NLO Predictions for VBS at the (HL-/HE-)LHC

$pp \to VV + jj + X$ with leptonic decays for the LHC at $\sqrt{s} = 13$, 14, and 27 TeV

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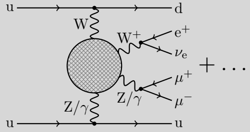
Vector-boson scattering (VBS): VBF's big(ger) brother

VBS process class: $\mathrm{pp} \to \mathrm{VV} + \mathrm{jj} + X$ with lept. dec.:

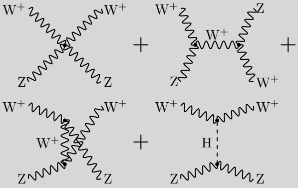
1. $\mathrm{W^+W^+}$: $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^- + \mathrm{jj} + X$
2. $\mathrm{W^+Z}$: $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\nu_\mu + \mathrm{jj} + X$
3. ZZ : $\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\,\mu^+\mu^- + \mathrm{jj} + X$
4. $\mathrm{W^+W^-}$: $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^-\overline{\nu}_\mu + \mathrm{jj} + X$

\+ processes related by cc. and different lep. comb.

VBS@LHC: $\mathrm{W^+Z}$ scattering



$\mathrm{W^+Z} \to \mathrm{W^+Z}$



What can we learn?

- nature of QGCs (with triple-boson prod.)
- form of Higgs-vector-vector couplings

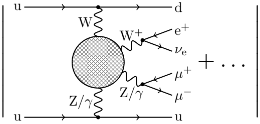
→ EWSB: interplay between QGC, TGC, and Higgs boson(s)

Coupling structure of VBS: LOs

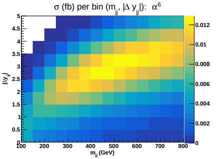
LO: $\mathcal{O}(\alpha_s^2\alpha^4)$ $\mathcal{O}(\alpha_s^1\alpha^5)$ $\mathcal{O}(\alpha_s^0\alpha^6)$

M_{jj}-$|\Delta y_{jj}|$ plots for W^+W^+ [Ballestrero, et al.]:

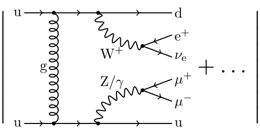
$|\mathcal{M}_{\mathrm{EW}}|^2 = \Big|$



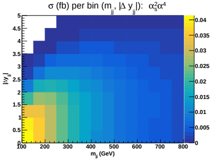
$+ \dots \Big|^2 \longrightarrow$



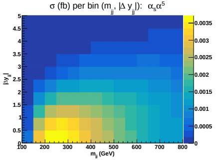
$\Big|\mathcal{M}_{\mathrm{QCD}}\Big|^2 = \Big|$



$+ \dots \Big|^2 \longrightarrow$



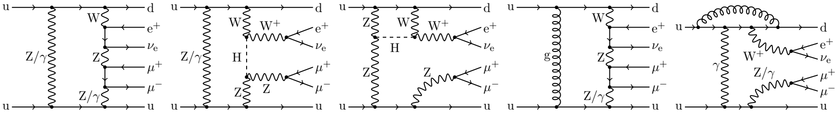
$2\,\Re\mathfrak{e}\left(\mathcal{M}_{\mathrm{EW}}\mathcal{M}^*_{\mathrm{QCD}}\right) \longrightarrow$



Coupling structure of VBS: NLOs

$$\begin{tikzcd}
\textbf{LO}: & & \mathcal{O}(\alpha_s^2\alpha^4) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^1\alpha^5) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^0\alpha^6) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & \\
\textbf{NLO}: & \mathcal{O}(\alpha_s^3\alpha^4) & & \mathcal{O}(\alpha_s^2\alpha^5) & & \mathcal{O}(\alpha_s^1\alpha^6) & & \mathcal{O}(\alpha_s^0\alpha^7)
\end{tikzcd}$$

A few diagrams ($\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\nu_\mu + \mathrm{jj} + X$):



NLO doable, but only most important are done by now:

- [B. Biedermann, A. Denner, M. Pellen]: complete NLO tower for $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\nu_\mu + \mathrm{jj} + \mathrm{X}$
- [A. Denner, S. Dittmaier, P. Maierhöfer, M. Pellen, C.S.]: next-to-leading $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ for WZ
- ZZ and opposite-sign WW will be ***very expensive*** computationally
- QCD corrections are available, sometimes approximated

Coupling structure of VBS: NNLOs

$$\begin{tikzcd}
\textbf{LO}: & & & \mathcal{O}(\alpha_s^2\alpha^4) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^1\alpha^5) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^0\alpha^6) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \\
\textbf{NLO}: & & \mathcal{O}(\alpha_s^3\alpha^4) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^2\alpha^5) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^1\alpha^6) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & & \mathcal{O}(\alpha_s^0\alpha^7) \arrow[dl, "\text{QCD}"'] \arrow[dr, "\text{EW}"] & \\
\textbf{NNLO}: & \mathcal{O}(\alpha_s^4\alpha^4) & & \mathcal{O}(\alpha_s^3\alpha^5) & & \mathcal{O}(\alpha_s^2\alpha^6) & & \mathcal{O}(\alpha_s^1\alpha^7) & & \mathcal{O}(\alpha_s^0\alpha^8)
\end{tikzcd}$$

→ NNLO will be hopeless for a long time (but probably not needed antime soon?)

WW like-sign scattering: A few references

- [B. Jäger, C. Oleari, D. Zeppenfeld], [A. Denner, L. Hošeková, S. Kallweit]: approx. $\mathcal{O}(\alpha_\mathrm{s}\alpha^6)$
- [T. Melia, K. Melnikov, R. Röntsch, G. Zanderighi]: $\mathcal{O}(\alpha_\mathrm{s}^3\alpha^4)$
- [M. Rauch]: VBF and VBS review
- [B. Biedermann, A. Denner, M. Pellen]: $\mathcal{O}(\alpha^7)$

→ [B. Biedermann, A. Denner, M. Pellen]: complete NLO tower

- [A. Ballestrero et al.]: Tool/approximation comparison for $\mathcal{O}(\alpha_\mathrm{s}\alpha^6)$ and PS, PDF uncertainties

→ [HL-LHC Collaboration and HE-LHC Working Group; A. Denner, M. Pellen]: 14 and 27 TeV results

WW like-sign: 13 TeV [B. Biedermann, A. Denner, M. Pellen]

Leading orders:

$\mathcal{O}(\alpha^6)$ [fb]	$\mathcal{O}(\alpha_s\alpha^5)$ [fb]	$\mathcal{O}(\alpha_s^2\alpha^4)$ [fb]	Sum [fb]
1.4178(2)	0.04815(2)	0.17229(5)	1.6383(2)

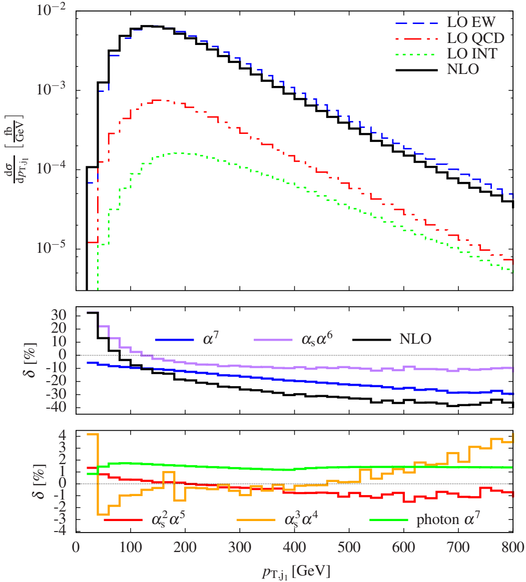
- $\mathcal{O}(\alpha_s^2\alpha^4)$ kinematically supressed
- $\mathcal{O}(\alpha_s\alpha^5)$ also colour supressed

Next-to-leading orders:

$\mathcal{O}(\alpha^7)$ [fb]	$\mathcal{O}(\alpha_s\alpha^6)$ [fb]	$\mathcal{O}(\alpha_s^2\alpha^5)$ [fb]	$\mathcal{O}(\alpha_s^3\alpha^4)$ [fb]	Sum [fb]
−0.2169(3)	−0.0568(5)	−0.000 32(13)	−0.0063(4)	−0.2804(7)
−13.2 %	−3.5 %	0.0 %	−0.4 %	−17.1 %

- large EW corrections, only weakly dep. on M_{jj} and Δy_{jj} cuts
- small QCD corrections
- both mixed correction tiny/negligable

WW like-sign: 13 TeV [B. Biedermann, A. Denner, M. Pellen]

