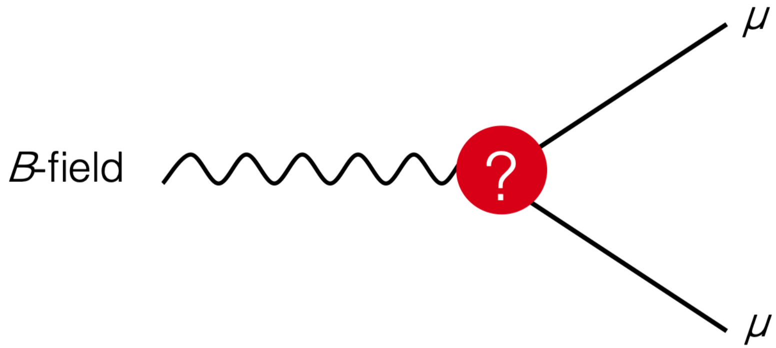
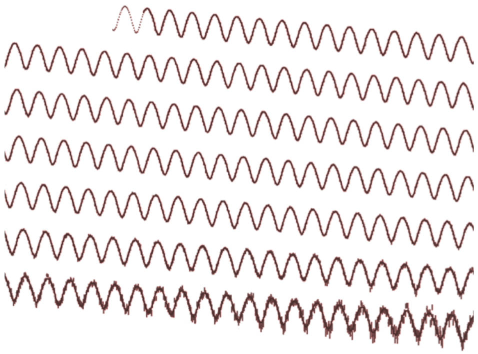
Hadronic vacuum polarization contributions to the muon g–2 and to $\alpha_{QED}(m_Z)$

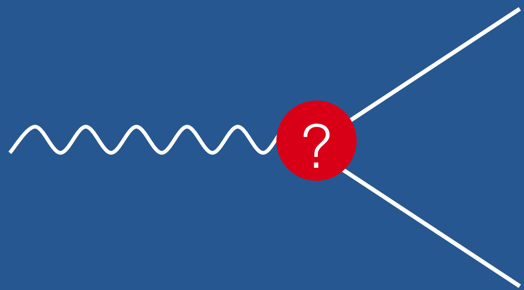




Andreas Hoecker (CERN)

Ultimate Precision at Hadron Colliders Workshop, Paris-Saclay, Nov 29, 2019

Work in collaboration with M. Davier, B. Malaescu, Z. Zhang (DHMZ) [arXiv:1908.00921 (2019)]



Lepton g factor

Dirac's relativistic theory of the electron (1928) naturally accounted for quantized particle spin and described elementary spin-1/2 particles

Paul Dirac

In the classical limit, one finds the Pauli equation with magnetic moment:

$$\vec{\mu} = -g_\ell \frac{e}{2m_\ell} \vec{S} \,, \quad \text{with } |g_\ell| = 2 \text{ the gyromagnetic factor}$$

(and radius $R_\ell = 0$, ie, elementary !)

Where $g_p/g_e = 2.8$ hinted that the proton is not elementary

Today, everyone knows that the proton is composite, and leptons are point-like particles …

But – are they really ?

→ Precise g_ℓ measurement and prediction are key !

The anomalous magnetic moment of the muon

BNL-E821 final result (1997–2001 data):

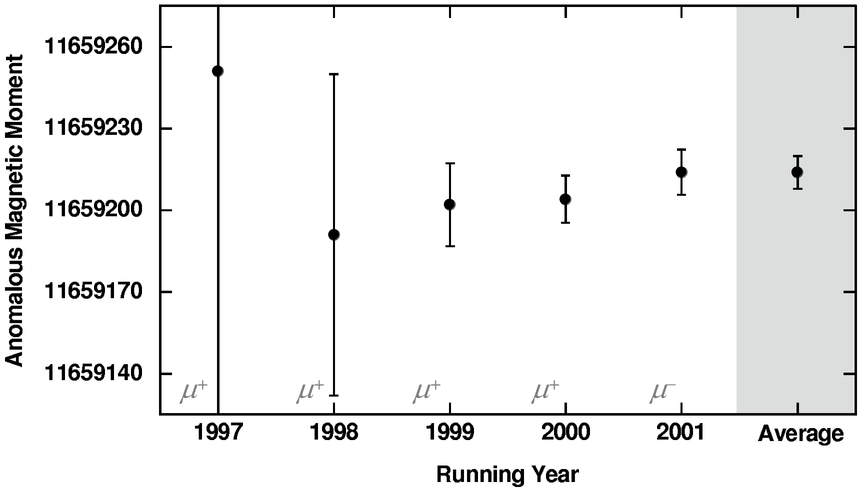
$$a_\mu = \frac{g_\mu - 2}{2} = 11\,659\,209.1\,(5.4)(3.3)\ \cdot 10^{-10}$$

(0.54 ppm precision, assumes CPT invariance)

[Muon g–2, E821, hep-ex/0602035 with updated value for λ]

Agreement between μ^+ and μ^- results

Evolution versus running year [hep-ex/0602035]



Evolution of experimental sensitivity:

[See, eg, Miller, de Rafael, Roberts, hep-ph/0703049]

Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required theor. terms
Columbia-Nevis ('57)	μ^+	g = 2.00 (σ = 0.10)		$g = 2$
Columbia-Nevis ('60)	μ^+	0.001 13 (+16)(–12)	12 %	$\alpha/2\pi$
CERN 1 (SC, 1961)	μ^+	0.001 145 (22)	1.9 %	$\alpha/2\pi$
CERN 1 (SC, 1962)	μ^+	0.001 162 (5)	0.43 %	$(\alpha/\pi)^2$
CERN 2 (PS, 1968)	μ^+	0.001 166 16 (31)	266 ppm	$(\alpha/\pi)^3$
CERN 3 (PS, 1979)	$\mu^\pm$	0.001 165 923 0 (84)	7.2 ppm	$(\alpha/\pi)^3$ + had (60 ppm)
BNL E821 (1997–2001)	$\mu^\pm$	0.001 165 920 91 (63)	0.54 ppm	$(\alpha/\pi)^4$ + had + weak + ?

Muon behaves like heavy electron

Electrostatic focusing, magic γ

BNL-E821 final result (1997–2001 data):

$$a_\mu = \frac{g_\mu - 2}{2} = 11\,659\,209.1\,(5.4)(3.3)\;\cdot 10^{-10}$$

(0.54 ppm precision, assumes CPT invariance)

[Muon g–2, E821, hep-ex/0602035 with updated value for λ]

Evolution versus running year [hep-ex/0602035]

A

Next is Fermilab's new Muon g–2 Experiment

Columbia-Nevis ('60)	μ^+	0.001 13 (+16)(–12)	12 %	$\alpha/2\pi$	behaves like heavy electron
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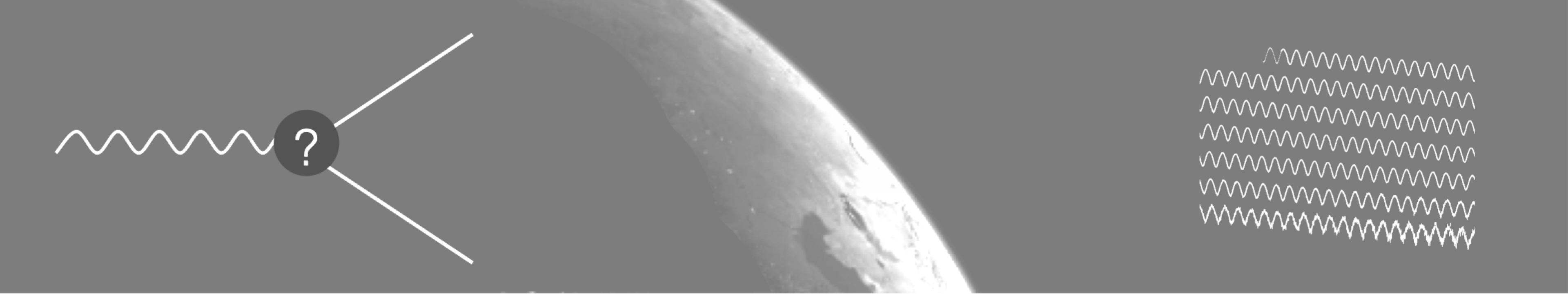
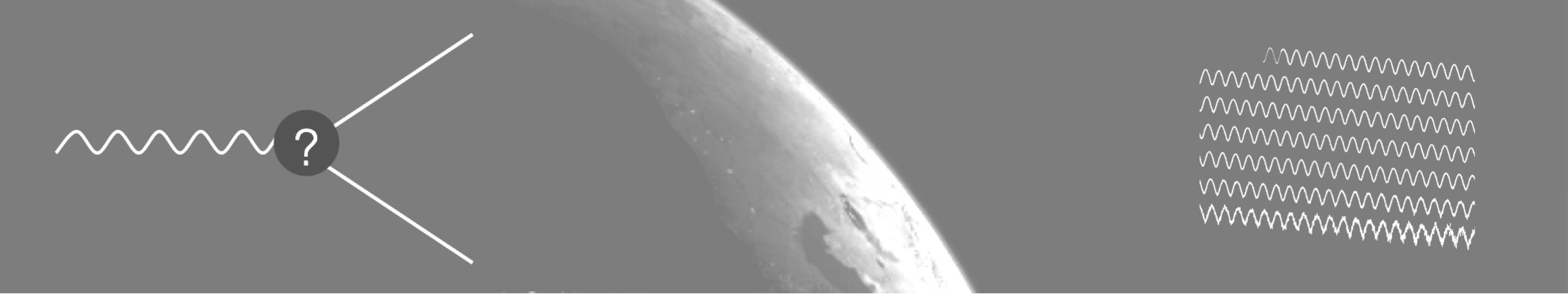


Confronting Experiment with Theory

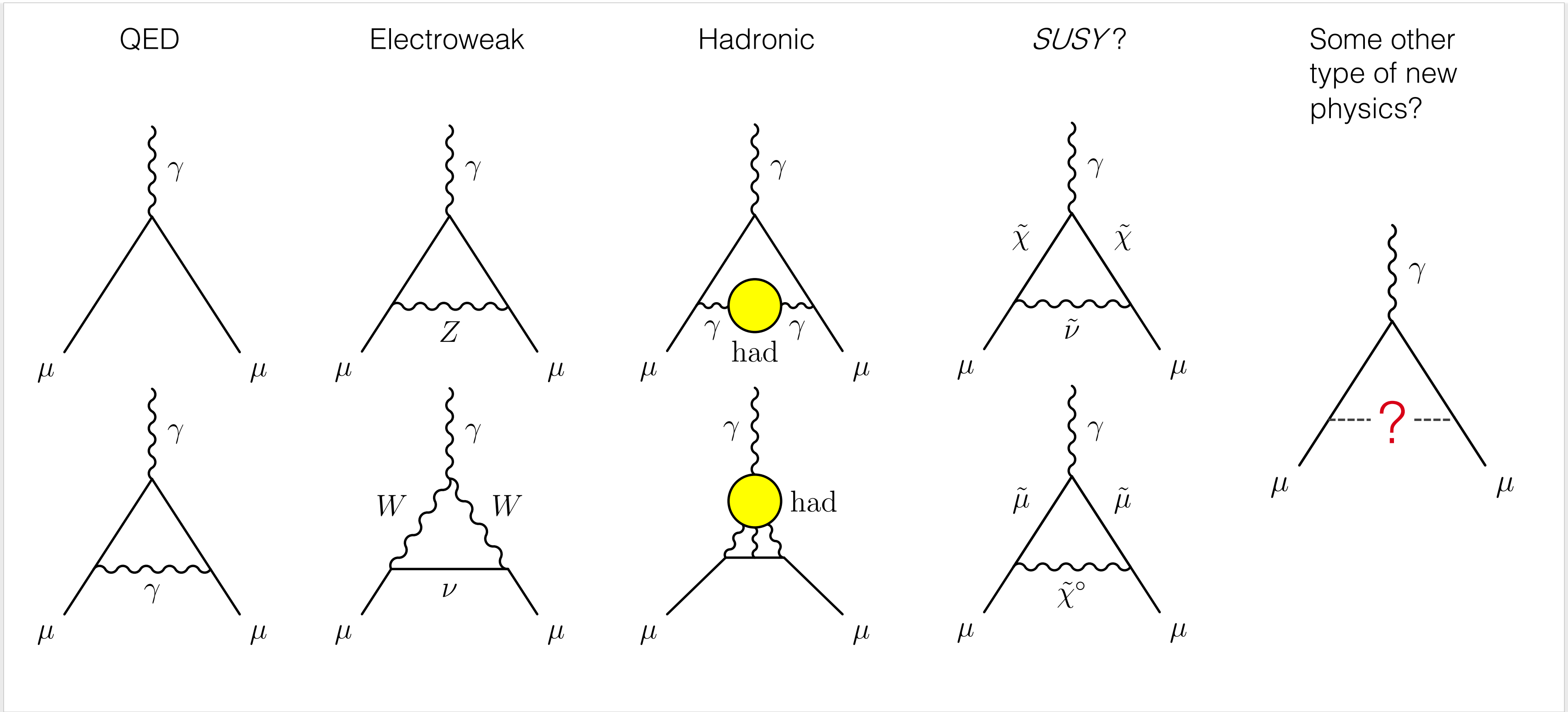
The Standard Model prediction of a_μ is traditionally decomposed in its main contributions:

$$a_\mu^{\mathrm{SM}} = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}}$$

of which the hadronic contribution has the largest uncertainty



Confronting Experiment with Theory

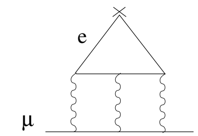


QED contribution

Known to 5 loops, good convergence, diagrams with internal electron loops enhanced:

$$a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + 0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.880(6)\left(\frac{\alpha}{\pi}\right)^4 + 752.2(1.0)\left(\frac{\alpha}{\pi}\right)^5$$

[Schwinger term]



3-loop light-by-light scattering with electron loop

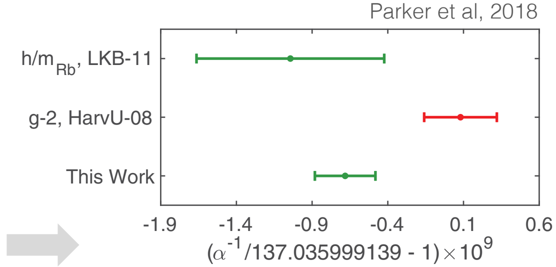
[5-loop: Aoyama, Hayakawa, Kinoshita, Nio, 1205.5370 (2012)]

Using α^{-1} = 137.035 999 046 (27) from a precise measurement* of h/m_{Cs} (0.2 ppb!), gives:

$$a_\mu^{\mathrm{QED}} = 11\,658\,471.892(0.003)\cdot 10^{-10}$$

with negligible uncertainty compared to experimental error of $6.3 \cdot 10^{-10}$

Parker et al, 2018



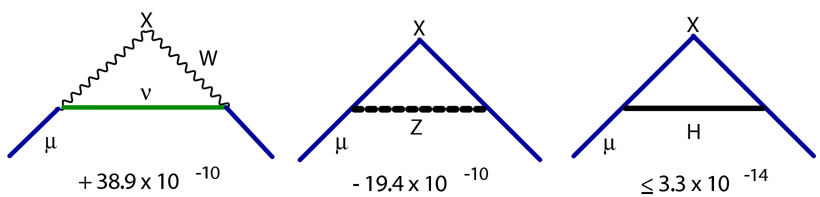
*From Parker et al., Science Vol. 360, Issue 6385, 191 (2018), reporting 2.4σ tension with α from electron g–2

Electroweak contribution

EW contribution involving *W*, *Z* or Higgs is suppressed at least by a factor: $\frac{\alpha}{\pi}\frac{m_\mu^2}{m_W^2} \approx 4 \cdot 10^{-9}$

The first loop gives: [Jackiw, Weinberg and others 1972]

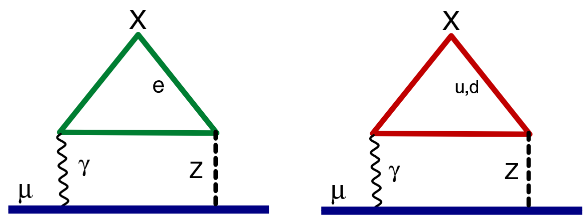
$$a_\mu^{\mathrm{EW,1-loop}} = \frac{G_F m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1 - 4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right] = 19.48 \cdot 10^{-10}$$



1-loop diagrams (some cancellation between *W*/*Z* graphs)

Two-loop contribution surprisingly large due to large $\ln(m_Z/m_\mu)$: [Czarnecki, Krause, Marciano, 1995, and others]

