

# Revisiting the role of bin-bin correlations in PDF uncertainties for the $M_W$ measurement

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Ultimate precision at hadron colliders workshop  
Orsay, France

# Introduction and motivations

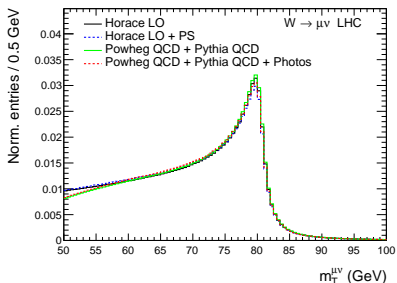
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# Introduction and motivations

- Study the role of bin-bin correlations in the procedure used to estimate/include PDF uncertainty in the extraction of  $M_W$  at the LHC, with a specific focus on the long term perspectives.
- Three sets of uncertainties linked to PDFs:
  1. **Uncertainty in the PDFs from the experimental uncertainty of the dataset used in the fit.**
  2. Different fit methodologies (i.e. differences between PDF sets of different collaborations).
  3. Theoretical uncertainties of the predictions used in PDF fits. Concerning Missing Higher Order Uncertainties (MHOUs), their inclusion is starting to be addressed systematically only recently ([L. A. Harland-Lang, R. S. Thorne – 1811.08434], [R. A. Khalek et al. (NNPDF) – 1906.10698]).

# Measuring the W mass at the LHC

Three observables sensitive to the W mass:  $M_T^W$ ,  $p_{\perp}^l$ ,  $p_T(\text{missing})$ .



[Carloni Calame et al '16]

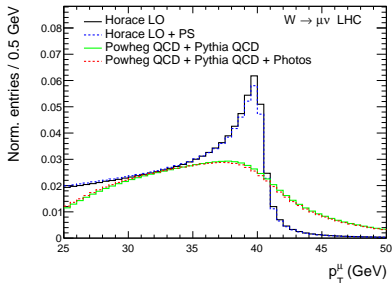
- Peak around  $m_W$ .
- $M_T = \sqrt{2p_T^l p_T^{\text{miss}}(1 - \cos \Delta\phi)}$
- Suffer from pileup and detector effects since it relies on  $\vec{E}_T$ .
- Stability under QCD radiative corrections.

W-boson charge Kinematic distribution	$W^+$		$W^-$		Combined	
	$p_T^l$	$m_T$	$p_T^l$	$m_T$	$p_T^l$	$m_T$
$\delta m_W$ [MeV]						
$\langle \mu \rangle$ scale factor	0.2	1.0	0.2	1.0	0.2	1.0
$\Sigma \vec{E}_T$ correction	0.9	12.2	1.1	10.2	1.0	11.2
Residual corrections (statistics)	2.0	2.7	2.0	2.7	2.0	2.7
Residual corrections (interpolation)	1.4	3.1	1.4	3.1	1.4	3.1
Residual corrections ( $Z \rightarrow W$ extrapolation)	0.2	5.8	0.2	4.3	0.2	5.1
Total	2.6	14.2	2.7	11.8	2.6	13.0

[ATLAS 1701.07240]

# Measuring the W mass at the LHC

Three observables sensitive to the W mass:  $M_T^W$ ,  $p_{\perp}^l$ ,  $p_T(\text{missing})$ .



[Carloni Calame et al '16]

- Peak around  $m_W/2$ .
- Detector modeling under control.
- High sensitivity to radiative corrections.
- We focus on  $p_{\perp}^l$ .

W-boson charge Kinematic distribution	$W^+$		$W^-$		Combined	
	$p_{\perp}^l$	$m_T$	$p_{\perp}^l$	$m_T$	$p_{\perp}^l$	$m_T$
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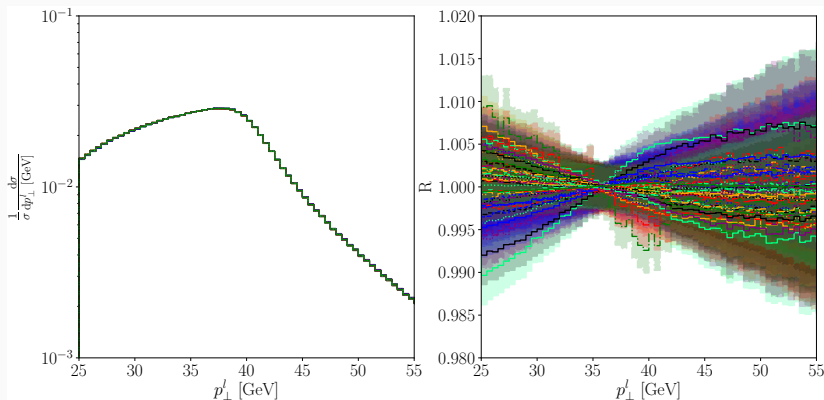
[ATLAS 1701.07240]

# The study

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# Monte-Carlo setup

- $W^+$  generated with POWHEG-BOX-v2 W\_ew-BMNNP,  $\sqrt{S} = 13$  TeV,  $\mu_r = \mu_f = m_W$ .
- Accuracy: NLO-QCD+PS, showered with PYTHIA82.
- Cuts:  $|\eta_l| < 2.5$ ,  $p_l > 25$  GeV,  $\cancel{E}_T > 25$  GeV.
- 15 million events; reweighted to the full set of 1000 replicas of NNPDF30-1000.