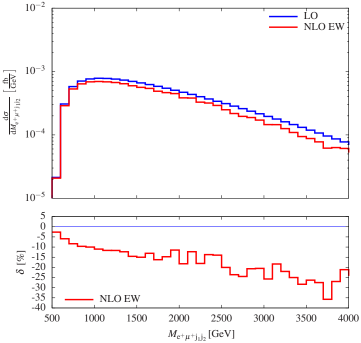


- large EW corrections
- even larger EW corrections towards higher transverse momenta (Sudakov logs)
- $\mathcal{O}(\alpha_s\alpha^5)$: large at small p_{j_1} (only soft jets); shape-changing

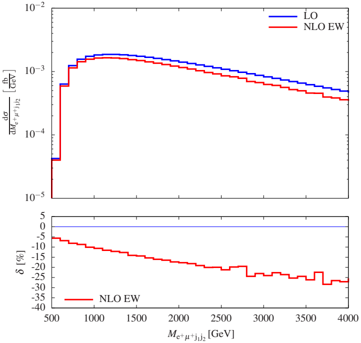
- $\mathcal{O}(\alpha_s^3\alpha^4)$ corrections small
- $\mathcal{O}(\alpha_s^2\alpha^5)$ corrections tiny

WW like-sign: 14 and 27 TeV [A. Denner, M. Pellen]

14 TeV:



27 TeV:



- slightly different setup w.r.t. 13 TeV
- Large EW corrections
- Slightly increasing NLO EW for 27 TeV
- 14 TeV to 27 TeV: Increase by a factor ∼3

	LO[1] [fb]	NLO EW [fb]
14 TeV	1.4282 100 %	1.213 −15.1 %
27 TeV	4.7848 100 %	3.881 −18.9 %

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[1] only $\mathcal{O}(\alpha^6)$

WZ scattering: references

- [G. Bozzi, B. Jäger, C. Oleari, D. Zeppenfeld]: Approx. $\mathcal{O}(\alpha_\mathrm{s}\alpha^6)$
- [F. Campanario, M. Kerner, L.D. Ninh, D. Zeppenfeld]: $\mathcal{O}(\alpha_\mathrm{s}^3\alpha^4)$
- [J. Bendavid et al.] (SM Les Houches 2017 report, Sec. V.3): LOs, Tool comparison
- [B. Jäger, A. Karlberg, J. Scheller]: Parton-shower effects to $\mathcal{O}(\alpha^6)$

→ [A. Denner, S. Dittmaier, P. Maierhöfer, M. Pellen, C.S.]: NLO $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_\mathrm{s}\alpha^6)$

- [A. Ballestrero, E. Maina, G. Pelliccioli]: Polarisation studies

LO integrated cross sections

Integrated xs for $\mathrm{pp} \to \mathrm{e}^{+}\nu_{\mathrm{e}}\,\mu^{+}\mu^{-}\mathrm{jj}$ @ $\sqrt{s} = 13\,\mathrm{TeV}$:

Sum [fb]	EW [fb]	QCD [fb]	Int. [fb]
1.55	$0.255^{+9.03\,\%}_{-7.75\,\%}$	$1.10^{+37.0\,\%}_{-24.9\,\%}$	$0.006\,82^{+18.4\,\%}_{-14.4\,\%}$
100 %	16.4 %	70.6 %	0.439 %

Photons [fb]	Bottom-quarks [fb]
$0.000\,988^{+11.5\,\%}_{-9.47\,\%}$	$0.195^{+3.59\,\%}_{-7.22\,\%}$
0.0636 %	12.5 %

- very large QCD contributions mainly due to gluon-PDF
- small interference (colour and kinematical suppression)
- smaller EW contribution compared to like-sign VBS (→ Z-boson)
- photon contributions completely irrelevant → leave out photon-initiated at NLO
- important: bottom-quark contributions

NLO integrated cross section

Integrated xs for $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\mathrm{jj} + X$:

$\sqrt{s}$	LO[2] [fb]	NLO EW [fb]	NLO QCD [fb]	NLO EW+QCD [fb]
13 TeV	$0.2551^{+9.0\,\%}_{-7.8\,\%}$ 100.0 %	0.2142 −16.0 %	$0.2506^{+1.0\,\%}_{-1.0\,\%}$ −1.8 %	$0.2097^{+1.3\,\%}_{-2.2\,\%}$ −17.8 %
14 TeV	$0.299^{+8.5\,\%}_{-7.4\,\%}$ 100.0 %	0.251 −16.1 %	$0.294^{+0.7\,\%}_{-1.0\,\%}$ −1.8 %	$0.245^{+0.8\,\%}_{-0.8\,\%}$ −17.9 %
27 TeV	$1.031^{+4.6\,\%}_{-4.3\,\%}$ 100.0 %	? ?	? ?	? ?

- Uncertainties are 7-point QCD-scale variations → NLO EW not accounted for!
- Large corrections on the integrated cross section, very similar to like-sign scattering
- QCD corrections small, large EW corrections
- 14 TeV to 27 TeV: Increase by a factor ∼3

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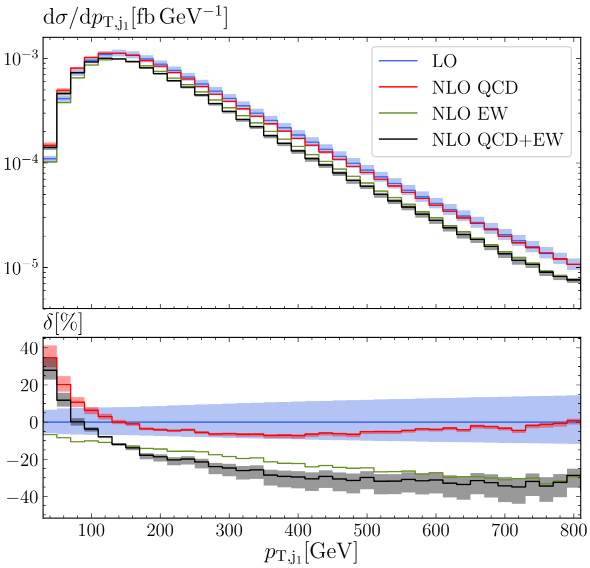
[2] only $\mathcal{O}(\alpha^6)$

Jet observables 13 TeV



- Leading jet p_T peaks around 140 GeV
- Note that

$$p_{T,j_1} > p_{T,j_2} > p_{T,j_3} > 30\text{ GeV}$$

→ Large positive QCD corrections for small p_{T,j_1} (*all* jets have small transv. momentum)

- EW corr. become increasingly negative; Sudakov logs
- QCD uncertainty band small for large p_{T,j_1} due to

$$\mu = \sqrt{p_{T,j_1} \cdot p_{T,j_2}}$$

Technical challenges of the calculation of pp $\rightarrow e^{+}\nu_{e}\mu^{+}\mu^{-}jj + X$

NLO calculation straightforward thanks to OPENLOOPS and RECOLA, but . . .

Virtuals:

- Up to 83 000 Feynman diagrams for each partonic channel
- ME evaluation expensive: evaluation of each partonic process takes about ten seconds → parallelisation with MPI [MPI Forum]

→ Computational costs: $\sim 8 \times 10^{5}$ CPU hours for each one of $\left\{\mathcal{O}(\alpha^{7}), \mathcal{O}(\alpha_{s}\alpha^{6})\right\}$; comparable to NNLO QCD calculations, see T. Gehrmann's talk last Thursday

Reals:

- Most complicated example: $\mathcal{O}(\alpha_{s}\alpha^{6})$
  - 40 qq partonic processes for the QCD reals, each has 12–14 dipoles
  - 14 qg partonic processes for the QCD reals, each has 10–11 dipoles
  - 16 qq partonic processes for the EW reals, each has 42 dipoles

→ around 1500 dipoles: automation is key → most complicated NLO computation to date

- phase space integration complicated: 5×10^{10} points before cuts (eff.: 14 %) → MPI

In general:

- Time consuming checks of two calculations against each other, but neccessary!

ZZ and WW opposite-sign scattering

- [B. Jäger, C. Oleari, D. Zeppenfeld]: WW approx. $\mathcal{O}(\alpha_s \alpha^6)$
- [B. Jäger, C. Oleari, D. Zeppenfeld]: ZZ approx. $\mathcal{O}(\alpha_s \alpha^6)$
- [T. Melia, K. Melnikov, R. Röntsch, G. Zanderighi]: WW $\mathcal{O}(\alpha_s^3 \alpha^4)$
- [B. Jäger, G. Zanderighi]: WW approx. $\mathcal{O}(\alpha_s \alpha^6)$ matched with PS
- ZZ $\mathcal{O}(\alpha_s^3 \alpha^4)$?

- Any EW corrections are missing

Conclusions

Like-sign WW:

- NLO tower available for like-sign WW scattering
- small QCD, large EW corrections: ∼16 % (13 and 14 TeV)
- for WW like-sign 27 TeV slightly larger: ∼19 %

WZ:

- for WZ full $\mathcal{O}(\alpha_s\alpha^6)$ and $\mathcal{O}(\alpha^7)$ now known, results similar to like-sign WW,
- $\mathcal{O}(\alpha_s^2\alpha^5)$ still is missing

ZZ/opposite WW:

- for ZZ and opposite-sign WW only QCD corrections are known
- full NLO will be expensive

→ QCD $\ll$ EW: FO uncertainty estimation?