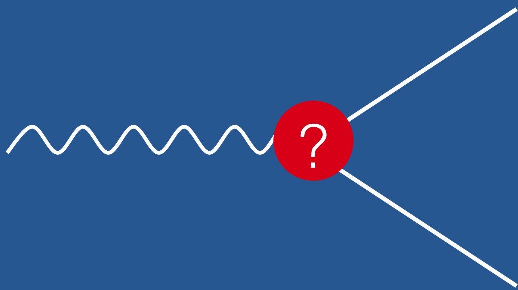
$$a_\mu^{\mathrm{EW,2-loop}} = -4.12(0.10) \cdot 10^{-10}$$



2-loop diagrams (+ Higgs exchange)

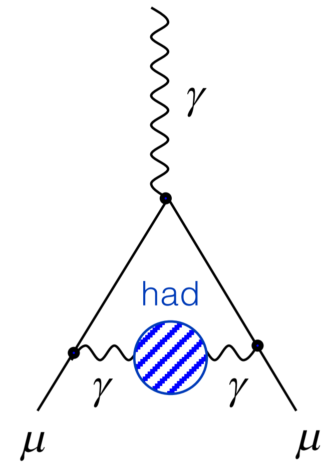
$$\Longrightarrow a_\mu^{\mathrm{EW,1+2-loop}} = 15.36(0.10) \cdot 10^{-10}$$

Three-loop leading logarithms are found to be small ($\sim 10^{-12}$) [Degrassi, Giudice, hep-ph/9803384, and others]



Hadronic contribution

The dominant hadronic contribution and uncertainty stems from the lowest order contribution, $a_\mu^{\mathrm{Had,LO}}$, which cannot be calculated from perturbative QCD as it is in the nonperturbative regime



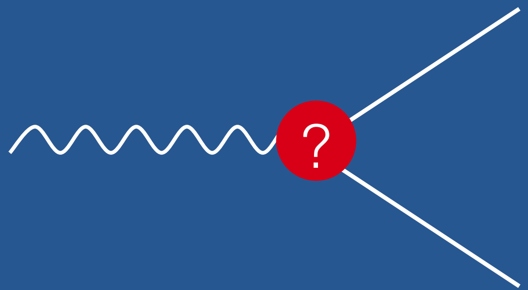
Tools to approach low-energy QCD:

1. Lattice QCD (encouraging results, but precision is challenging; *prediction of broad range of dispersion relations prior to* $a_\mu^{Had,LO}$ *needed to build confidence*)

2. Effective QFT with hadrons such as chiral perturbation theory (limited validity range)

3. Hadronic models (hard to estimate robust uncertainties)

4. Dispersion relations and experimental data …



Lowest-order hadronic contribution

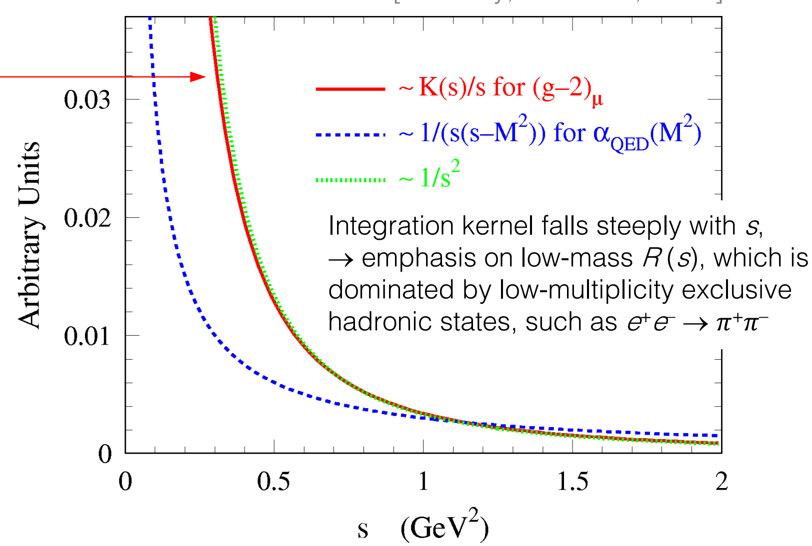
The lowest-order hadronic contribution to a_μ can be obtained from a dispersion relation:

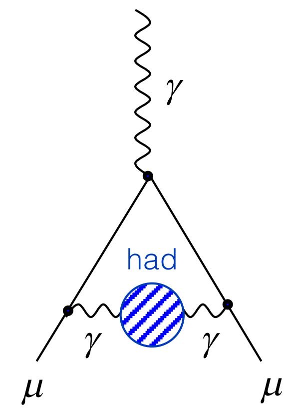
[Bouchiat, Michel, 1961]

$$a_\mu^{\mathrm{Had,LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Where: $R(s) = \dfrac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

[Brodsky, de Rafael, 1968]



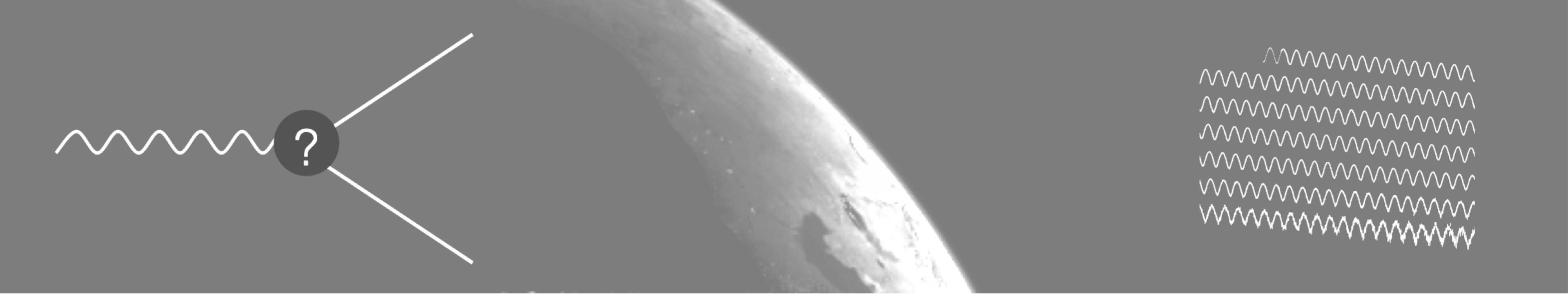


Recent estimate (2019):

$$\Longrightarrow a_\mu^{\mathrm{Had,LO}} = 693.9(4.0) \cdot 10^{-10}$$

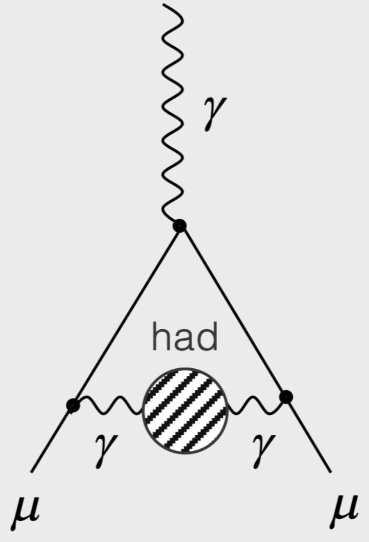
[Davier-Hoecker-Malaescu-Zhang (DHMZ), 1908.00921 (2019)]

→ dominant uncertainty in SM prediction

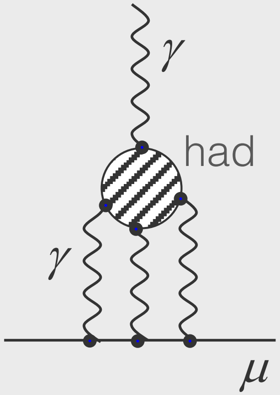


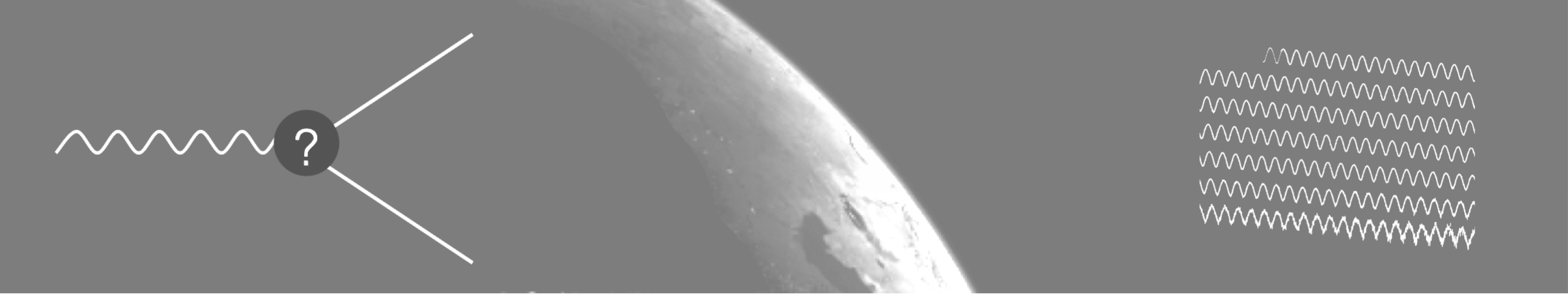
The hadronic contribution to the muon g–2

All hadronic contributions (LO, NLO, NNLO), except for light-by-light scattering (LBLS), can be obtained via dispersion relations using a mix of experimental data and perturbative QCD



The LBLS contribution is a four-point function that is currently estimated using meson models





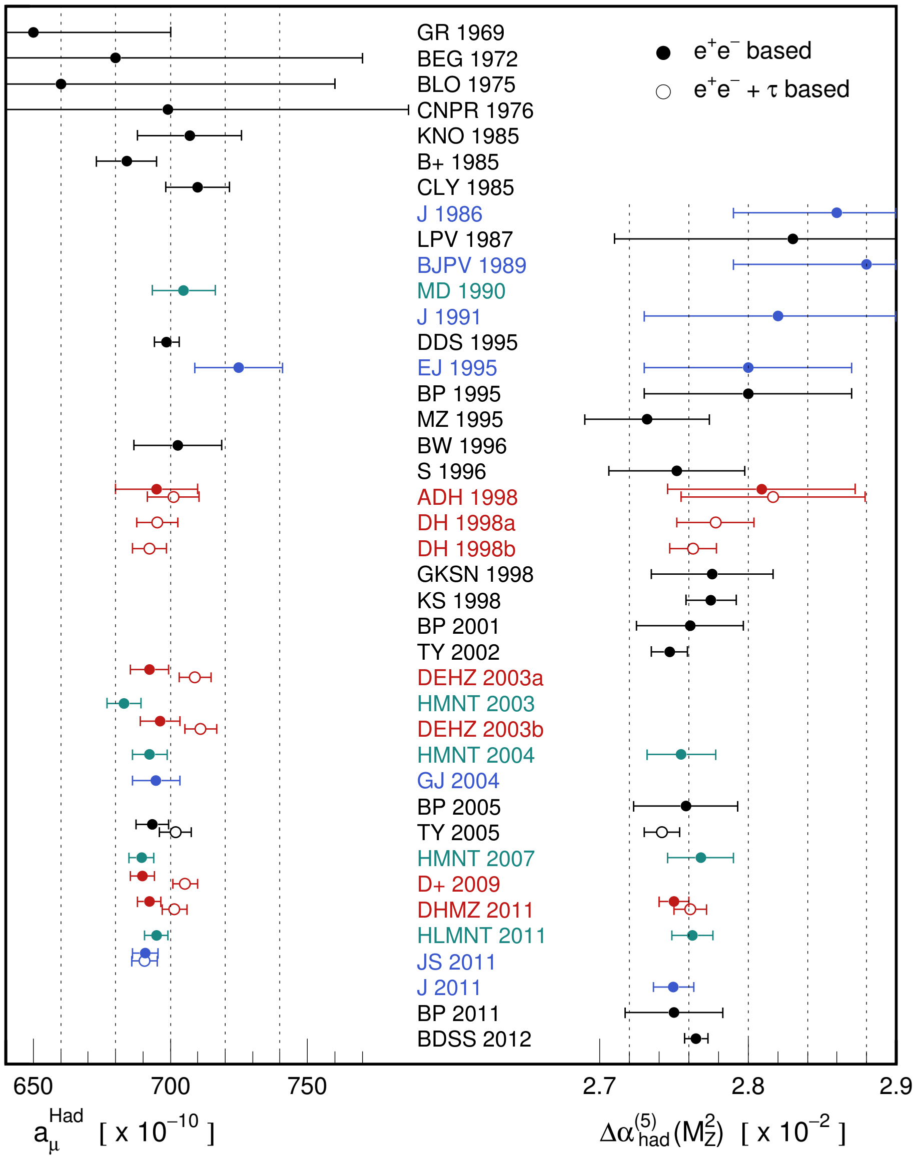
The hadronic contribution to the muon *g*–2

In the following, all a_μ numbers are given in units of 10^{-10}

Long history of $a_\mu^{\text{Had,LO}}$ determinations involving theorists and experimentalists

$$a_\mu^{\text{Had,LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

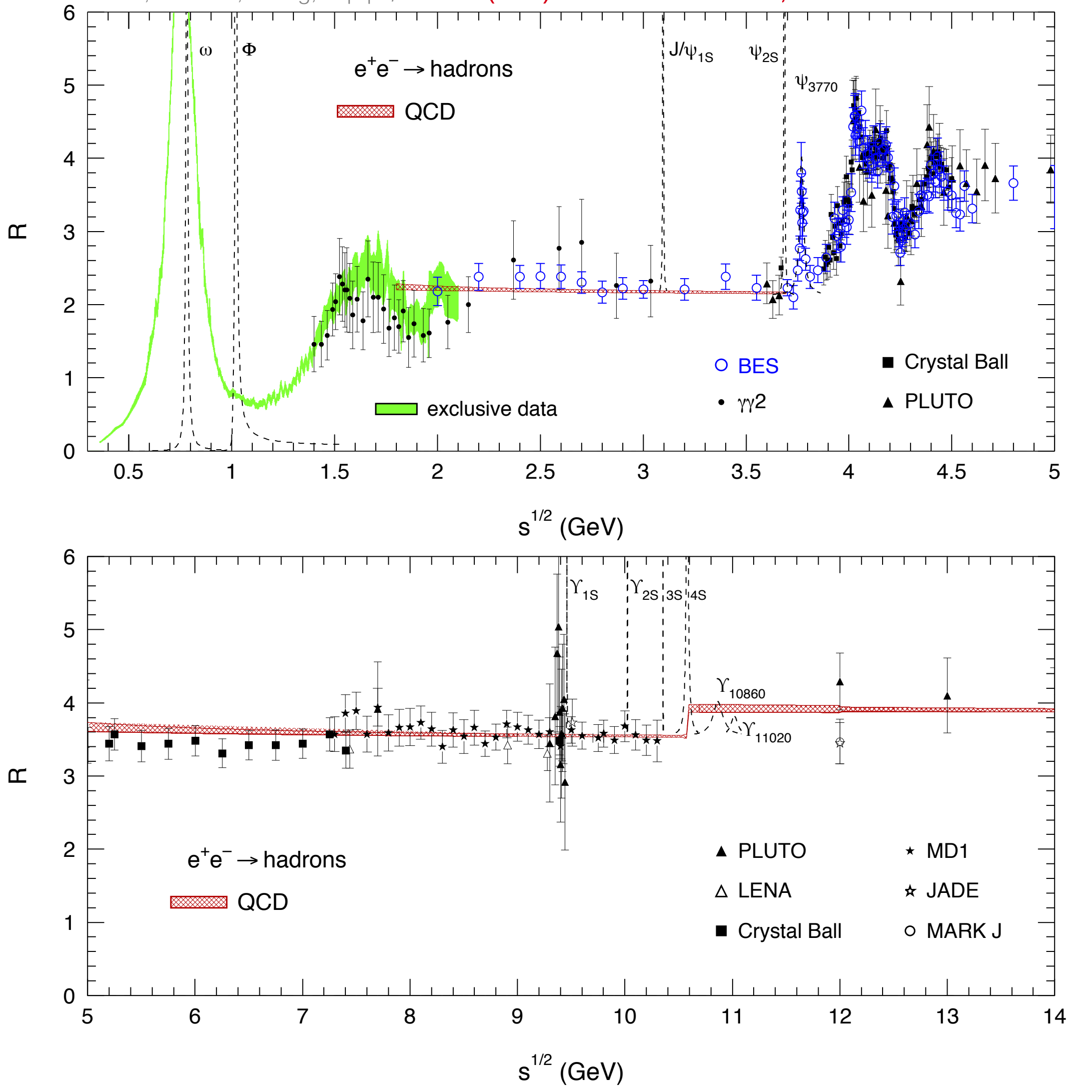
- Improvement mostly driven by better e⁺e⁻ → hadrons data (intermittently also hadronic tau decays used to improve over insufficient-quality low-mass e⁺e⁻ data)
- The understanding of the data and the treatment of their uncertainties improved over time
- Sum-rule tests allowed to expand the use of perturbative QCD to predict *R*(*s*)
- Fairly consistent picture reached



The challenge

The dispersion relation is solved using a mix of $e^+e^- \to$ had data and QCD, depending on $\sqrt{s}$

Davier, Hoecker, Zhang, hep-ph/0507078 (2005) — FIGURE OUTDATED, FOR ILLUSTRATION ONLY



- **[$\pi^0\gamma$ – 1.8 GeV]**: sum of 32 exclusive channels; very few unmeasured channels are estimated using isospin symmetry
- **[1.8 – 3.7 GeV]**: agreement between data and QCD for *uds* continuum → more precise QCD NNNLO used; J/ψ & $\psi(2S)$ resonances from Breit-Wigner forms
- **[3.7 – 5.0 GeV]**: open charm pair production: use of data
- **[5.0 GeV – ∞]**: NNNLO QCD (assuming global quark-hadron duality to hold across *bb* threshold)

Data combination procedure

The combination of data with dominant systematic uncertainties and sometimes discrepancies among datasets is a delicate procedure that requires care. The stake (new physics or not in the muon g–2) is high, so a (reasonably) conservative approach is mandatory.