# Previous studies for $M_W$

- Tevatron collaborations [0707.0085,0708.3642,0908.0766,1203.0275,1203.0293,1307.7627].
- Comprehensive study on the PDF uncertainty on  $M_T^W$  using modern matched MCs (see also [Bozzi, Rojo, Vicini – 1104.2056]), however with inaccurate  $M_T^W$  modeling.
- Subsequent study on  $p_T^l$  presented in [Bozzi, Citelli, Vicini – 1501.05587] and extended to the study of a high-rapidity lepton in [Bozzi, Citelli, Vesterinen, Vicini – 1508.06954].

## Prescription for the estimation of the uncertainty in those studies

- Generate  $M_W$ -templates using the central replica of the NNPDF set.
  - $\chi_{k,r}^2 = \sum_{i \in bins} (\mathcal{T}_{0,k} - \mathcal{D}_r)_i^2 / \sigma_i^2$ .
  - Fit other NNPDF replica; compute the standard deviation of the  $M_W$  corresponding to minima of the replica  $\chi^2$  and take it as a proxy of the PDF uncertainty.
  - Neglect the value of the  $\chi^2$ .
  - Fixed fit range,  $p_{\perp}^l \in [29, 49]$  GeV.
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- ATLAS [1701.07240], [Kotwal PRD 98, 033008].
  - Other recent studies: [E. Manca, O. Cerri, N. Foppiani, L. Rolandi – 1707.09344], [L. Bianchini and G. Rolandi – 1902.03028], [S. Farry, O. Lupton, M. Pili, M. Vesterinen – 1902.04323], [M. Hussein, J. Isaacson, J. Huston – 1905.00110].



# The role of bin-bin PDF correlations

## Experimental side

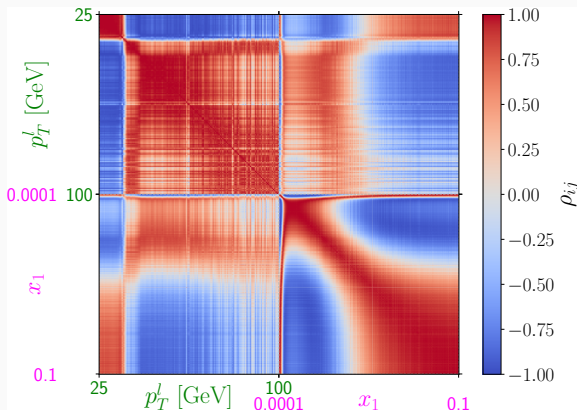
- They were *not* included in the published  $M_W$  measurement from ATLAS, though the effect has been partially included through the combination of different categories.
- They will be included in future measurements both from ATLAS and CMS.
- They were included in other measurements (e.g.  $\sin^2 \theta_1^{eff}$ , or  $\alpha_s$ ).

## Phenomenological studies

- Included in the recent [S. Farry, O. Lupton, M. Pili, M. Vesterinen – 1902.04323], through a reweighting procedure.

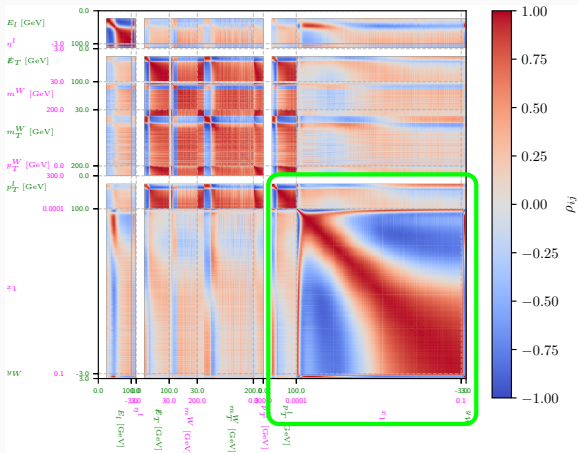
- What is the structure and origin of the bin-bin  $p_T^l$  correlations?
- What is the perspective for a measurement with a large integrated luminosity?

# $p'_\perp$ and PDF correlations



- Different elements drive correlation between replicae (QCD framework)
- $(\Sigma_{PDF})_{rs} = \langle (T - \langle T \rangle_{PDF})_r (T - \langle T \rangle_{PDF})_s \rangle_{PDF}$
- Block-structure in the  $p'_\perp$  self-correlation (top-left corner).
- Interplay in the hadron level cross-section between the parton-level cross-section and the luminosity.