Longitudinal VBS and Tree-Level Unitarity (I)

1. EW symmetry breaking makes $\mathrm{W}^{\pm}$, Z massive
2. Massive vector boson have one additional polarisation: longitudinal
3. In the high energy limit (Goldstone-Boson equivalence theorem), longitudinal polarisation $\varepsilon_{\mathrm{L}}(p)$ behave as $\varepsilon_{\mathrm{L}}(p) \to p$
4. Longitudinal VBS: Four longitudinal polarisations, does $\mathcal{M}_{\mathrm{LLLL}}$ blow up in the high energy limit (pert. unitarity violation)?

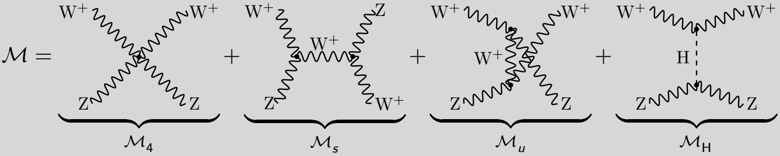
For $\mathrm{W}^{+}(p_1)\mathrm{Z}(p_2) \to \mathrm{W}^{+}(p_3)\mathrm{Z}(p_4)$ and $t = (p_1 - p_3)^2$ choose

$$\varepsilon_{\mathrm{L}}^{\mu}(p_i) = \frac{1}{N}\left(\frac{p_i^{\mu}}{M_i} - \frac{2M_i}{t - 2M_i^2}p_j^{\mu}\right), \quad j = (i+2) \bmod 4$$

such that $p \cdot \varepsilon_{\mathrm{L}}(p) = 0$, with normalisation N so that $\varepsilon_{\mathrm{L}}^{*} \cdot \varepsilon_{\mathrm{L}} = -1$.

Longitudinal VBS and Tree-Level Unitarity (II)

$W^+Z \to W^+Z$ @ $M_H = 125$ GeV



$$\mathcal{M}_4 \propto -s^2 - u^2 - 4su + 2(M_W^2 + M_Z^2)\frac{s^2 + 6su + u^2}{s+u} + \dots$$

$$\mathcal{M}_s \propto s^2 + 2su - 2M_W^2\frac{3su + u^2}{s+u} - 2M_Z^2\frac{2u^2 + 3su - s^2}{s+u} - \frac{M_Z^4}{M_W^2}s + \dots$$

$$\mathcal{M}_u \propto u^2 + 2su - 2M_W^2\frac{3su + s^2}{s+u} - 2M_Z^2\frac{2s^2 + 3su - u^2}{s+u} - \frac{M_Z^4}{M_W^2}u + \dots$$

$$\mathcal{M}_H \propto -\frac{M_Z^4}{M_W^2}\frac{t^2(t - 4M_W^2)(t - 4M_Z^2)}{(t - M_H)(t - 2M_W^2)(t - 2M_Z^2)} = \frac{M_Z^4}{M_W^2}s + u + \dots$$

$$\mathcal{M} = \mathcal{M}_4 + \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_H \propto 0 + \dots$$

- SM VVH vertex crucial to the cancellation
- different Higgs sector: enhancements in intermediate regions possible

Calculation from [Schwartz]

Why are the EW corrections so large?

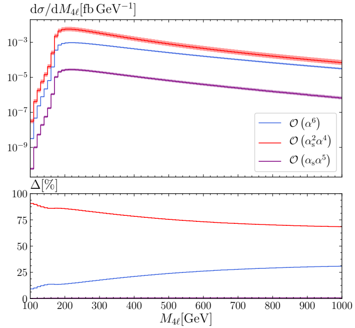
Approximate EW correction using most important EW logs [Denner, Pozzorini] [Denner, Pozzorini] to the VBS subprocess:

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}}(1 + \delta_{\mathrm{EW,LL}})$$

with

$$\delta_{\mathrm{EW,LL}} = \frac{\alpha}{4\pi}\left(-\frac{8}{\mathrm{s}_{\mathrm{w}}^2}\log^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{19}{3\,\mathrm{s}_{\mathrm{w}}^2}\log\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right)$$

with a characteristic scale Q chosen as $M_{4\ell} := M_{\bar{\mathrm{e}}\nu_{\mathrm{e}}\bar{\mu}\mu}$ (s-invariant of the VBS):



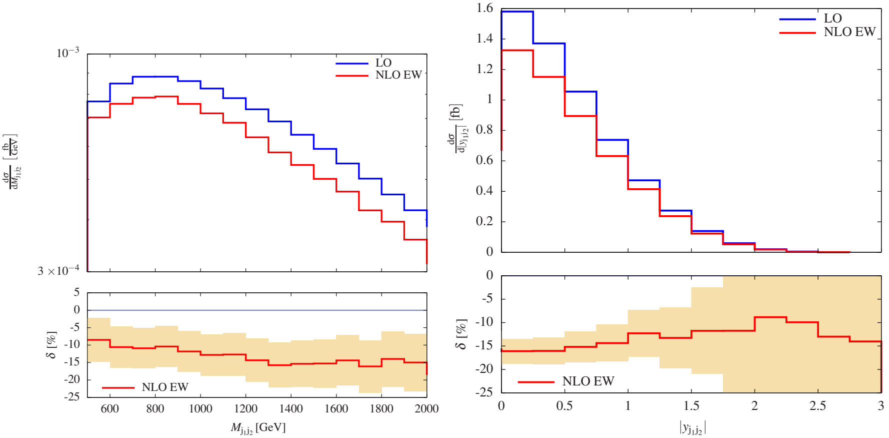
For WZ VBS:

- $Q = \langle M_{4\ell}\rangle \approx 413\,\mathrm{GeV} \Rightarrow \delta_{\mathrm{EW,LL}} = -17.5\,\%$
- $Q = M_{4\ell}$, binwise with distribution on the left $\Rightarrow \delta_{\mathrm{EW,LL}} = -16.4\,\%$

For comparison [Biedermann, Denner, Pellen]:

- like-sign-W VBS: $Q \approx 390\,\mathrm{GeV}$
- diboson production: $Q \approx 250\,\mathrm{GeV}$

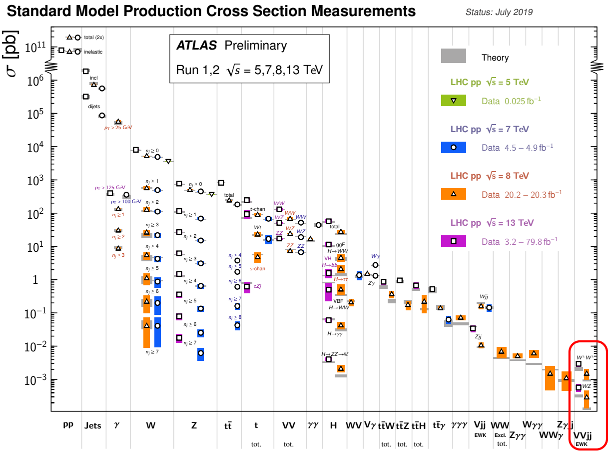
14 TeV with HL-LHC statistical uncertainties [A. Denner, M. Pellen]



Yellow band estimates the statistical uncertainties for 3000 fb^{-1} using $\pm 1/\sqrt{N_{\text{obs}}}$

VBS at the LHC, $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^+\mu^-\mathrm{jj} + \mathrm{X}$

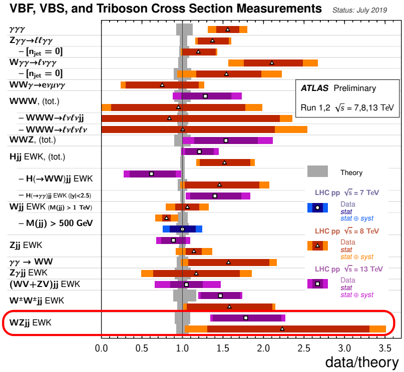
VBS: $\sigma \approx \mathcal{O}(1\,\mathrm{fb}) \to$ need large $\sqrt{s}$ and $\mathcal{L}$: new class of rare processes accessible in run II

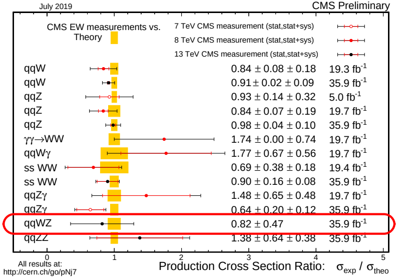


(staircase plot from the [ATLAS Collaboration])

