Radiative Corrections in Precision Experiments: Some Recent Progress

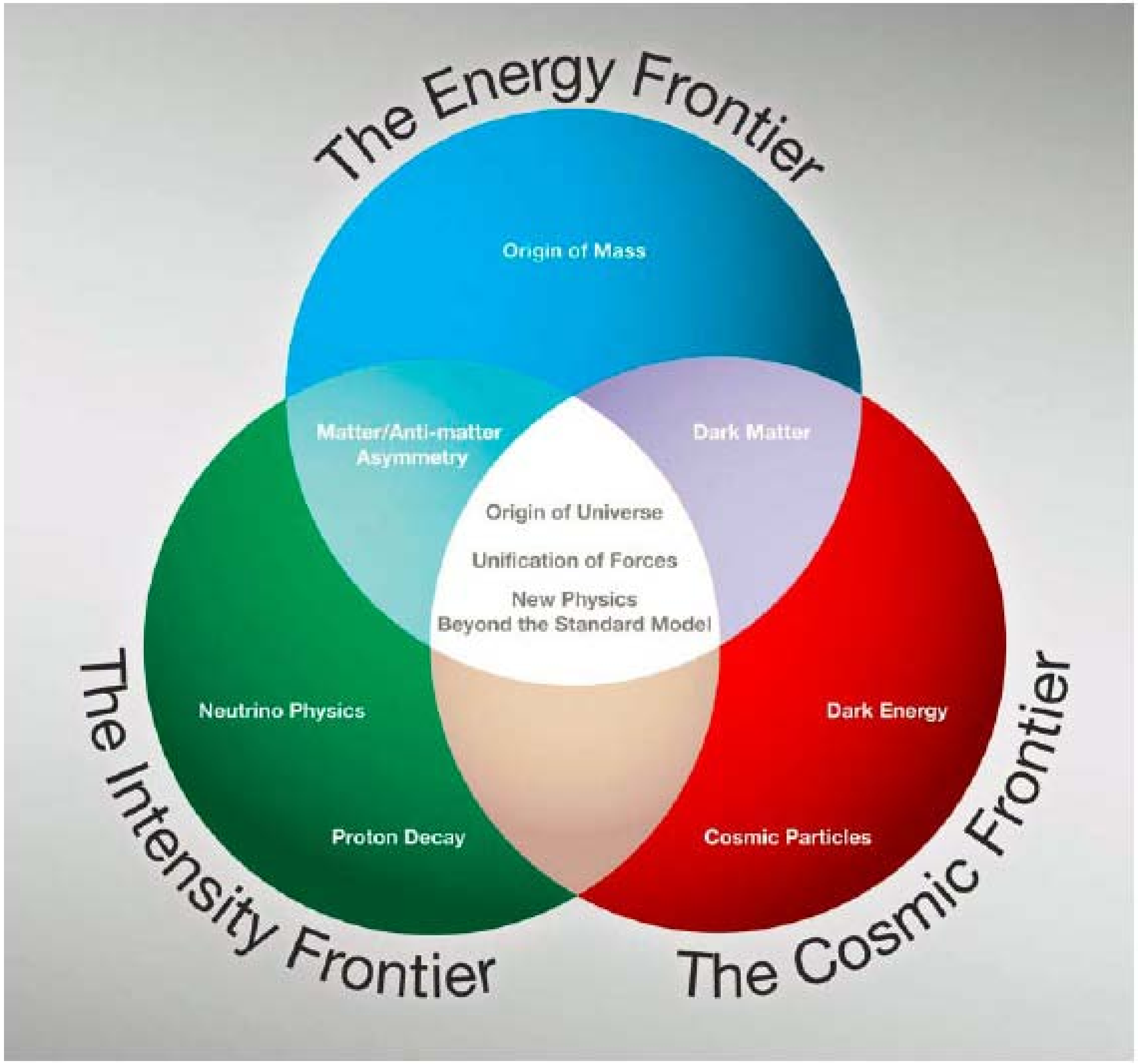
Chien-Yeah Seng

Helmholtz-Institut für Strahlen- und Kernphysik
and
Bethe Center for Theoretical Physics,
Universität Bonn

“Ultimate Precision at Hadron Colliders” workshop,
Institut Pascal, Orsay, France