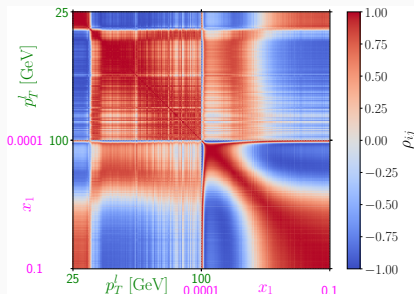
# Other observables



(caveat: *only* this plot at NNPDF30-100/LHEF)

- Shapes of differential observables non-trivially correlated under PDF variation

# Fitting methodology

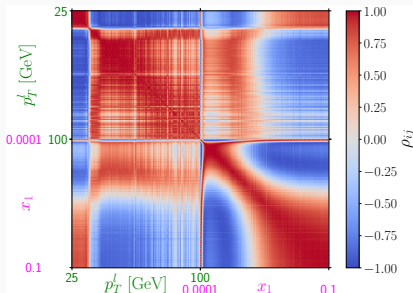


$$\chi_k^2 = \sum_{i \in \text{bins}} \frac{((\mathcal{T}_{0,k})_i - (\mathcal{D}^{\text{exp}})_i - \sum_{r \in \mathcal{R}} \alpha_r (\mathcal{S}_{r,k})_i)^2}{\sigma_i^2} + \sum_{r \in \mathcal{R}} \alpha_r^2$$

$$\mathcal{S}_{r,k} \equiv \mathcal{T}_{r,k} - \mathcal{T}_{0,k}$$

- Treat PDF uncertainty in a frequentist framework, associating a nuisance parameter to each replica → covariance matrix (for the best-fit values of the nuisances).
- Fit the (pseudo)data using the templates (in our case the central replica in both cases), introducing a covariance matrix in the  $\chi^2$  definition.
- Estimate the PDF uncertainty as the half-width of the  $\Delta\chi^2 = 1, 4, 9$  interval.
- The covariance matrix shows a non-trivial structure that has an impact in reducing the sensitivity to the PDF in the fit.

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$$\chi_{k,min}^2 = \sum_{(r,s) \in bins} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_r (\mathcal{C}^{-1})_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_s$$

$$\mathcal{C} = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp, syst}$$

$$(\Sigma_{PDF})_{rs} =$$

$$\langle (\mathcal{T} - \langle \mathcal{T} \rangle_{PDF})_r (\mathcal{T} - \langle \mathcal{T} \rangle_{PDF})_s \rangle_{PDF}$$

$$\langle \mathcal{O} \rangle_{PDF} \equiv \frac{1}{N_{cov}} \sum_{l=1}^{N_{cov}} \mathcal{O}^{(l)}$$

# Results

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# Numerical results: without any covariance

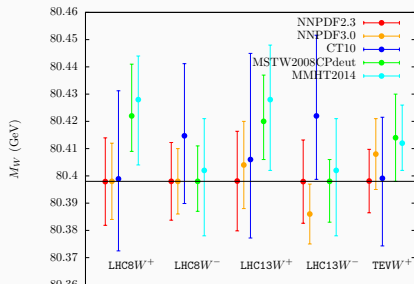
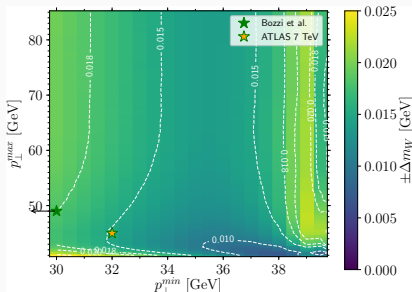


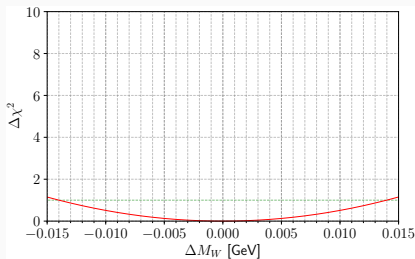
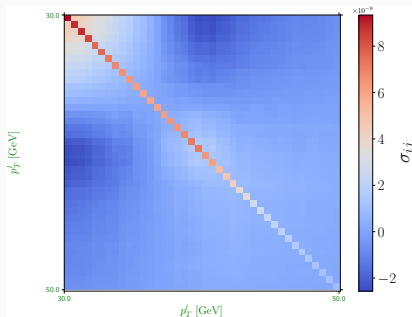
Fig. 4 left from [BCV - 1501.05587]

- $\chi_{k,r}^2 = \sum_{i \in bins} (\mathcal{T}_{0,k} - \mathcal{D}_r)_i^2 / \sigma_i^2$ .
- Compatible results for (nearly) the same fit window.
- The study shows a sizable variability on the fit range.



# Numerical results: with stat+PDF covariance

- PDF covariance +  $1\text{fb}^{-1}$  stat. included.



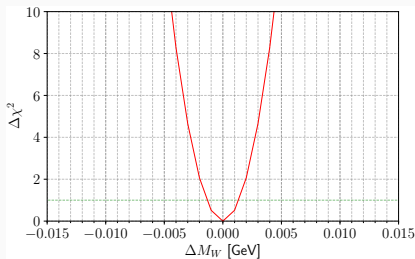
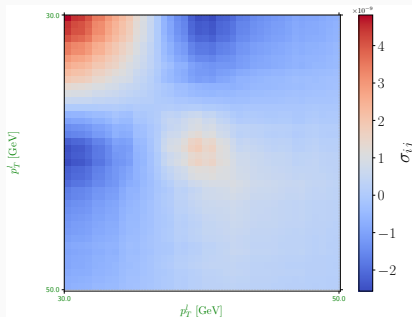
$$\chi_{k,min}^2 = \sum_{(r,s) \in bins} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_r \left( C^{-1} \right)_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_s$$

$$C = \Sigma_{PDF} + \Sigma_{stat}$$



# Numerical results: with stat+PDF covariance

- PDF covariance +  $300\text{fb}^{-1}$  stat. included.

