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TMD physics at the LHC and LHeC

LHeC2019 Workshop “Electrons for the LHC”

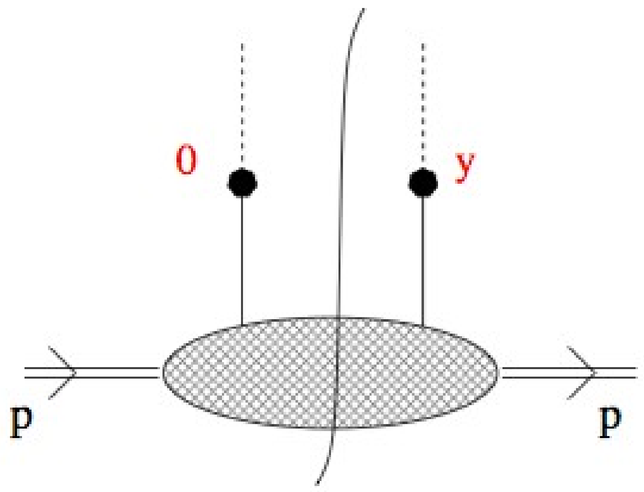
Chavannes-de-Bogis, October 2019

Contents

- TMD effects from low-x dynamics
- TMD effects from low-qT dynamics
- Toward TMD Monte Carlo generators:
  Progress in Parton Branching including TMDs

UNINTEGRATED, OR TRANSVERSE MOMENTUM DEPENDENT (TMD), PARTON DISTRIBUTION FUNCTIONS

- Parton correlation functions at non-lightlike distances:



$$p = (p^+ , m^2 / 2 p^+ , 0_\perp)$$

$$\widetilde{f}(y) = \langle\, P \mid \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0) \mid P\, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

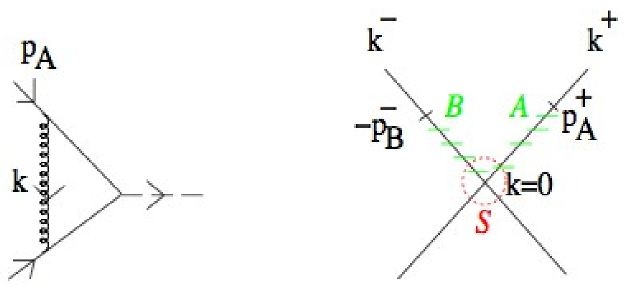
$$V_y(n) = \mathcal{P} \exp\left(i g_s \int_0^\infty d\tau\ n \cdot A(y + \tau\ n) \right)$$

- TMD pdfs:

$$f(x, k_\perp) = \int \frac{dy^-}{2\pi} \frac{d^{d-2} y_\perp}{(2\pi)^{d-2}} e^{-ixp^+ y^- + ik_\perp \cdot y_\perp} \tilde{f}(y)$$

TMDs from low-qT QCD resummation and from low-x (high-energy) QCD resummation

low q_T : $q_T \ll Q$



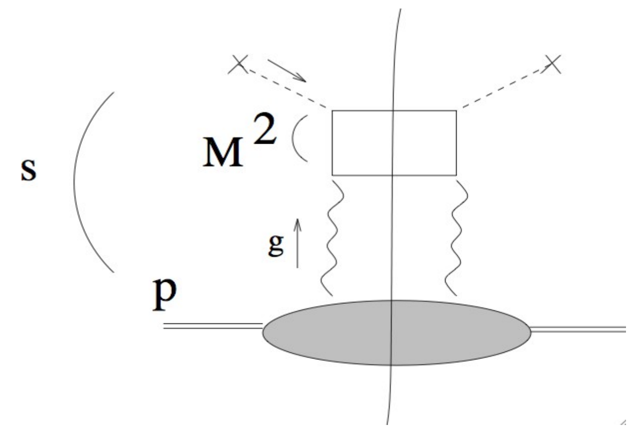
(a) (b)

$$\alpha_s^n \ln^m Q/q_T$$

CSS evolution equation

(or variants – SCET, ...)

high $\sqrt{s}$: $\sqrt{s} \gg M$



$$(\alpha_s \ln \sqrt{s}/M)^n$$

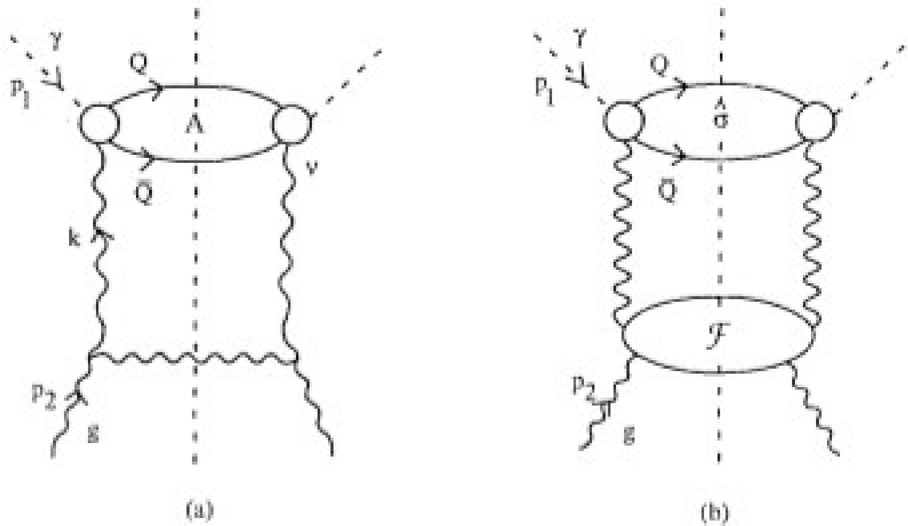
BFKL evolution equation

(or CCFM, BK, JIMWLK, …)

R. Angeles-Martinez et al., “Transverse momentum dependent (TMD) parton distribution functions: status and prospects”, Acta Phys. Polon. B46 (2015) 2501

I.TMDs at high energies

Ex.: heavy flavor electroproduction for $s \gg M^2 \gg \Lambda^2_{\rm QCD}$



$$\gamma + h \to Q + \bar{Q} + X$$

$$4M^2\,\sigma(x, M^2) = \int d^2\mathbf{k}_\perp \int_x^1 \frac{dz}{z}\,\hat{\sigma}_{\gamma g}(x/z, \mathbf{k}_\perp^2/M^2, \alpha_{\rm S}(M^2))\,\mathcal{A}_{g/h}(z, \mathbf{k}_\perp)$$

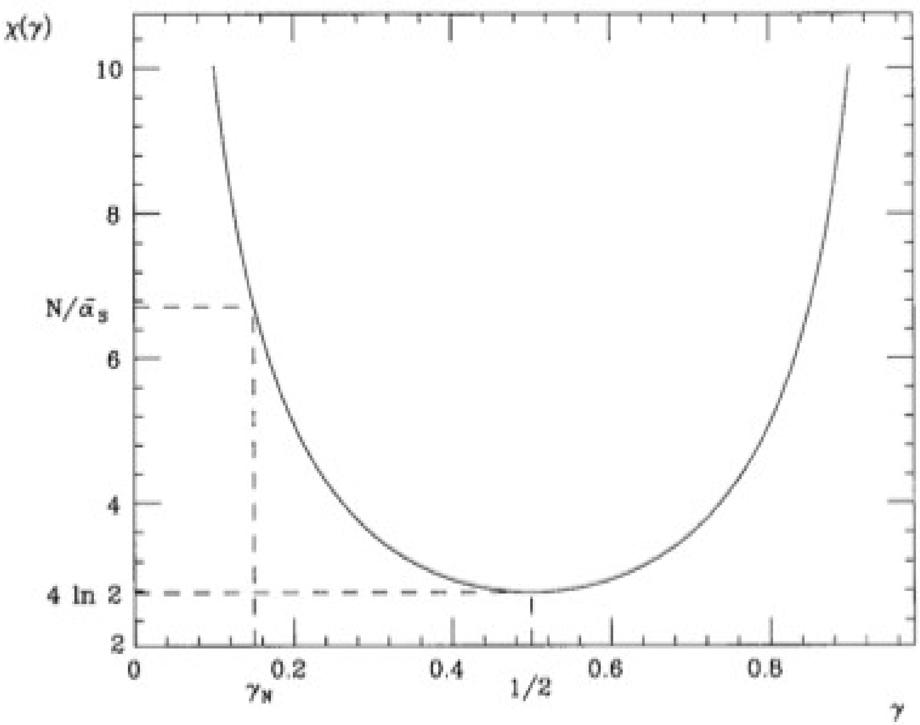
where TMD gluon distribution is given by
Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution:

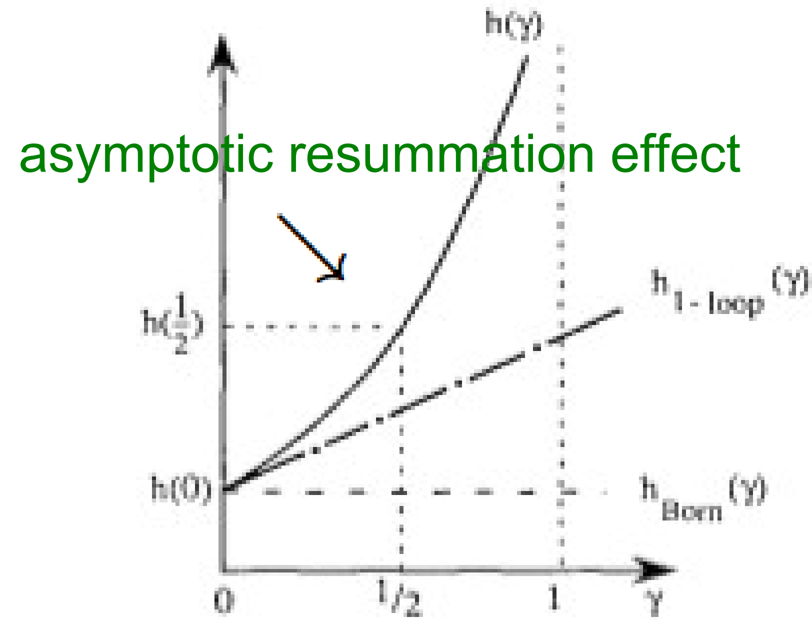
$$\mathcal{A}_{g/h}(x, \mathbf{k}_\perp) \sim \frac{1}{2\pi}\, e^{-\lambda\,\ln x}\,(\mathbf{k}_\perp^2)^{\gamma-1} \quad , \qquad \lambda \to 4\,C_A\,\frac{\alpha_{\rm S}}{\pi}\,\ln 2 \qquad \gamma \to \frac{1}{2}$$