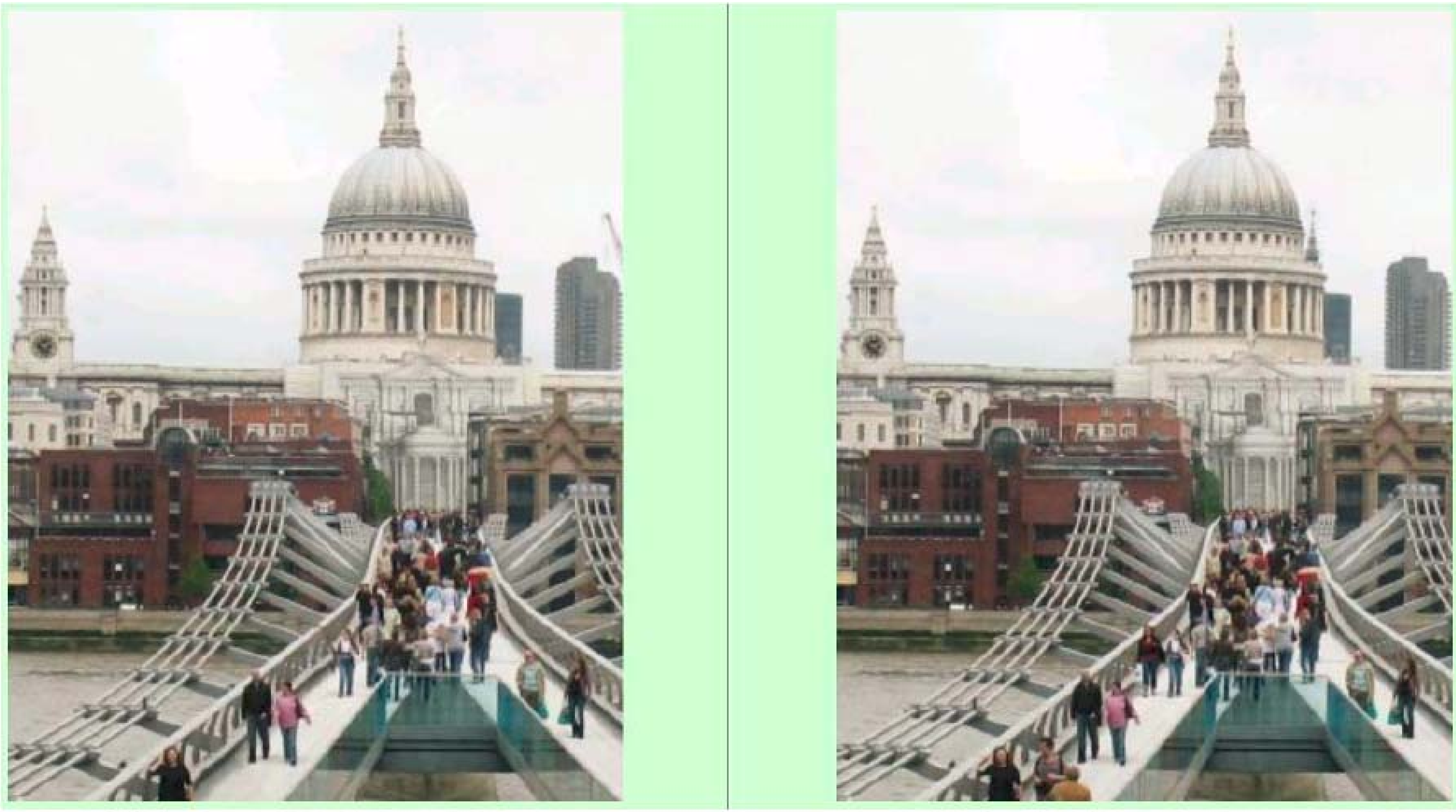
29 November, 2019

Searches of BSM Physics on the Precision (Intensity) Frontier



Searches of BSM Physics on the Precision (Intensity) Frontier

- **Precision frontier:** seek for **small deviations** from SM predictions.

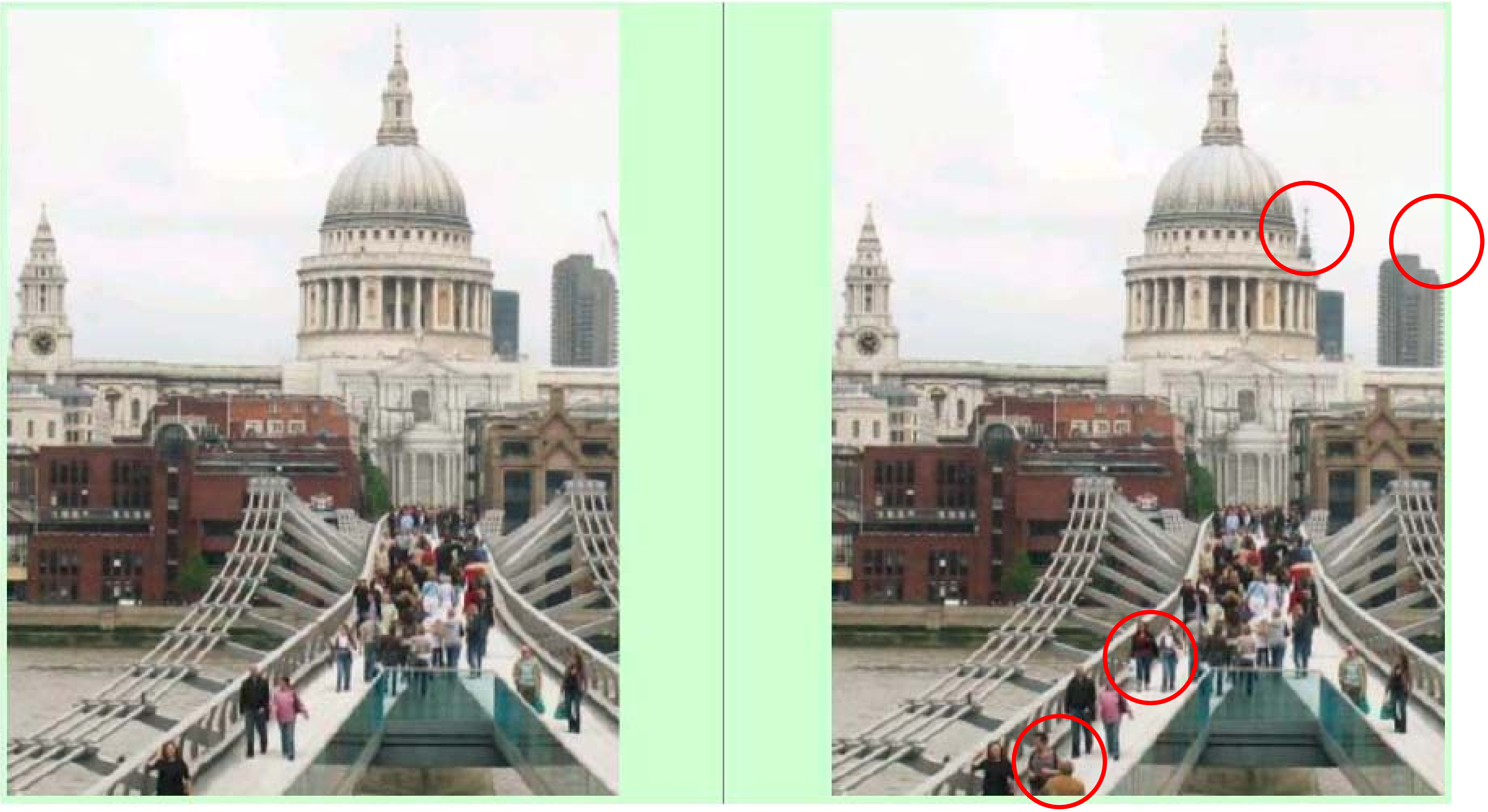


Standard Model Prediction

Experimental Result

Searches of BSM Physics on the Precision (Intensity) Frontier

- **Precision frontier:** seek for **small deviations** from SM predictions.