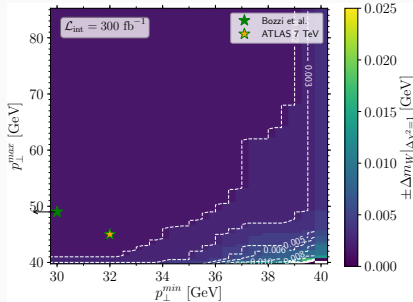
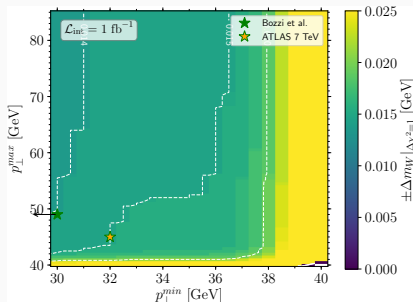


$$\chi_{k,min}^2 = \sum_{(r,s) \in bins} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_r \left( C^{-1} \right)_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_s$$

$$C = \Sigma_{PDF} + \Sigma_{stat}$$

# Numerical results: with MC+stat+PDF covariance

- No MC uncertainty.
- Add MC uncertainty corresponding to  $10^{10}$  events.



- Large statistics is needed but it does not seem a limiting factor.
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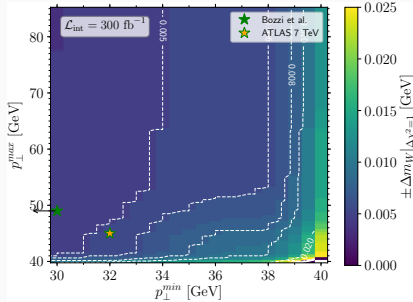
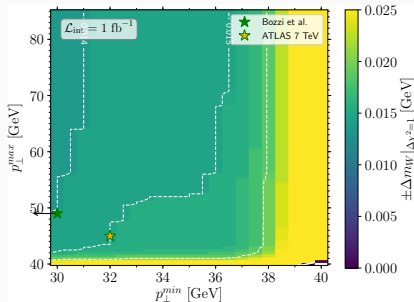
$$\chi_{k,min}^2 = \sum_{(r,s) \in bins} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_r \left( \mathbf{C}^{-1} \right)_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_s$$

$$\mathbf{C} = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC}$$

- What about other source of uncertainties?

# Numerical results: with MC+stat+PDF covariance

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$$\chi_{k,min}^2 = \sum_{(r,s) \in bins} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_r \left( \mathbf{C}^{-1} \right)_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{exp})_s$$

$$\mathbf{C} = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC}$$

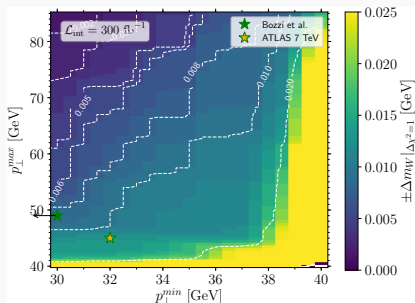
- What about other source of uncertainties?

# Numerical results: with sys+stat+PDF covariance

- We tried to qualitative understand the impact of detector effects on  $p_{\perp}^l$ .
- We used the model proposed by E. Manca (CMS) [[CERN-THESIS-2016-173](#)].

$$\left(\frac{\sigma_{p_T^l}}{p_T^l}\right)^2 = a^2(\eta_l) \cdot r_L^2(\eta_l) + c^2(\eta_l) p^2 \cdot r_L^4(\eta_l) + \frac{b^2(\eta_l) \cdot r_L^2(\eta_l)}{1 + \frac{d^2(\eta_l)}{p^2} \cdot \frac{1}{r_L^2(\eta_l)}}$$

- Uncertainty of  $10^{-4}$  GeV on the overall muon scale.



- We compute a “CMS-covariance matrix” using 100 toys. We sum it to the PDF+stat covariance matrix.
- Detector effects reduce the efficacy of the method.
- A quantitative precise statement on the PDF uncertainty depends on the details of the all the systematics of the measurements.

## **Conclusions and outlook**

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# Conclusions and outlook

## Summary

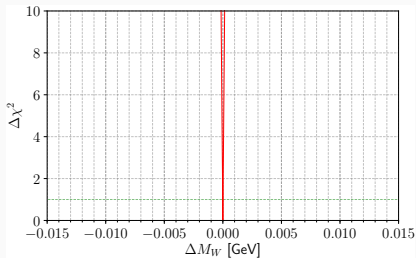
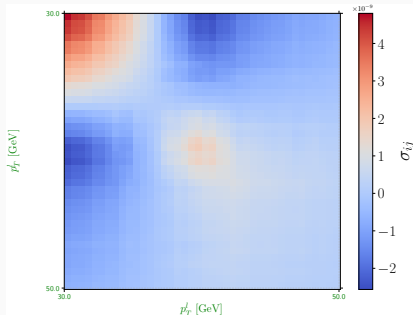
- Treat PDF uncertainty in a frequentist framework as nuisances → covariance matrix.
- Correlation structure of bin above/below the Jacobian peak non-trivial.
- Fitting including the full covariance matrix shows a reduced sensitivity to the PDF uncertainty, if other source of errors are under control.
- Inclusion of bin-bin correlations especially beneficial with large integrated luminosity and good control over the systematics.

