$$\langle \bar{c}_1 \gamma^i c_2(T) \bar{c}_2 \Gamma^J h(t) \bar{h} \gamma^5 c_1(0) \rangle$$



introduce momentum by imposing twisted boundary conditions $c_2(x+L) = e^{i\pi\theta} c_2(x)$ on the charm quark attached to the current. This gives the J/ψ momentum $\frac{\theta\pi}{L_x}$

Extracting Matrix Elements

Fit these correlation functions using Bayesian methods and extract the relevant matrix elements and energies.

$$\langle 0|\mathcal{O}_{J/\psi}(t)\mathcal{O}^{\dagger}_{J/\psi}(0)|0\rangle = \sum_{n}\left((A^n)^2 e^{-tE_n} + (-1)^t (A^n_o)^2 e^{-tE^o_n}\right)$$

$$\langle 0|\mathcal{O}_{H_c}(t)\mathcal{O}^{\dagger}_{H_c}(0)|0\rangle = \sum_{n}\left((B^n)^2 e^{-tM_n} + (-1)^t (B^n_o)^2 e^{-tM^o_n}\right)$$

$$\langle 0|\mathcal{O}_{J/\psi}(T)\mathcal{J}_{h\to c}(t)\mathcal{O}^{\dagger}_{H_c}(0)|0\rangle = \sum_{n,m}\left(A^n B^m J^{nm} e^{-(T-t)E_n - tM_m} + \ldots\right.$$

$$\frac{1}{\sqrt{2E_n}}\langle 0|\mathcal{O}_{J/\psi}(0)|J/\psi^n\rangle = A^n$$

$$\frac{1}{\sqrt{2M_n}}\langle 0|\mathcal{O}_{H_c}(0)|H^n_c\rangle = B^n$$

$$\frac{1}{\sqrt{2E_n M_m}}\langle J/\psi^n|\mathcal{J}_{b\to c}(0)|H^m_c\rangle = J^{nm}$$

Heavy Quarks on the Lattice

To simulate precisely need $am_q < 1$, but typical modern lattices have lattice spacing $a > 1/m_b$, on these lattices cannot simulate physical b quarks directly.

- Instead, use unphysically light heavy masses am_h on multiple lattices with $a > 1/m_b$
- fit data to polynomial in Λ_{QCD}/m_h and extrapolate to $m_h = m_b$
- use Highly Improved Staggered Quarks (HISQ) $\rightarrow$ very small discretisation errors, crucial for calculations involving heavy quarks.

Heavy-HISQ

The procedure the extract physical form factors is then as follows:

- Compute form factors for multiple heavy masses and momenta using HISQ
- Use the z-expansion to fit the q^2 dependence of data with coefficients polynomial in $\frac{\Lambda_{QCD}}{m_h}$ and including discretisation terms going like a^n
- Set $m_h = m_b$ and take the coefficients of discretisation terms to zero to obtain form factor as a function of $z(q^2)$ at the physical point.

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

where

$$t_\pm = (M_{H_c} \pm M_{J/\psi})^2$$

Heavy-HISQ

The fit function is then

$$F(q^2) = P(q^2) \sum_{n=0}^{3} a_n z^n (1 + \delta_n)$$

where

$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \left(\frac{\Lambda}{m_h}\right)^j \left(\frac{am_c^{\mathrm{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi}\right)^{2l},$$

$P(q^2)$ includes the $b\overline{c}$ (axial-)vector meson pole from the current, and δ_n captures quark mass mistuning errors.

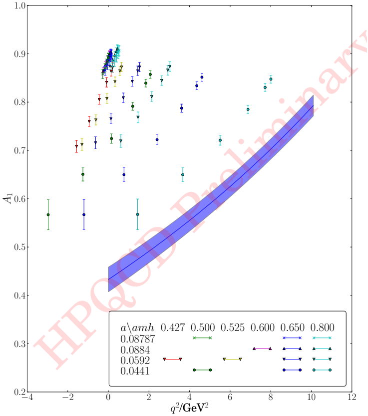
Details

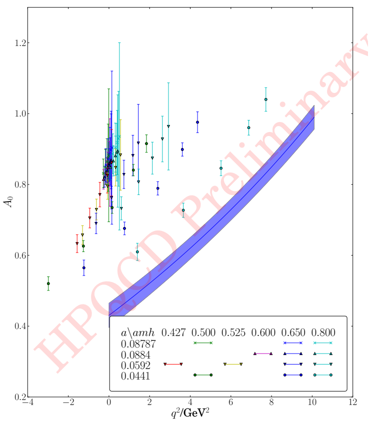
Details of the lattices we use in this analysis.