Standard Model Prediction   Experimental Result

Searches of BSM Physics on the Precision (Intensity) Frontier

- But if you don't understand SM precise enough…

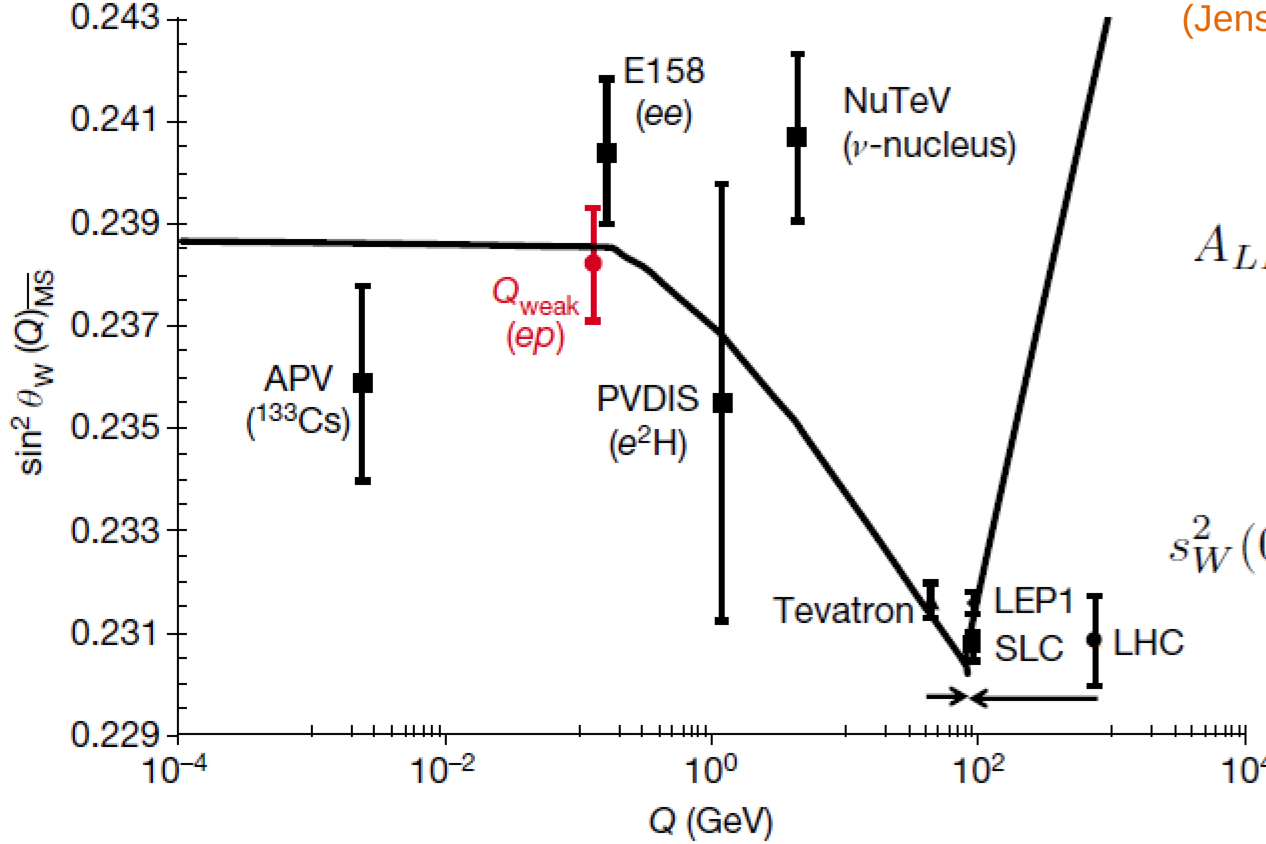
Standard Model Prediction

Experimental Result

Radiative Corrections: Overview

- **Semi-leptonic processes** are often great venues for **precision SM tests**.
- Measurement of the **weak mixing angle** in **ep-scatteing**:

(Jens Erler's talk)



Qweak Collaboration, Nature 557, 207-211 (2018)

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Q_{weak}:

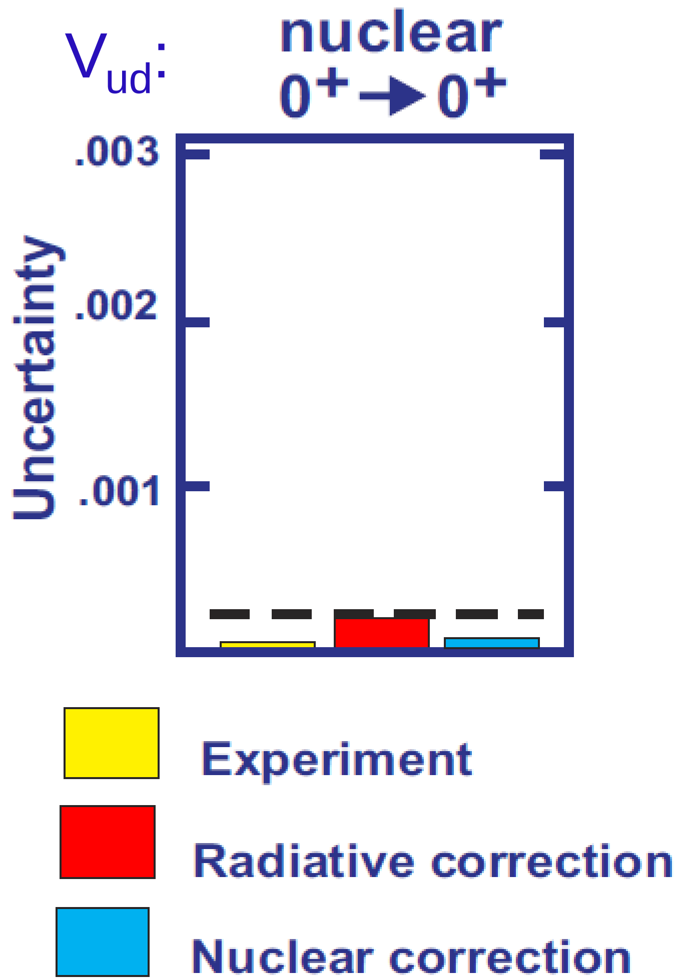
$$s_W^2(0)_{\overline{MS}} = 0.2383(11)$$

- Precision reaches **10^{-3} level**.
- **P2 experiment**: **1.4%** precision for proton weak charge, **0.15%** for s_W^2.

(Frank Maas's talk)

Radiative Corrections: Overview

- Test of the **first-row CKM unitarity** by precise measurement of $\mathbf{V_{ud}}$:



John Hardy, April 2018

Superallowed beta decay:

$$|V_{ud}|^2 = \frac{2984.432(3)\ \mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Half-life +N.C → $\mathcal{F}t$; R.C → Δ_R^V

Situation in early 2018:

$$|V_{ud}| = 0.97420(21) \quad \text{Superallowed}$$

$$|V_{us}| = 0.2243(5) \quad K_{l2} + K_{l3}$$

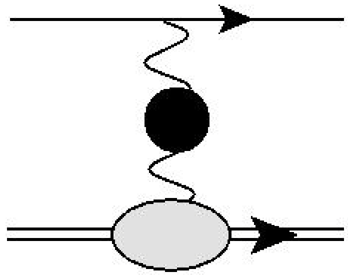
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$$

Experimental precision reaches **$10^{-3} \sim 10^{-4}$**.
All **SM effects** at this order need to be taken into account.

