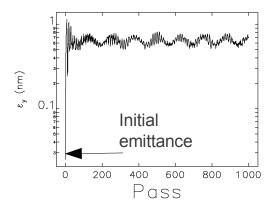# Containing emittance blowup in APS storage ring with RF deflection scheme

V. Sajaev
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V. Sajaev

History

- Soon after we started simulating the deflecting cavities, we have found significant emittance blowup in both planes[1]

Vertical emittance increased from 30 pm to 600 pm

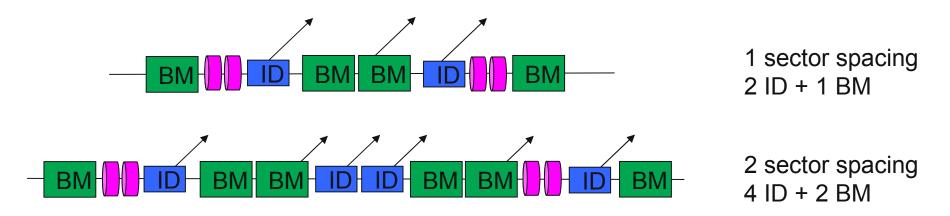- This blowup undermines the whole concept – which takes advantage of the small vertical beam size

$$\sigma_{t,xray} = \frac{E}{V\omega}\sqrt{\sigma^2_{y',e} + \sigma^2_{y',rad}}$$

- It required us to understand the sources of the blowup before we could find ways of minimizing it

[1]M. Borland

Deflecting/Crabbing Cavity Applications in Accelerators

Possible configurations

- At minimum two cavities are required to create a chirp
- Below are possible configurations of the deflecting insertion:

1 sector spacing
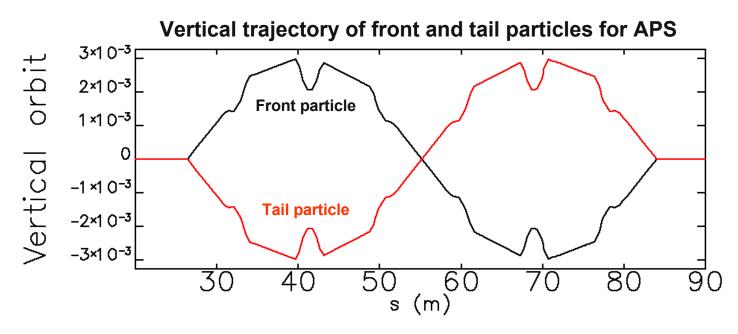2 ID + 1 BM

2 sector spacing
4 ID + 2 BM

- Every APS sector contains 2 dipoles, 10 quadrupoles, and 7 sextupoles

Beam trajectories

- After kick in the first cavity, front particles start moving up and tail particles start moving down with amplitude proportional to their longitudinal position in the bunch

- In an ideal linear system with zero energy spread, the second cavity completely cancels vertical motion of all particles

Vertical trajectory of front and tail particles for APS

Various effects

- In a real machine, many effects could lead to non complete trajectory closure, and therefore emittance degradation
  - Various errors and imperfections are first things coming to mind
- However, even in a perfect machine the emittance can increase many fold
  - Path length dependence on the particle energy leads to incomplete kick canceling in the second cavity
  - Betatron phase advance dependence on energy (chromaticity) leads to closed bump condition breaking
  - Sextupoles between cavities introduce nonlinearities that generate betatron phase advance dependence on amplitude and linear coupling between horizontal and vertical planes
- Here we will discuss the effects that always exist even in a perfect machine

Momentum compaction

- This effect is present even if there are no errors and nonlinearities (but usually small)
- It comes from the path length difference between the cavities for particles with different energy
- Additional kick after the second cavity is

$$\Delta y' = \frac{-V \omega \Delta t}{E}$$

Which gives emittance increase of

$$\frac{\Delta \epsilon_y}{\epsilon_y} = \frac{\sqrt{\sigma_{y'}^2 + \sigma_{\Delta y'}^2}}{\sigma_{y'}} - 1$$

- For extreme case of V=6MV and h=8, it gives about 6% increase of emittance in a single turn