Set	a	$N_x \times N_t$	n_{configs}	cpu hours
1	0.0884	32×96	980	0.94×10^6
2	0.05922	48×144	500	2.1×10^6
3	0.0441	64×192	374	2.6×10^6
4	0.08787	64×96	300	1.2×10^6

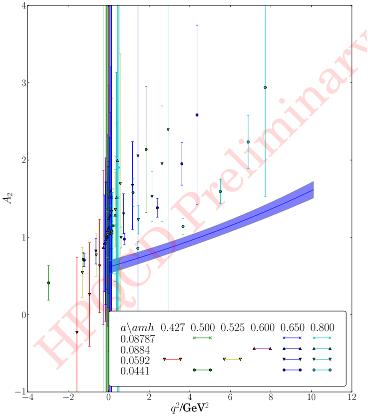
Utilised nearly 7 million cpu hours in total.

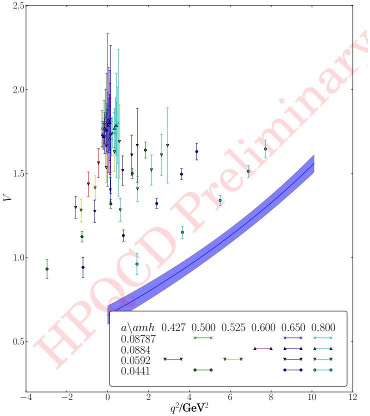
Results for $B_c \to J/\psi$, Form Factors



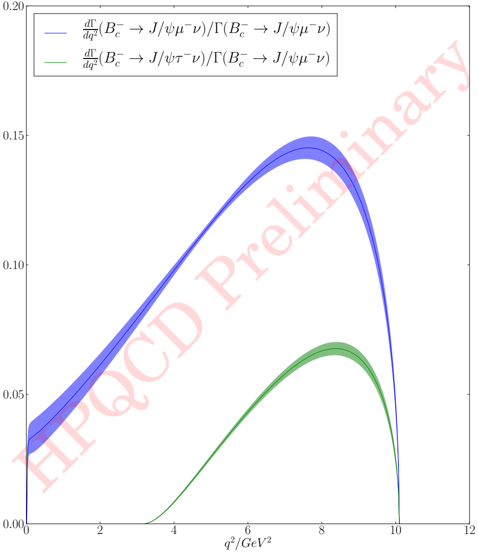


Results for $B_c \to J/\psi$, Form Factors





Results for $B_c \to J/\psi$, $d\Gamma$



We find for the total rates

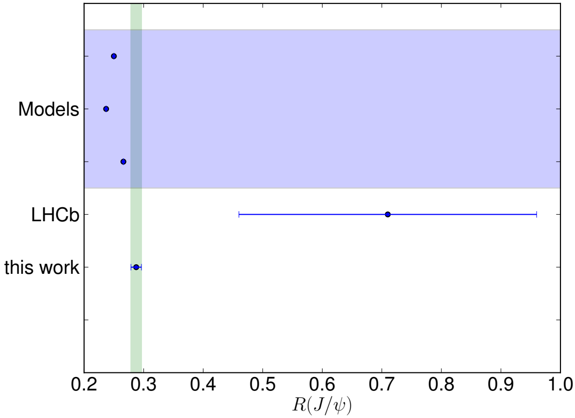
$$\Gamma(\ell = \mu) = 2.39(20) 10^{10} s^{-1} (\textit{preliminary})$$
$$\Gamma(\ell = \tau) = 6.86(41) 10^{9} s^{-1} (\textit{preliminary})$$

and the ratio

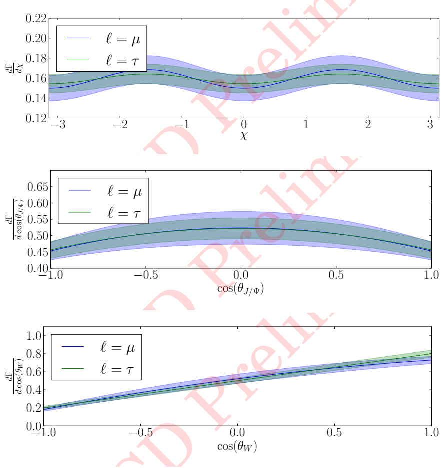
$$R(J/\psi) = 0.2875(88)(\textit{preliminary})$$

- First ever lattice QCD calculation of this quantity. (Paper in preparation)
- Precision comensurate with projected experimental uncertainty.