TMDs at high energies

$$\Rightarrow 4M^2\sigma(x,M^2) \sim x^{-\lambda}\,(M^2)^{\frac{1}{2}}\;\; h(1/2)\;,$$

$$\text{where}\;\; h(1/2) = \frac{1}{2}\int_0^\infty \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2}\left(\frac{\mathbf{k}_\perp^2}{M^2}\right)^{\frac{1}{2}}\int_0^1 \frac{dx}{x}\;\; \hat{\sigma}_{\gamma g}(x,\mathbf{k}_\perp^2/M^2,\alpha_{\rm S})$$

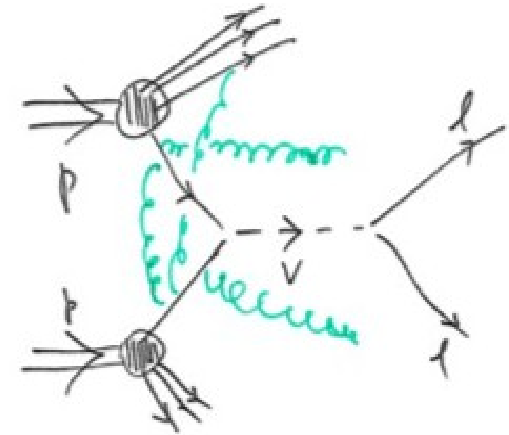




- needed to estimate realistic effects at LHeC, FCC-eh

NB:
- incorporate sub-asymptotic, finite-x terms → CCFM evolution
- dense-medium modifications in nucleons and nuclei → nonlinear evolution

II. TMDs for low qT



Ex.: Drell-Yan production qT spectra for Q >> qT

$$\frac{d\sigma}{d^2\mathbf{q}_T dQ^2 dy} = \sum_{i,j} \frac{\sigma^{(0)}}{s} H(\alpha_S) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_T\cdot\mathbf{b}} \mathcal{A}_i(x_1,\mathbf{b},\mu,\zeta)\mathcal{A}_j(x_2,\mathbf{b},\mu,\zeta) + \{\mathbf{q}_T - \text{finite}\} + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)$$

where $\frac{\partial \ln \mathcal{A}}{\partial \ln \sqrt{\zeta}} = K(\mathbf{b},\mu)$ Collins-Soper-Sterman (CSS) evolution

and $\frac{d \ln \mathcal{A}}{d \ln \mu} = \gamma_f(\alpha_s(\mu), \zeta/\mu^2)$, $\frac{dK}{d\ln\mu} = -\gamma_K(\alpha_s(\mu))$ RG evolution

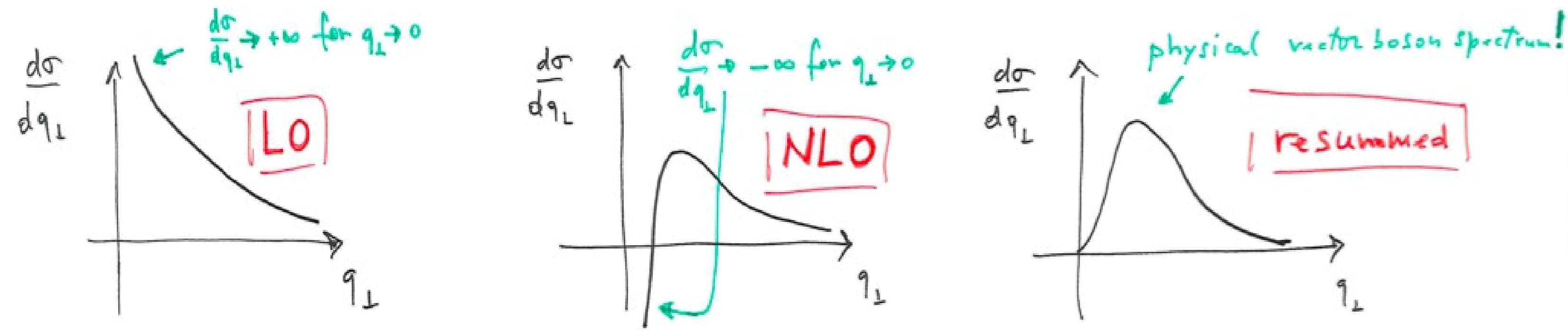
cusp anomalous dimension

$$\Rightarrow \quad -\gamma_K = \frac{\partial}{\partial \ln\sqrt{\zeta}} \gamma_f \qquad \text{i.e.} \quad \gamma_f(\alpha_s(\mu), \zeta/\mu^2) = \gamma_f(\alpha_s(\mu), 1) - \frac{1}{2}\gamma_K \ln \zeta$$

- Soft Collinear Effective Theory (SCET) provides alternative approach leading to same results

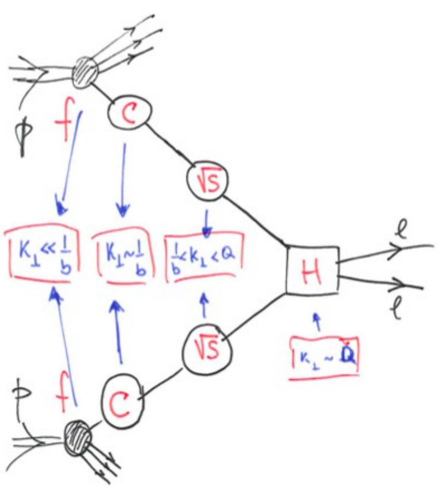
TMDs for low qT

- OUTCOME: SUM $\alpha_s^m \ln^k Q^2/q_\perp^2$ TO ALL ORDERS IN α_s



NOTE: SHOWER MONTE CARLO GENERATORS DO THIS "EFFECTIVELY"

"Parton-like" formulation by decomposing the TMD pdfs in terms of ordinary pdfs ("OPE"):

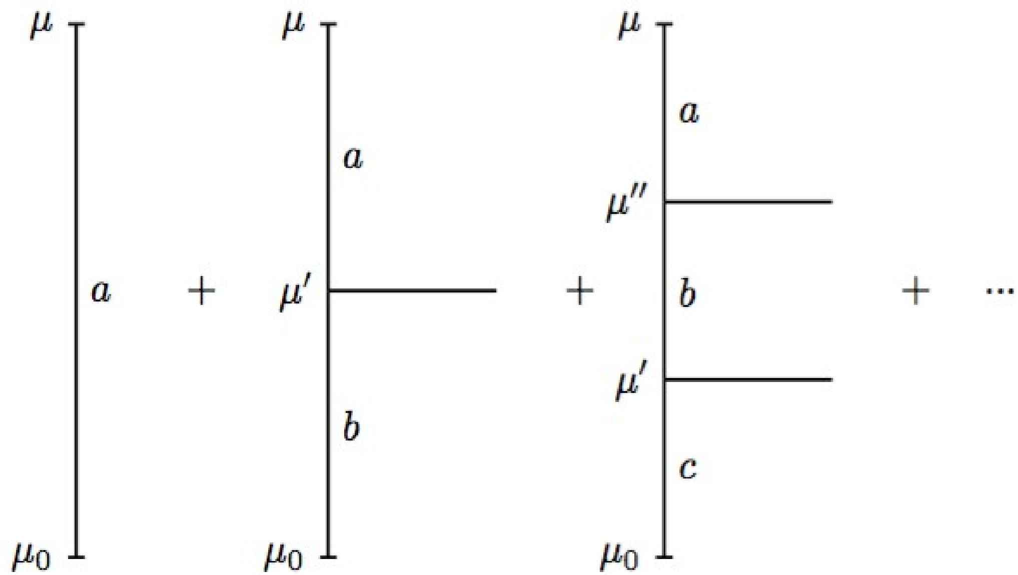


$$\mathcal{A}_i(\mathbf{b},\mu) \sim \sum_k S_i \otimes C_{ik} \otimes f_k$$

Sudakov form factor — evolution coefficients — pdfs

- can be studied in semi-inclusive DIS at future eh colliders

III. Toward TMD Monte Carlo Event Generators: Parton Branching (PB) including TMDs



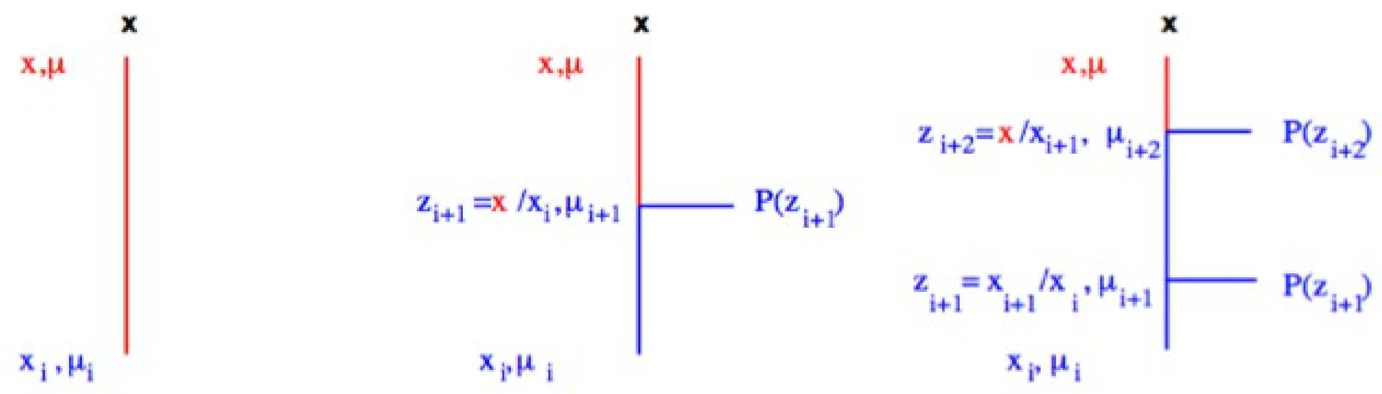
– how to describe TMD evolution in a PB formalism?