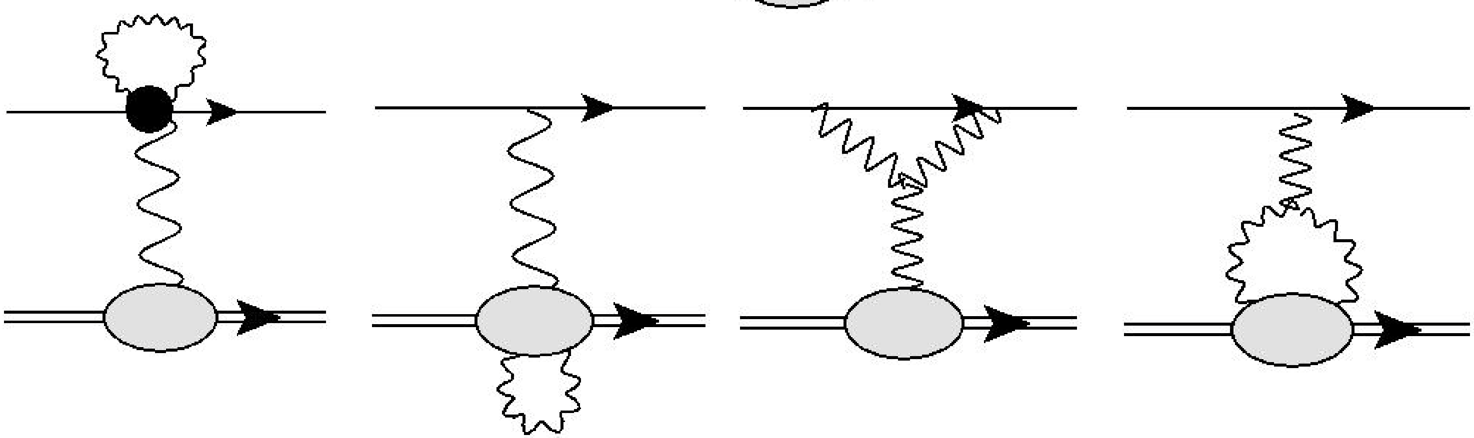
$$\frac{\alpha}{\pi} \sim 10^{-3}$$

Radiative Corrections: Overview

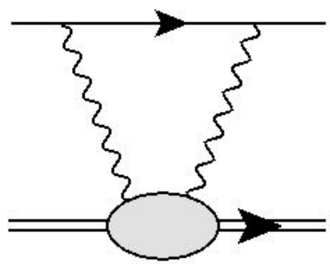
- Generic electroweak radiative corrections to semileptonic processes:



Boson self energy



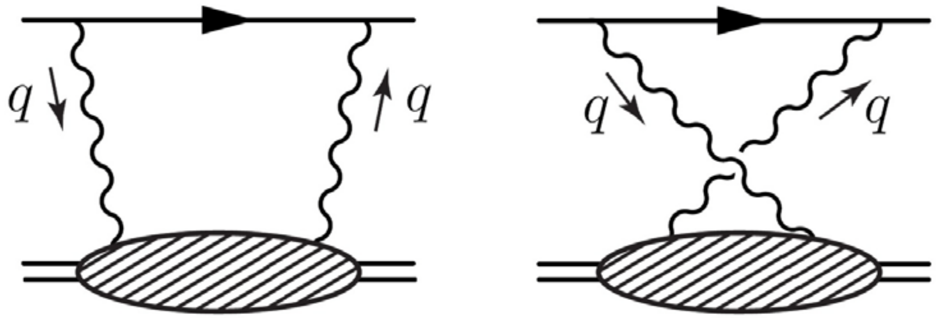
Vertex Corrections



Box diagrams

Radiative Corrections: Overview

- **Electroweak box diagrams**: Feynman diagrams involving the exchange of a **pair of EW gauge bosons** between a **lepton** and a **QCD bound state**.



Forward EW boxes

- General structure (in forward limit):

$$\Box \sim \int \frac{d^4 q}{(2\pi)^4} L^{\mu\nu} \frac{1}{q^2 - m_{B_1}^2} \frac{1}{q^2 - m_{B_2}^2} T_{\mu\nu}(p, q)$$

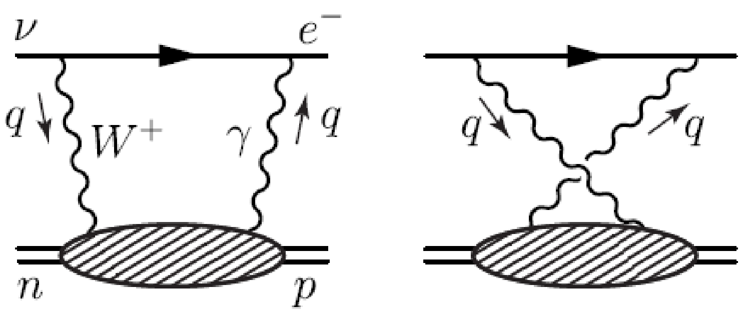
Lepton piece — Hadron piece

$$T^{\mu\nu}(p, q) = \int d^4 x e^{iq\cdot x} \langle N(p)| T[J_1^\mu(x) J_2^\nu(0)] |N(p)\rangle$$

Generalized forward Compton tensor

Radiative Corrections: Overview

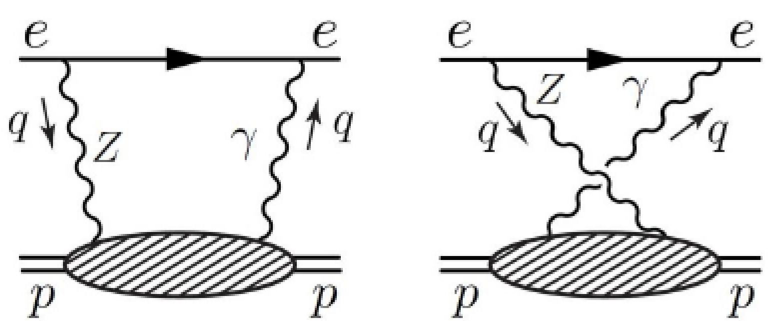
- Examples of **Non-Perturbative EW Boxes**:
  - (1) **γW-box** in **neutron/nuclear beta decay**:



V_{ud} from superallowed beta decay

$$|V_{ud}|^2 = \frac{2984.432(3)\ \mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

  - (2) **γZ-box** in **P-odd ep-scattering**:



Proton weak charge

$$Q_W^p = (1+\Delta\rho+\Delta_e)(1-4s_W^2(0)+\Delta_e') + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0).$$

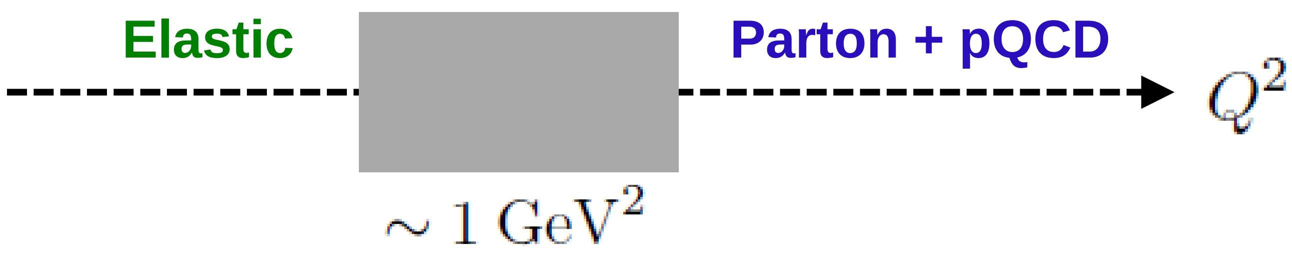
They represent one of the main contributors of theoretical uncertainty in their respective processes!

Radiative Corrections: Overview

- $T_{\mu\nu}(p,q)$ involves **off-shell intermediate hadron states** --- hard to model
- Earlier treatments:
  - **Low-Q^2**: Consider **elastic** contributions only
  - **High-Q^2**: Consider **free-field OPE** only, with **pQCD** corrections
  - Uncertainty obtained by varying the matching point!