## Future developments

- What happens to the correlations if we fix the PDF methodology but we change data sets? Disentangle theory vs experimental effects.
- Correlation structure in the other (Hessian) PDF sets.
- Differences between different sets.
- Scale/smearing/MC-modelling dependence of the covariance matrix?

**Backup slides**

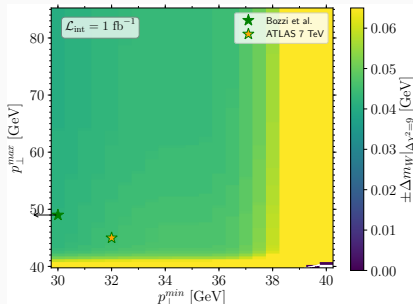
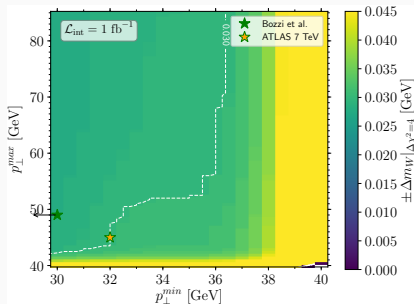
# Numerical results: with PDF covariance



- Shape fit in  $p_{\perp}' \in [30, 50]$  GeV.
- Only PDF covariance included.



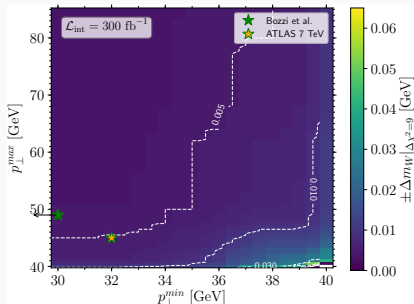
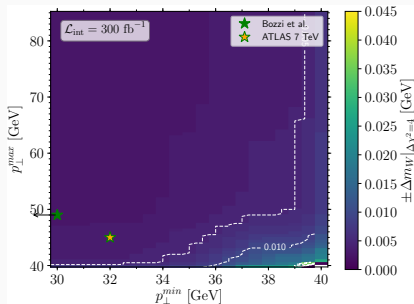
$\mathcal{L}_{\text{int}} = 1 \text{ fb}^{-1}$ ,  $2 \sigma$  and  $3 \sigma$  intervals



- $\Delta \chi^2 = 4$  half-interval.

- $\Delta \chi^2 = 9$  half-interval.

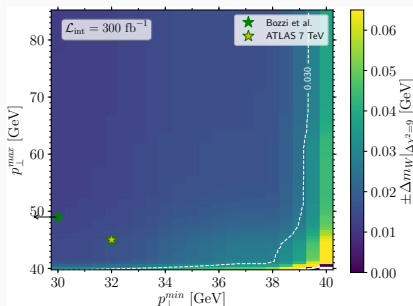
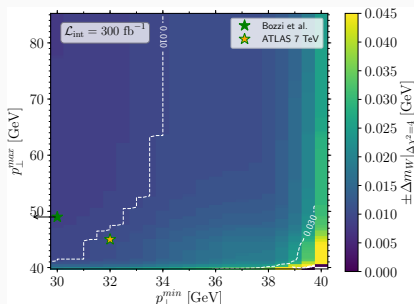
$\mathcal{L}_{\text{int}} = 300 \text{ fb}^{-1}$ ,  $2 \sigma$  and  $3 \sigma$  intervals



- $\Delta \chi^2 = 4$  half-interval.

- $\Delta \chi^2 = 9$  half-interval.

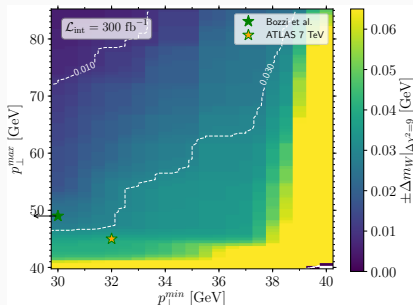
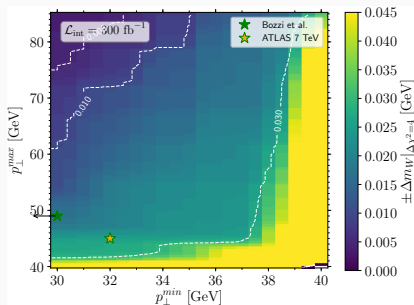
$\mathcal{L}_{\text{int}} = 300 \text{ fb}^{-1} + \text{MC } 10^{10} \text{ events, } 2 \sigma / 3 \sigma \text{ intervals}$