– construct the analogue of a parton shower for TMDs?

- connection with DGLAP collinear evolution?

TMDs from Parton Branching (PB)

Jung, Lelek, Radescu, Zlebcik & H, "Collinear and TMD quark and gluon densities from parton branching", JHEP 1801 (2018) 070



PB evolution equation motivated by

– applicability over large kinematic range from low to high transverse momenta

– applicability to exclusive final states and Monte Carlo event generators

PB method:

A new evolution equation for TMDs

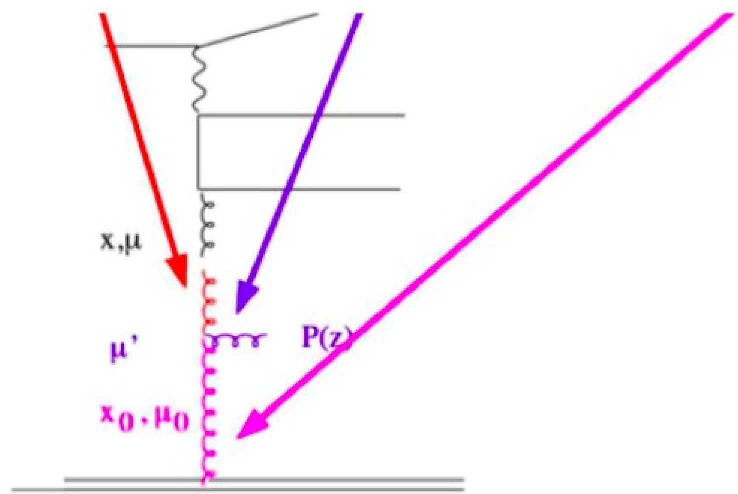
$$\widetilde{A}_a\left(x,\boldsymbol{k},\mu^2\right) = \Delta_a\left(\mu^2,\mu_0^2\right)\widetilde{A}_a\left(x,\boldsymbol{k},\mu_0^2\right) + \sum_b \int \frac{\mathrm{d}^2\boldsymbol{\mu}'}{\pi\mu'^2}\Theta\left(\mu^2-\mu'^2\right)\Theta\left(\mu'^2-\mu_0^2\right)$$

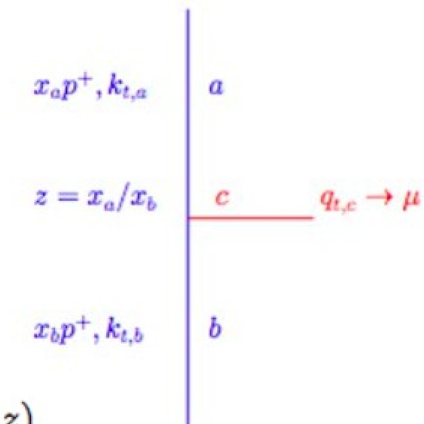
$$\times \int_x^1 \mathrm{d}z\ \Theta\left(z_M(\mu')-z\right)\frac{\Delta_a\left(\mu^2,\mu_0^2\right)}{\Delta_a\left(\mu'^2,\mu_0^2\right)}\ P_{ab}^R\left(z,\alpha_s(b(z)^2\mu'^2)\right)\widetilde{A}_b\left(\frac{x}{z},\boldsymbol{k}+a(z)\boldsymbol{\mu}',\mu'^2\right)$$

$z_M(\mu') = 1 - q_0/\mu'$ $\qquad b(z) = 1 - z$ $\qquad a(z) = 1 - z$

NB: angular ordering

- solvable by iterative MC technique





$\mu = |\mathbf{q}_c|/(1-z)$

where

$$\Delta_a(\mu^2,\mu_0^2) = \exp\left[-\sum_b \int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^1 \mathrm{d}z\ \Theta\left(z_M(\mu')-z\right)\ z\,P_{ba}^R\left(z,\alpha_s\left(b(z)^2\mu'^2\right)\right)\right],\qquad P_{ba}^{(R)}(\alpha_s,z) = \delta_{ba}k_b(\alpha_s)\,\frac{1}{1-z} + R_{ba}(\alpha_s,z)$$

$$k_b(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n k_b^{(n-1)},\quad R_{ba}(\alpha_s,z) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n R_{ba}^{(n-1)}(z)$$

Integrated PB-TMD with angular ordering:

$$\widetilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\widetilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^1 dz$$
$$\times \quad \Theta(1 - q_0/\mu' - z)\, \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a(\mu'^2,\mu_0^2)}\, P^R_{ab}\left(z, \alpha_s\left((1-z)^2\mu'^2\right)\right) \widetilde{f}_b\left(\frac{x}{z},\mu'^2\right)$$

- coincide with CMW result for coherent branching

[Catani-Marchesini-Webber, Nucl. Phys. B349 (1991) 635;

Marchesini-Webber, Nucl. Phys. B310 (1988) 461.]

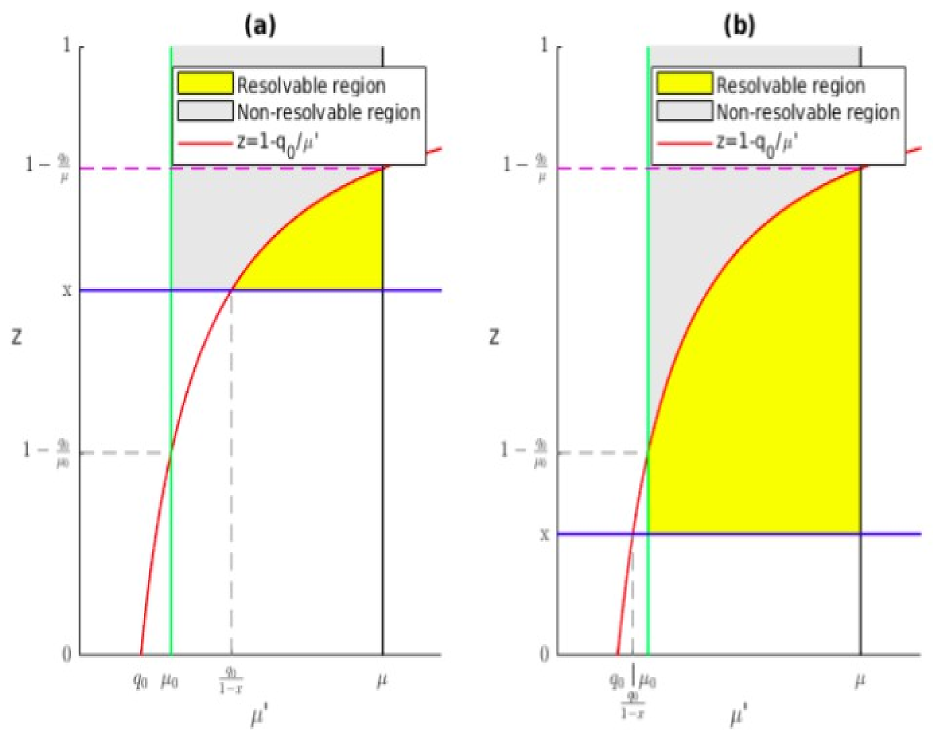
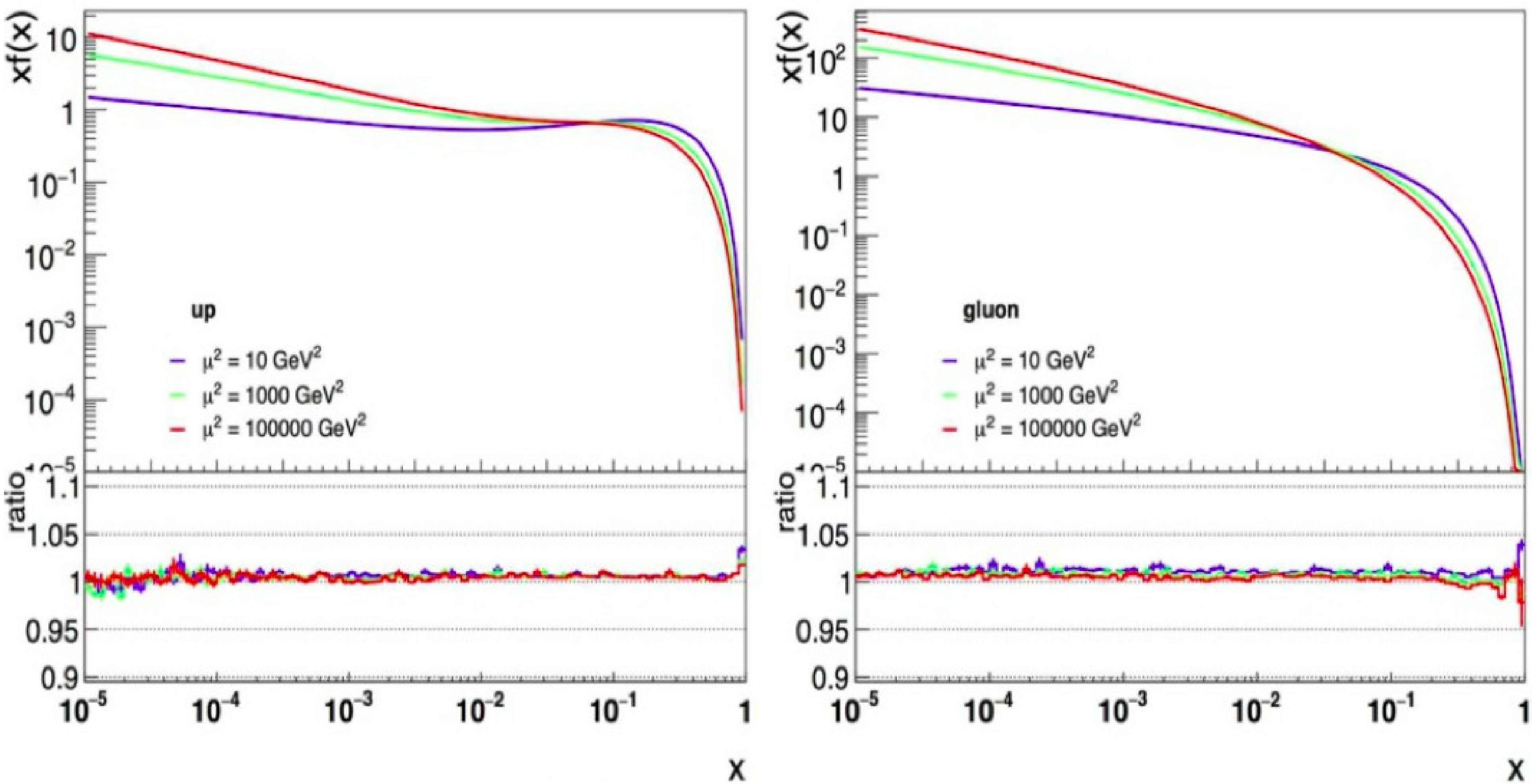


Figure 2: The angular ordering condition $z_M(\mu') = 1 - q_0/\mu'$ with the resolvable and non-resolvable emission regions in the (μ', z) plane: a) the case $1 > x \geq 1 - q_0/\mu_0$; b) the case $1 - q_0/\mu_0 > x > 0$.