- Largest VBS channel is $\mathrm{W}^+\mathrm{W}^+ \to \mathrm{W}^+\mathrm{W}^+$, full NLO corrections available [Biedermann, Denner, Pellen]
- Second largest channel: $\mathrm{W}^+\mathrm{Z} \to \mathrm{W}^+\mathrm{Z}$

Experiment: pp $\rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$





- ATLAS 8 TeV: [CERN-EP-2016-017]
- ATLAS 13 TeV: Obsers. with 5.6 σ sig. ($\mathcal{L} = 36.1\,\text{fb}^{-1}$) [ATLAS-CONF-2018-033]
- ATLAS 13 TeV: Obsers. with 5.3 σ sig. ($\mathcal{L} = 36.1\,\text{fb}^{-1}$) [CERN-EP-2018-286]

- CMS 13 TeV: Meas. with 1.9 σ sig. ($\mathcal{L} = 35.9\,\text{fb}^{-1}$) [CMS-PAS-SMP-18-001]
- CMS 13 TeV: Meas. with 2.2 σ sig. ($\mathcal{L} = 35.9\,\text{fb}^{-1}$) [CMS-SMP-18-001]

Fiducial phase space volume and parameters

Cuts are exactly the "loose fiducial" cuts defined by CMS [CMS-SMP-18-001]:

- At least two $R = 0.4$ anti-k_{t} jets with $p_{\mathrm{T}} > 30\,\mathrm{GeV}$, $|\eta| < 4.7$, and $\Delta R_{\mathrm{j}\ell} > 0.4$
- $M_{\mathrm{j_1 j_2}} > 500\,\mathrm{GeV}$, $\Delta\eta_{\mathrm{j_1 j_2}} > 2.5$
- $p_{\mathrm{T},\ell} > 20\,\mathrm{GeV}$ and $|y_\ell| < 2.5$
- $|M_{\mu\bar{\mu}} - M_Z| < 15\,\mathrm{GeV}$
- $M_{\ell\ell} > 4.0\,\mathrm{GeV}$ and $M_{3\ell} > 100.0\,\mathrm{GeV}$

Other:

- Photons recombined with charged particles using anti-k_{t} algorithm with $R = 0.4$
- PDFs: `NNPDF31_nlo_as_0118_luxqed`

Complex mass scheme [A. Denner, S. Dittmaier, M. Roth, D. Wackeroth][A. Denner, S. Dittmaier, M. Roth, L.H. Wieders], input parameters:

- $G_\mu = 1.6638 \times 10^{-5}\,\mathrm{GeV}^{-2}$
- $M_{\mathrm{W}} = 80.3530\,\mathrm{GeV}$, $\Gamma_{\mathrm{W}} = 2.0843\,\mathrm{GeV}$
- $M_{\mathrm{Z}} = 91.1535\,\mathrm{GeV}$, $\Gamma_{\mathrm{Z}} = 2.4943\,\mathrm{GeV}$
- $M_{\mathrm{H}} = 125.0\,\mathrm{GeV}$, $\Gamma_{\mathrm{H}} = 4.07 \times 10^{-3}\,\mathrm{GeV}$

with EW coupling calculated as:

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_{\mathrm{W}}^2 \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right)$$

Scale choice:

- $\mu = \sqrt{p_{\mathrm{T,j_1}} \cdot p_{\mathrm{T,j_2}}}$ [A. Denner, L. Hošeková, S. Kallweit]
- 7-point scale variation to estimate pert. uncertainty

Overview: Leading orders

$\mathrm{pp} \to \mathrm{e}^+ \nu_\mathrm{e}\, \mu^+ \mu^- \mathrm{jj} + X$ has three LOs:

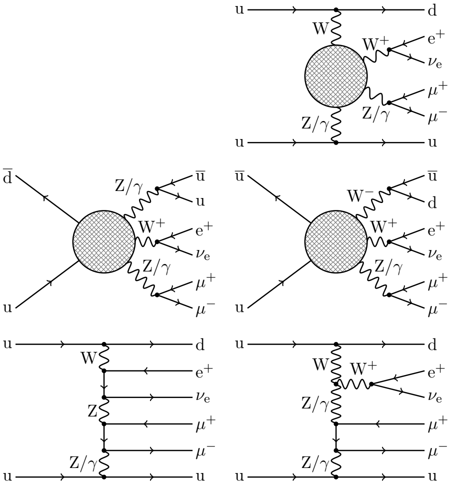
$$\begin{tikzcd}
& \boxed{\mathcal{O}(\alpha^6)} \arrow[dl, "\text{EW}"'] \arrow[dr, "\text{QCD}"] & & \boxed{\mathcal{O}(\alpha_\mathrm{s}\alpha^5)} \arrow[dl, "\text{EW}"'] \arrow[dr, "\text{QCD}"] & & \boxed{\mathcal{O}(\alpha_\mathrm{s}^2\alpha^4)} \arrow[dl, "\text{EW}"'] \arrow[dr, "\text{QCD}"] & \\
\mathcal{O}(\alpha^7) & & \mathcal{O}(\alpha_\mathrm{s}\alpha^6) & & \mathcal{O}(\alpha_\mathrm{s}^2\alpha^5) & & \mathcal{O}(\alpha_\mathrm{s}^3\alpha^4)
\end{tikzcd}$$

We divided them into five (mutually exclusive) classes:

1. $\mathcal{O}(\alpha^6)$ electroweak production with quark-quark initial state (but without bottom-quarks)
2. $\mathcal{O}(\alpha_\mathrm{s}^2\alpha^4)$ strong production (without bottom-quarks)
3. $\mathcal{O}(\alpha_\mathrm{s}\alpha^5)$ quark-quark interference

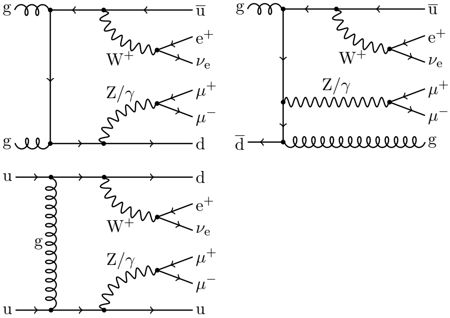
4. $\mathcal{O}(\alpha^6)$ double-photon initiated and $\mathcal{O}(\alpha_\mathrm{s}\alpha^6)$ single-photon initiated
5. $\mathcal{O}(\alpha^6)$ and $\mathcal{O}(\alpha_\mathrm{s}^2\alpha^4)$ with bottom-quarks

Electroweak production: $\mathcal{O}(\alpha^6)$



- 40 different partonic channels at $\mathcal{O}(\alpha^6)$
- contain the vector-boson scattering subdiagrams,
- and “semi-leptonic triple-gauge-boson production” processes ($W^{\pm}ZZ$, $W^{+}W^{-}Z$), suppressed at LO by $M_{jj} > 500\,\mathrm{GeV}$
- and other double-, single, non-resonant diagrams

Strong production: $\mathcal{O}(\alpha_s^2\alpha^4)$

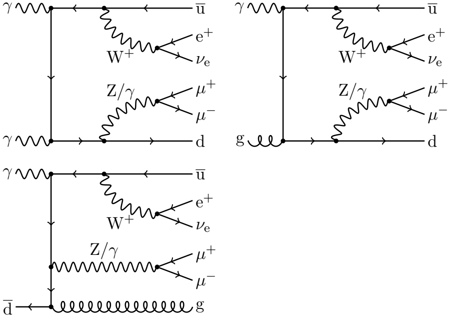


→ In comparison to like-sign W-scattering gluons are possible at LO (charge)

- 8 additional MEs with two gluons, making up 66 % of the cross section

→ in total the $\mathcal{O}(\alpha_s^2\alpha^4)$ is 4.3 times larger than the electroweak LOs

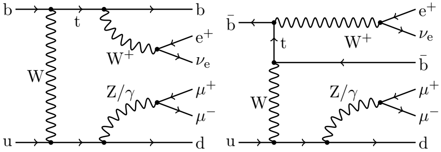
Photon-initiated: $O(\alpha^6)$, $\mathcal{O}(\alpha_s\alpha^5)$



→ Every channel with an initial-state photon

- 2 double photon MEs at $\mathcal{O}(\alpha^6)$ (tiny contribution)
- 12 photon–gluon MEs at $\mathcal{O}(\alpha_s\alpha^5)$ (very small contribution)
- remember: no final state photons at LO because of $n_j \geq 2$

Bottom-quark LOs: pp → tZj



12 MEs with bottom-quarks:

1. $\mathrm{bu} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\mathrm{bd}$
2. $\bar{\mathrm{b}}\mathrm{u} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\bar{\mathrm{b}}\mathrm{d}$
3. $\mathrm{b}\bar{\mathrm{d}} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\mathrm{b}\bar{\mathrm{u}}$
4. $\bar{\mathrm{b}}\bar{\mathrm{d}} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\bar{\mathrm{b}}\bar{\mathrm{u}}$
5. $\mathrm{u}\bar{\mathrm{d}} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\bar{\mathrm{b}}\mathrm{b}$
6. $\mathrm{b}\bar{\mathrm{b}} \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\bar{\mathrm{u}}\mathrm{d}$

\+ MEs with 2nd gen. quark line

- also contain VBS, but dominant contributions are . . .