Our procedure is as follows:

- **Quadratic interpolation** (splines) of adjacent data points is performed for each experiment
- A **local combination** of the interpolations of different datasets is computed in bins of 1 MeV, or in narrower bins for the ω, ϕ resonances
- The test statistic for the local combination (conservatively) uses local information only, **avoiding constraints from potentially badly controlled long-range correlations** of systematic uncertainties
- Where data are locally inconsistent, the **uncertainty of the combination is rescaled following the PDG prescription**
- The uncertainties on the combined dataset and dispersion integral are computed using **pseudo-experiments generated taking into account all known correlations between datapoints and datasets**
- The **full procedure has been validated using pseudo-experiments with known truth**

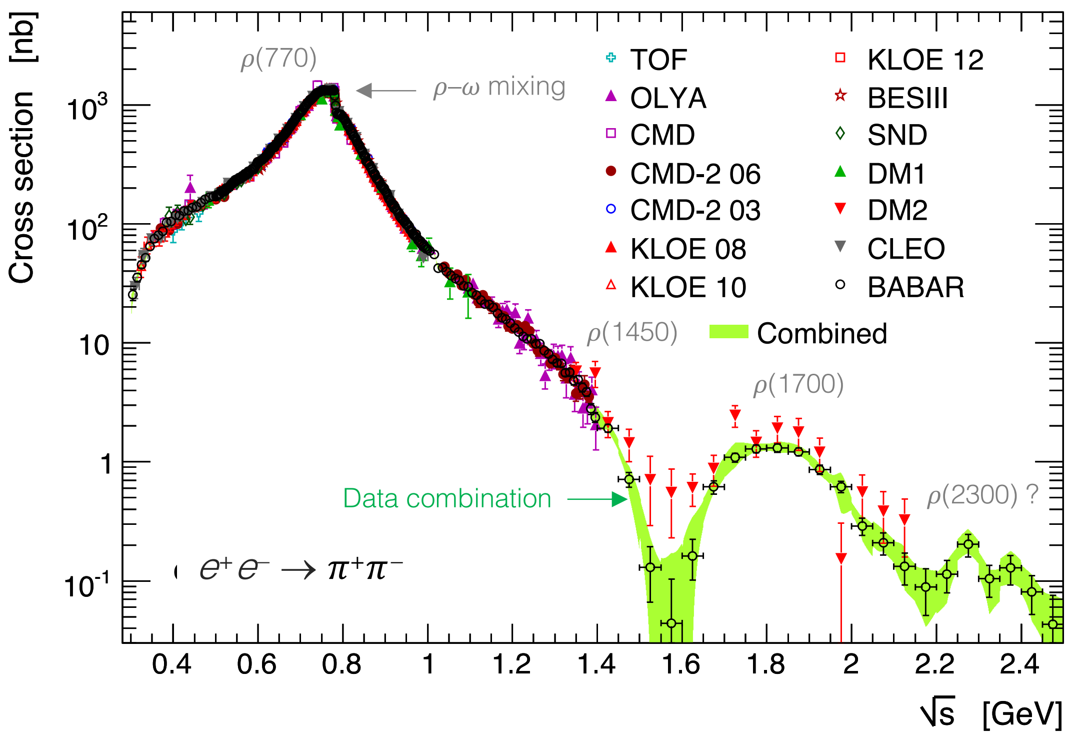
The dominant $e^+e^- \to \pi^+\pi^-$ contribution

$e^+e^- \to \pi^+\pi^-$ contributes 73% to $a_\mu^{\mathrm{Had,LO}}$ and 71% to total uncertainty-squared

Relative to uncertainty^2 due to quadratic addition

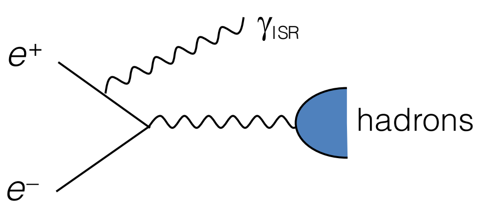
Many of the efforts in the last twenty years concentrated on that channel.
Measurements dominated by systematic uncertainties

Compilation and combination: DHMZ, arXiv:1908.00921 (2019)



Three types of input data:

- **Energy scans**: CMD-2 ($\delta_{syst} \sim 0.8\%$), SND ($\delta_{syst} \sim 1.5\%$), + DM1, DM2, OLYA, TOF
- **ISR-based measurements**: BABAR ($\delta_{syst} \sim 0.5\%$), BES-III ($\delta_{syst} \sim 0.9\%$), KLOE ($\delta_{syst} \sim 0.8$–1.4%)



- **Hadronic tau decay data** via isospin symmetry (CVC): ALEPH, OPAL, CLEO, Belle ($\delta_{syst\text{-}combined} \sim 0.7\%$),

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Measurements dominated by **systematic uncertainties**

Dominant systematic uncertainties / challenges:
(in parentheses uncertainties for best measurements)

- **Energy scan measurements** (ex. CMD-2 / 0.8%): detection efficiency, radiative corrections (0.4%), beam energy (0.3%), …
- **ISR-based measurements** (ex. BABAR / >0.5%): pion identification (0.3%), $\mu^+\mu^-$ reference (0.4%), …
- **Tau data** (ALEPH, 0.3% on normalisation): π^0 and photon reconstruction (0.2%), hadronic interactions (0.2%), isospin-violating effects → not used anymore

Huge amount of precision data, but — with a close look — one notices issues…

Three types of input data:

- **Energy scans**: CMD-2 ($\delta_{\mathrm{syst}} \sim 0.8\%$), SND ($\delta_{\mathrm{syst}} \sim 1.5\%$), + DM1, DM2, OLYA, TOF
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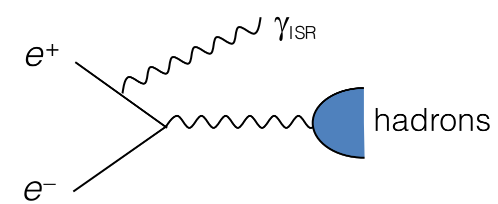

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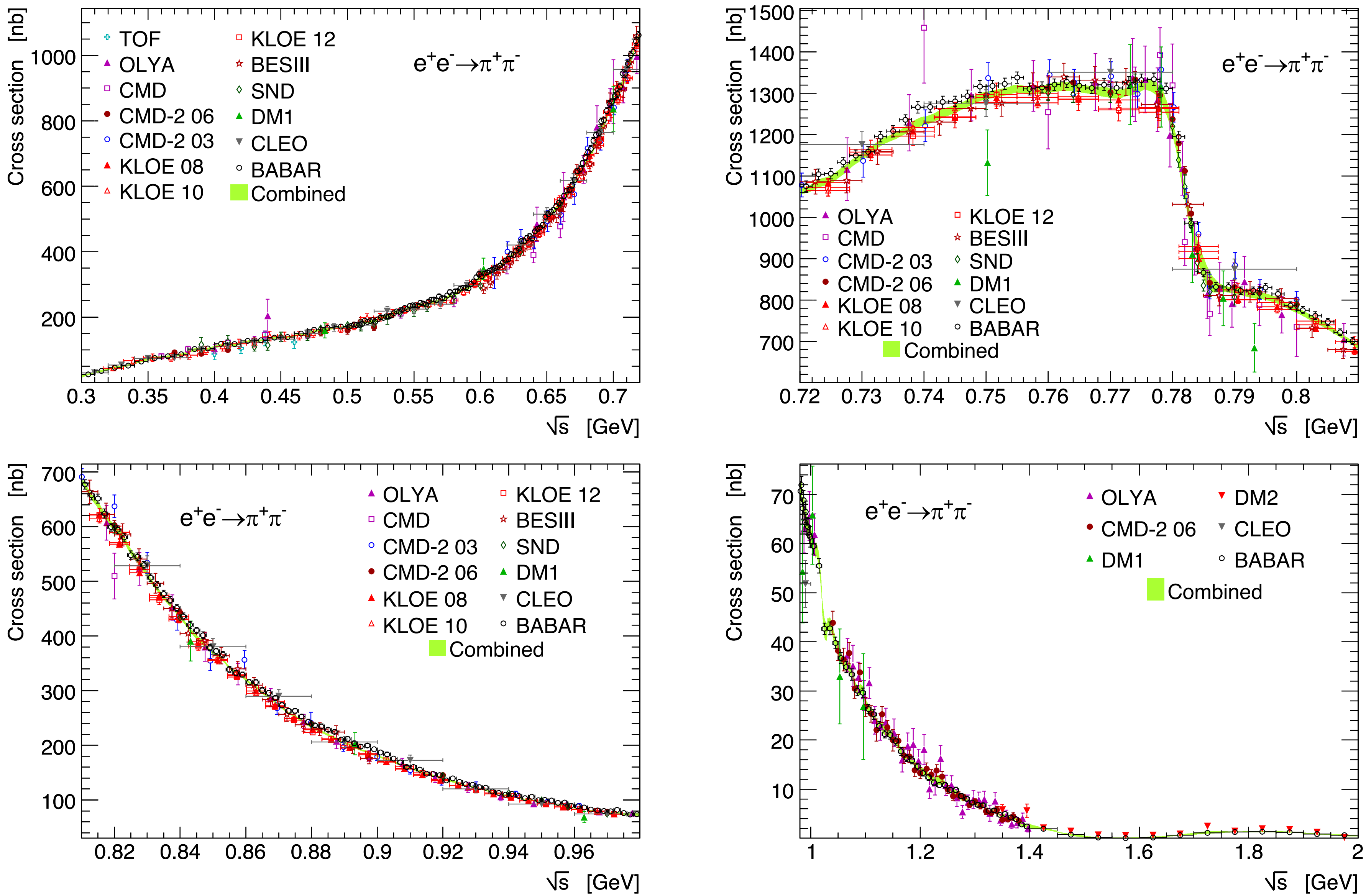
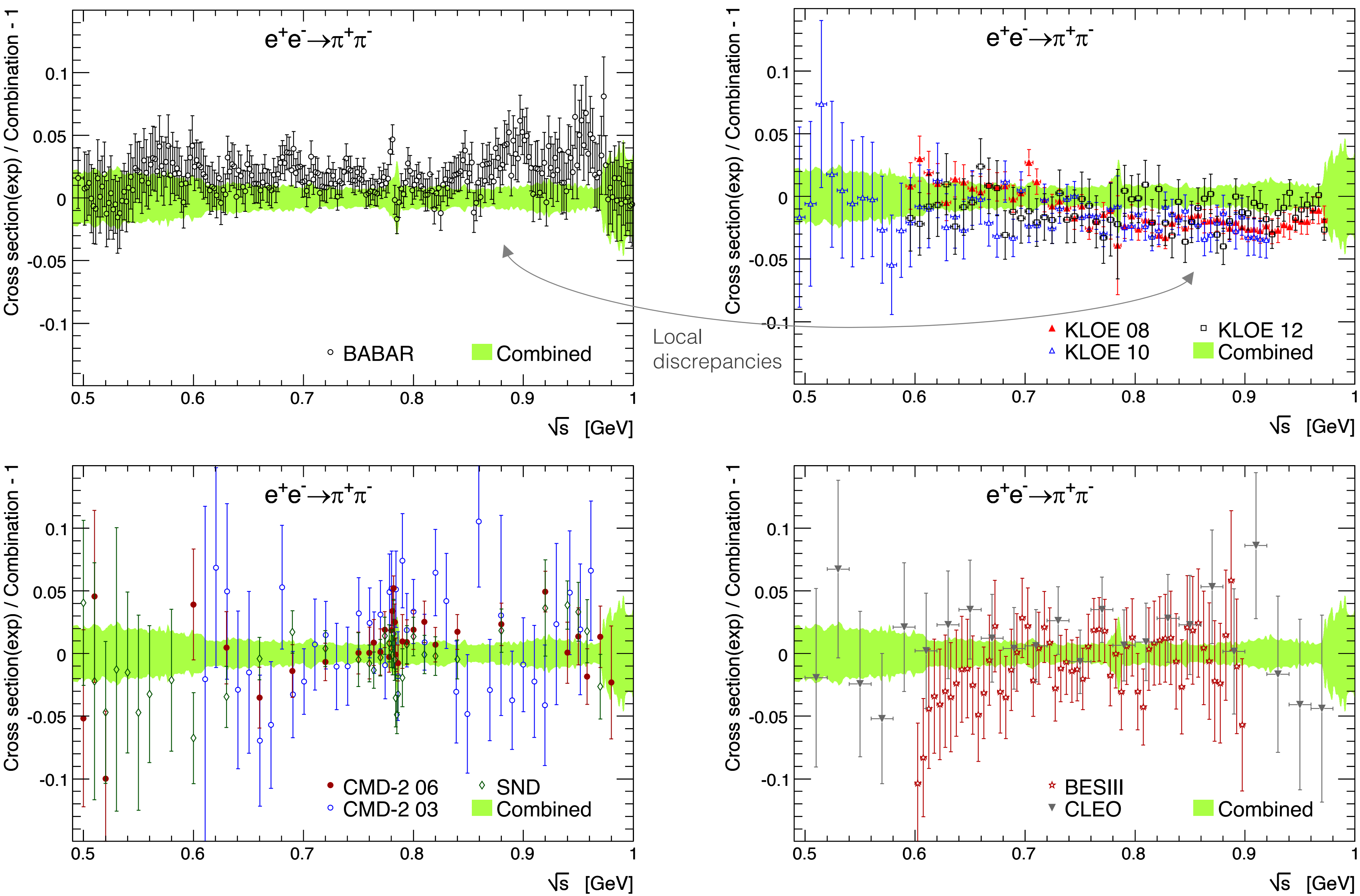
The dominant $e^+e^- \to \pi^+\pi^-$ contribution

Compilation and combination: DHMZ, arXiv:1908.00921 (2019)