Marciano and Sirlin, Phys.Rev.D27, 552(1983)
Marciano and Sirlin, Phys.Rev.D29, 75(1983)

- Later improvements: construct an "**interpolation function**" between the two regions. Marciano and Sirlin, Phys.Rev.Lett 96(2006) 032002

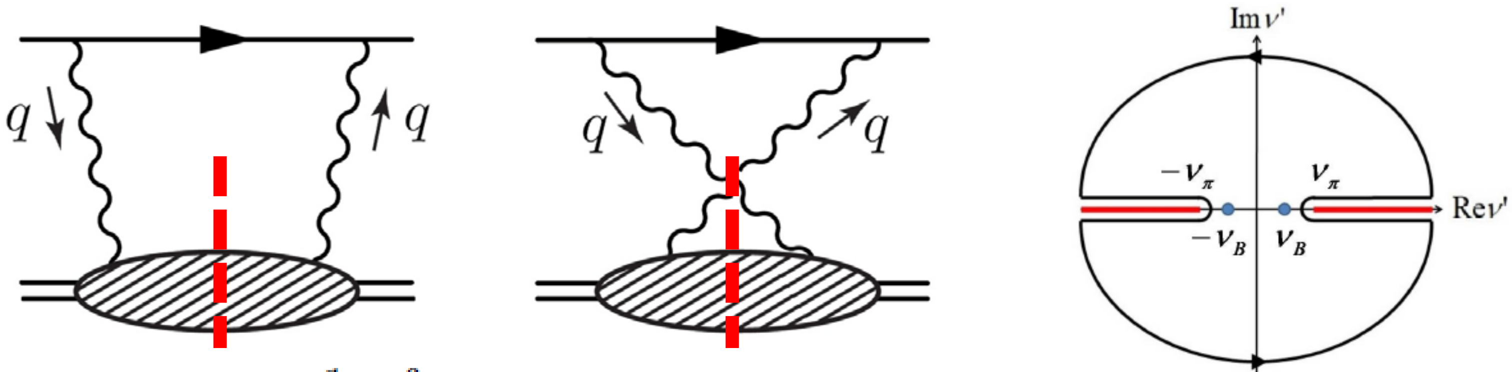


Dispersive Approach

- **Dispersive treatments** to box diagrams are developed since the decade, relating the former to matrix elements of **on-shell intermediate states** Gorchtein and Horowitz, Phys.Rev.Lett,102, 091806 (2009)

$$T^{\mu\nu}(p,q) = \int d^4x e^{iq\cdot x} \langle N(p)| T[J_1^\mu(x) J_2^\nu(0)] |N(p)\rangle$$

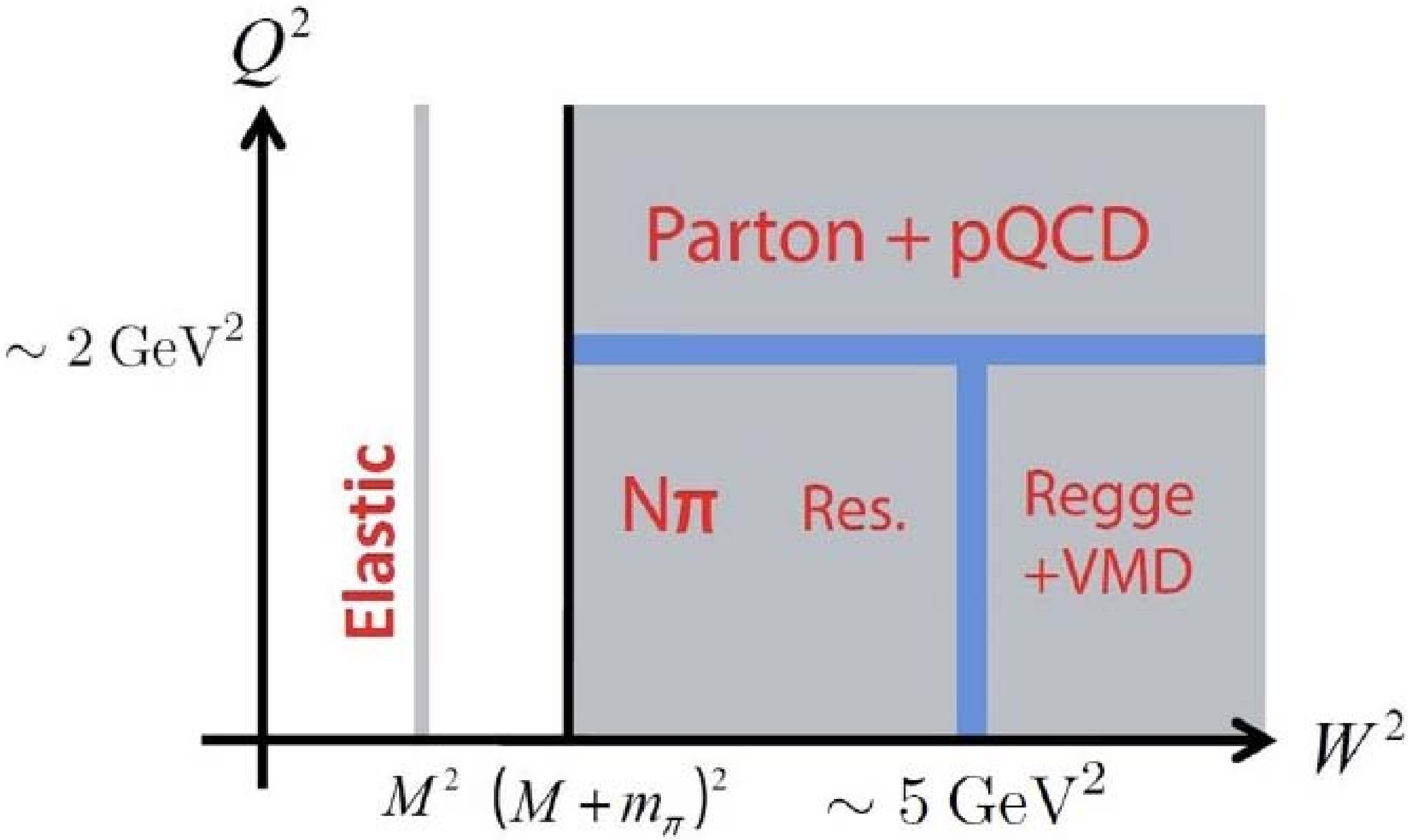
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} T_2 - i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} T_3 + \dots$$



$$W^{\mu\nu}(p,q) = \frac{1}{4\pi} \int d^4x e^{iq\cdot x} \langle N(p)| [J_1^\mu(x), J_2^\nu(0)] |N(p)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} F_2 - i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} F_3 + \dots$$