Validation at NLO for zM -- > 1 against DGLAP result from QCDNUM

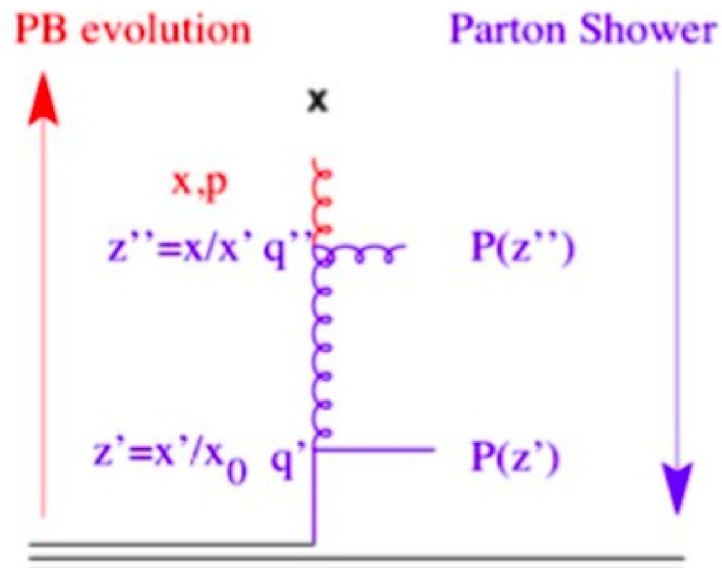


Very good agreement at NLO over all x and mu.
NB: the same approach is designed to work at NNLO.

3D Imaging and Monte Carlo

- **Parton Branching evolution**
  - start from hadron side and evolve from small to large scale μ^2

$$\Delta_s = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P(z) \right)$$



- **Parton Shower**
  - backward evolution from hard scale μ^2 to hadron scale $\mu^2{}_0$ (for efficiency reasons)

$$\Delta_s = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P(z) \frac{\frac{x}{z}\mathcal{A}\left(\frac{x}{z}, k'_\perp, \mu'\right)}{x\mathcal{A}(x, k_\perp, \mu')} \right)$$

➔ in backward evolution, parton density (TMD) imposed further constraint !

APPLICATIONS to DIS and DY

PB method in xFitter

TMD distributions from fits to precision inclusive-DIS data from HERA using the open source QCD platform

xFitter [*S. Alekhin et al., E. Phys. J. C 75 (2014) 304*]

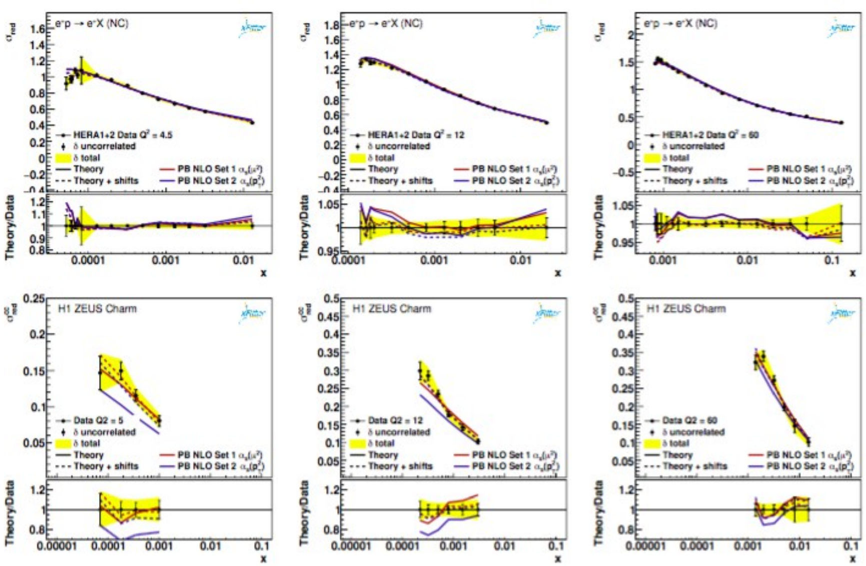


Figure 5: Measurement of the reduced cross section obtained at HERA compared to predictions using Set 1 and Set 2. Upper row: inclusive DIS cross section [11], lower row: inclusive charm production [38]. The dashed lines include the systematic shifts in the theory prediction.

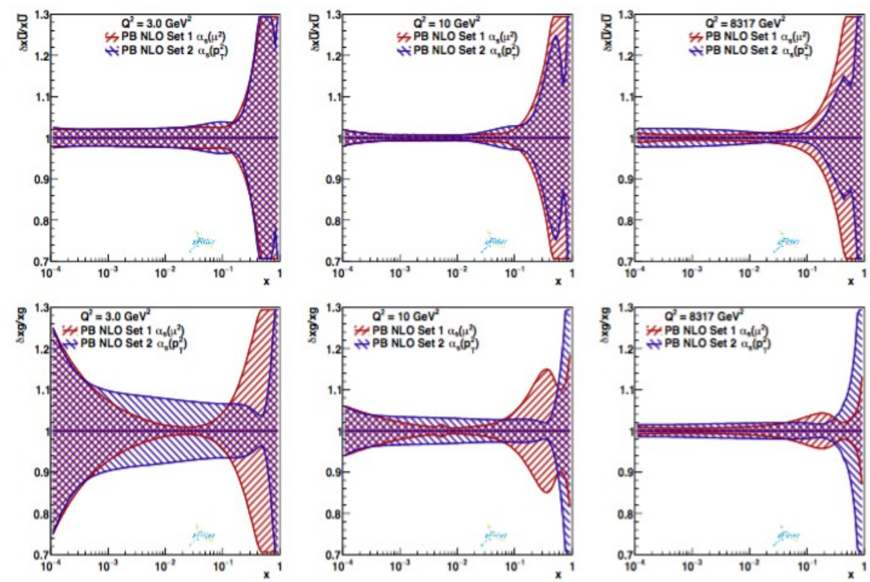


Figure 4: Total uncertainties (experimental and model uncertainties) for the two different sets at different values of the evolution scale μ^2.

A. Bermudez et al., Phys. Rev. D99 (2019) 074008

- NLO determination of TMDs including uncertainties

TMD distribution functions from precision DIS data fits

- NLO determination of TMDs including uncertainties

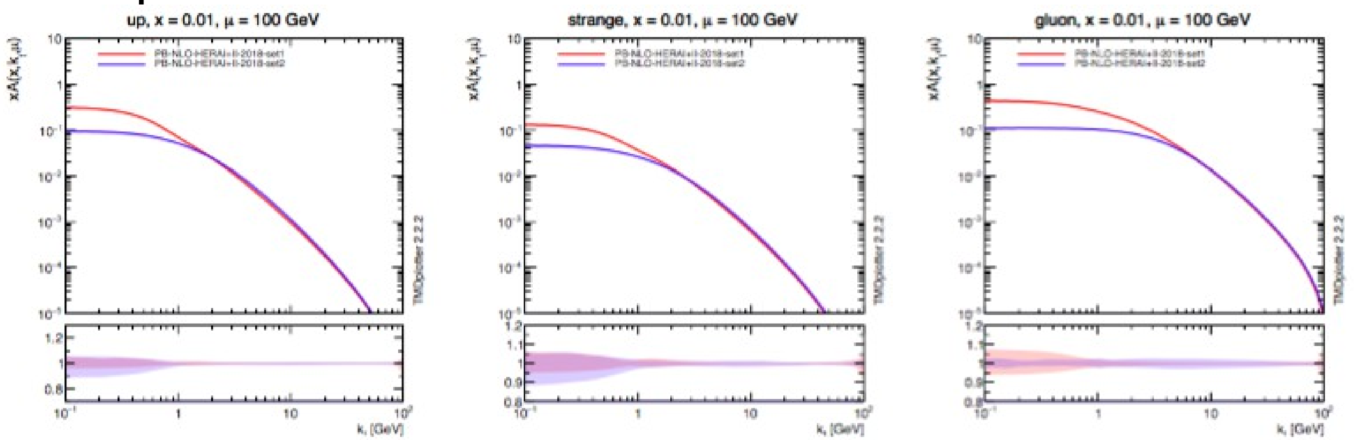
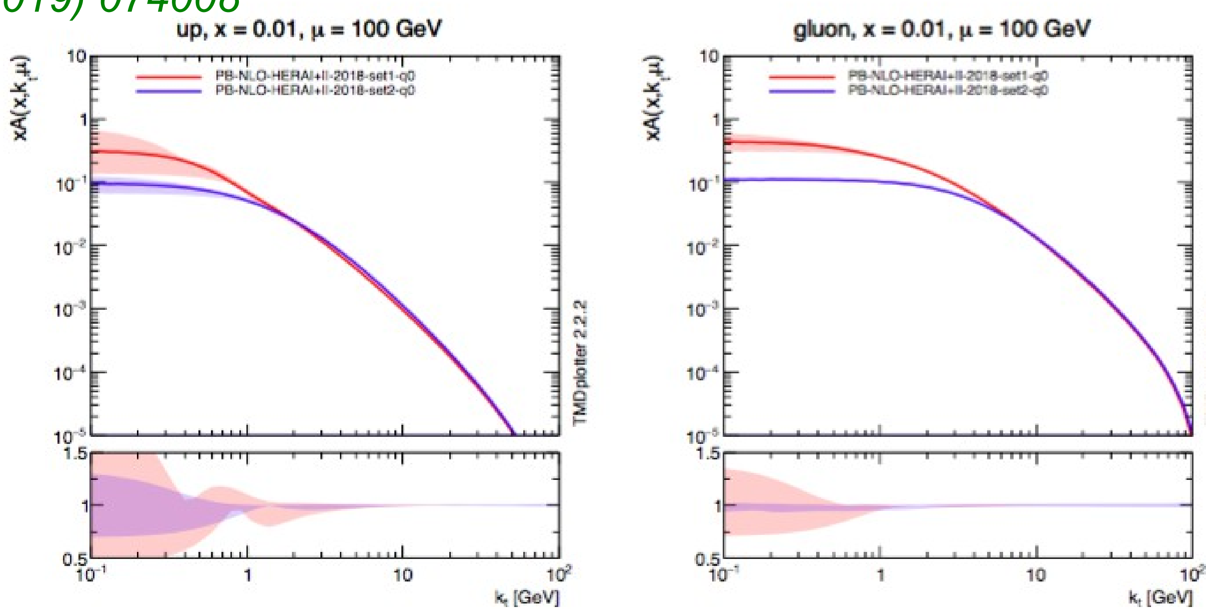