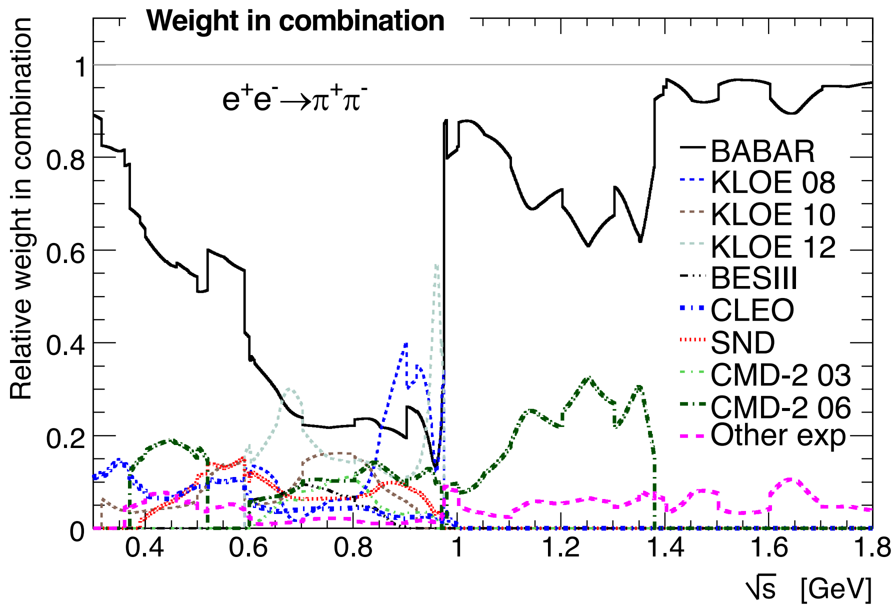


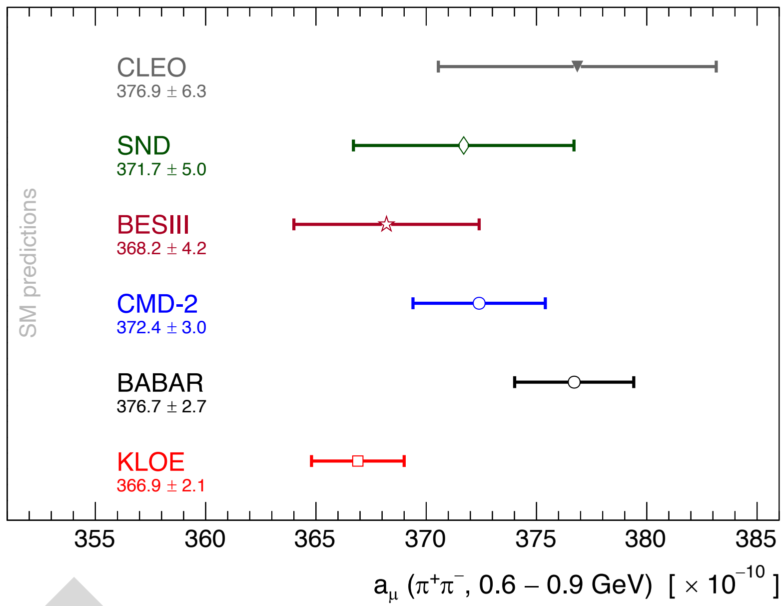
The dominant $e^+e^- \to \pi^+\pi^-$ contribution

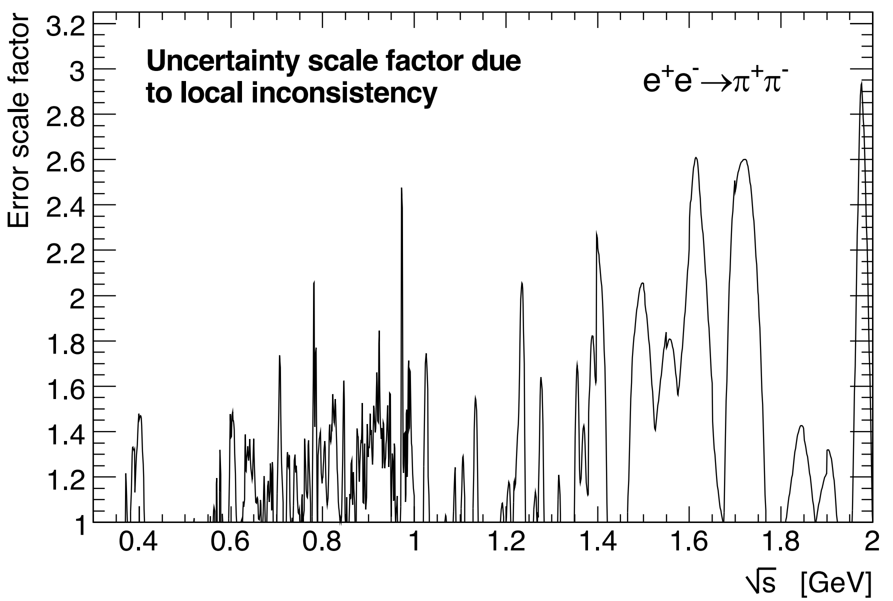
Compilation and combination: DHMZ, arXiv:1908.00921 (2019)



The dominant $e^+e^- \to \pi^+\pi^-$ contribution — impact on a_μ







Comparison of results between 0.6–0.9 GeV for the various experiments.
In case of CMD-2 all available measurements have been combined using HVPTools. For KLOE the result from the public combination is displayed.

Removing from full combination BABAR or KLOE, respectively, leads to a difference of 5.6×10^{-10}, which (despite local error rescaling) is not covered by uncertainty of 2.1×10^{-10}

→ Add half of difference as additional uncertainty to $a_\mu[\pi^+\pi^-]$

Phenomenological fit of $\pi^+\pi^-$ threshold region

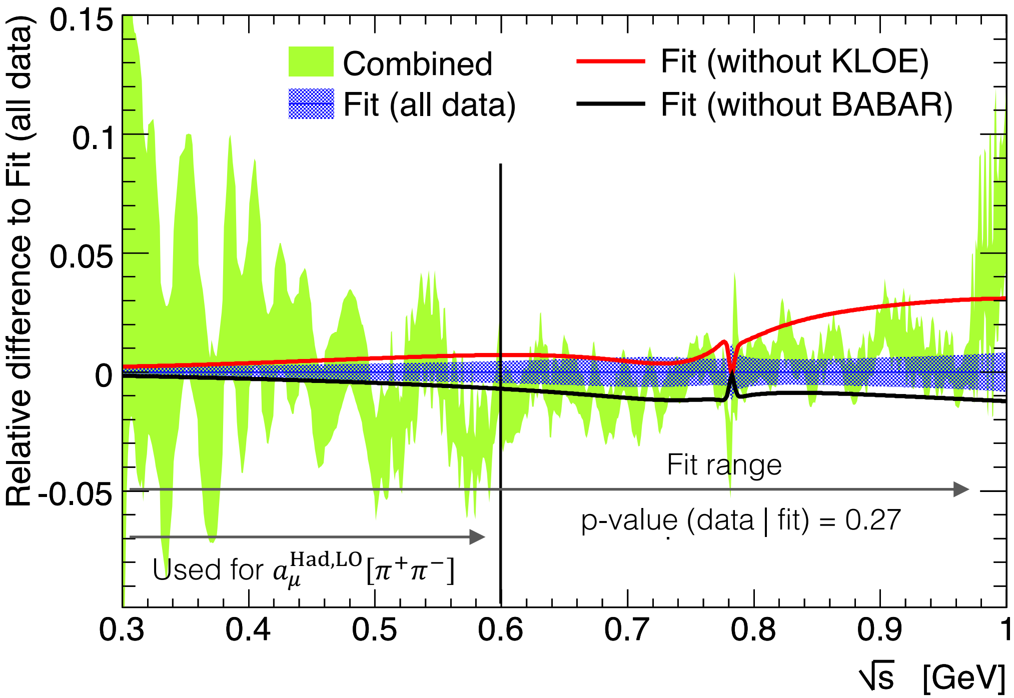
Use phenomenological fit to supplement less precise data in the low-energy domain < 0.6 GeV

$$\sigma^{(0)}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3s}\beta_0^3(s) \cdot |F_\pi^0(s)|^2 \cdot \mathrm{FSR}(s)$$

Following approach of Hanhart et al, arXiv:1611.09359

Form factor expressed as unitary Breit-Wigner incl. $\rho - \omega$ mixing with 6 fit parameters

Fit performed using conservative diagonal test statistic to avoid biased central values. Parameter and integration uncertainties determined via pseudo-experiments taking into account all known correlations



Evaluation between 0.3 – 0.6 GeV gives:

$$\sigma(a_\mu^{\mathrm{Had,LO}})[\pi^+\pi^-] = 109.8 \pm 0.4_{\mathrm{exp}} \pm 0.4_{\mathrm{model}}$$

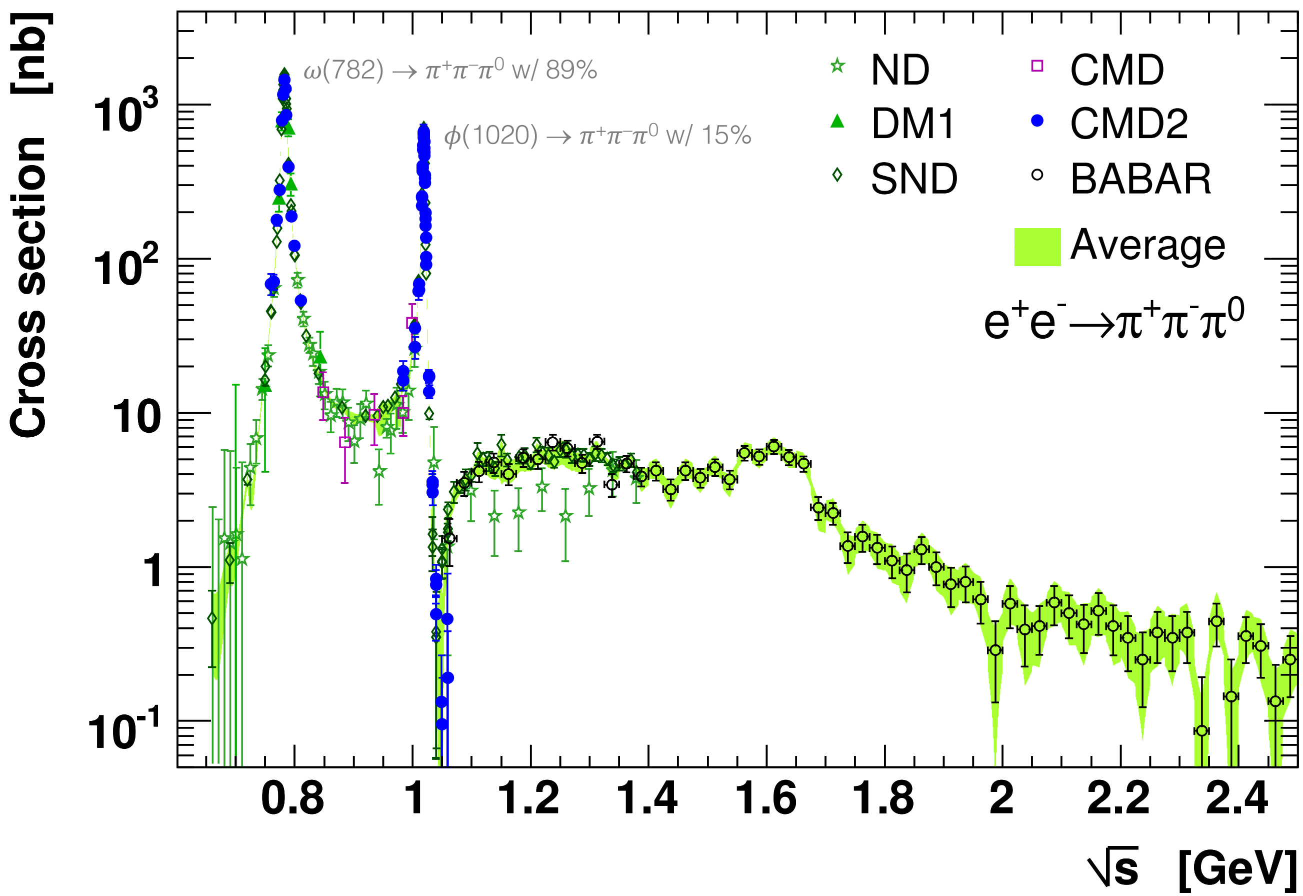
Compared to 109.6 ± 1.0 from direct data integration

Note: the two above estimates do not include the additional systematic error from the overall BABAR-KLOE discrepancy, which is however included in the full $a_\mu^{\mathrm{Had,LO}}[\pi^+\pi^-]$ evaluation.

$e^+e^- \to \pi^+\pi^-\pi^0$ contributes with 6.6% to $a_\mu^{\text{Had,LO}}$ and 19% to its uncertainty-squared

Good agreement among precision data (no BABAR data yet below 1.04 GeV)

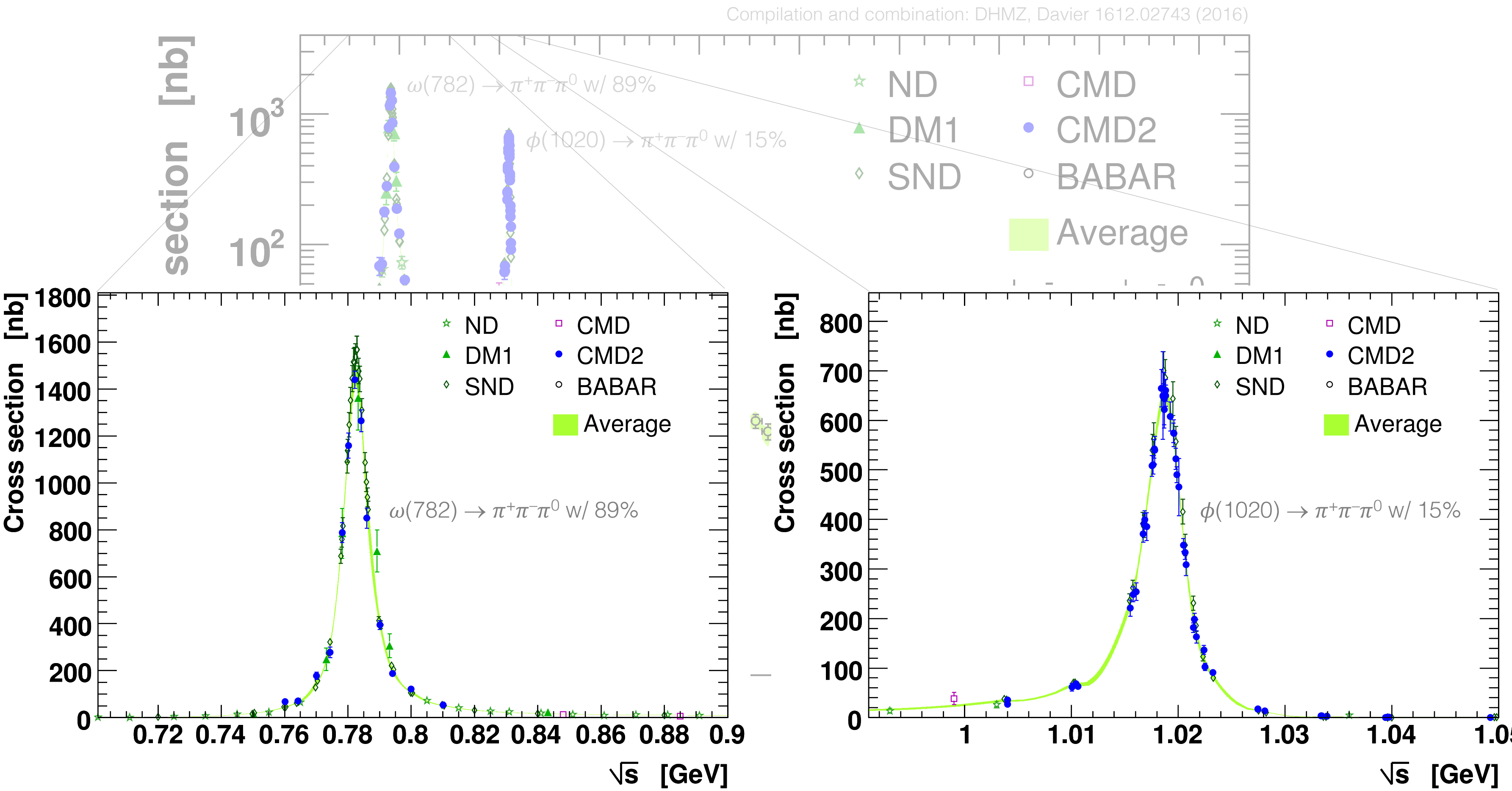
Compilation and combination: DHMZ, Davier 1612.02743 (2016)



The e⁺e⁻ → $\pi^+\pi^-\pi^0$ contribution

e⁺e⁻ → $\pi^+\pi^-\pi^0$ contributes with 6.6% to $a_\mu^{\mathrm{Had,LO}}$ and 19% to its uncertainty-squared

Good agreement among precision data (no BABAR data yet below 1.04 GeV)