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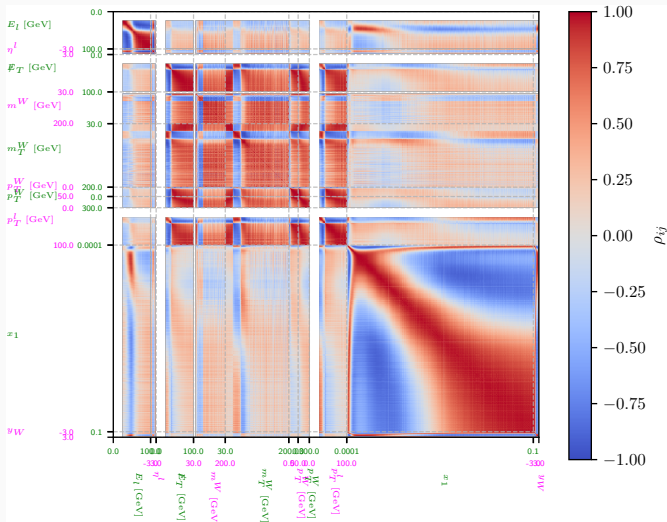
$\mathcal{L}_{\text{int}} = 300 \text{ fb}^{-1} + \text{smearing } 2 \sigma \text{ and } 3 \sigma \text{ intervals}$



- $\Delta\chi^2 = 4$  half-interval.

- $\Delta\chi^2 = 9$  half-interval.

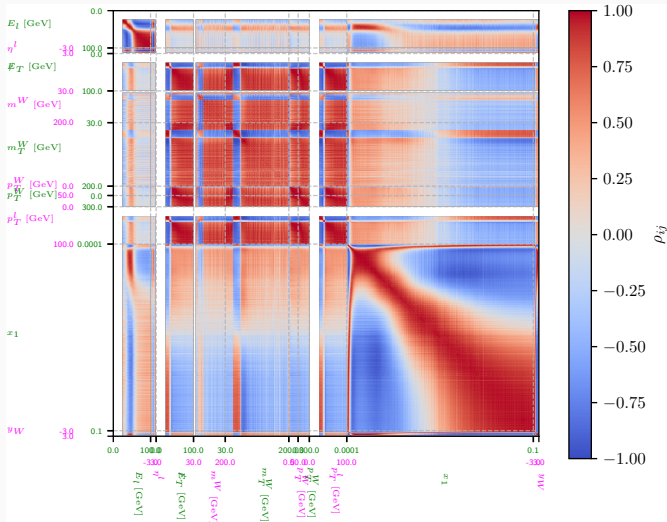
# Bin-bin PDF correlation and partonic channels



- all-channels

(caveat: this plot at  
NNPDF30-100/LHEF)

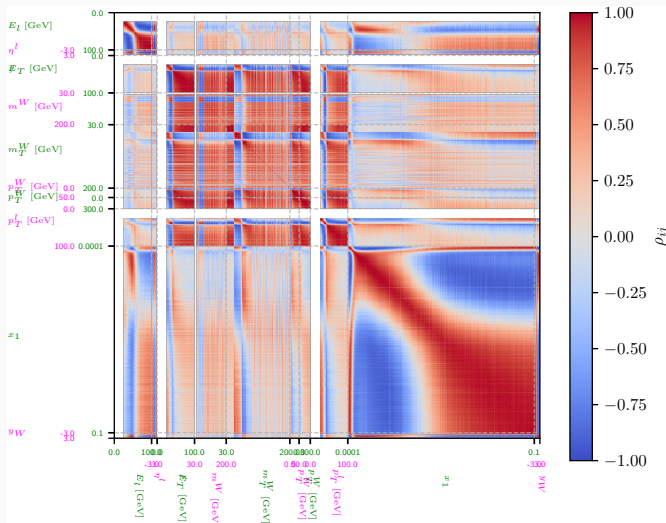
# Bin-bin PDF correlation and partonic channels



- qq'-channels

(caveat: this plot at  
NNPDF30-100/LHEF)

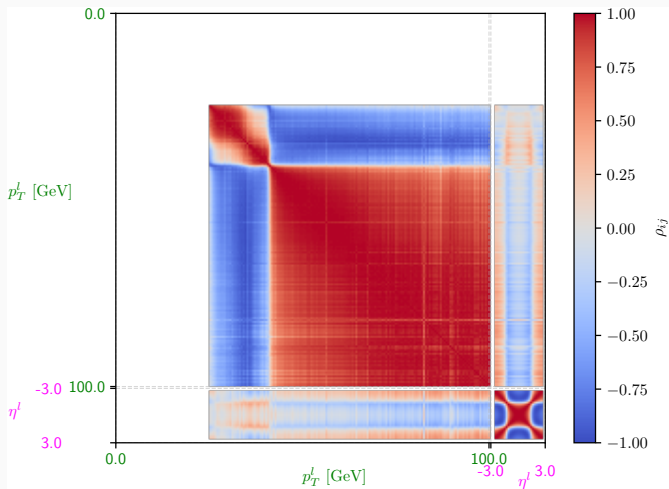
# Bin-bin PDF correlation and partonic channels



- qg-channels

(caveat: this plot at  
NNPDF30-100/LHEF)