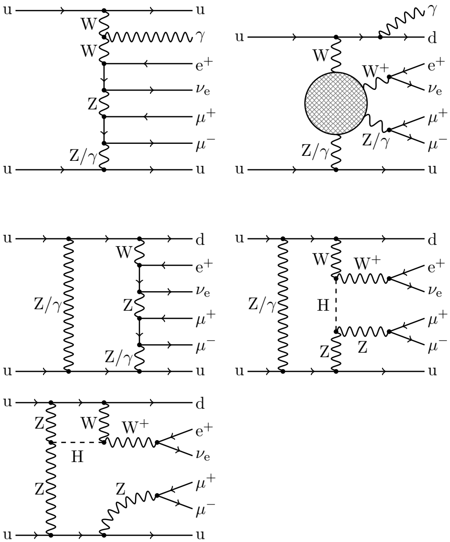
→ "top-Z-jet production" for $\mathrm{b}(\mathrm{u}/\mathrm{c}) \to \mathrm{e}^+\nu_\mathrm{e}\,\mu^+\mu^-\mathrm{b}(\mathrm{d}/\mathrm{s})$

- no resonant *anti*-tops because of W^+ → up-bottom contribution dominates over all others (90 %)
- separable with b-tagging in principle, except for n. 6 (very small)
- contribution comparable in size with the EW LOs

top-Z-jet analyses:

- ATLAS: [arXiv:1710.03659]
- CMS: [arXiv:1712.02825]

$\mathcal{O}(\alpha^7)$ real and virtual correction diagrams

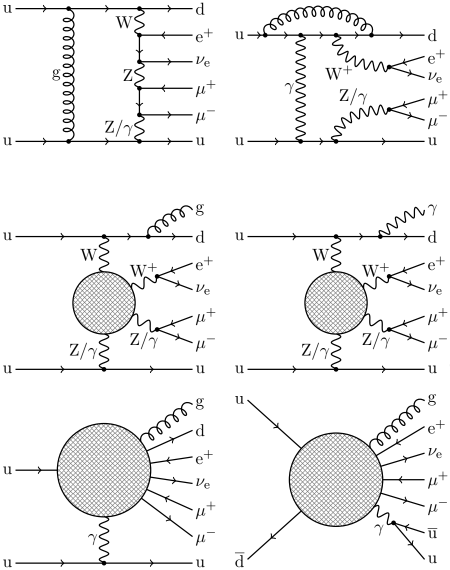


$$\begin{tikzcd}
 & \mathcal{O}(\alpha^6) \arrow[dl, "\text{EW}"'] \arrow[dr, "\text{“QCD”}"] & & \mathcal{O}(\alpha_s\alpha^5) \arrow[dl, "\text{EW}"'] \\
\boxed{\mathcal{O}(\alpha^7)} & & \mathcal{O}(\alpha_s\alpha^6) &
\end{tikzcd}$$

- No b-quark contributions (b-tagging)
- No $\gamma q/\gamma\gamma$ contributions (expectation 1–2 %) at NLO

- Loops with 8-point functions, different complex masses
- More diagrams with Higgs bosons!
- Up to 83,000 diagrams per partonic channel

$\mathcal{O}(\alpha_s\alpha^6)$ mixed corrections: “QCD”



$$\begin{tikzcd}
 & \mathcal{O}(\alpha^6) \arrow[dl, "\text{EW}"'] \arrow[dr, "\text{“QCD”}"] & & \mathcal{O}(\alpha_s\alpha^5) \arrow[dl, "\text{EW}"'] \\
\mathcal{O}(\alpha^7) & & \boxed{\mathcal{O}(\alpha_s\alpha^6)} &
\end{tikzcd}$$

QED singularities in “QCD”:

- initial state: cancelled with collinear counterterm (PDFs)
- final state: photon-to-jet conversion function [Denner, Dittmaier, Pellen, C.S.]

→ Correction is neither purely QCD/EW, it is mixed

Checks and validation

We performed two independent calculations for all leading- and next-to-leading orders:

BONSAY+OpenLoops

- General purpose Monte Carlo [CS]
- MEs from OpenLoops 1 [Cascioli, Maierhöfer, Pozzorini]
- Loops evaluated with DD (COLI fallback) from COLLIER
- Dipole subtraction [Catani, Seymour] to regularize IR singularities
- PDFs from LHAPDF 6 [Buckley, et al.]

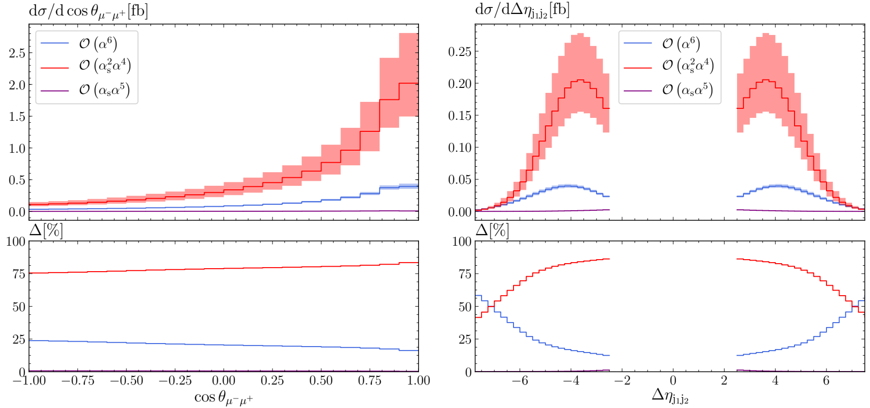
MoCaNLO+RECOLA

- MoCaNLO [Feger] used by M. Pellen
- MEs from RECOLA [Actis, Denner, Hofer, Scharf, Uccirati]
- Loops evaluated with COLI (and DD) from COLLIER [Denner, Dittmaier, Hofer]
- CS dipole subtraction with α-dependent dipoles [Nagy]
- PDFs from LHAPDF 6

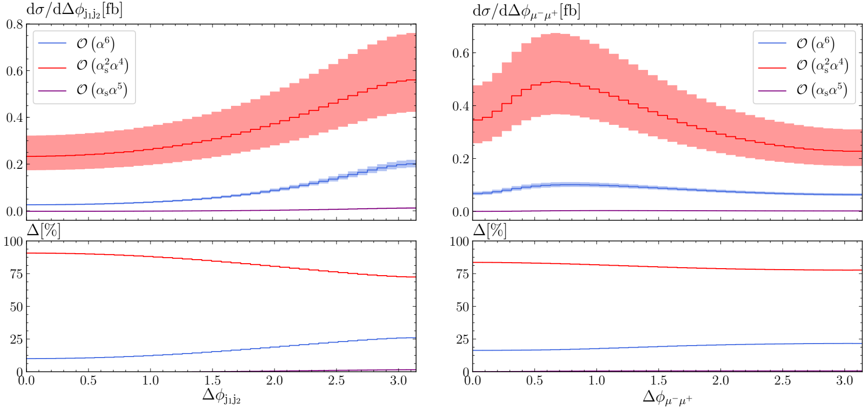
Checks

- NLO virtuals checked against each other for 1000 PS points passing the cuts
- Integrated cross sections
- Each bin of 26 differential distributions within stat. unc., ca. 8000 bins for each order

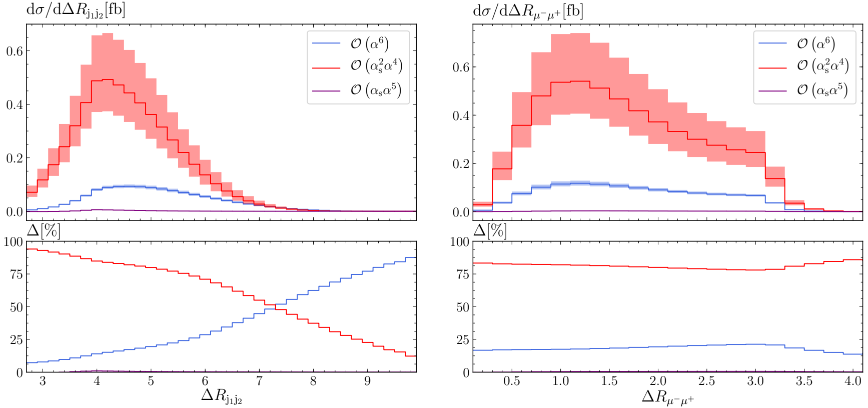
LO distributions (I)