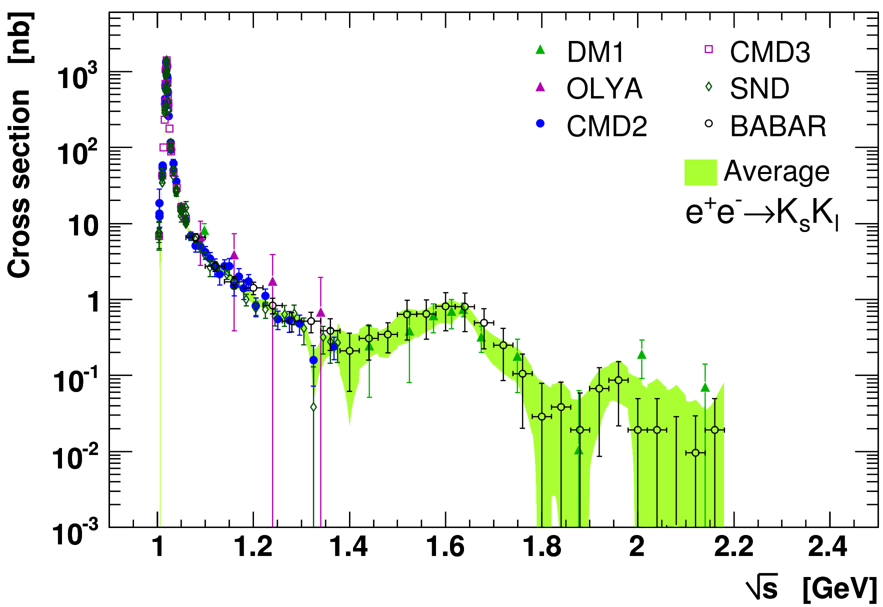


The $e^+e^- \to K_S K_L$, K^+K^- contributions

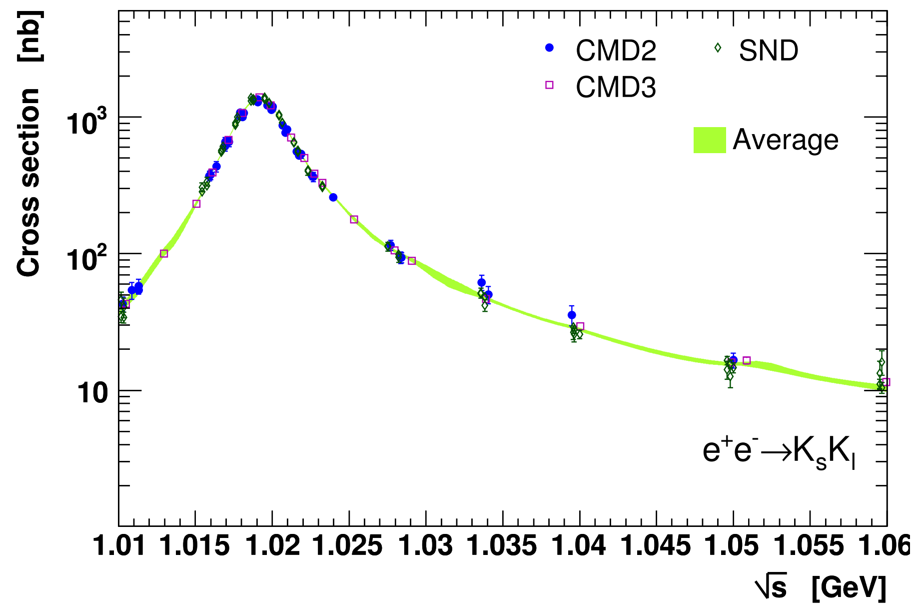
$e^+e^- \to K_S K_L$, K^+K^- contribute to 5.1% to $a_\mu^{\mathrm{Had,LO}}$ and 2.3% to its uncertainty-squared

Good consistency in $K_S K_L$ final state, newer data from CMD-3 and BABAR

BABAR reconstructed K_L directly via their nuclear interactions in the electromagnetic calorimeter



Compilation and combination: DHMZ, Davier 1612.02743 (2016)

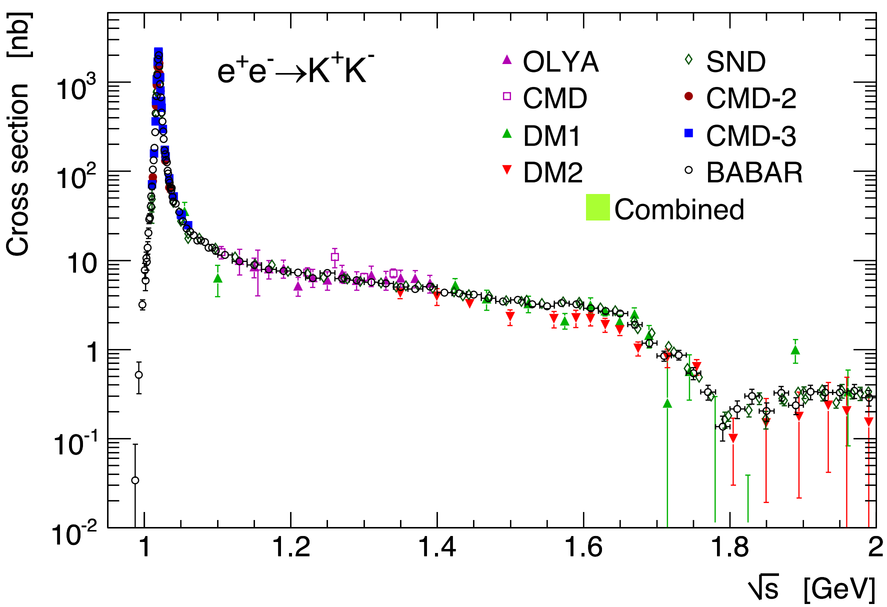


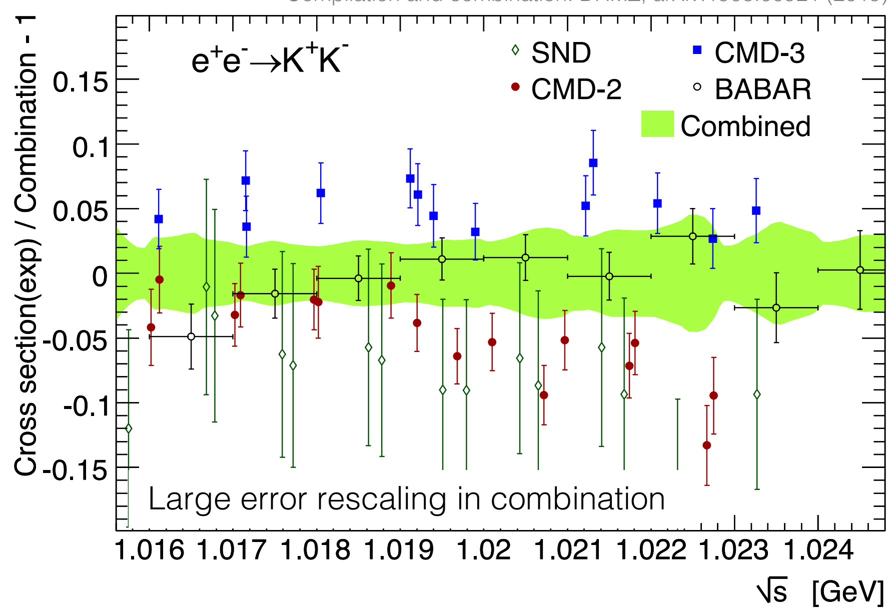
$e^+e^- \to K_S K_L$, K^+K^- contribute to 5.1% to $a_\mu^{\mathrm{Had,LO}}$ and 2.3% to its uncertainty-squared

Problems in K^+K^- channel, discrepancy between BABAR and SND (VEPP-2000) resolved with new SND dataset. Remaining discrepancy between BABAR vs CMD-2 vs CMD-3.

K^+K^- final state with low kaons at threshold hard to reconstruct for energy-scan experiments. Easier in BABAR due to ISR boost

Compilation and combination: DHMZ, arXiv:1908.00921 (2019)





BABAR (σ_{syst} = 0.7%) **higher** by 5.1% than CMD-2 (σ_{syst} = 2.2%), but 5.5% **lower** than CMD-3 (σ_{syst} = 2.2%)

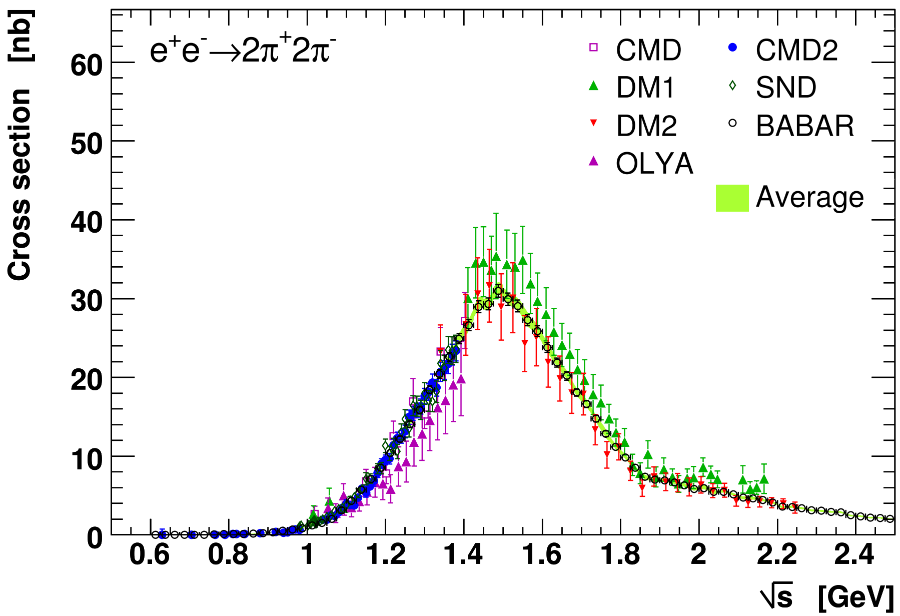
Overall difference of 11%, not covered by CMD-2/3 systematic uncertainties. Deterioration by factor of 2 of combined dataset due to local uncertainty rescaling

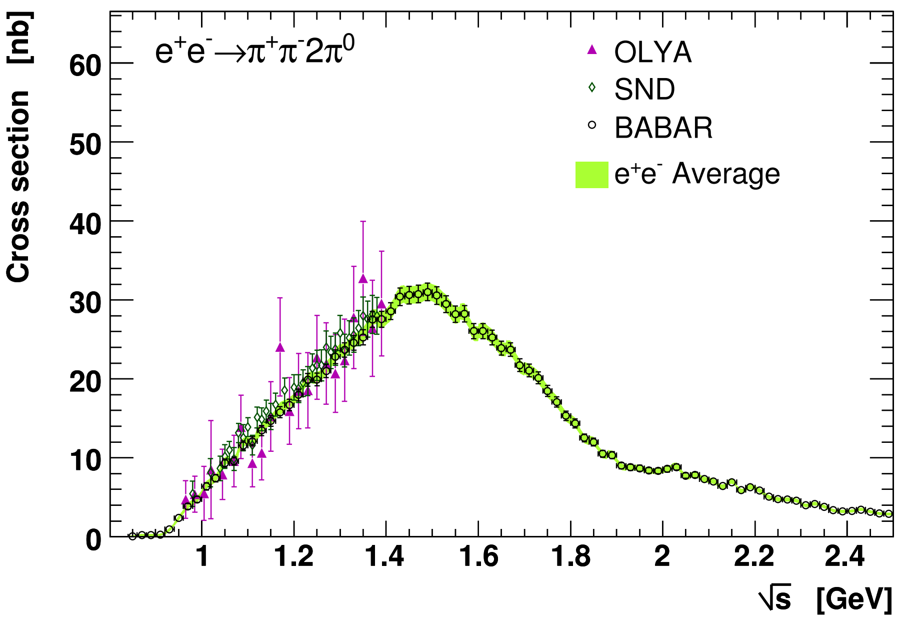
The $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ contributions

The four pion channels contribute with 4.5% to $a_\mu^{\mathrm{Had,LO}}$ and 3.7% to its uncertainty-squared

$\pi^+\pi^-\pi^+\pi^-$ channel pretty well known since long, but $\pi^+\pi^-\pi^0\pi^0$ challenging. Discrepancies in earlier data, but recent precise (~3.1% systematic) measurement from BABAR much improving

Compilation and combination: DHMZ, Davier 1612.02743 (2016)

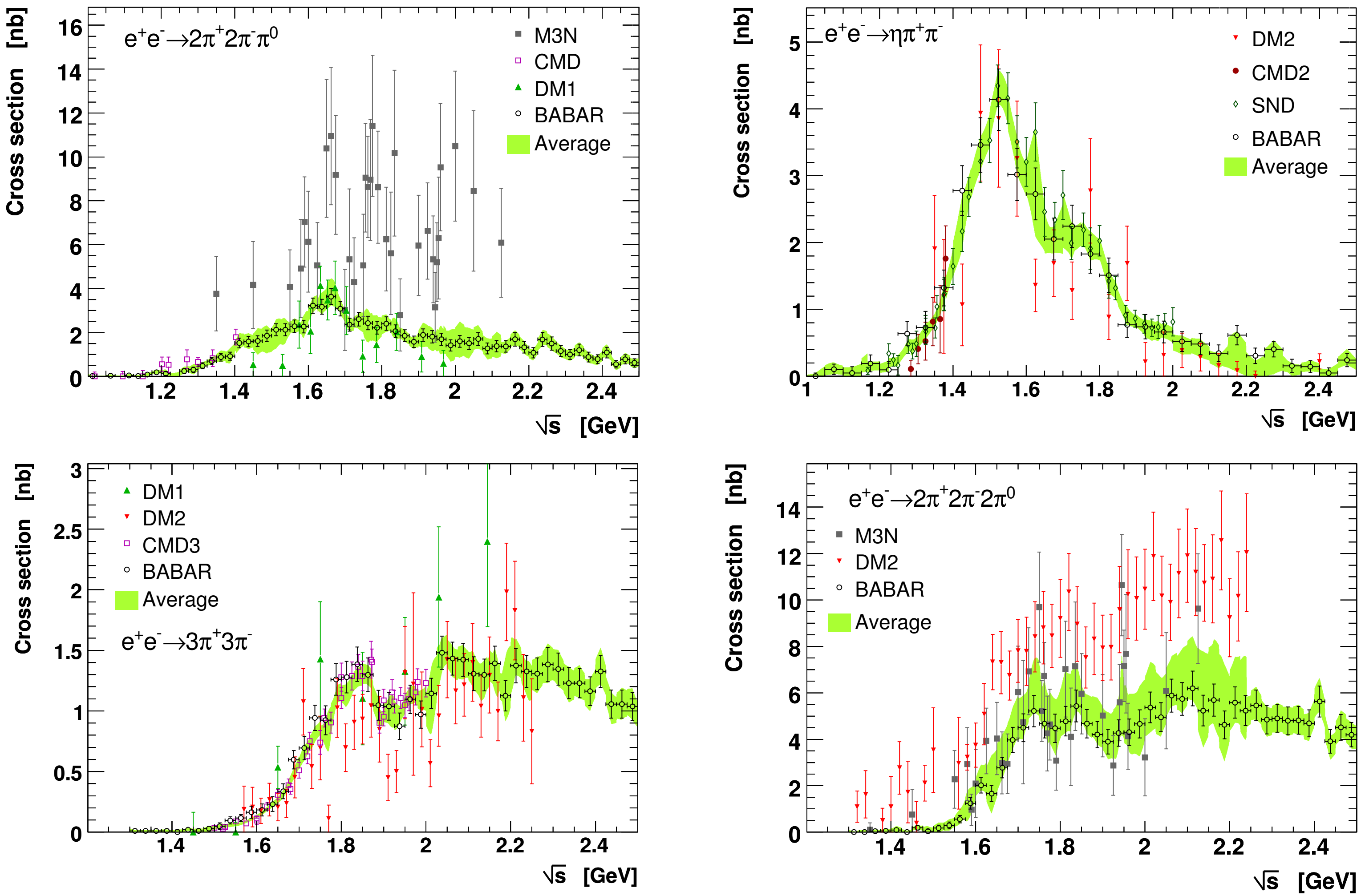




The $e^+e^- \to \geq 5\pi$ contributions

$\geq 5\pi$ channels (incl. $\eta\pi\pi$) contribute with 0.5% to $a_\mu^{\mathrm{Had,LO}}$ and 1.5% to its uncertainty-squared