Chromaticity and energy spread

- The second cavity is placed at $n\pi$ phase advance to cancel the kick of the first cavity
- If there is chromaticity ξ_y between the cavities, the phase advance of a particle with δ_i is changed by $-2\pi\xi_y\delta_i$ which leads to a particle position change at the second cavity

$$y_2 = \beta\, y'_1 \sin\left(2\pi\xi_y\delta_i\right)$$
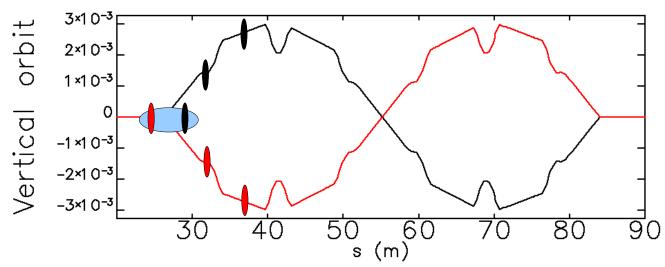
- The rms value of the residual amplitude is

$$\sigma_{y_2} = 2\pi\,\xi_y\beta\,\frac{V\omega}{E}\sigma_\delta\sigma_t$$

- For APS parameters with uncompensated chromaticity, this works out to a number that is almost 4 times as large as the nominal vertical beam size of 11 μm
- Therefore you absolutely need to correct chromaticity between the cavities

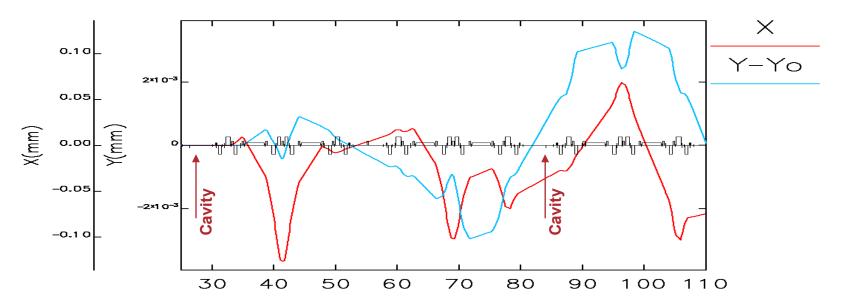Deflecting/Crabbing Cavity Applications in Accelerators

Sextupole nonlinearities

- Sextupoles can affect in two ways:

- By introducing amplitude-dependent focusing
    - for particles going off-axis the kick cancellation at the second cavity is not perfect
- By introducing transverse coupling
    - deflecting cavities generate non-zero vertical trajectories in sextupoles
    - Creates coupling between large horizontal and small vertical emittances

Deflecting/Crabbing Cavity Applications in Accelerators

Sextupole nonlinearities – dipole kicks

- In the first sextupole, a particle experiences only vertical dipole field due to non-zero vertical orbit, which generates horizontal trajectory

- At the second sextupole, the particle has both vertical and horizontal coordinates and experiences both horizontal and vertical dipole kick

Large linear part of the vertical trajectory is subtracted

Derivation of emittance oscillations

- First, we study the effect of an orbit bump produced by corrector magnets:

- A particle experiences a vertical kick when displaced vertically in a sextupole:

$$\Delta y' = b_s (y_0 + y) x$$

- This leads to an emittance increase:

$$\Delta \epsilon_y = b_s y_0 \sqrt{\epsilon_x \epsilon_y \beta_x \beta_y}\, e^{\frac{-\theta^2}{2\tau_y^2}}\, e^{-2\sigma_E^2 \frac{(C_x - C_y)^2}{Q_s^2} \sin^2 \frac{Q_s \theta}{2}} \sin(\Delta Q \theta + \Delta \chi)$$

- In case of several sextupoles, it transforms to:

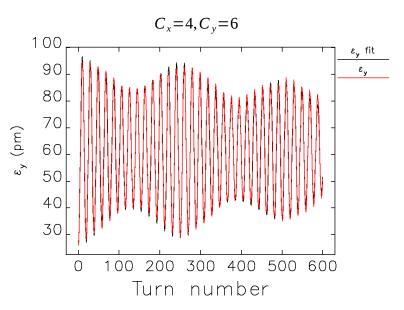$$\Delta \epsilon_y = \sqrt{\epsilon_x \epsilon_y}\, e^{\frac{-\theta^2}{2\tau_y^2}}\, e^{-2\sigma_E^2 \frac{(C_x - C_y)^2}{Q_s^2} \sin^2 \frac{Q_s \theta}{2}} \Im\left(\sum_j b_{sj} y_{0j} \sqrt{\beta_{xj} \beta_{yj}}\, e^{i\,\Delta \chi_j} \right) \sin(\Delta Q \theta + \Delta \psi)$$
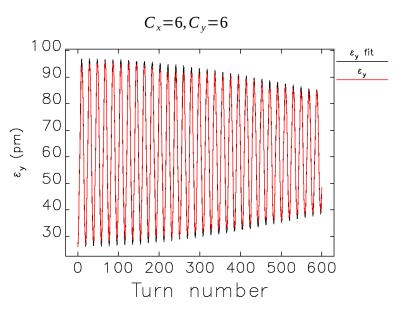
Emittance oscillations for dipoles