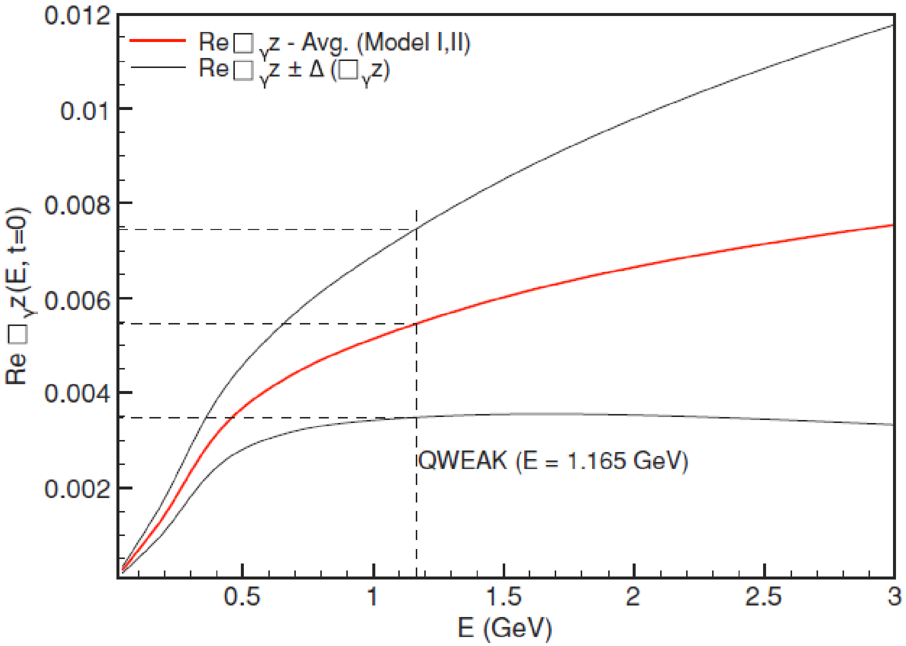
Dispersive Approach



Dispersive Approach

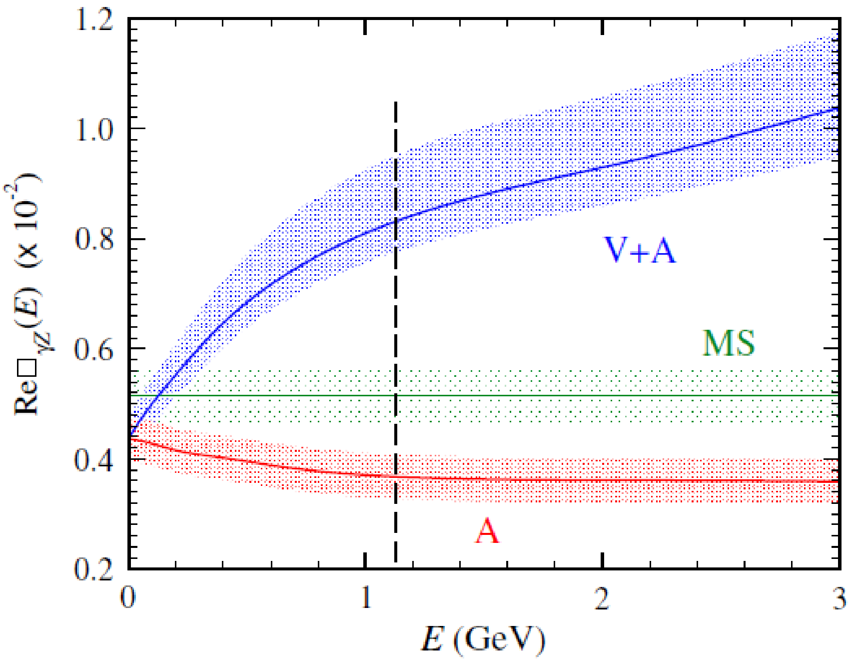
- First applications in vector and axial **γZ box** in forward ep-scattering:

Vector box



Gorchtein, Horowitz and Ramsey-Musolf, Phys.Rev.C84, 015502 (2011)

Axial box



Blunden, Melnitchouk and Thomas, Phys.Rev.Lett.,107,081801(2011)

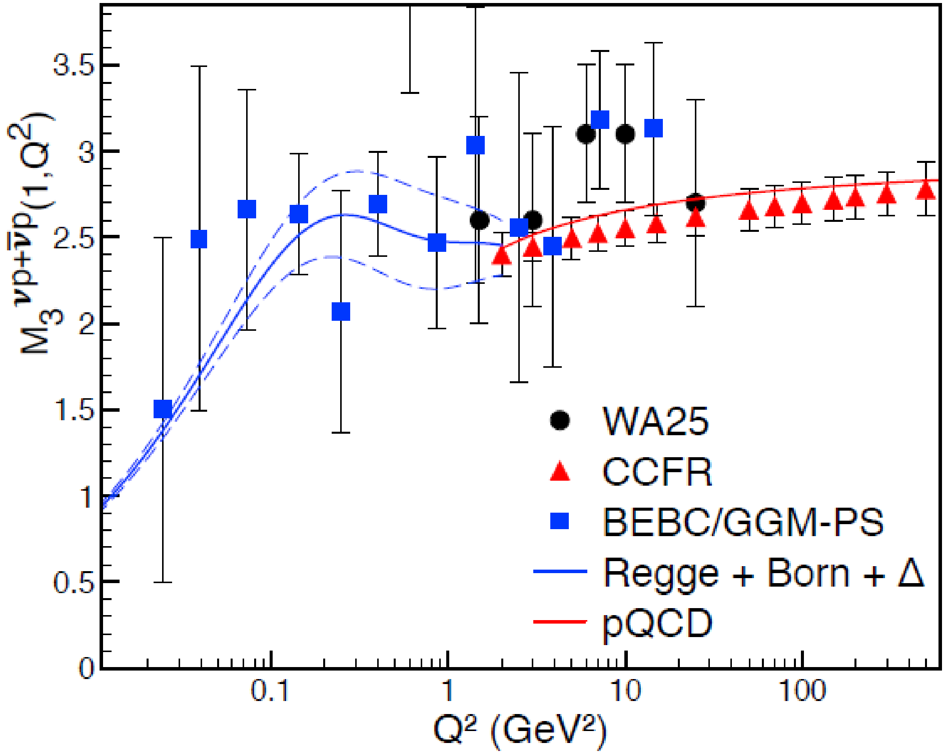
Rislow and Carlson: 2011, 2012, 2013

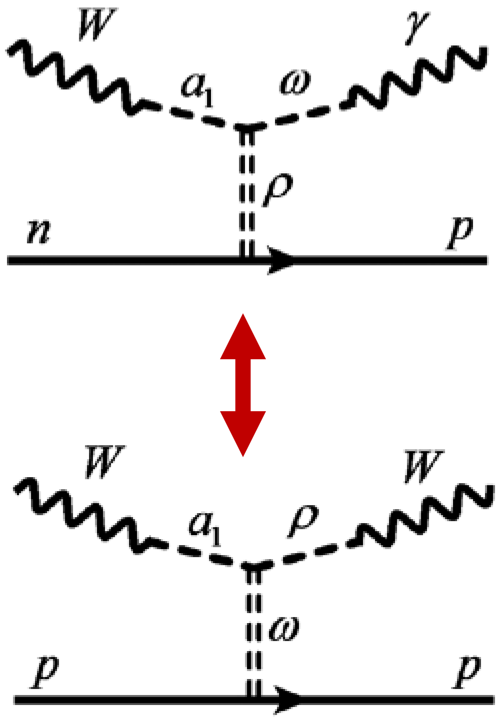
Unveils the underestimated theoretical uncertainty at the Q_{weak} energy, stimulates new experiment (P2) at lower energy, E=155 MeV