Also here, large improvement from BABAR ISR data, problems in older datasets

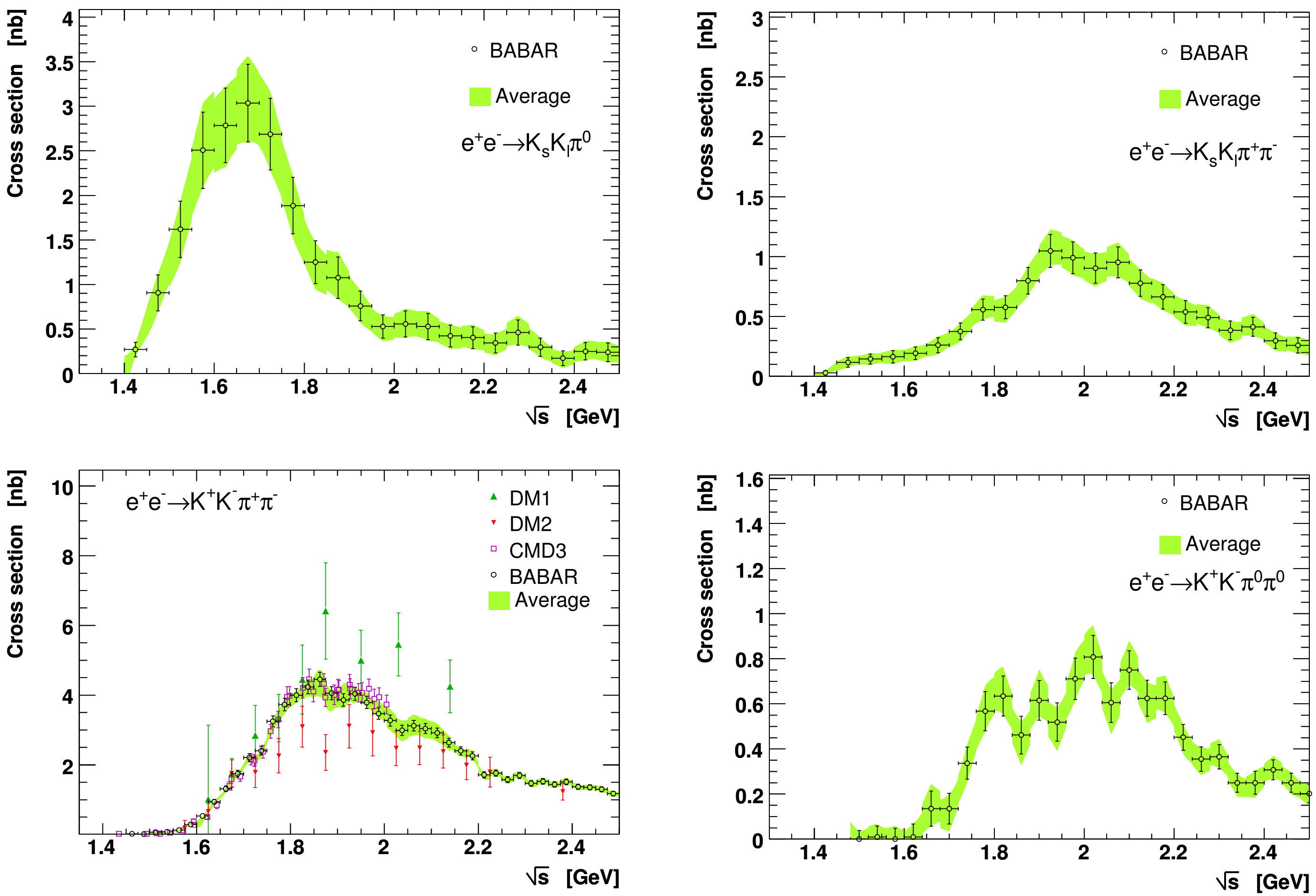


Compilation and combination: DHMZ, Davier 1612.02743 (2016)

The $e^+e^- \to KK\,\pi(\pi\pi)$ contributions (many charge combinations)

Past analyses suffered from missing final states that were estimated by symmetry arguments

Systematic measurement of exclusive processes by BABAR completes the $KK\pi$ and (almost) all $KK\pi\pi$ final states. Their sum contributes 0.5% to $a_\mu^{\mathrm{Had,LO}}$ and 0.2% to uncertainty-squared

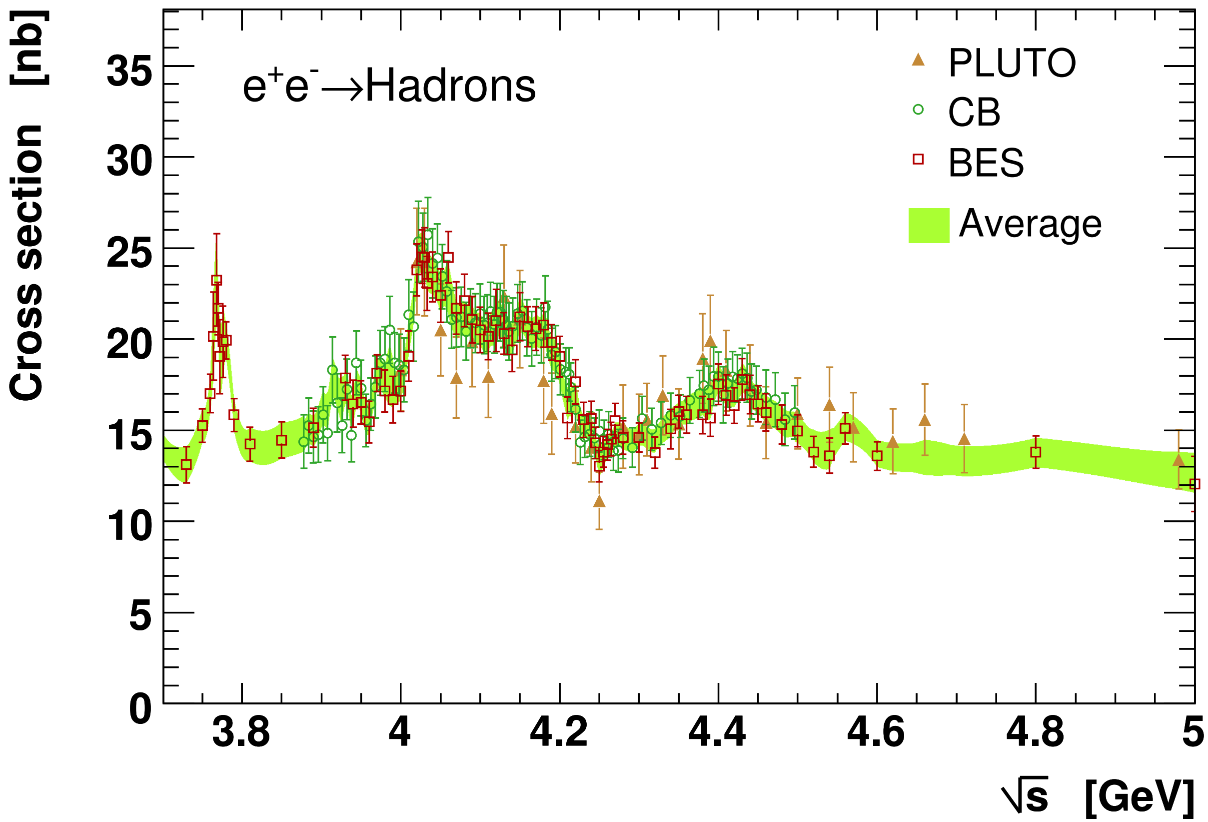


Selection of measurements by BABAR and others

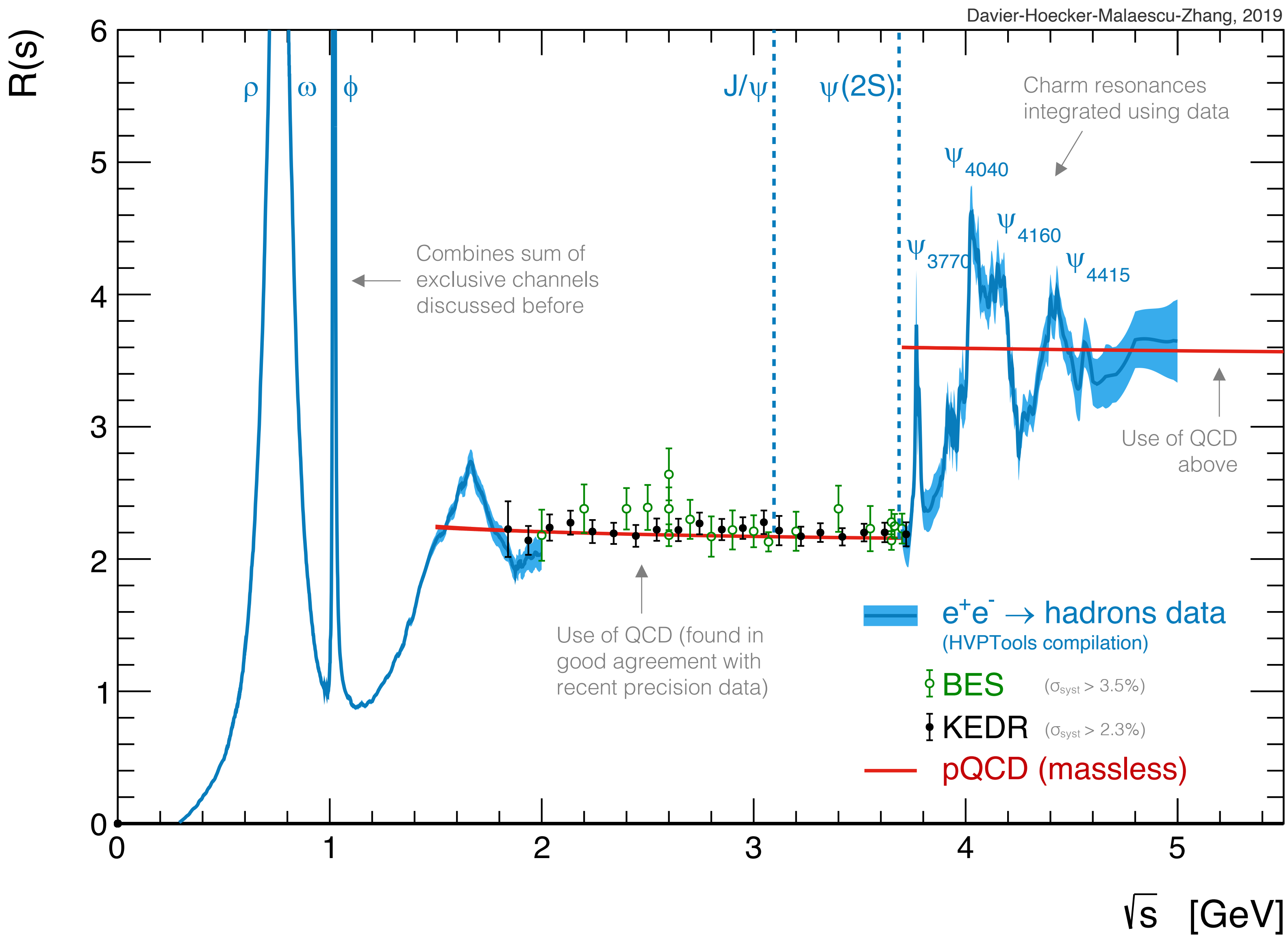
Charm resonance region (above $D\bar{D}$ threshold)

3.7–5.0 GeV region contributes with 1.1% to $a_\mu^{\mathrm{Had,LO}}$ and 0.8% to its uncertainty-squared

Good agreement between measurements. Precision dominated by BES (σ_{syst} ~ 3.5%)



Data, QCD and the big picture



Full compilation in numbers

DHMZ, arXiv:1908.00921 (2019)

Legend: First error statistical, second channel-specific systematic, third common systematic (correlated)
For R_{QCD}, uncertainties are due to: α_S, NNNLO truncation, resummation (FOPT vs. CIPT), quark masses

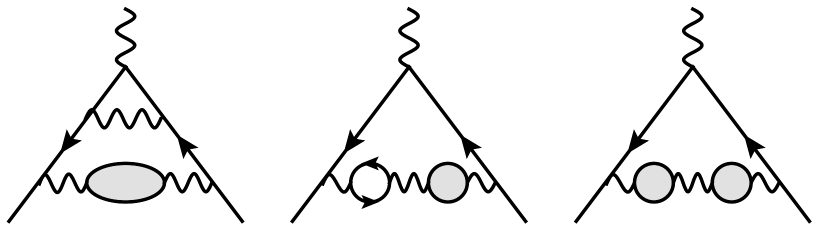
Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$

Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8 - 3.7\ \mathrm{GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\mathrm{dual}}$
R_{QCD} $[5.0 - 9.3\ \mathrm{GeV}]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3 - 12.0\ \mathrm{GeV}]_{udscb}$	1.21 ± 0.01
R_{QCD} $[12.0 - 40.0\ \mathrm{GeV}]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{t}$	0.00 ± 0.00

$$a_\mu^{\mathrm{Had,LO}} = 693.9 \pm 1.0_{\mathrm{stat}} \pm 3.8_{\mathrm{syst}} \pm 0.1_{\psi} \pm 0.7_{\mathrm{QCD}} = 693.9 \pm 4.0$$

NLO two-point correlation contributions to $a_\mu^{\mathrm{Had,NLO}}$ can be computed akin to the LO part via (a sum of) dispersion relations

$$a_\mu^{\mathrm{Had,NLO}(i)} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K^{(i)}(s)}{s} R(s)$$



Each diagram corresponds to specific kernel function $K^{(i)}$

$$\Longrightarrow a_\mu^{\mathrm{Had,NLO}} = (-9.87 \pm 0.09) \cdot 10^{-10}$$

Kurz et al, 1511.08222.

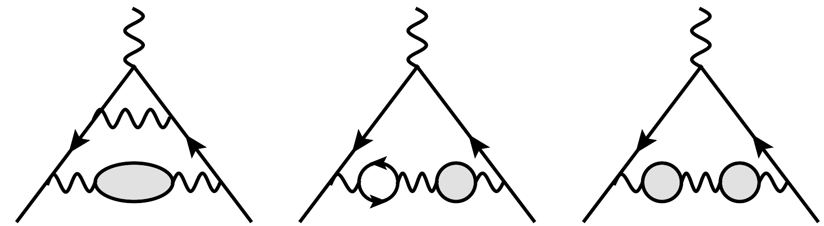
Kurz et al, 1511.08222.

NNLO two-point function corrections have also been computed: $a_\mu^{\mathrm{Had,NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10}$

Higher order hadronic terms

NLO two-point correlation contributions to $a_\mu^{\text{Had,NLO}}$ can be computed akin to the LO part via (a sum of) dispersion relations

$$a_\mu^{\text{Had,NLO}(i)} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K^{(i)}(s)}{s} R(s)$$



Each diagram corresponds to specific kernel function $K^{(i)}$

$$\Longrightarrow a_\mu^{\text{Had,NLO}} = (-9.87 \pm 0.09) \cdot 10^{-10}$$

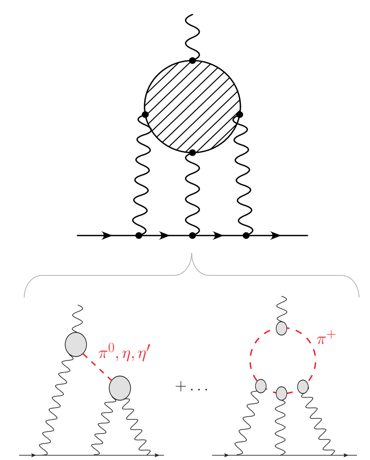
Kurz et al, 1511.08222.