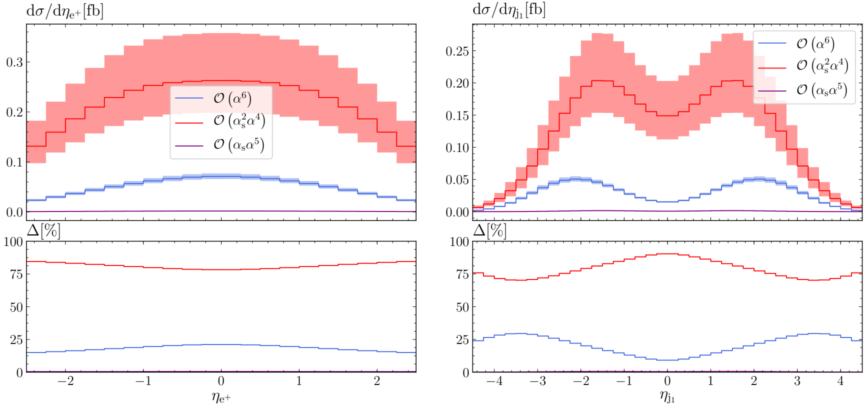
LO distributions (II)



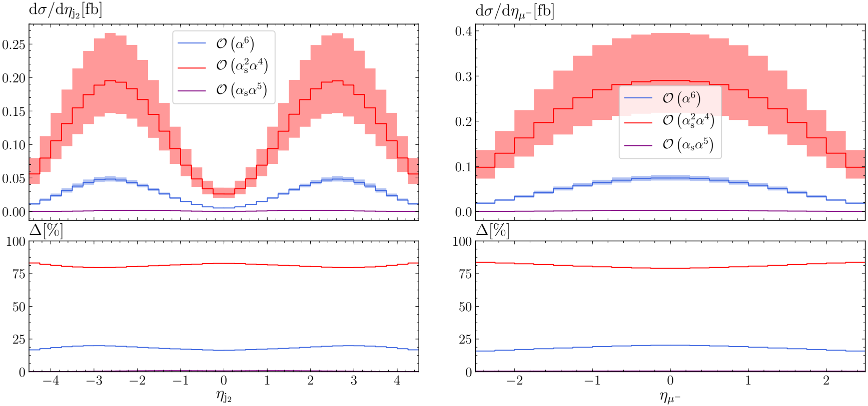
LO distributions (III)



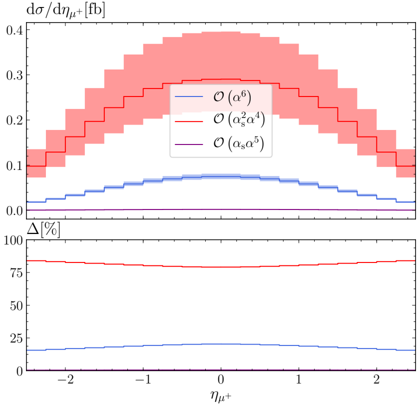
LO distributions (IV)

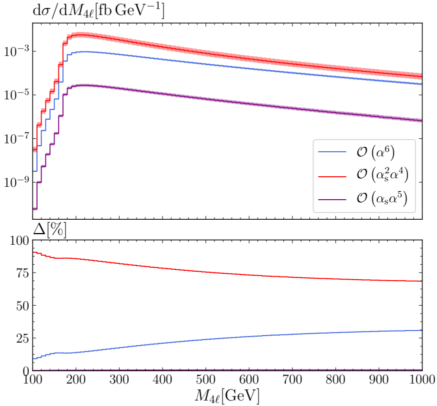


LO distributions (V)

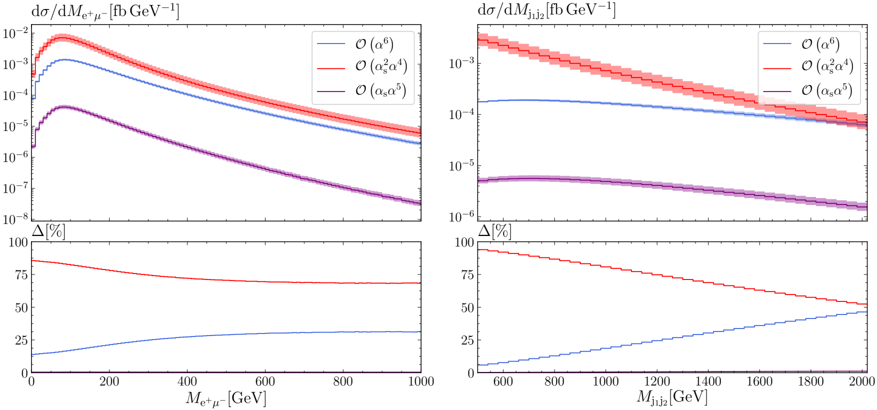


LO distributions (VI)

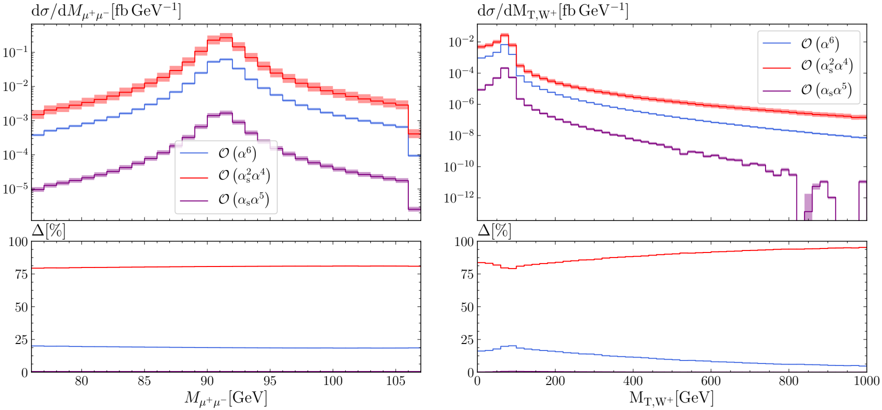




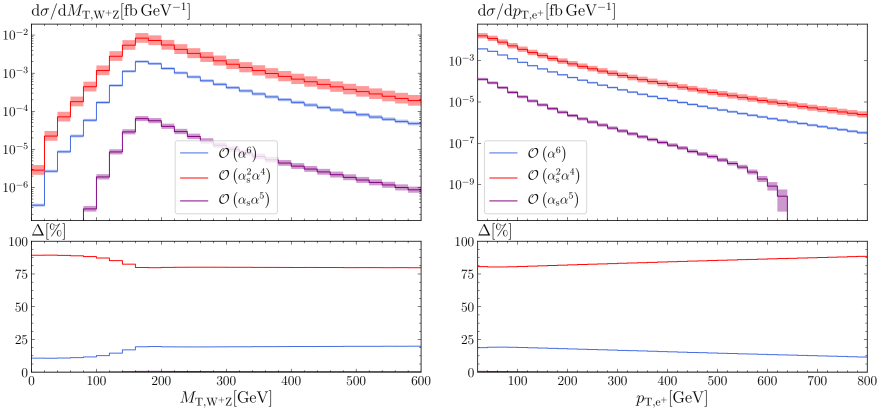
LO distributions (VII)



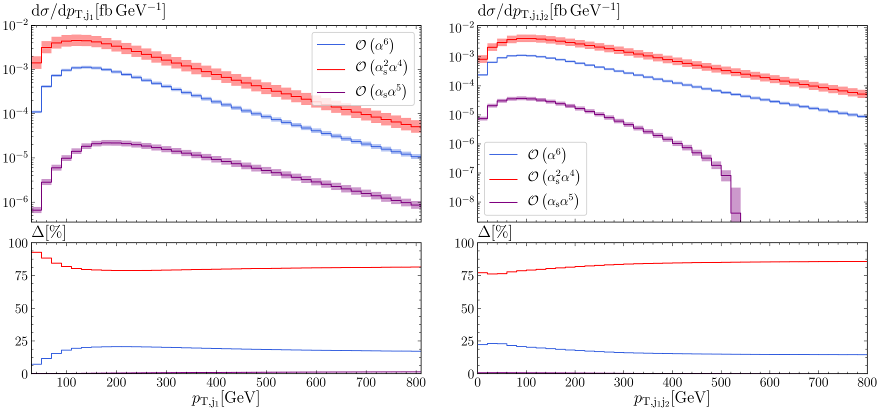
LO distributions (VIII)



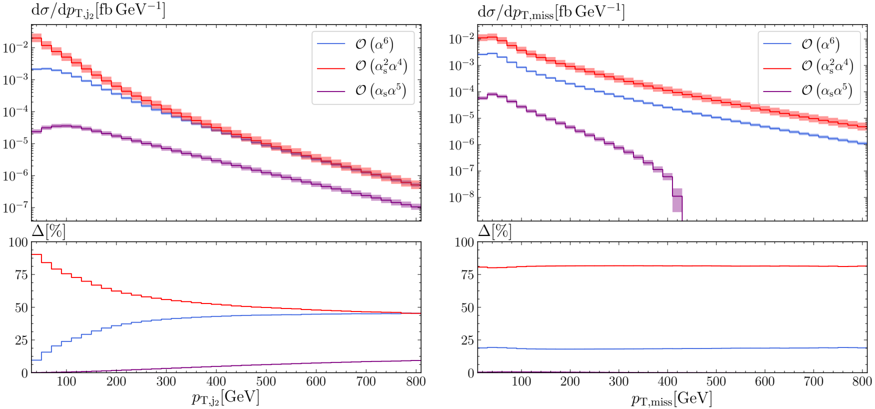
LO distributions (IX)