# $p_T^l - \eta_l$ correlation

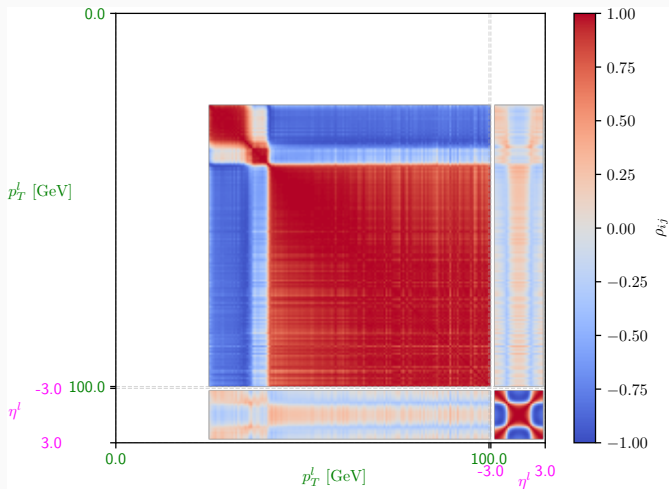


■ all-channels

(caveat: this plot at  
NNPDF30-100/LHEF)



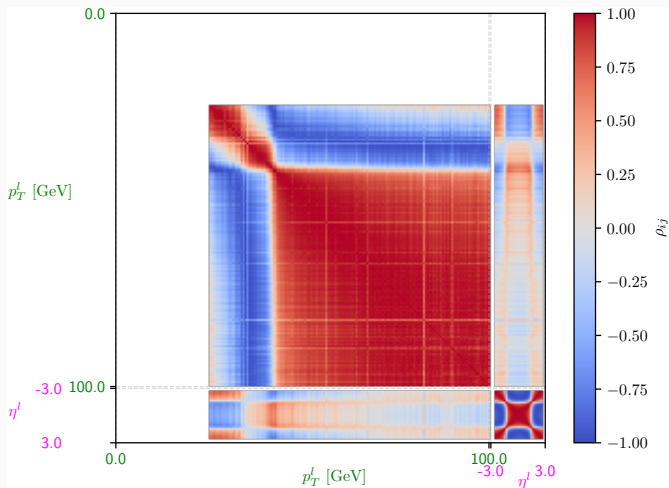
# $p_T^l - \eta_l$ correlation



- qq'-channels

(caveat: this plot at  
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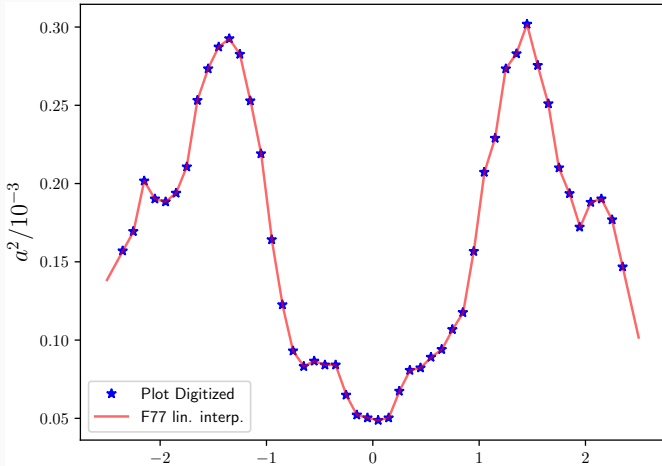
# $p_T^l - \eta_l$ correlation



- qg-channels

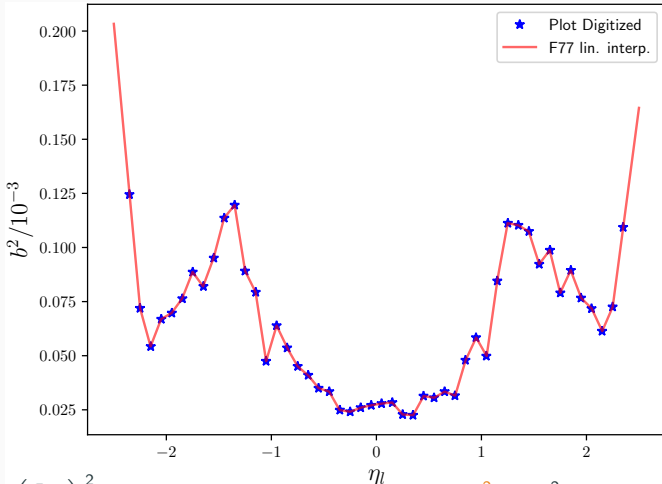
(caveat: this plot at  
NNPDF30-100/LHEF)

# $p'_\perp$ smearing



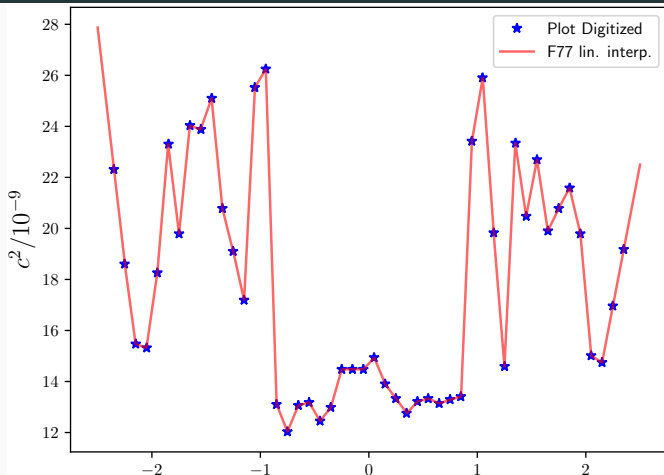
$$\left(\frac{\sigma_{p'_T}}{p'_T}\right)^2 = a^2(\eta_l) \cdot r_L^2(\eta_l) + c^2(\eta_l) p_T^2 \cdot r_L^4(\eta_l) + \frac{b^2(\eta_l) \cdot r_L^2(\eta_l)}{1 + \frac{d^2(\eta_l)}{p^2} \cdot \frac{1}{r_L^2(\eta_l)}}$$