- To compare with simulations, we re-write the expression to:

$$\epsilon_y(\theta) = \epsilon_{final} - (\epsilon_{final} - \epsilon_{initial}) e^{\frac{-\theta^2}{2\tau_y^2}} e^{-2\sigma_E^2 \frac{(C_x - C_y)^2}{Q_s^2} \sin^2 \frac{Q_s \theta}{2}} \sin(\Delta Q \theta)$$

$C_x=4, C_y=6$

$C_x=6, C_y=6$

- The expression above describes oscillation of the emittance after sudden coupling kick

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Oscillations of the horizontal emittance

- When electron is displaced vertically in a sextupole, it experiences a kick in horizontal plane:

$$\Delta x' = \frac{b_s}{2}(y_0+y)^2 \approx b_s y y_0 + \frac{b_s}{2}y_0^2$$
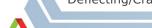
- This gives the following emittance increase:

$$\Delta \epsilon_x \approx -b_s y_0 \sqrt{\epsilon_x \epsilon_y \beta_x \beta_y}\sin(\phi_x-\phi_y)+b_s y_0^2 \sqrt{\epsilon_x \beta_x}\sin(\phi_x)$$

- For many sextupoles we get the following emittance oscillations:

$$\Delta \epsilon_x \approx \sqrt{\epsilon_x}\, e^{\frac{-\theta^2}{2\tau_y^2}}\, e^{-2\sigma_E^2 \frac{C_x^2}{Q_s^2}\sin^2 \frac{Q_s\theta}{2}}\, \Im\left(\sum_j b_{sj} y_{0j}^2 \sqrt{\beta_{xj}}\, e^{i\Delta x_{xj}}\right)\sin(Q_x\theta+\psi_x)$$
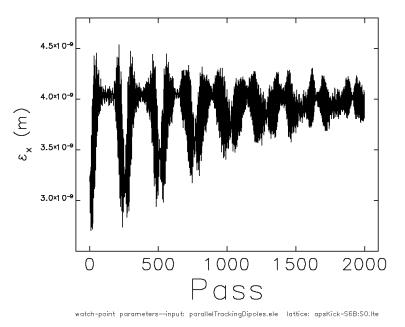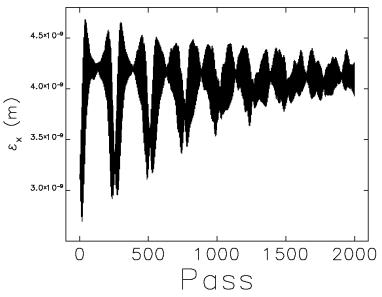
Equilibrium emittances with dipoles

- Long enough after the coupling kick, we have just the lattice with local vertical orbit bump
- Equilibrium vertical emittance will be defined by the new coupling resulting from the vertical orbit bump
- Horizontal emittance should not be changed at all !?

- What is the source of the horizontal emittance oscillations in the first place?
  - Vertical orbit bump generates small kick in the horizontal plane which changes the horizontal orbit slightly
  - We do tracking starting from the orbit the beam had before the dipoles were turned on
  - Turning the dipoles on is equivalent to a sudden kick in horizontal plane which leads to betatron oscillations and emittance growth due to decoherence
- This means that the horizontal emittance will recover due to SR damping

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Horizontal emittance oscillations

- Below is the comparison of emittance oscillations due to dipoles creating the vertical orbit bump (left) and due to simple horizontal kick (right)
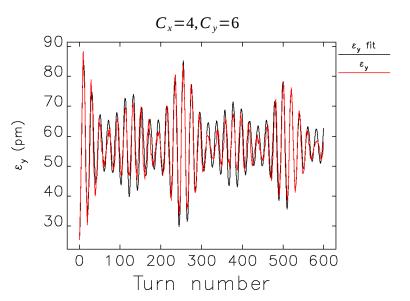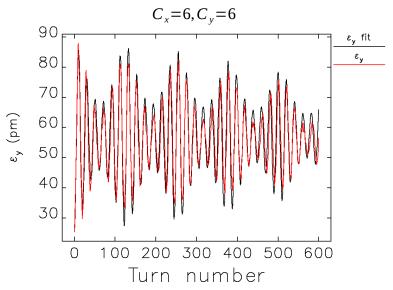
Emittance oscillations with cavities

- To account for a slice-to-slice variation of the amplitude of the orbit bump within the electron bunch caused by a time-dependent kick, we modify the earlier expression:

$$\epsilon_y(\theta) = \epsilon_{final} - (\epsilon_{final} - \epsilon_{initial}) e^{\frac{-\theta^2}{2\tau_y^2}} e^{-2\sigma_E^2 \frac{(C_x - C_y)^2}{Q_s^2} \sin^2 \frac{Q_s\theta}{2}} e^{-\sin^2(\frac{Q_s\theta}{2})} \sin(\Delta Q\theta)$$