Figure 2: TMD parton distributions for up, strange and gluon (PB-NLO-2018-Set1 and PB-NLO-2018-Set 2) as a function of k_t at $\mu = 100$ GeVand $x = 0.01$.

A. Bermudez et al., Phys. Rev. D99 (2019) 074008



- can improve sensitivity at low kT by precise semi-inclusive DIS measurements at future eh colliders

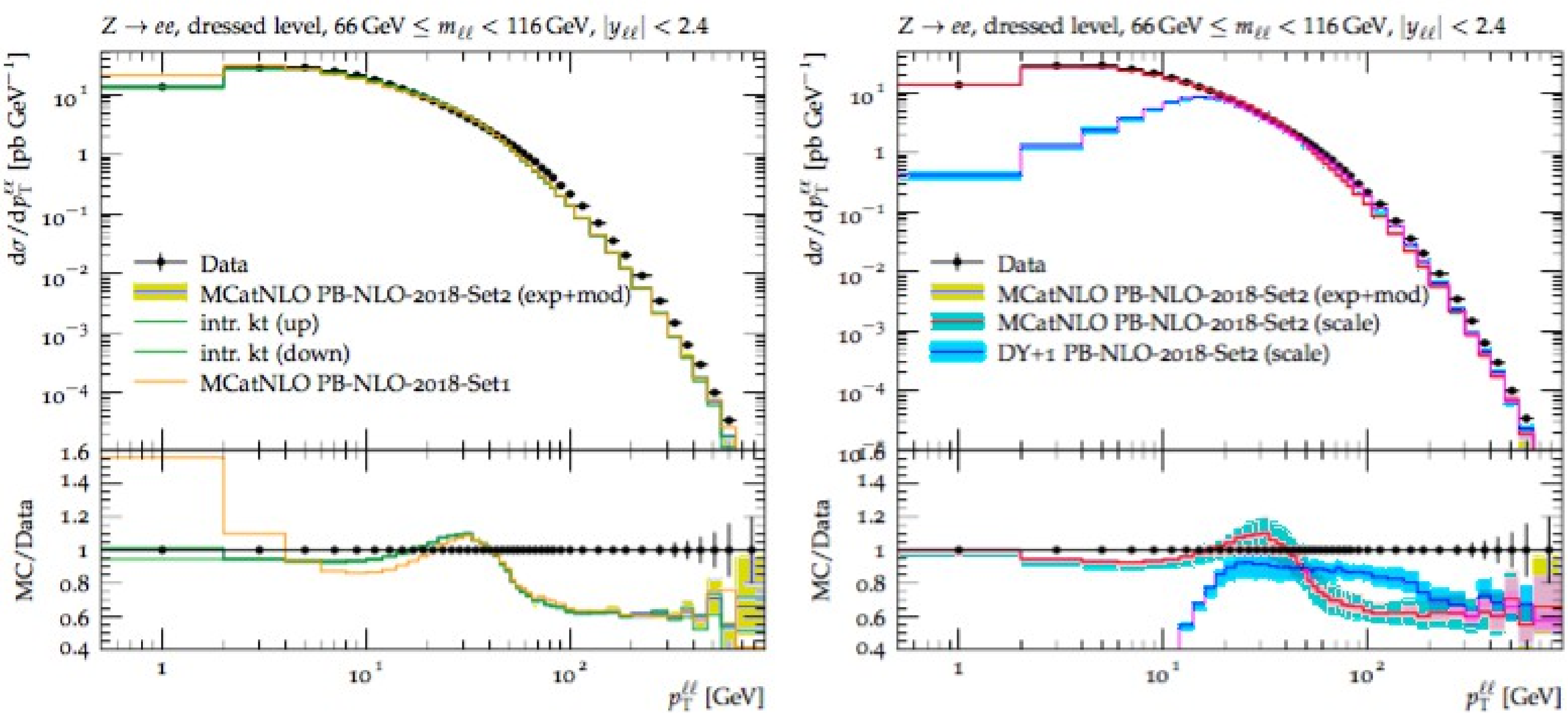
Figure 3: TMD parton distributions for up-quark and gluon (PB-NLO-2018-Set1 and PB-NLO-2018-Set 2) as a function of k_t at $\mu = 100$ GeVand $x = 0.01$ with a variation of the mean of the intrinsic k_t distribution.

Z-boson DY production at the LHC:
TMDs fitted to inclusive DIS + NLO DY calculation

A Bermudez et al, arXiv:1906.00919

- Use MadGraph5_aMC-at-NLO
- Apply PB-TMD
- Set matching scale mu_m (kT < mu_m)

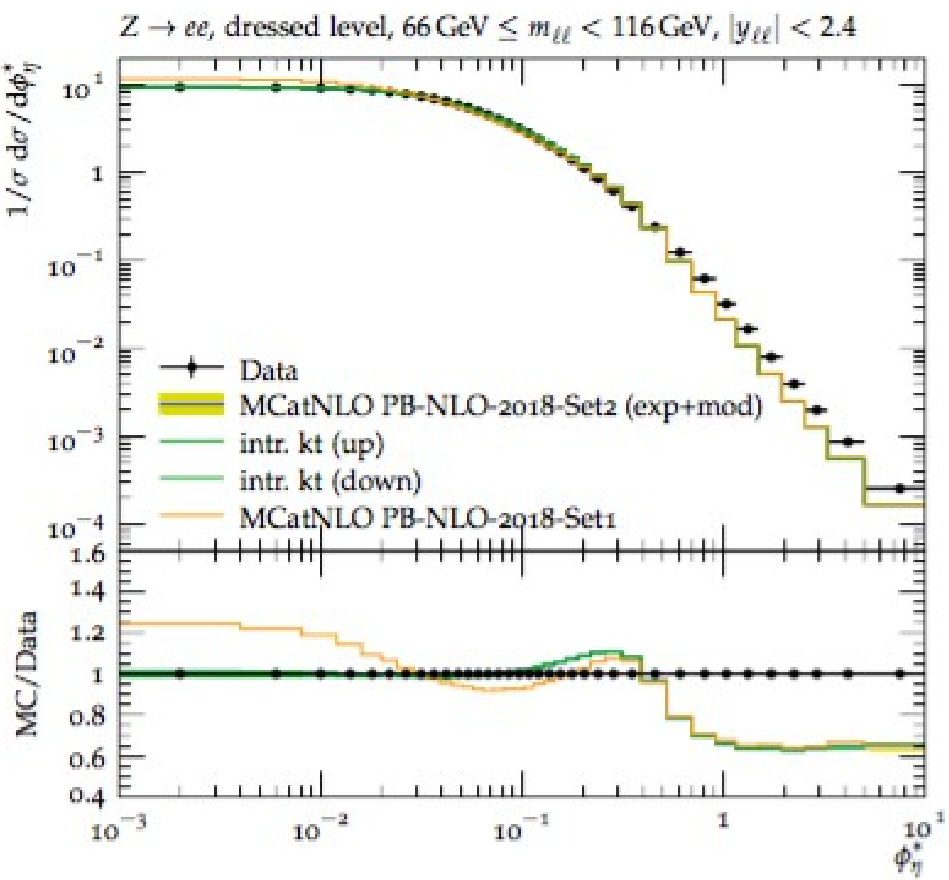
ATLAS 8 TeV data [E. Phys. J. C76 (2016) 291]