Dispersive Approach

- "Transplantation" into **neutron and nuclear beta decay**:

CYS, M.Gorchtein, H.H.Patel and M.J.Ramsey-Musolf, Phys.Rev.Lett. 121 (2018) no. 24, 241804





Matching the axial γW box **to ν p/νbar p scattering data**

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 m_N E}{\pi(1+Q^2/m_W^2)^2}\left[xy^2F_1^{\nu(\bar{\nu})} + \left(1-y-\frac{m_N xy}{2E}\right)F_2^{\nu(\bar{\nu})} \pm x\left(y-\frac{y^2}{2}\right)F_3^{\nu(\bar{\nu})}\right]$$

Dispersive Approach

- Recent success of the **dispersion relation (DR)** approach:
  - Reduced hadronic uncertainty in the **determination of V_{ud}**:

DR+data

$$\Delta_R^V: \quad 0.024(8) \longrightarrow 0.02361(38) \longrightarrow 0.02467(22)$$

1986 2006 2018

- Impact on the **first-row CKM unitarity**:

$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

	V_{ud}	$\Delta_{\rm CKM}$ with K_{l2}	$\Delta_{\rm CKM}$ with K_{l3}
2006 MS	0.97420(21)	-0.0002(5)	-0.0011(5)
2018 DR	0.97370(15)	-0.0012(4)	-0.0021(4)

Tension with first-row CKM unitarity!

- Shift in V_{ud} also confirmed in later (non-DR) studies

A. Czarnecki, W. Marciano and A. Sirlin, Phys.Rev.D 100 (2019) 073008

- DR treatment also leads to identification of **new nuclear-structure effects**

CYS, M.Gorchtein and M.J.Ramsey-Musolf, Phys.Rev.D100(2019) 013001
M.Gorchtein, Phys.Rev.Lett. 123 (2019) 042503

- Work in progress to apply dispersive approach in **kaon semileptonic decay**

CYS, D.Galviz and U.G.Meißner, arXiv: 1910.13208[hep-ph]

Dispersive Approach

- Recent success of the **dispersion relation (DR)** approach:
  - Refining the **axial γZ-box in ep-scattering** with neutrino data:

$$\Box^{A}_{\gamma Z}(0): \quad \underset{2003}{0.052(5)} \xrightarrow{\text{DR}} \underset{2011}{0.0044(4)} \xrightarrow{\text{DR+data}} \underset{2019}{0.0045(2)}$$

J.Erler, M.Gorchtein, O.Koshchii, CYS, H.Spiesberger, Phys.Rev.D100(2019)053007

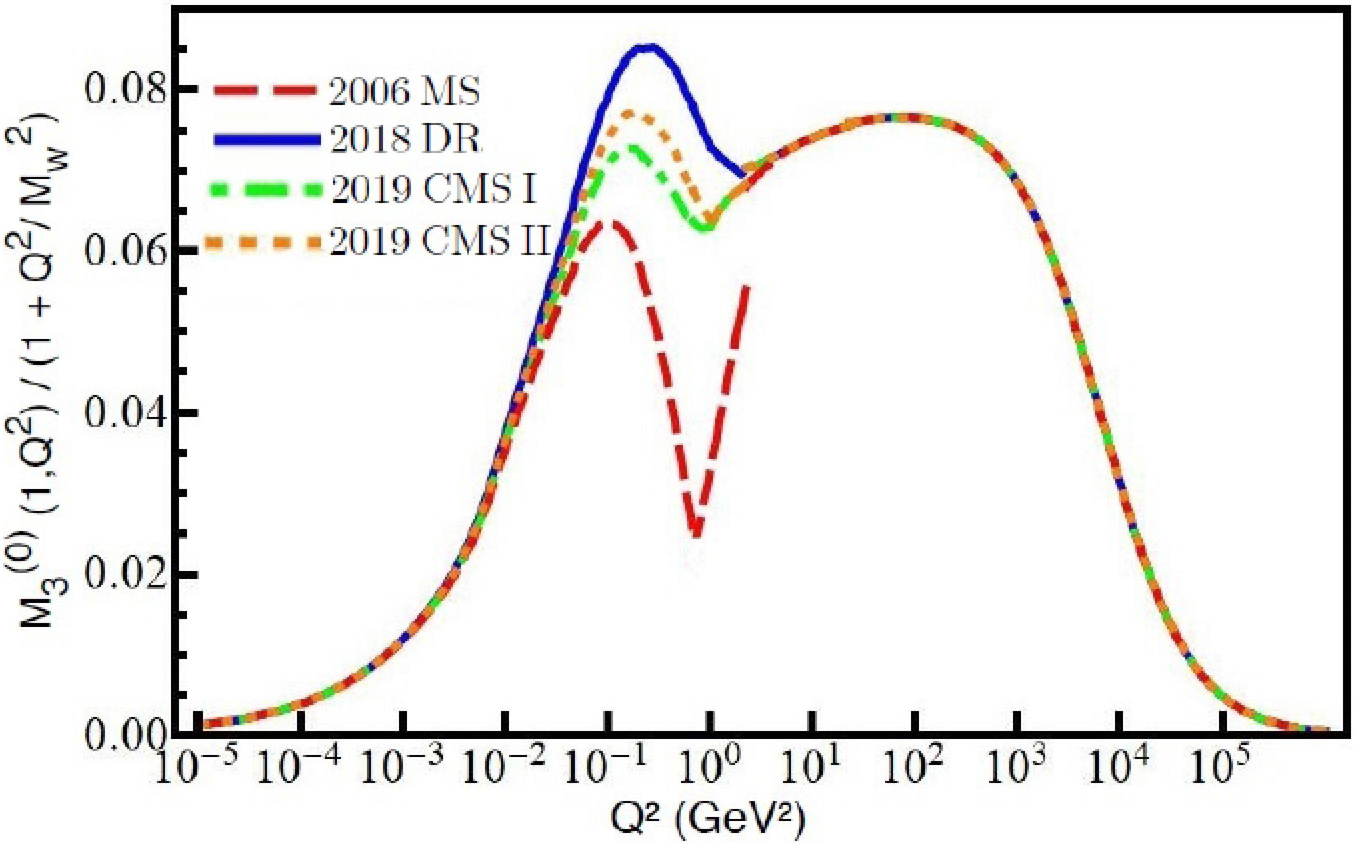
Accuracy requirement of P2: $Q_w^p \times 1.4\% \sim 6 \times 10^{-4}$

  - Our understanding of all one-loop effects have **reached the targeted accuracy for the P2 experiment**. Future theoretical efforts should shift to two-loop effects.