# $p'_\perp$ smearing



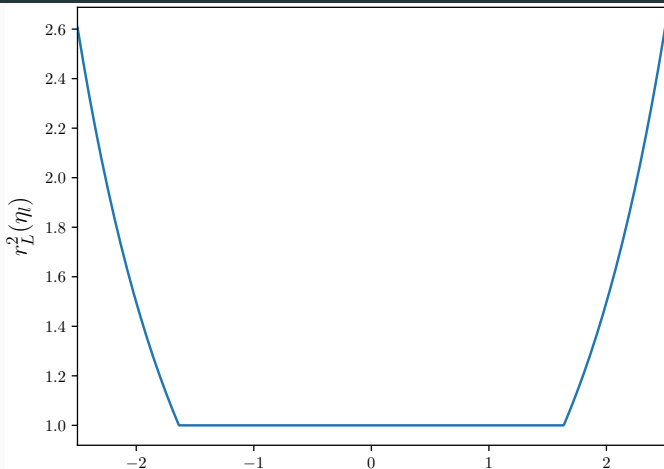
$$\left(\frac{\sigma_{p'_T}}{p'_T}\right)^2 = a^2(\eta_l) \cdot r_L^2(\eta_l) + c^2(\eta_l) p_T^2 \cdot r_L^4(\eta_l) + \frac{b^2(\eta_l) \cdot r_L^2(\eta_l)}{1 + \frac{d^2(\eta_l)}{p^2} \cdot \frac{1}{r_L^2(\eta_l)}}$$

# $p'_\perp$ smearing



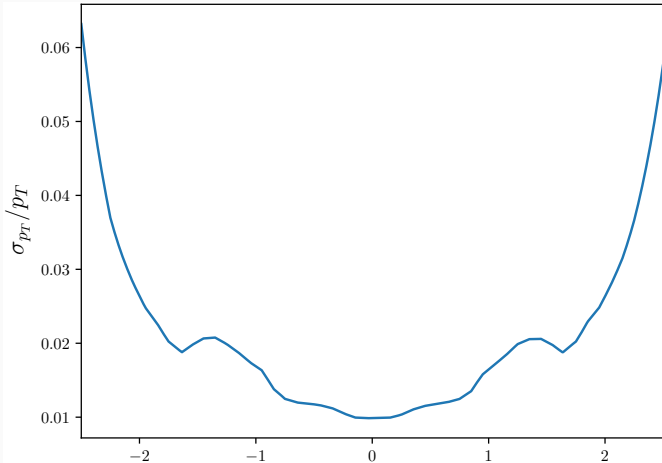
$$\left(\frac{\sigma_{p'_T}}{p'_T}\right)^2 = a^2(\eta_l) \cdot r_L^2(\eta_l) + c^2(\eta_l) p_T^2 \cdot \frac{\eta_l}{r_L^4(\eta_l)} + \frac{b^2(\eta_l) \cdot r_L^2(\eta_l)}{1 + \frac{d^2(\eta_l)}{p^2} \cdot \frac{1}{r_L^2(\eta_l)}}$$

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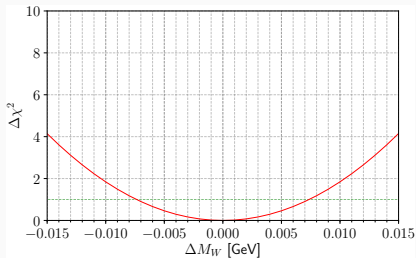
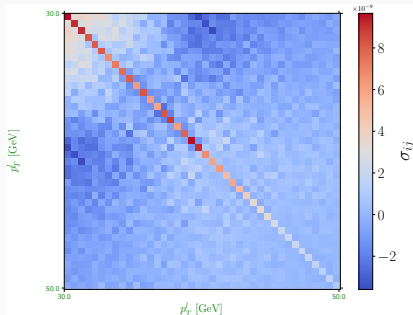
$$\left(\frac{\sigma_{p'_T}}{p'_T}\right)^2 = a^2(\eta) \cdot r_L^2(\eta) + c^2(\eta) p_T^2 \cdot r_L^4(\eta) + \frac{b^2(\eta) \cdot r_L^2(\eta)}{1 + \frac{d^2(\eta)}{p^2} \cdot \frac{1}{r_L^2(\eta)}}$$

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$$\left( \frac{\sigma_{p'_T}}{p'_T} \right)^2 = a^2(\eta) \cdot r_L^2(\eta) + c^2(\eta) p_T^2 \cdot r_L^4(\eta) + \frac{b^2(\eta) \cdot r_L^2(\eta)}{1 + \frac{d^2(\eta)}{p^2} \cdot \frac{1}{r_L^2(\eta)}}$$

# Covariance-enabled fit



- Shape fit in  $p_T' \in [30, 50]$  GeV.
- PDF covariance +  $300\text{fb}^{-1}$  stat. + smearing included.