Kurz et al, 1511.08222

NNLO two-point function corrections have also been computed: $a_\mu^{\text{Had,NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10}$

The four-point hadronic LBL scattering contribution, however, cannot be obtained this way and models are used instead

Calculation uses hadronic models with π^0, $\eta^{(')}$, … pole insertions and $\pi^{\pm}$ loops in the large-N_C limit (*Lattice QCD offers promising alternative*)



$$\Longrightarrow a_\mu^{\text{Had,LBL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$

Prades, de Rafael, Vainshtein, 0901.0306

Educated guess (other groups find smaller / larger uncertainty)

Muon $g-2$ summary

Summing all contributions [· 10^{-10}]:

$$a_\mu^{\mathrm{QED}} = 11\,658\,471.892 \pm 0.003$$

$$a_\mu^{\mathrm{EW}} = 15.36 \pm 0.10$$

$$a_\mu^{\mathrm{Had,LO}} = 693.9 \pm 4.0$$

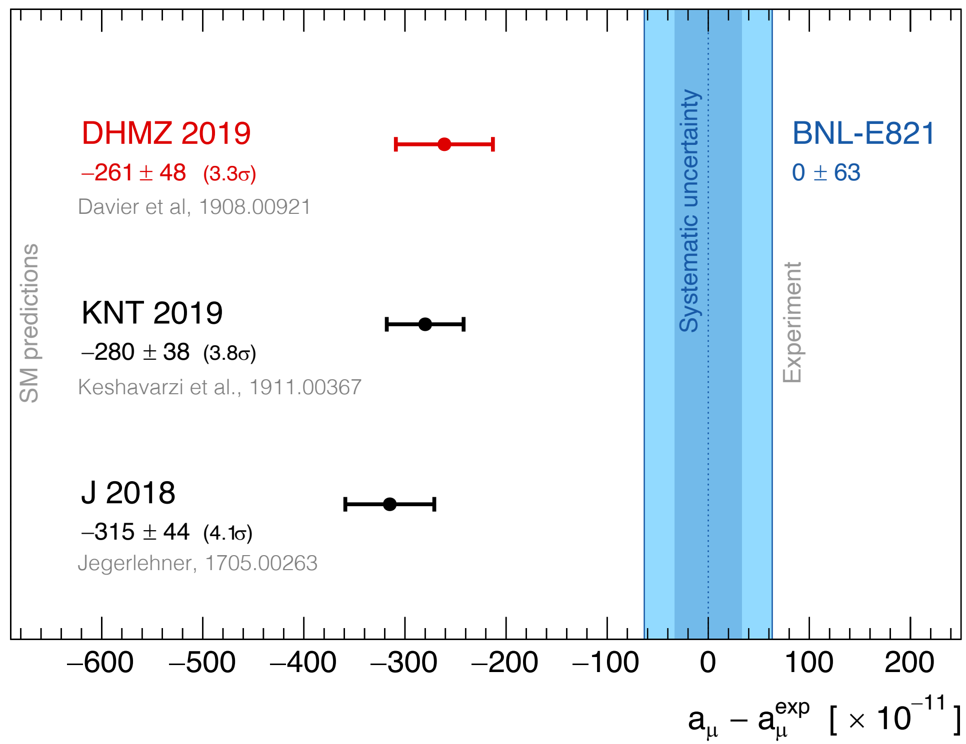
$$a_\mu^{\mathrm{Had,NLO}} = -9.87 \pm 0.09$$

$$a_\mu^{\mathrm{Had,NNLO}} = 1.24 \pm 0.01$$

$$a_\mu^{\mathrm{Had,LBL}} = 10.5 \pm 2.6$$

$$\boldsymbol{a_\mu^{\mathrm{SM}} = (11\,659\,183.0 \pm 4.8) \cdot 10^{-10}}$$

$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = 26.1 \pm 6.3_{\mathrm{exp}} \pm 4.8_{\mathrm{SM}} \;\; (\pm 7.9_{\mathrm{tot}})$$ → 3.3σ level



Compilation of contributions to $\alpha_{QED}(m_Z)$

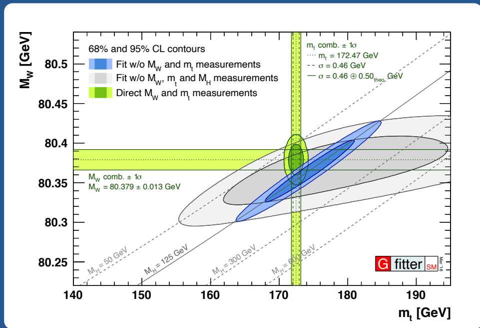
DHMZ, arXiv:1908.00921 (2019)

Legend: First error statistical, second channel-specific systematic, third common systematic (correlated)
For R_{QCD}, uncertainties are due to: α_S, NNNLO truncation, resummation (FOPT vs. CIPT), quark masses

Channel	$\Delta\alpha_{had}(m_Z^2)$ $[10^{-4}]$
$\pi^0\gamma$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$

Channel	$\Delta\alpha_{had}(m_Z^2)$ $[10^{-4}]$
K^+K^-	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	7.09 ± 0.08
$\psi(2S)$ (BW integral)	2.50 ± 0.04
R data $[3.7 - 5.0]$ GeV	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
R_{QCD} $[1.8 - 3.7$ GeV$]_{uds}$	$24.27 \pm 0.18 \pm 0.28_{dual}$
R_{QCD} $[5.0 - 9.3$ GeV$]_{udsc}$	34.89 ± 0.17
R_{QCD} $[9.3 - 12.0$ GeV$]_{udscb}$	15.56 ± 0.04
R_{QCD} $[12.0 - 40.0$ GeV$]_{udscb}$	77.94 ± 0.12
R_{QCD} $[> 40.0$ GeV$]_{udscb}$	42.70 ± 0.06
R_{QCD} $[> 40.0$ GeV$]_t$	-0.72 ± 0.01

$$\Delta\alpha_{had}(m_Z^2) = \left(275.43 \pm 0.15_{stat} \pm 0.76_{syst} \pm 0.09_{\psi} \pm 0.55_{QCD}\right) \cdot 10^{-4} = (275.4 \pm 1.0) \cdot 10^{-4}$$



$\alpha_{\mathrm{QED}}(m_Z)$

Summing all contributions

$$\Delta\alpha_{\mathrm{had}}(m_Z^2) = (275.4 \pm 1.0)\cdot 10^{-4}$$ DHMZ, arXiv:1908.00921 (2019)

$$\Delta\alpha_{\mathrm{lep}}(m_Z^2) = (314.979 \pm 0.002)\cdot 10^{-4}$$ 4-loop QED, Sturm, arXiv: 1305.0581 (2013)

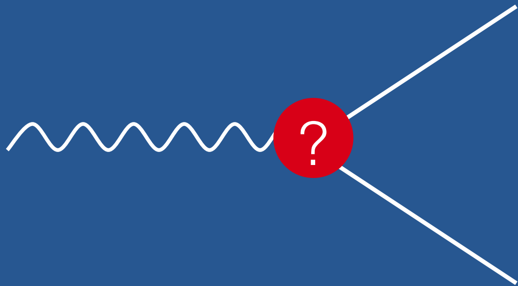
$$\boldsymbol{\alpha_{\mathrm{QED}}\left(m_Z^2\right) = \frac{\alpha_{\mathrm{QED}}(0)}{1-\Delta\alpha_{\mathrm{lep}}\left(m_Z^2\right)-\Delta\alpha_{had}\left(m_Z^2\right)} = 1/(128.946 \pm 0.013)}$$

The uncertainty on $\Delta\alpha_{\mathrm{had}}(m_Z^2)$ is (currently) subdominant in global electroweak fit, eg:

$$\begin{aligned} M_W &= 80.3535 \pm 0.0027_{m_t} \pm 0.0030_{\delta_{\mathrm{theo}} m_t} \pm 0.0026_{M_Z} \pm 0.0026_{\alpha_S} \\ &\qquad \boxed{\pm 0.0024_{\Delta\alpha_{\mathrm{had}}}} \pm 0.0001_{M_H} \pm 0.0040_{\delta_{\mathrm{theo}} M_W}\ \mathrm{GeV}\,, \\ &= 80.354 \pm 0.007_{\mathrm{tot}}\ \mathrm{GeV} \end{aligned}$$

(to be compared with exp. uncertainty of 13 MeV)

Gfitter Group, arXiv:1803.01853 (2018)



Conclusions

Long standing > 3 sigma discrepancy between data and SM on a_μ

Huge improvement in experimental data during the last ~20 years, but to match the precision of the new Fermilab muon g–2 experiment further progress is needed on the SM calculation of the hadronic contribution

For LO:

- BABAR-KLOE discrepancy in $\pi^+\pi^-$ channel unresolved, limiting improvement of LO evaluation: $\sigma(a_\mu^{\mathrm{Had,LO}})[\pi^+\pi^-] = 3.4 \cdot 10^{-10}$
  New $\pi^+\pi^-$ data from SND & CMD-3 expected (systematic error < 0.5% possible), also new BABAR analysis underway
- The K^+K^- data data discrepancy between CMD-2/3/BABAR needs to be understood: $\sigma(a_\mu^{\mathrm{Had,LO}})[K^+K^-] = 0.44 \cdot 10^{-10}$
- Also $\pi^+\pi^-\pi^0$ contribution needs more precision: $\sigma(a_\mu^{\mathrm{Had,LO}})[\pi^+\pi^-\pi^0] = 1.5 \cdot 10^{-10}$

Beyond LO, a robust estimate (and uncertainty) of the LBLS contribution is most crucial

Conclusions

Accurate, minute measurement seems to the non-scientific imagination, a less lofty and dignified work than looking for something new.

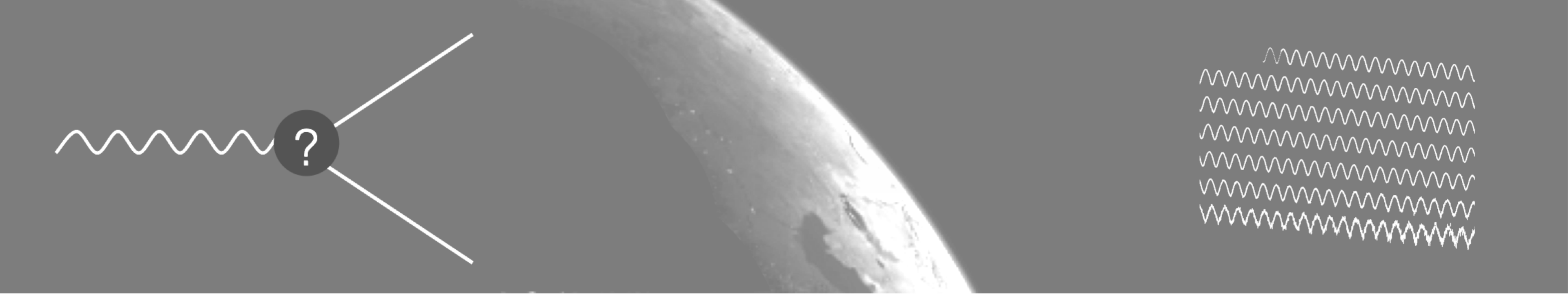
But [many of] the grandest discoveries of science have been but the rewards of accurate measurement and patient long-continued labour in the minute sifting of numerical results.

Said to originate from: William Thomson Kelvin

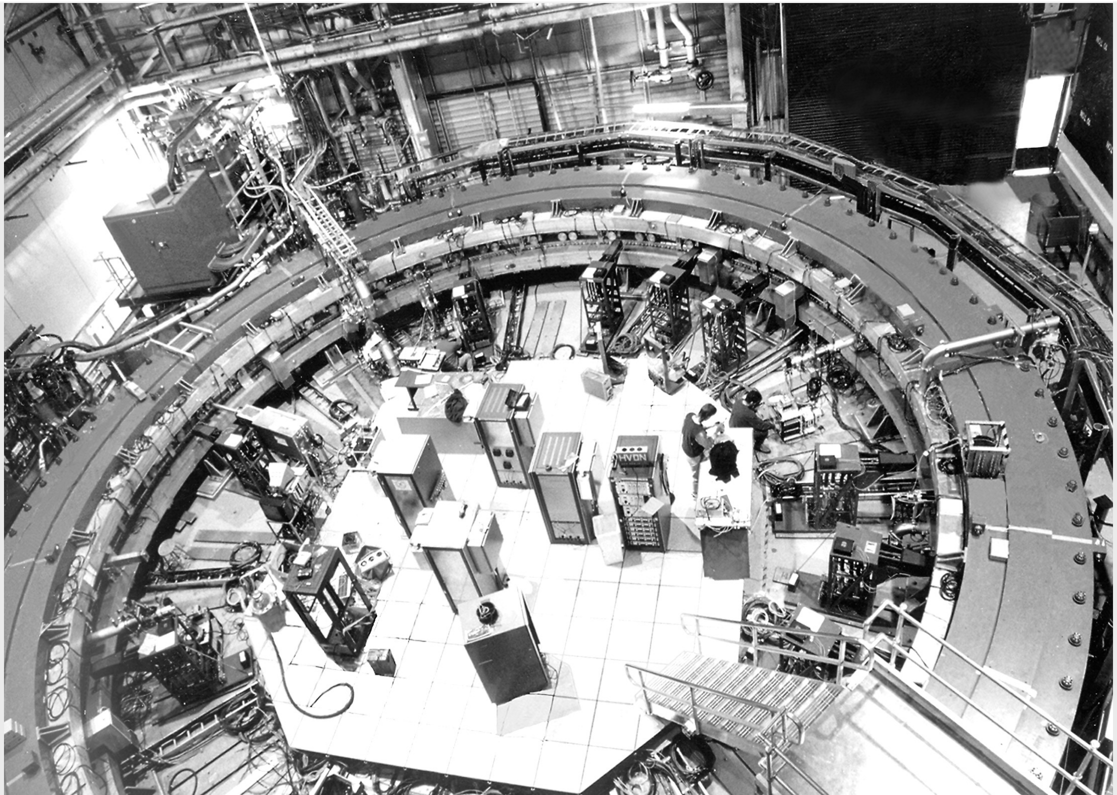
2 Aug 1871 in a speech to the British Association for the Advancement of Science

Lord Kelvin

Additional slides



Measuring the muon $g-2$

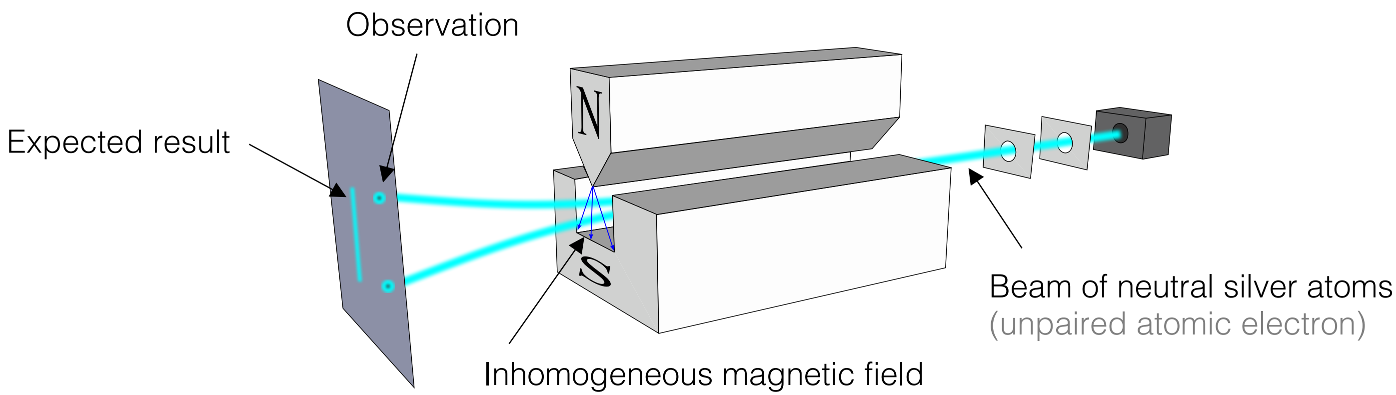


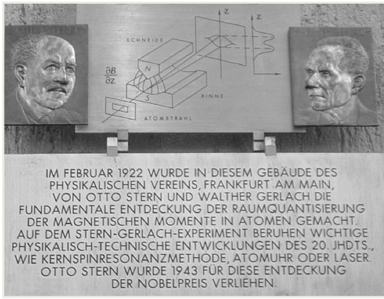
The BNL muon g–2 experiment (E821), 1997–2001

Magnetic moment

The magnetic dipole moment of a particle can be observed from its motion in a magnetic field

Intrinsic magnetic moment discovered in Stern-Gerlach experiment, 1922:

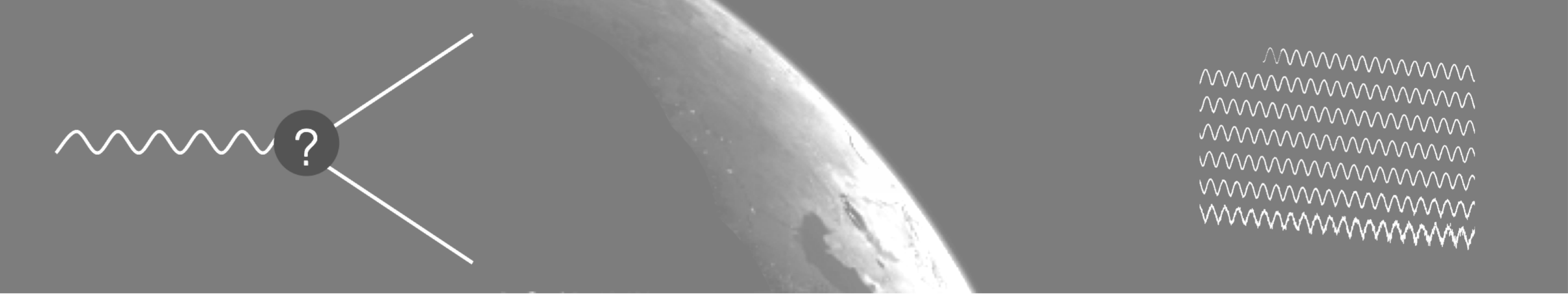




A commemorative plaque at the Frankfurt physics institute

→ atoms have intrinsic and quantised angular momentum

Uhlenbeck & Goudsmit postulated in 1925 that electrons have spin angular momentum with magnetic dipole moment: $e/2m_e$ (Bohr magneton)



Measuring the muon $g-2$

Analogous approach as for electron: search for discrepancy between the frequencies of cyclotron motion and spin precession

For polarised muons moving in a uniform B field (perp. to muon spin and orbit plane), and focused in an electric quadrupole field, the observed difference between spin precession and cyclotron frequency (= "anomalous frequency"), ignoring μEDM, is:

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

Motional magnetic field

With electrostatic focusing, no gradient B field focusing needed so that B can be made as uniform as possible !