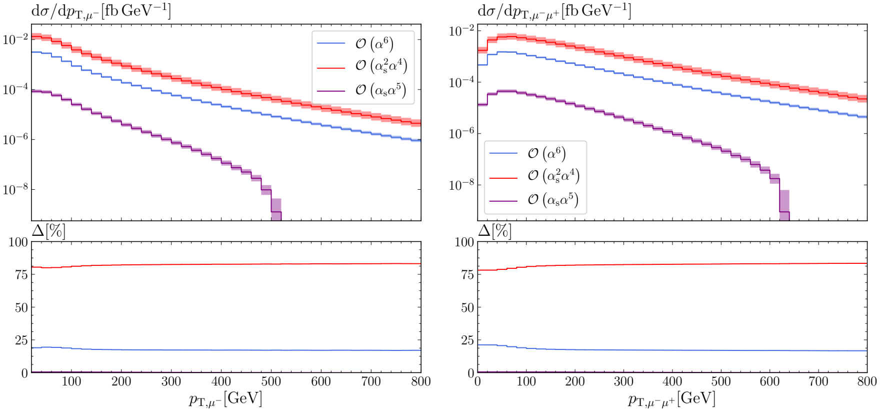
LO distributions (X)



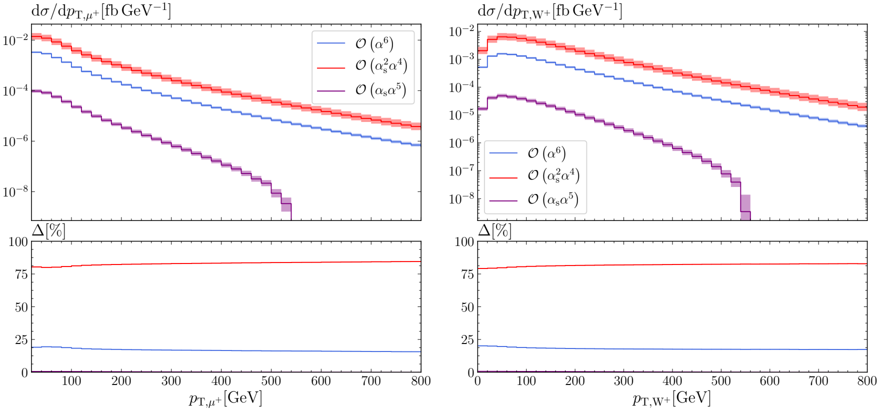
LO distributions (XI)



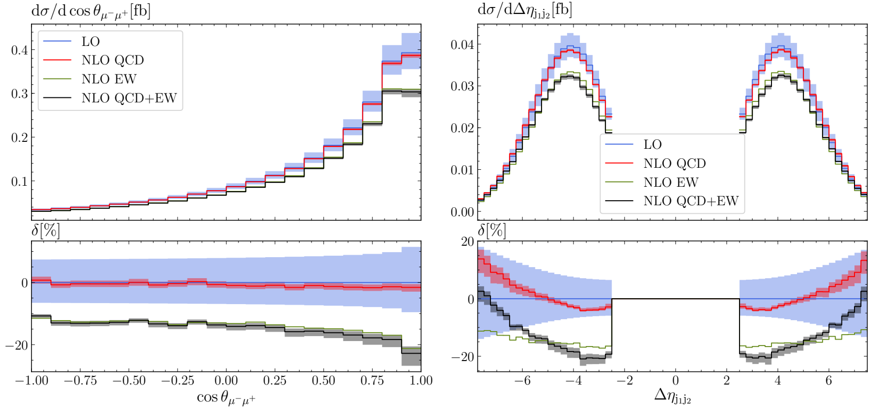
LO distributions (XII)



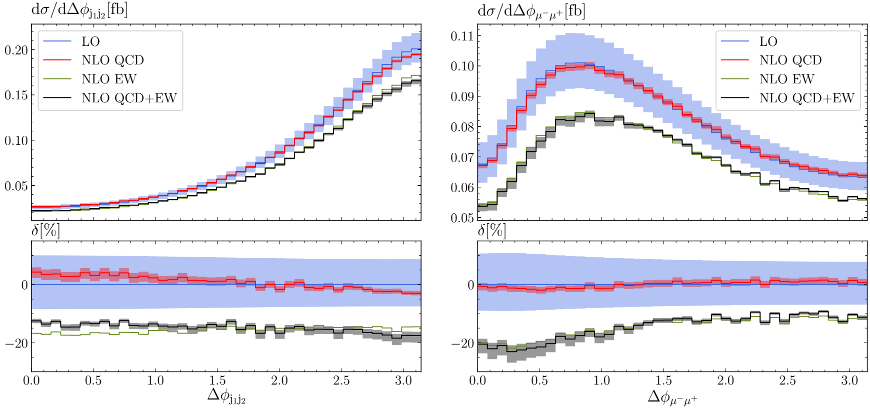
LO distributions (XIII)



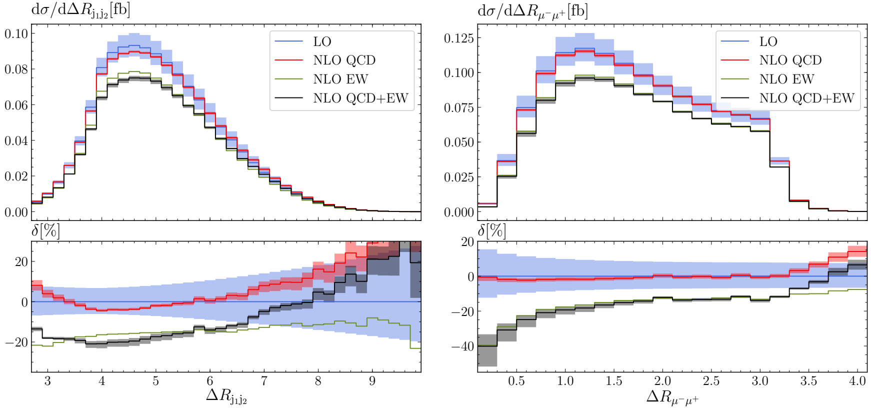
NLO distributions (I)



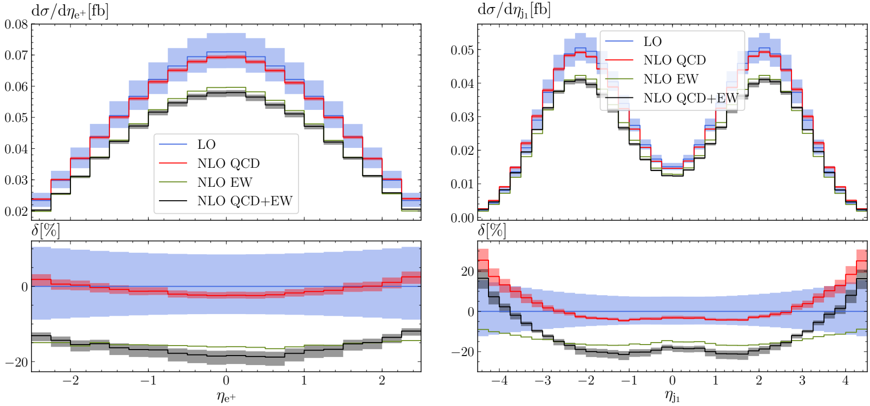
NLO distributions (II)



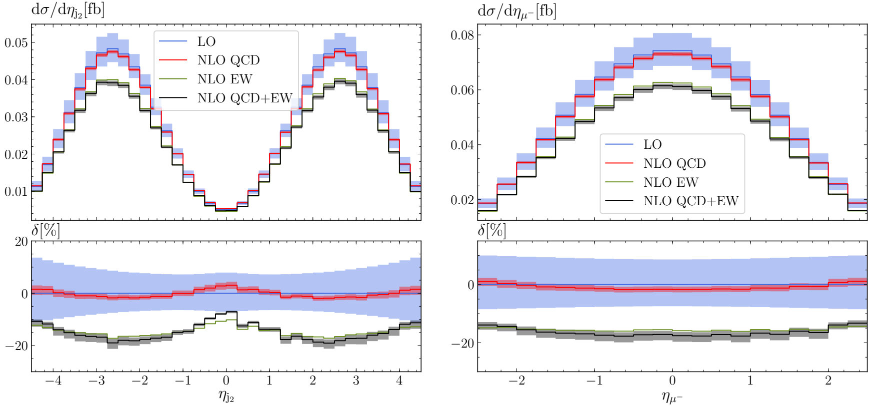
NLO distributions (III)