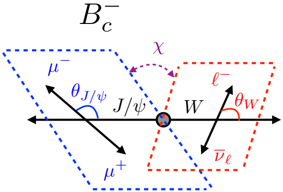
Results for $B_c \to J/\psi$, $d\Gamma$



Results for $B_c \to J/\psi$, Angular Rates

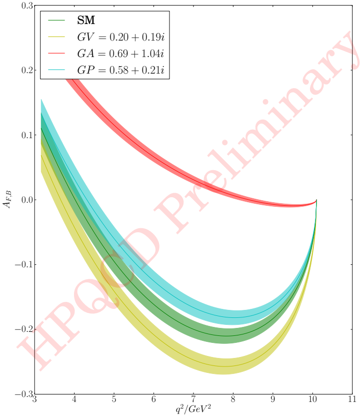


Can also construct angular differential rates



Results for $B_c \to J/\psi$, Angular Asymmetries

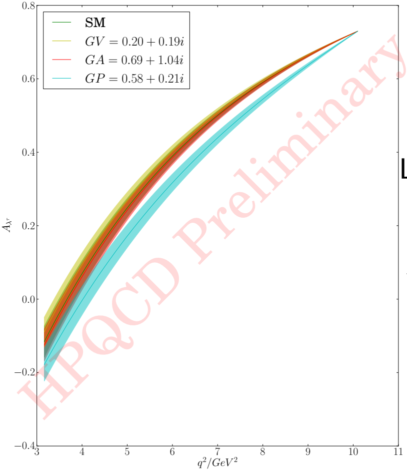
Following arXiv:1907.02257v2 we can also construct angular asymmetry ratios relevant to LFUV for the case $B_c \to J/\psi\tau\overline{\nu}_\tau$.



Forward-Backward Asymmetry

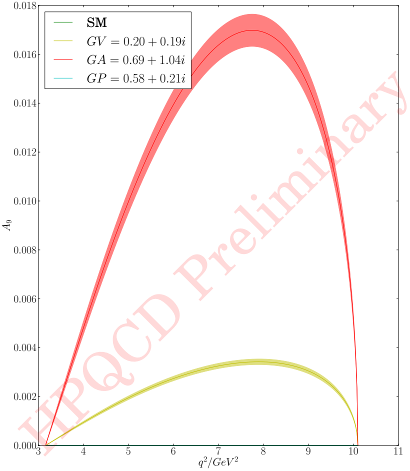
$$A_{FB}(q^2) = \frac{\frac{d\Gamma}{dq^2 d\cos(\theta_W)}\Big|_{\cos(\theta_W)}}{\frac{d\Gamma}{dq^2}}$$

The different G correspond to NP coefficients appearing in the effective $b \to c\tau\nu_\tau$ hamiltonian, taken from fits to $R(D)$ and $R(D^*)$ in arXiv:1907.02257v2.

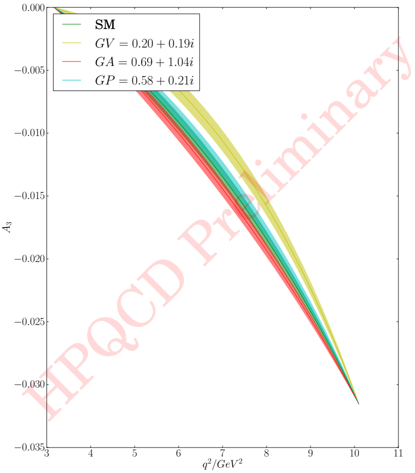


Lepton polarisation asymmetry

$$A_{\lambda_\ell}(q^2) = \frac{\frac{d\Gamma^{\lambda_\ell=-1/2}}{dq^2} - \frac{d\Gamma^{\lambda_\ell=+1/2}}{dq^2}}{\frac{d\Gamma}{dq^2}}$$



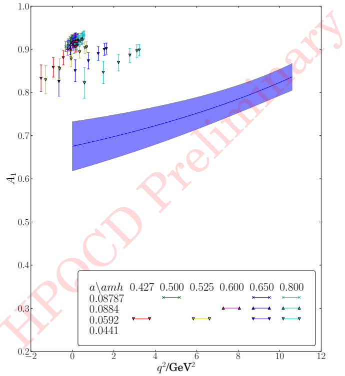
$$A_9(q^2) = \frac{\frac{d\Gamma}{dq^2 d\chi}\Big|_{\sin(2\chi)}}{\frac{d\Gamma}{dq^2}}$$

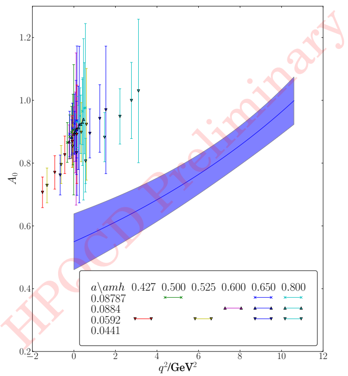


$$A_3(q^2) = \frac{\frac{d\Gamma}{dq^2 d\chi}\Big|_{\cos(2\chi)}}{\frac{d\Gamma}{dq^2}}$$