$\vec{\omega}_a$ Independent of muon momentum

The E field dependence is eliminated at the "magic γ": $\gamma = 29.3 \rightarrow p_\mu = 3.09$ GeV

The experiment measures $(g_\mu - 2)/2$ directly

[J. Bailey et al., NP B150, 1 (1979)]

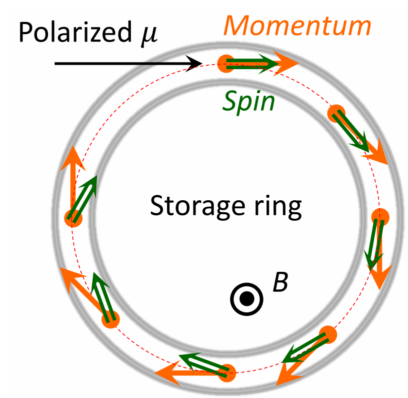
Exploit muon properties in experiment

1. Parity violation polarizes muons in pion decay

   Pions from proton-nucleon collision (AGS)

   spin orientation

   $$\bar{\nu}_\mu \longleftarrow \pi^- \longrightarrow \mu^-_{\text{polarised}}$$



2. Anomalous frequency proportional to a_μ

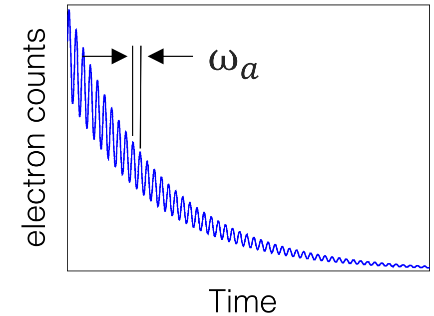
3. Magic γ :

$$\vec{\omega}_a = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right] \approx \frac{e}{m_\mu c} a_\mu \vec{B}$$

4. Again parity violation in muon decay

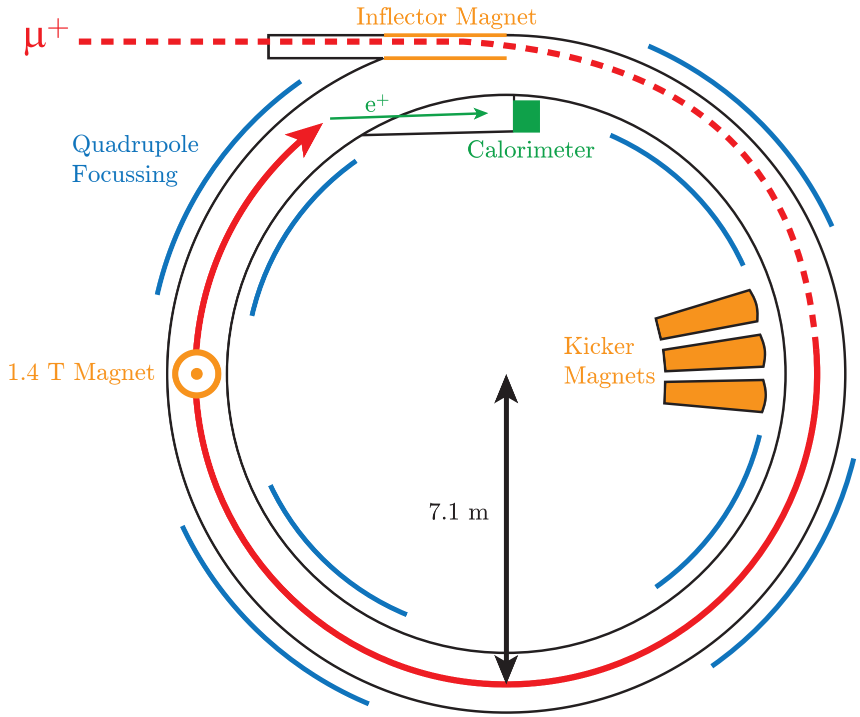
   $$\mu^-_{\text{polarised}} \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

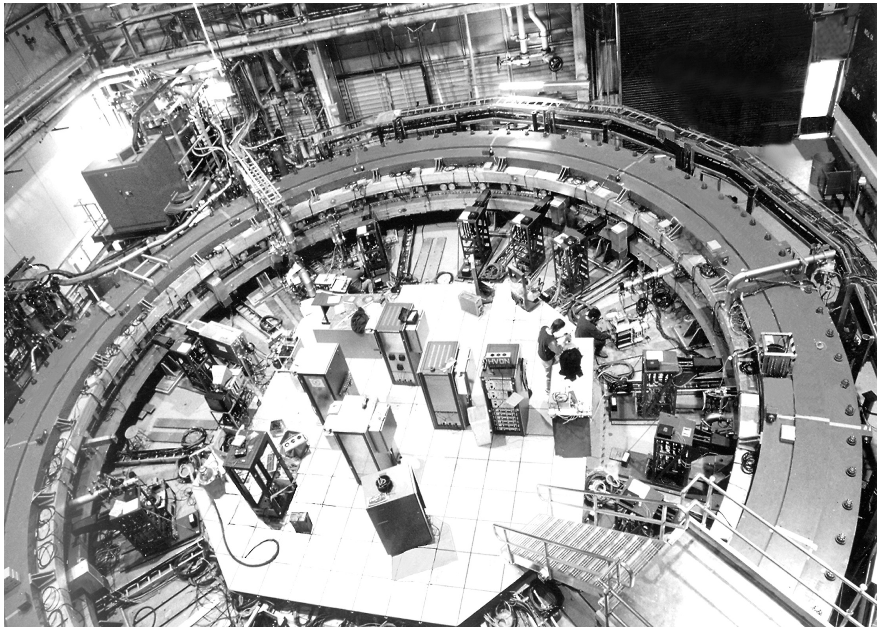
   fast electron emitted in direction opposite to muon spin



BNL E821: muon $g-2$ experiment

- A 24 GeV proton beam (AGS) incident on a target produces large number of pions that decay to muons
- The 3.1 GeV muon beam (relativistically enhanced lifetime of 64 μs) is injected into a 7.1 m radius ring with 1.4 T vertical magnetic field, which produces cyclotron motion matching the ring radius
- Electrostatic focusing of the beam is provided by a series of quadrupole lenses around the ring.

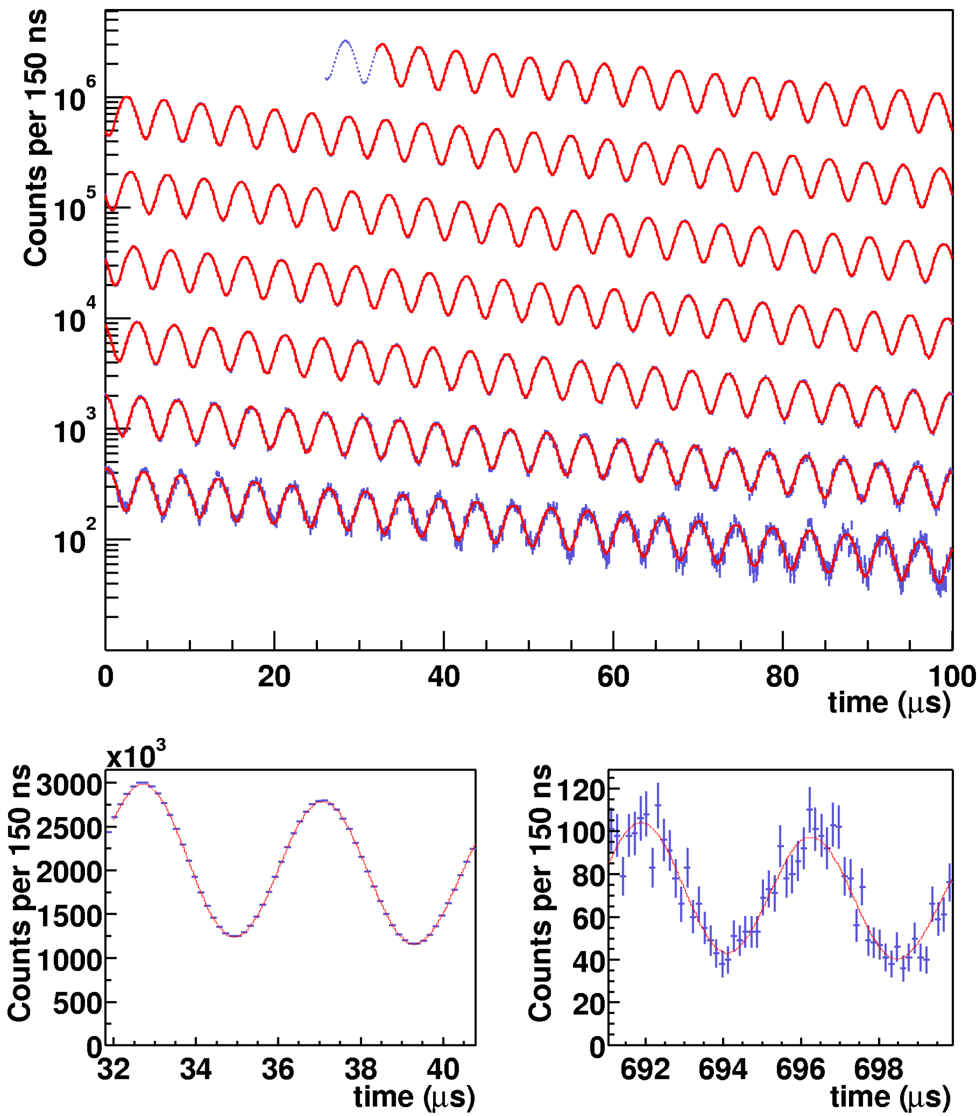




- Decay electrons (correlated with μ spin precession) counted vs. time in calorimeters inside ring ($\rightarrow \omega_a$)
- Precise measurement of ω_a and B allows to extract a_μ

BNL E821: muon $g-2$ experiment

E821 (g –2), hep-ex/0202024



Observed positron rate in successive 100 μs periods
~150 polarisation rotations during measurement period

Anomalous frequency:

$$\omega_a \approx \frac{e}{m_\mu c} a_\mu B$$

obtained from time-dependent fit to electron counts (for given energy E)

$$N(t) = N_0 e^{-t/\gamma\tau}[1 - A \cdot \sin(\omega_a t - \phi)]$$

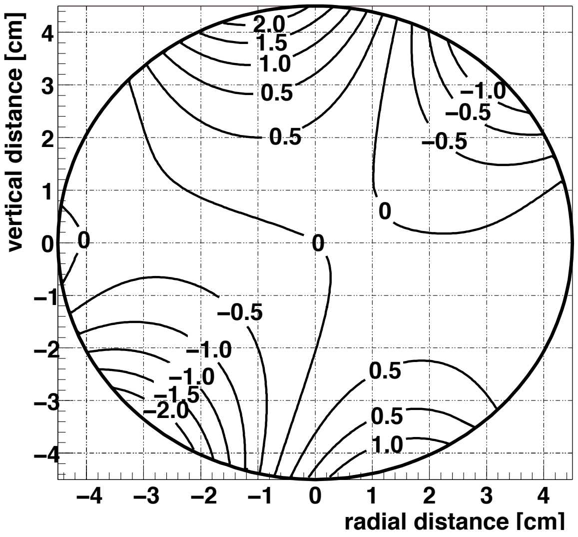
In blue: fit parameters

Total systematic uncertainty on ω_a: 0.2–0.3 ppm, with largest contributors:

- *pileup* (~in-time arrival of two low-E electrons)
- *muon losses*
- *coherent betatron* oscillation (muon loss and CBO amplitude [frequency: 0.48 MHz, compared to ω_a: 0.23 MHz] are part of fit)
- *calorimeter gain changes*

The B-field is mapped with 17 NMR probes mounted on a trolley pulled through the beampipe

hep-ex/0602035



Azimuthal average for one trolley run.
Contours are 0.5 ppm field differences.

B-field is proportional to free proton precession frequency ω_p ($B = \omega_p/\mu_p$) measured by NMR probes so one can write:

$$a_\mu = \frac{\frac{e}{m_\mu c} a_\mu B}{\frac{e}{m_\mu c}\frac{g}{2}B - \frac{e}{m_\mu c} a_\mu B} = \frac{\omega_a}{\omega_L - \omega_a}$$

$$= \frac{\omega_a/\omega_p}{\omega_L/\omega_p - \omega_a/\omega_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

where: ω_L is Larmor frequency of muon, $\mathcal{R}$ measured by E821, and the μ-to-p magnetic moment ratio is: λ = 3.183 345 107(84)

(λ is determined from muonium ($\mu^+ e^-$) hyperfine level structure measurements)

→ Systematic uncertainty on ω_p between 0.2 and 0.4 ppm

ω_a and ω_p measured independently in blind analyses → doubly blind experiment!

Digression: Running of $\alpha_{QED}(M_Z)$

Photon vacuum polarisation function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx}\langle 0|TJ^\mu_{em}(x)\left(J^\nu_{em}(0)\right)^\dagger|0\rangle = -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi_\gamma(q^2)$$

Only vacuum polarisation "screens" electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$$

with: $\Delta\alpha(s) = -4\pi\alpha\,\mathrm{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$

split into leptonic and hadronic contribution

Leptonic $\Delta\alpha_{lep}(s)$ calculable in QED (known to 3-loops). However, quark loops are modified by long-distance hadronic physics, cannot be calculated with perturbative QCD

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (unitarity)

$$12\pi\,\mathrm{Im}\,\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (analyticity)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\,\Pi_\gamma(s')}{s'(s'-s) - i\varepsilon} \quad \Rightarrow \quad \Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_0^\infty ds' \frac{R(s')}{s'(s'-s) - i\varepsilon}$$

Precise knowledge $\alpha(m_Z)$ important ingredient to global electroweak fit

$\Delta\alpha_{had}(s)$ uncertainty contributes 1.8 MeV to m_W SM prediction (total error of SM: 8 MeV), but dominant uncertainty to $\sin^2\theta_{eff}$ (SM)

Digression: Can it be real ?

The absolute size of the effect $\Delta a_\mu = 27.4 \pm 7.6$ is large compared to EW contribution of 15.4
(but some cancellation among bosons in latter contribution)

- **Generic decoupling new physics** predicts: $a_\mu^{\text{NP}} \sim \text{C} \cdot \left(\frac{m_\mu}{m_{\text{NP}}}\right)^2$ [Jegerlehner, Nyffeler, 0902.3360]

  Here: $m_{\text{NP}} \sim 2\ \text{TeV}$ for $C = 1$, $m_{\text{NP}} \sim 100\ \text{GeV}$ for $\text{C} = \frac{\alpha}{\pi}$ (natural strength), $m_{\text{NP}} \sim 5\ \text{GeV}$ for $C = \left(\frac{\alpha}{\pi}\right)^2$

- Generic **SUSY** predicts: $a_\mu^{\text{SUSY}} \sim \text{sign}(\mu) \cdot (13 \cdot 10^{-10}) \cdot \left(\frac{100\ \text{GeV}}{m_{\text{SUSY}}}\right)^2 \cdot \tan\beta$

  – In constrained SUSY models, Δa_μ cannot be reconciled with the non-observation of strongly produced sparticles at the LHC [de Vries et al, MasterCode, 1504.03260]

  – However, general models such as the pMSSM can still accommodate Δa_μ with light neutralinos, charginos and sleptons, not yet excluded by the LHC

- A "**dark photon**" (γ') coupling to SM via mixing with photon may give: $a_\mu^{\gamma'} \sim \frac{\alpha}{2\pi} \varepsilon F(m_{\gamma'})$

  – Δa_μ is accommodated for coupling strength $\varepsilon \sim 0.1\text{–}0.2\%$ and mass $m_{\gamma'} \sim 10 - 100\ \text{MeV}$

  – Searches for a dark photon have been performed (so far negative) or are planned at colliders (LHC, *B*-factories, KLOE, …) and fixed target experiments (Jefferson Lab, MAMI, …)