$C_x=4, C_y=6$

$C_x=6, C_y=6$

Equilibrium emittances with cavities

- If one compares the vertical emittance oscillation plots for dipoles and cavities, one would notice that they oscillate around approximately the same value of the final emittance
  - Reminder: dipole kick amplitudes correspond to cavity kicks for a particle at one sigma longitudinal position
- This observation makes sense however is it hard to prove it mathematically
- We checked cases with different sextupoles and found that the relation approximately stands

- This gives us the simple way to predict the equilibrium vertical emittance
- It is natural to expect that this relationship works only when the coupling is the dominating effect

Equilibrium horizontal emittance

- Extending the previous discussion to the horizontal emittance, we can conclude that the equilibrium horizontal emittance would not change with the cavities
- However, simulations with high RF voltage and not optimized sextupoles show possible emittance increase
  - We can say that when the orbit kick is strong enough, then the constant change of the amplitude due to synchrotron motion does not allow synchrotron radiation to damp the emittance completely
  - Our experience also shows that for all reasonably optimized sextupole configurations there is no horizontal emittance blowup
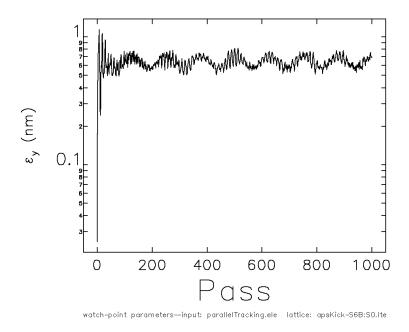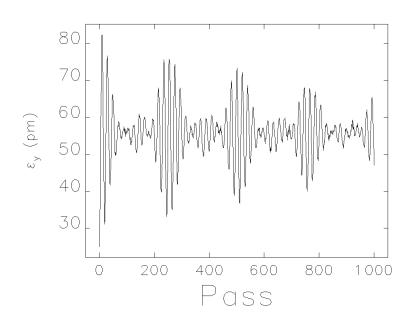
Sextupole optimization

- Finally we can state that the sextupoles between the cavities must:
  - Compensate natural chromaticity
  - Minimize coupling on the vertical trajectory
  - Minimize orbit bump leakage to the outside of the bump
  - Maintain satisfactory dynamic aperture (DA) (due to sextupole optimization inside the bump, the sextupole symmetry would be broken)
  - Maintain satisfactory momentum aperture (MA)

- First 3 items are required to keep emittance blowup under control
- Last 2 items are required to maintain good injection efficiency and lifetime of the storage ring
- We have found that if one would not care about the DA/MA, the emittance increase could be almost completely mitigated
- However, the DA/MA requirements limit the freedom of the sextupole changes

Sextupole optimization

- Sextupole optimization is done using genetic optimizer that varies sextupoles both inside and outside the cavity bump and on every iteration it calculates emittance increase and dynamic and momentum aperture using tracking

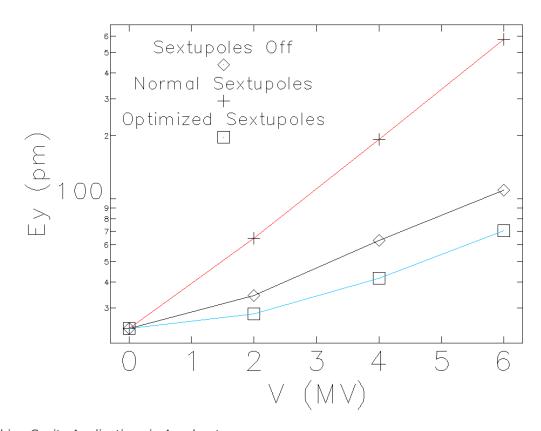- It is very CPU-hungry process, but it gives satisfactory results:

watch-point parameters--input: parallelTracking.ele   lattice: apsKick-S6B:S0.lte

Sextupole optimization results

Comparison of the vertical emittance growth for the three sextupole schemes: Normal sextupoles, Sextupoles off, and Optimized sextupoles (no synchrotron radiation)

Conclusions

- We explained the processes behind emittance increase in the deflecting cavity scheme
- We derived expressions for emittance oscillations due to sudden coupling kick
- We empirically found that the dipole kick can be used to predict equilibrium emittances
- Due to combination of multiple complicated processes, only numerical optimization of sextupoles is possible to reduce the emittance blowup
  - The optimization breaks sextupole symmetry and therefore the final results of the emittance blowup depend on how much one can sacrify DA/MA

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