Connection to Lattice

- γW box: need the **first Nachtmann moment** of the P-odd SF.

$$M_3^{(0)}(1,Q^2) = \frac{4}{3}\int_0^1 dx \frac{1+2r}{(1+r)^2} F_3^{(0)}(x,Q^2), \qquad r = \sqrt{1+4M^2x^2/Q^2}.$$



(R.C prop. to area under the curve)

- Value at (0.1-1) GeV^2 plays critical role in the first-row CKM unitarity

Connection to Lattice

- Recent years have seen a rapid growth in the **study of electromagnetic RC (EMRC) on lattice** Giusti et al., Phys.Rev.Lett., 120, 072001 (2018)
- Direct lattice calculation not the best choice for nuclear β–decay, because the ambiguous part ~ 5% of the total EMRC (lattice accuracy ~ 10%). Lattice calculation of the **integrand** is more preferred.
- Alternative approach: **Feynman-Hellmann Theorem** + **DR**

CYS and U.G-Meissner, Phys.Rev.Lett.,122(2019) 211802

$$H_\lambda = H_0 + 2\lambda_1 \int d^3x \cos(\vec{q}\cdot\vec{x}) J^2_{em}(\vec{x}) - 2\lambda_2 \int d^3x \sin(\vec{q}\cdot\vec{x}) J^3_A(\vec{x})$$

$$\left(\frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial\lambda_1 \partial\lambda_2}\right)_{\lambda=0} = \frac{i q_x}{Q^2 \omega} T_3^N(\omega, Q^2)$$

FHT

$$\left(\frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial\lambda_1 \partial\lambda_2}\right)_{\lambda=0} = \frac{4 q_x}{Q^2} \int_0^1 dx \frac{F_3^N(x, Q^2)}{1 - \omega^2 x^2} ,$$

DR

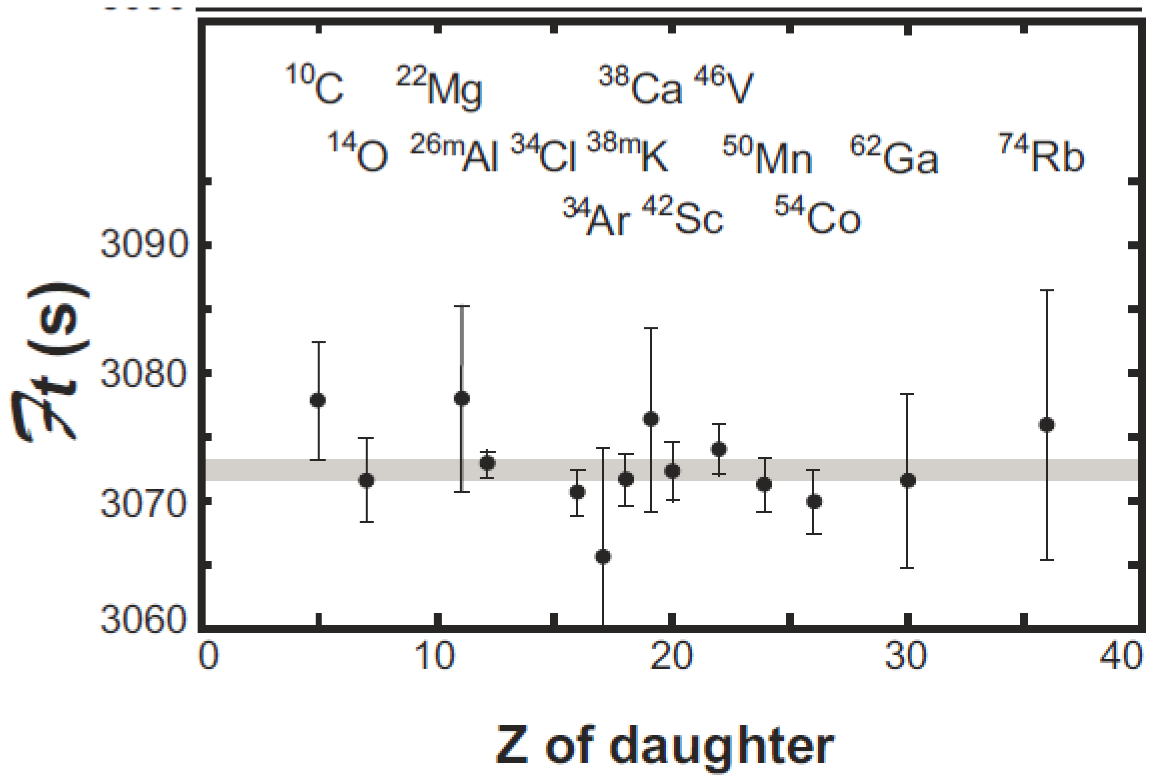
2^{nd} order Energy shift —**FHT**→ Generalized Forward Compton tensor —**DR**→ Structure Function

Summary

1. The applications of dispersion relation treatment to the electroweak box diagrams since year 2009 are discussed. An incomplete list of their impacts on SM precision tests is as follows:
   a) Better understanding of the theory uncertainty as function of E in ep-scattering, stimulating new experimental designs
   b) Reduction of the low-E theory uncertainty, meeting the P2 precision goal
   c) Reduction of theory uncertainty in V_{ud}, unveiling tensions in first-row CKM unitarity
   d) Identification of new nuclear-structure effects in beta decay
   e) Stimulating new lattice strategies for EMRC
   f) Possible future applications in kaon decay and V_{us} extraction

Backup Slides

Selected Superallowed beta decays



J. Hardy and I. Towner, Phys.Rev.C91 (2015) 025501

More details on V_{ud}, V_{us} and first-row CKM unitarity

$V_{us}(K_{l2}) = 0.2253(7)$ $\quad$ $V_{us}(K_{l3}) = 0.2233(6)$

	V_{ud}	$\Delta_{\rm CKM}$ with K_{l2}	$\Delta_{\rm CKM}$ with K_{l3}
2006 MS	0.97420(21)	-0.0002(5)	-0.0011(5)
2018 DR	0.97370(15)	-0.0012(4)	-0.0021(4)
2019 CMS average	0.97389(19)	-0.0008(5)	-0.0017(4)

Table I: V_{ud} and $\Delta_{\rm CKM}$ before including the new nuclear effects.

	V_{ud}	$\Delta_{\rm CKM}$ with K_{l2}	$\Delta_{\rm CKM}$ with K_{l3}
2006 MS	0.97421(37)	-0.0002(8)	-0.0011(8)
2018 DR	0.97371(33)	-0.0011(7)	-0.0020(7)
2019 CMS average	0.97390(35)	-0.0008(8)	-0.0017(7)

Table II: V_{ud} and $\Delta_{\rm CKM}$ after including the new nuclear effects.