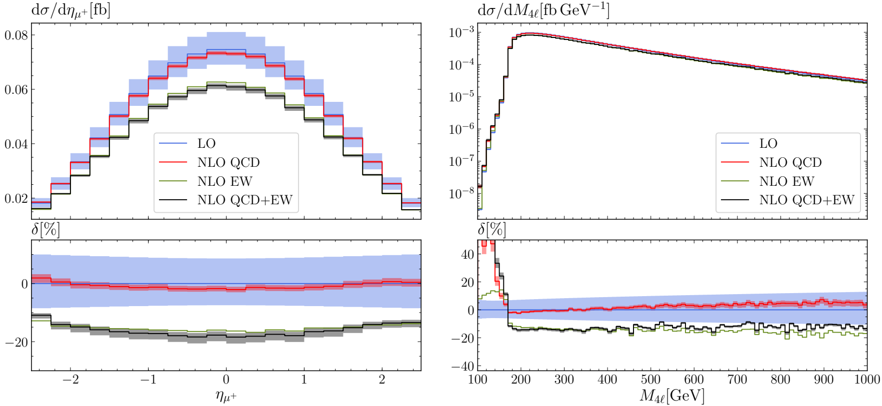
NLO distributions (IV)



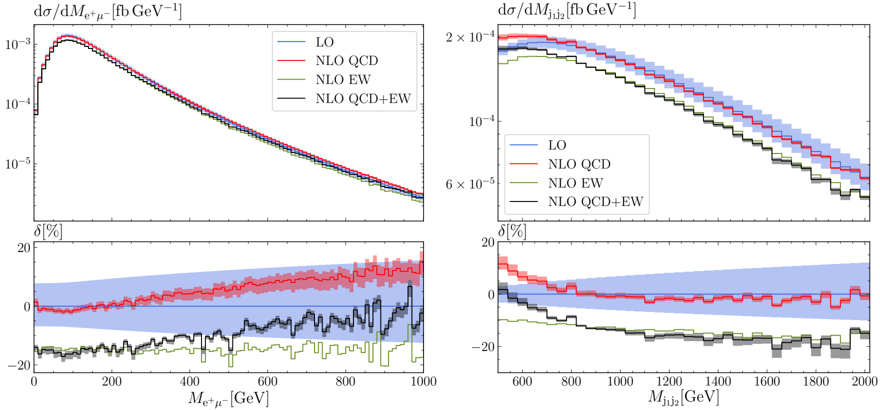
NLO distributions (V)



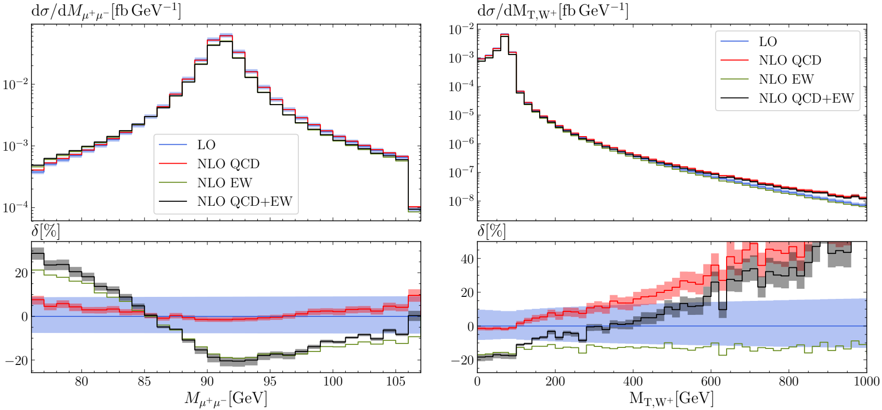
NLO distributions (VI)



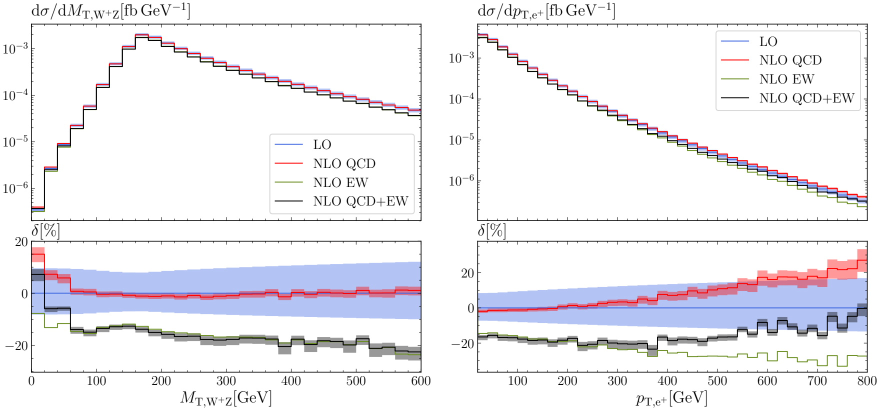
NLO distributions (VII)



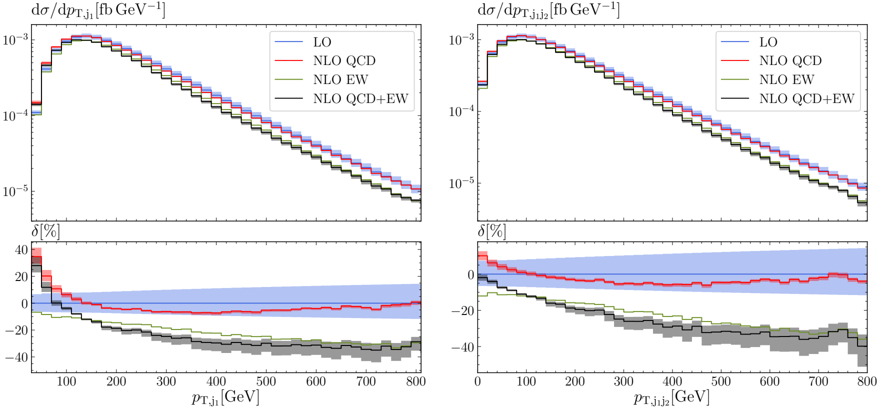
NLO distributions (VIII)



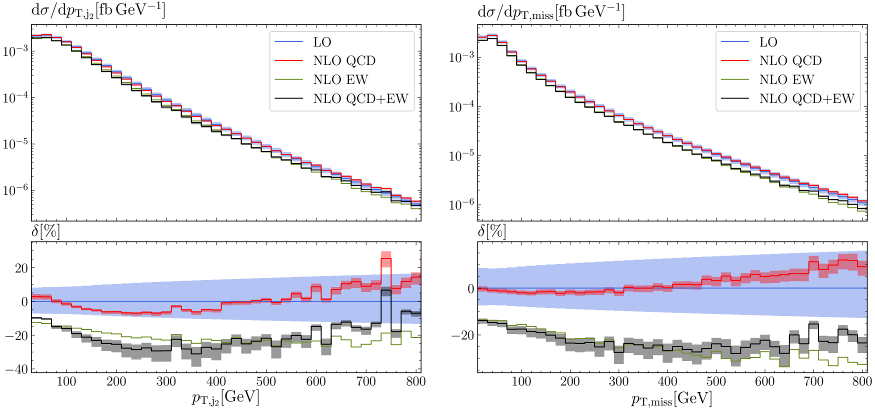
NLO distributions (IX)



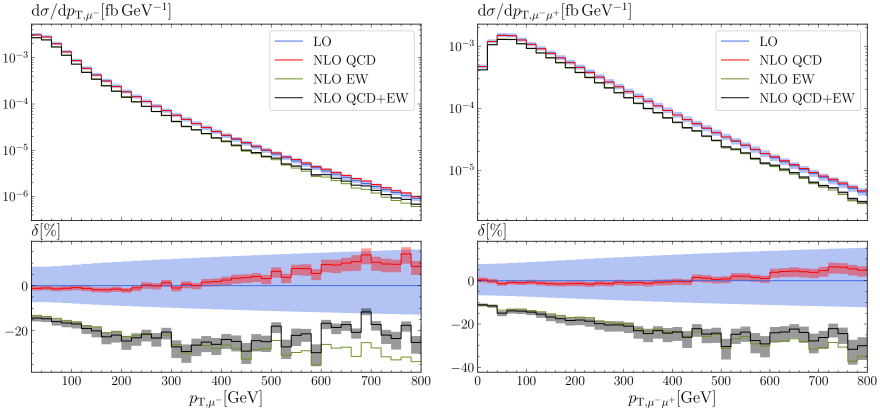
NLO distributions (X)



NLO distributions (XI)



NLO distributions (XII)



NLO distributions (XIII)