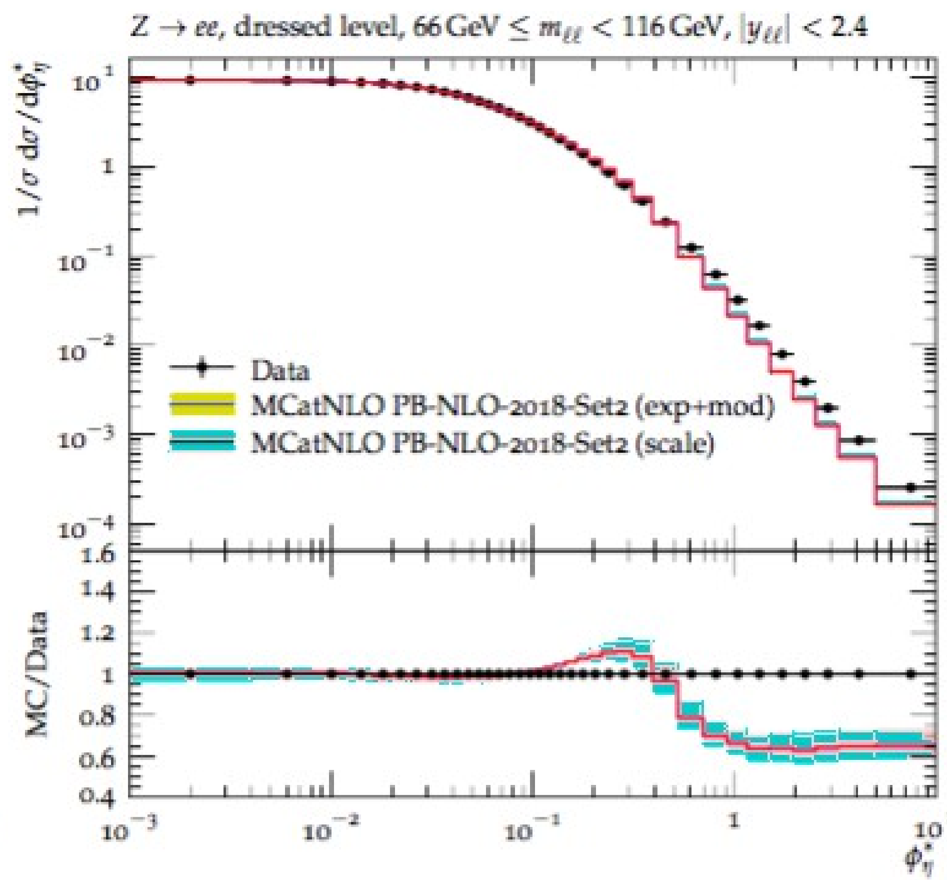


- Theoretical uncertainties dominated by scale dependences; TMD uncertainties moderate
- Low-pT spectrum sensitive to angular ordering (PB-TMD Set 2)
- Missing higher orders at high pT: see DY + 1 jet contribution

Z-boson DY production at the LHC:
TMDs fitted to inclusive DIS + NLO DY calculation

A Bermudez et al, arXiv:1906.00919





ATLAS 8 TeV data [E. Phys. J. C76 (2016) 291]

Conclusions

- TMD effects control QCD resummations for high energies and for high masses -- > needed for high-precision predictions

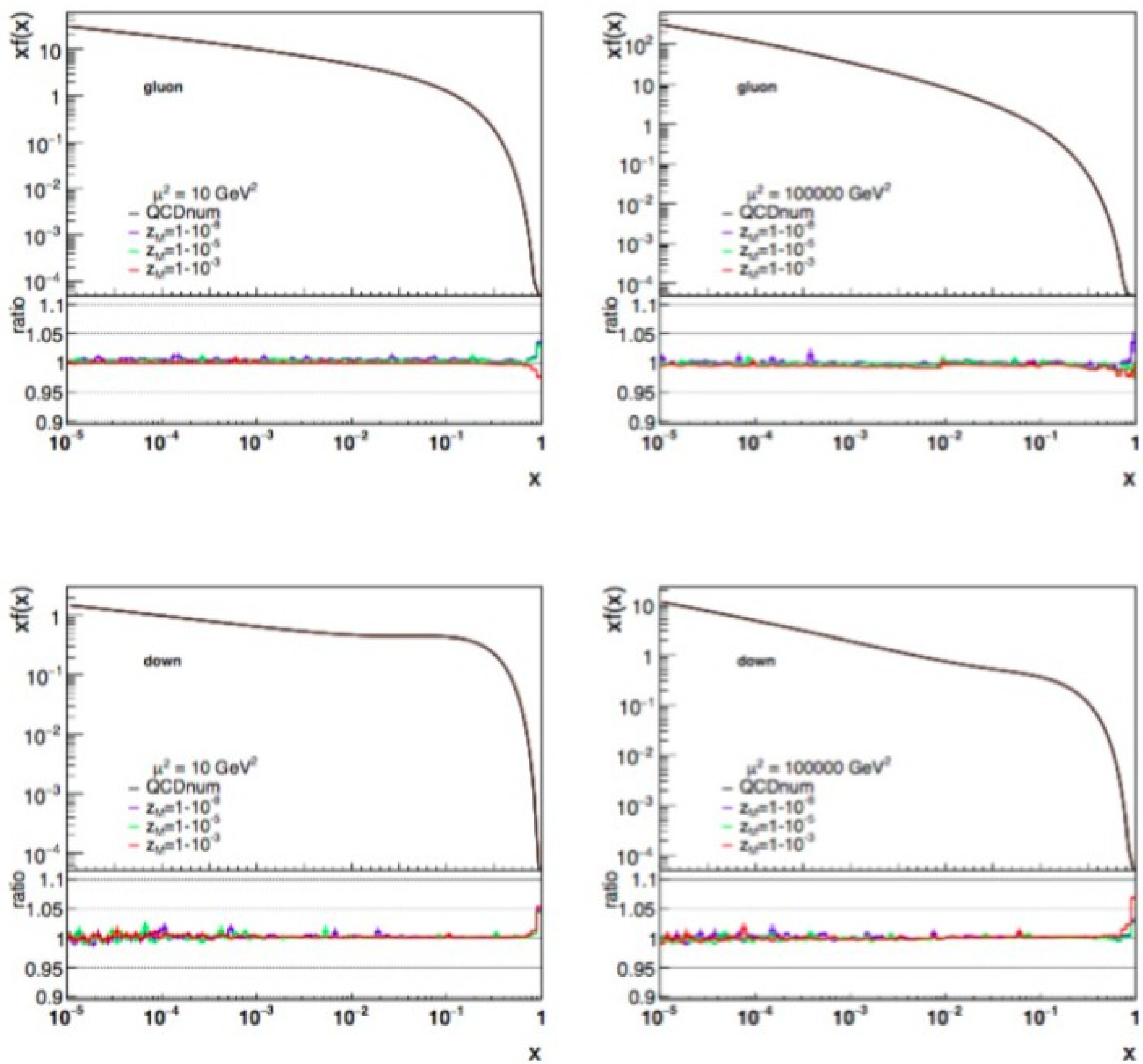
- 3D hadron structure impacts LHC and future hadron-hadron experiments

- Can be explored in semi-inclusive DIS processes at LHeC, FCC-eh

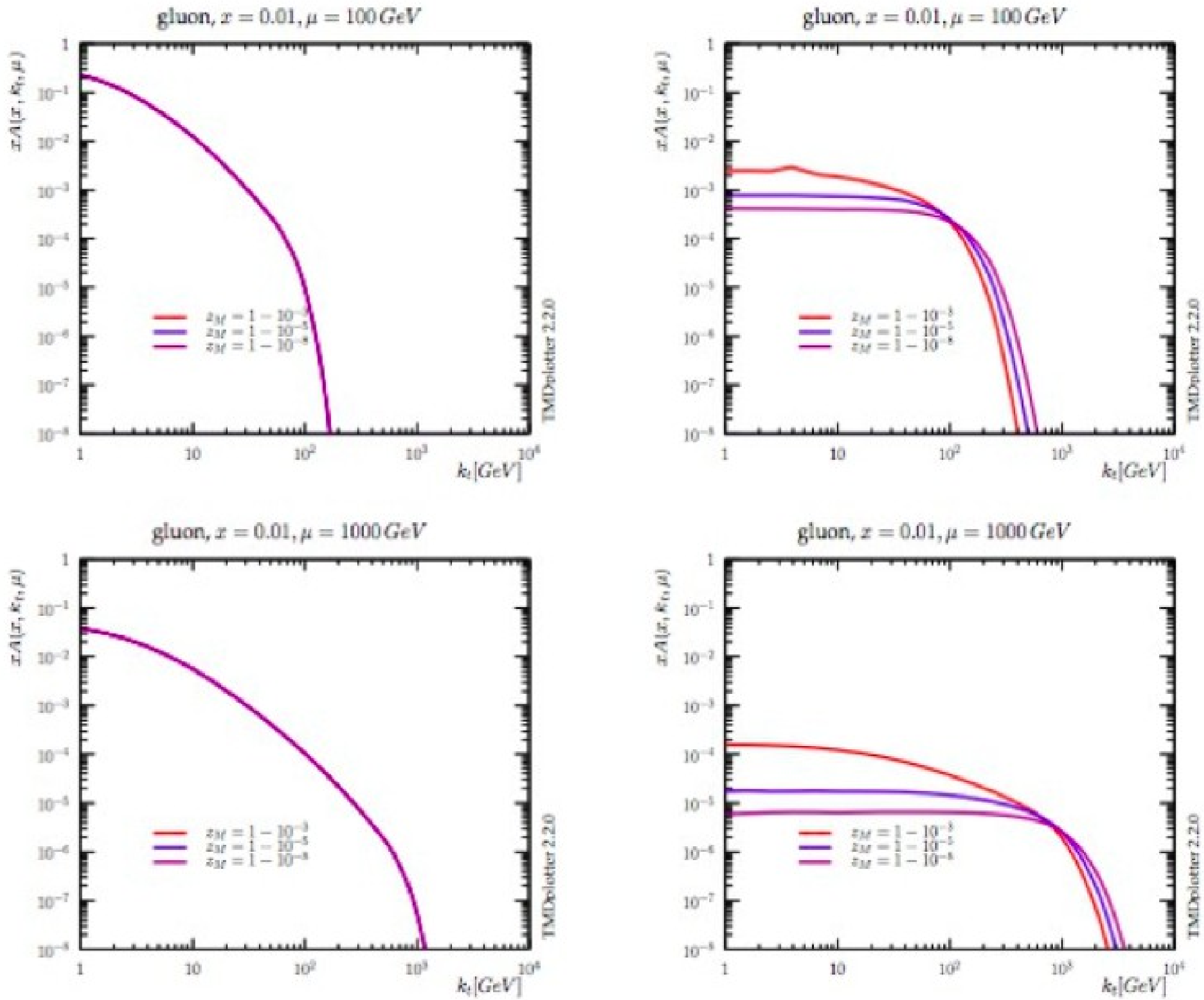
- PB formalism to take into account simultaneously soft-gluon emission at z -> 1 and transverse momentum qT recoils in the parton branchings along the QCD cascade
  - - > step toward Monte Carlo generators including TMDs