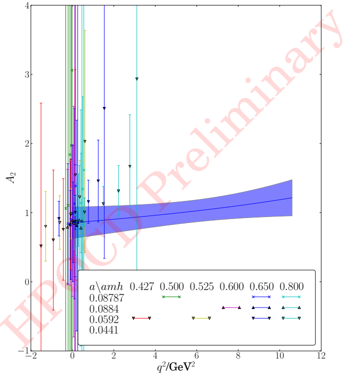
Results for $B_s \to D_s^*$

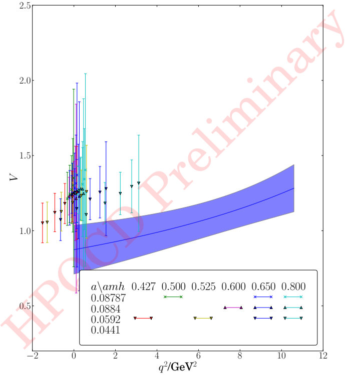
Similar calculation for $B_s \to D_s^*$ is underway, currently completed running on fine, physical fine and superfine lattices - enough for a preliminary Heavy-HISQ fit.



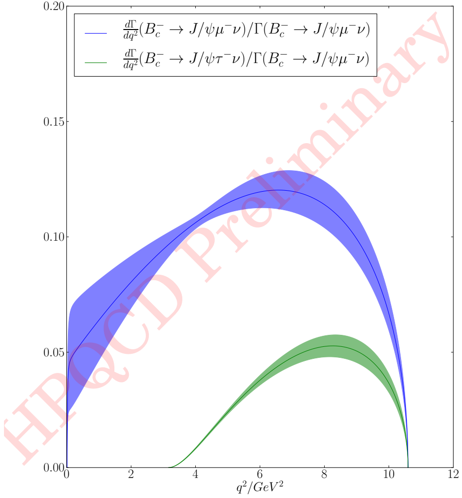


Results for $B_s \to D_s^*$





Results for $B_s \to D_s^*$



We find for the total rates

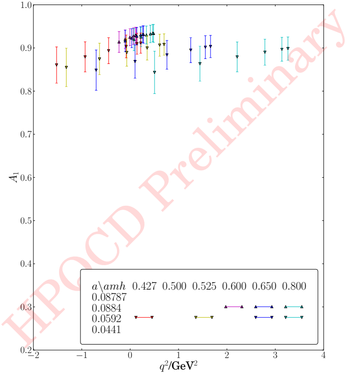
$$\Gamma(\ell = \mu) = 3.48(48)10^{10} s^{-1} (\textit{preliminary})$$
$$\Gamma(\ell = \tau) = 8.93(75)10^{9} s^{-1} (\textit{preliminary})$$

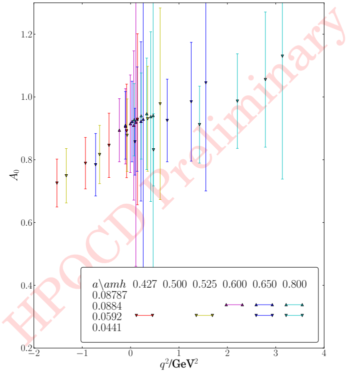
and the ratio

$$R(D_s^*) = 0.256(19)(\textit{preliminary})$$

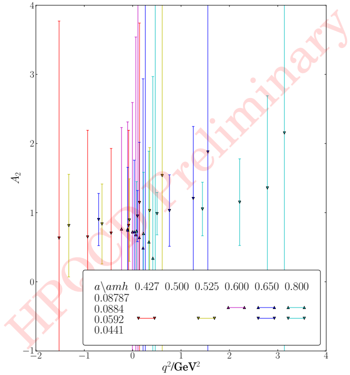
Results for $B \to D^*$

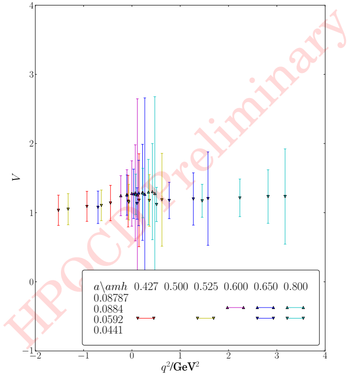
Calculation for $B \to D^*$ completed on fine and superfine lattices.





Results for $B \to D^*$





Conclusions

- We have computed $R(J/\psi)$ with precision which we expect to remain competitive with the experimental error for expected measurements.
- We have constructed angular asymmetry ratios for $B_c \to J/\psi$ for the SM and for different scenarios of NP arising in the axial, vector and pseudoscalar currents.
- Preliminary value of $R(D_s^*)$ is encouraging and suggests comparable precision may be reached once ultrafine run is completed.

Thank you