EXTRA SLIDES

Stability with respect to resolution scale z_M



TMDs and soft-gluon resolution effects



angular ordering

transverse momentum ordering

Well-defined TMDs require appropriate ordering condition

Comparison with
CSS (Collins-Soper-Sterman) resummation

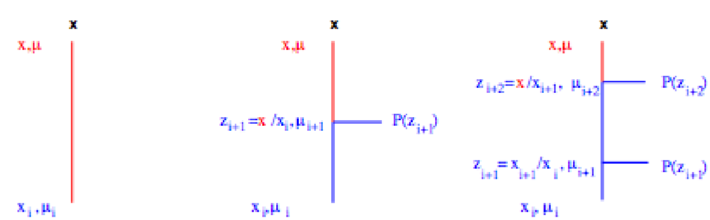
◇ The resummed DY differential cross section is given by

$$\frac{d\sigma}{d^2\mathbf{q}dQ^2dy} = \sum_{q,\bar{q}} \frac{\sigma^{(0)}}{s} H(\alpha_S) \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{q}\cdot\mathbf{b}} \mathcal{A}_q(x_1,\mathbf{b},Q)\mathcal{A}_{\bar{q}}(x_2,\mathbf{b},Q) + \mathcal{O}\left(\frac{|\mathbf{q}|}{Q}\right) \quad \text{where}$$

$$\begin{aligned} \mathcal{A}_i(x,\mathbf{b},Q) \quad = \quad & \exp\left\{\frac{1}{2}\int_{c_0/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2}\left[A_i(\alpha_S(\mu'^2))\ln\left(\frac{Q^2}{\mu'^2}\right) + B_i(\alpha_S(\mu'^2))\right]\right\} G_i^{(\mathrm{NP})}(x,\mathbf{b}) \\ & \times \sum_j \int_x^1 \frac{dz}{z} C_{ij}\left(z, \alpha_S\left(\frac{c_0}{\mathbf{b}^2}\right)\right) f_j\left(\frac{x}{z}, \frac{c_0}{\mathbf{b}^2}\right) \end{aligned}$$

and the coefficients H, A, B, C have power series expansions in α_S.

◇ The parton branching TMD is expressed in terms of real-emission $P^{(R)}$:



▷ via momentum sum rules, use unitarity to relate $P^{(R)}$ to virtual emission

▷ identify the coefficients in the two formulations, order by order in α_S, at LL, NLL, ...

Comparison with CSS (Collins-Soper-Sterman) resummation

More precisely:

▷ The parton branching TMD contains Sudakov form factor in terms of

$$P_{ab}^{(R)}(\alpha_s, z) = K_{ab}(\alpha_s) \frac{1}{1-z} + R_{ab}(\alpha_s, z) \quad \text{where}$$

$$K_{ab}(\alpha_s) = \delta_{ab} k_a(\alpha_s) , \quad k_a(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n k_a^{(n-1)} , \quad R_{ab}(\alpha_s, z) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n R_{ab}^{(n-1)}(z)$$

▷ Via momentum sum rules, use unitarity to re-express this in terms of

$$P^{(V)} = P - P^{(R)} , \quad \text{where}$$

$$P_{ab}(\alpha_s, z) = D_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s) \frac{1}{(1-z)_+} + R_{ab}(\alpha_s, z)$$

is full splitting function (at LO, NLO, etc.)

$$\text{with} \quad D_{ab}(\alpha_s) = \delta_{ab} d_a(\alpha_s) , \quad d_a(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n d_a^{(n-1)}$$

▷ Identify $d_a(\alpha_s)$ and $k_a(\alpha_s)$ with resummation formula coefficients (LL, NLL, . .)

Comparison with CSS (Collins-Soper-Sterman) resummation

- $d_a(\alpha_s)$ and $k_a(\alpha_s)$ perturbative coefficients

$$\text{one} - \text{loop} \ :$$

$$d_q^{(0)} = \frac{3}{2} C_F \quad , \quad k_q^{(0)} = 2 \ C_F$$

$$\text{two} - \text{loop} \ :$$

$$d_q^{(1)} = C_F^2 \left(\frac{3}{8} - \frac{\pi^2}{2} + 6 \ \zeta(3) \right) + C_F C_A \left(\frac{17}{24} + \frac{11\pi^2}{18} - 3 \ \zeta(3) \right) - C_F T_R N_f \left(\frac{1}{6} + \frac{2\pi^2}{9} \right) ,$$

$$k_q^{(1)} = 2 \ C_F \ \Gamma \ , \quad \text{where} \ \Gamma = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - T_R N_f \frac{10}{9}$$

- The k and d coefficients of the PB formalism match, order by order, the A and B coefficients of the CSS formalism:

$$\text{LL} \ : \quad k_q^{(0)} = 2 \ C_F = 2 \ A_q^{(1)}$$

$$\text{NLL} \ : \quad k_q^{(1)} = 2 \ C_F \ \Gamma = 4 \ A_q^{(2)} \ ; \quad d_q^{(0)} = \frac{3}{2} C_F = -B_q^{(1)}$$

$$\text{NNLL} \ : \quad \text{analysis in progress}$$

Where to find TMDs? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of the “Resummation, Evolution, Factorization” Workshop

- A library of parameterizations and fits of TMDs (LHAPDF-style)

  http://tmdlib.hepforge.org
  http://tmdplotter.desy.de

- Also contains collinear (integrated) pdfs

Eur. Phys. J. C (2014) 74:3220
DOI 10.1140/epjc/s10052-014-3220-9

THE EUROPEAN PHYSICAL JOURNAL C

Special Article - Tools for Experiment and Theory

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

F. Hautmann[1,2], H. Jung[3,4], M. Krämer[3], P. J. Mulders[5,6], E. R. Nocera[7], T. C. Rogers[8,9], A. Signori[5,6,a]

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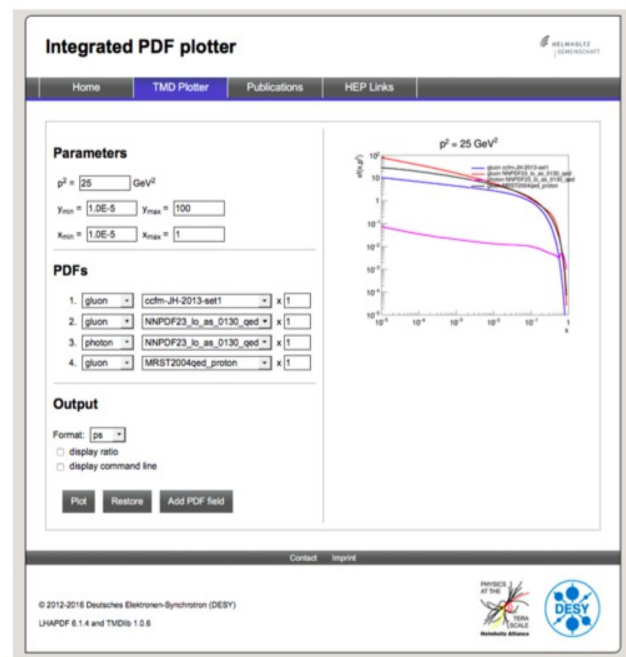
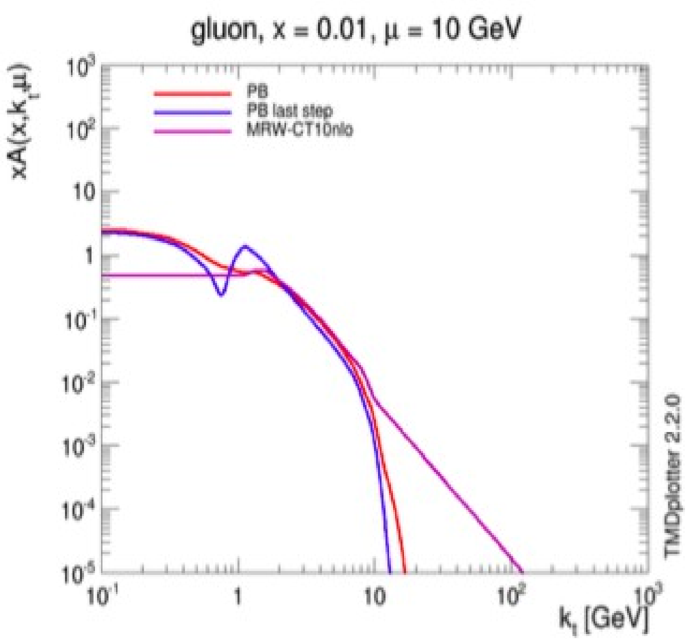
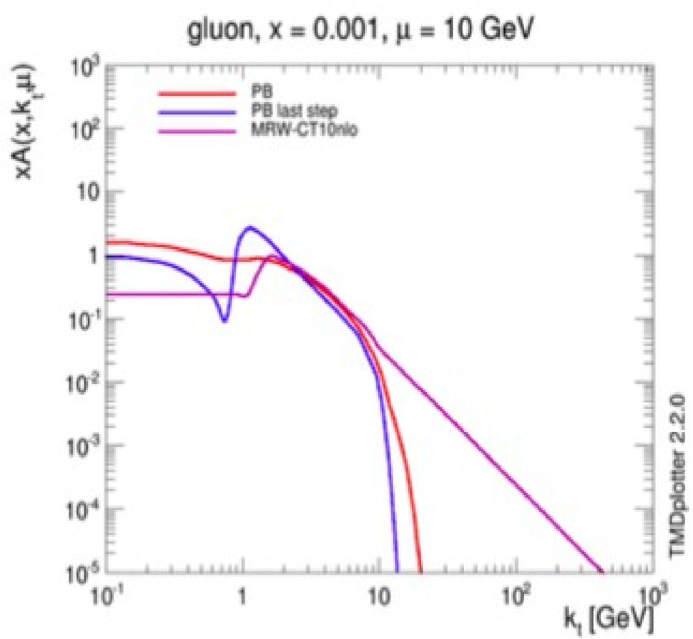


Comparison with KMRW unintegrated distributions

(Kimber-Martin-Ryskin-Watt)

KMRW : - transverse momentum generated by last emission
- radiation populates different phase space region
- no rescaling of transverse momenta in Sudakov form factor
- differs in treatment of non-resolvable processes



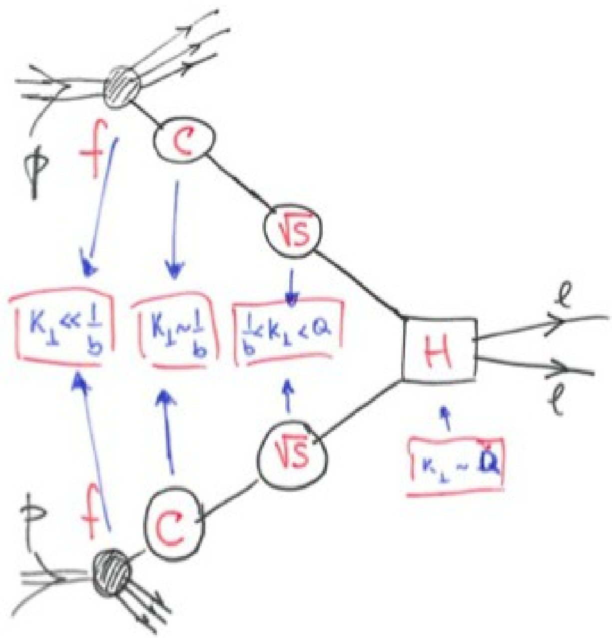


arXiv:1908.08524

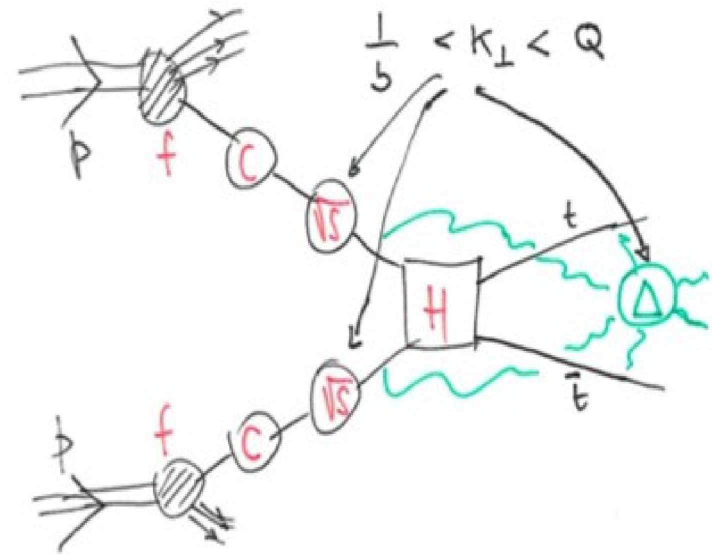
- low-kT kink from single-emission picture;
- high-kT tail from radiative effects + Sudakov

From color-neutral to color-charged final states

Color neutral:



Color charged:



- New long-time correlations in color-charged case:

$$\left(\frac{d\sigma}{d^4q}\right)_{t\bar{t}} = \sum_{ija_1a_2} \int d^2\mathbf{b}\; e^{i\mathbf{q}_T\cdot\mathbf{b}} \int dz_1 \int dz_2\; S(Q,\mathbf{b})\; f_{a_1} \otimes [\mathrm{Tr}(H\Delta)C_1C_2]_{ija_1a_2} \otimes f_{a_2}$$

- Generate azimuthal correlations

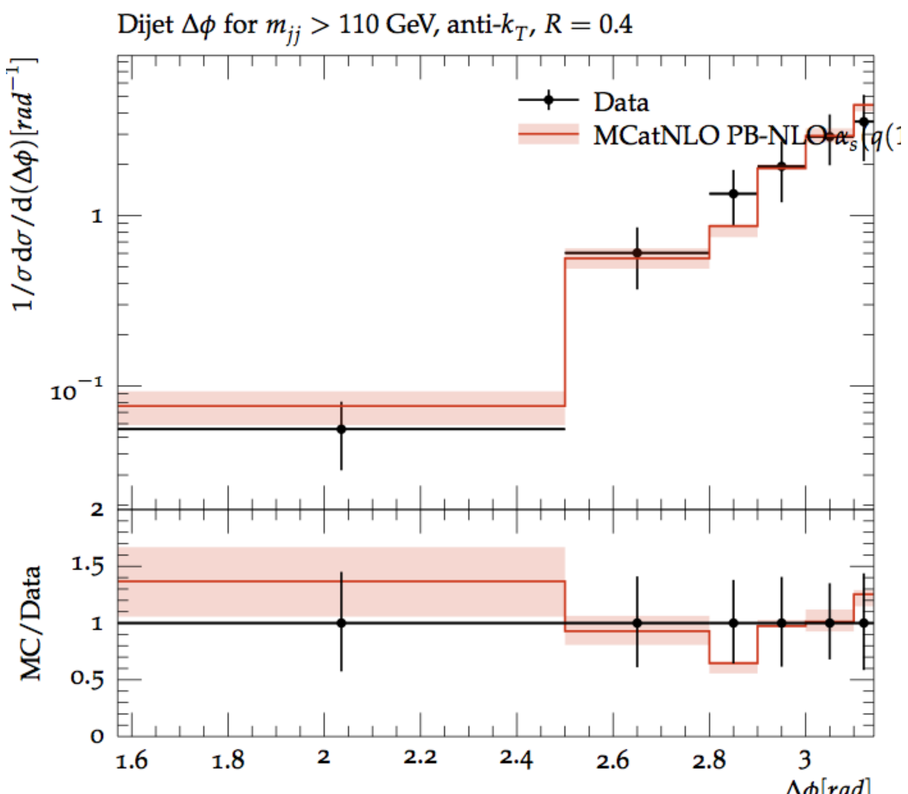
- Observable for $\Delta p_\perp$ high compared to $\Lambda_{\rm QCD}$?

soft gluons coupling initial and final states

Heavy quark hadroproduction:
b-jets at the LHC from PB TMDs + NLO

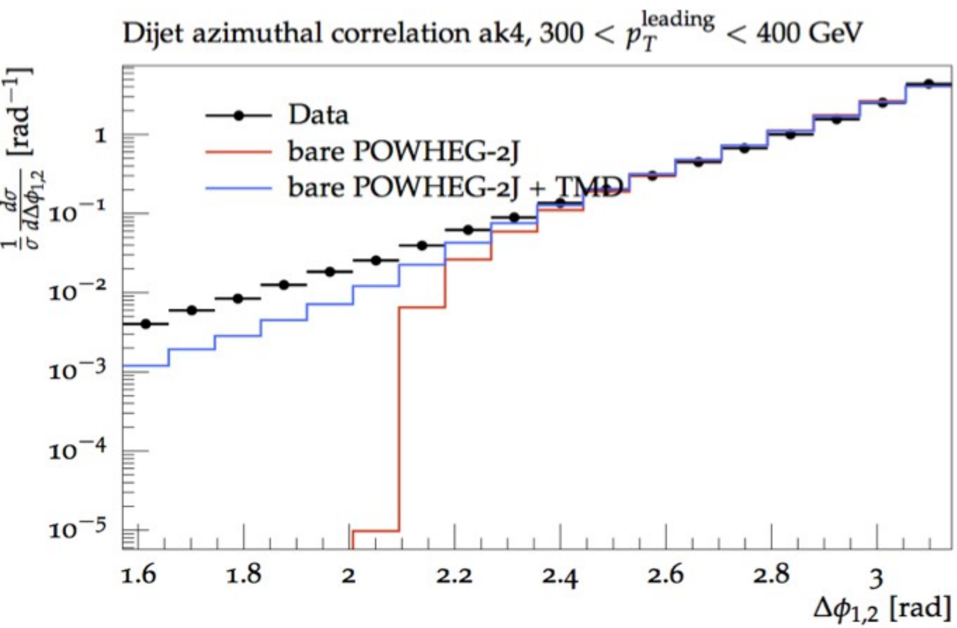
- Use MadGraph5_aMC-at-NLO
- Apply PB-TMD
- Set matching scale mu_m (kT < mu_m)

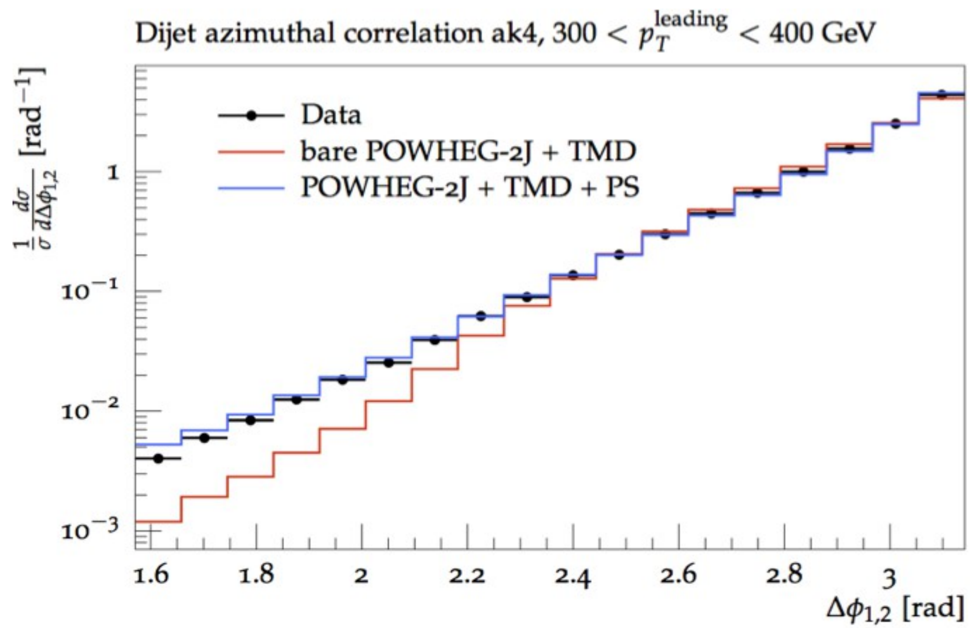
A Bermudez et al, in progress



ATLAS 7 TeV data [E. Phys. J. C71 (2011) 1846]

The role of TMD densities and TMD showers: inclusive jets



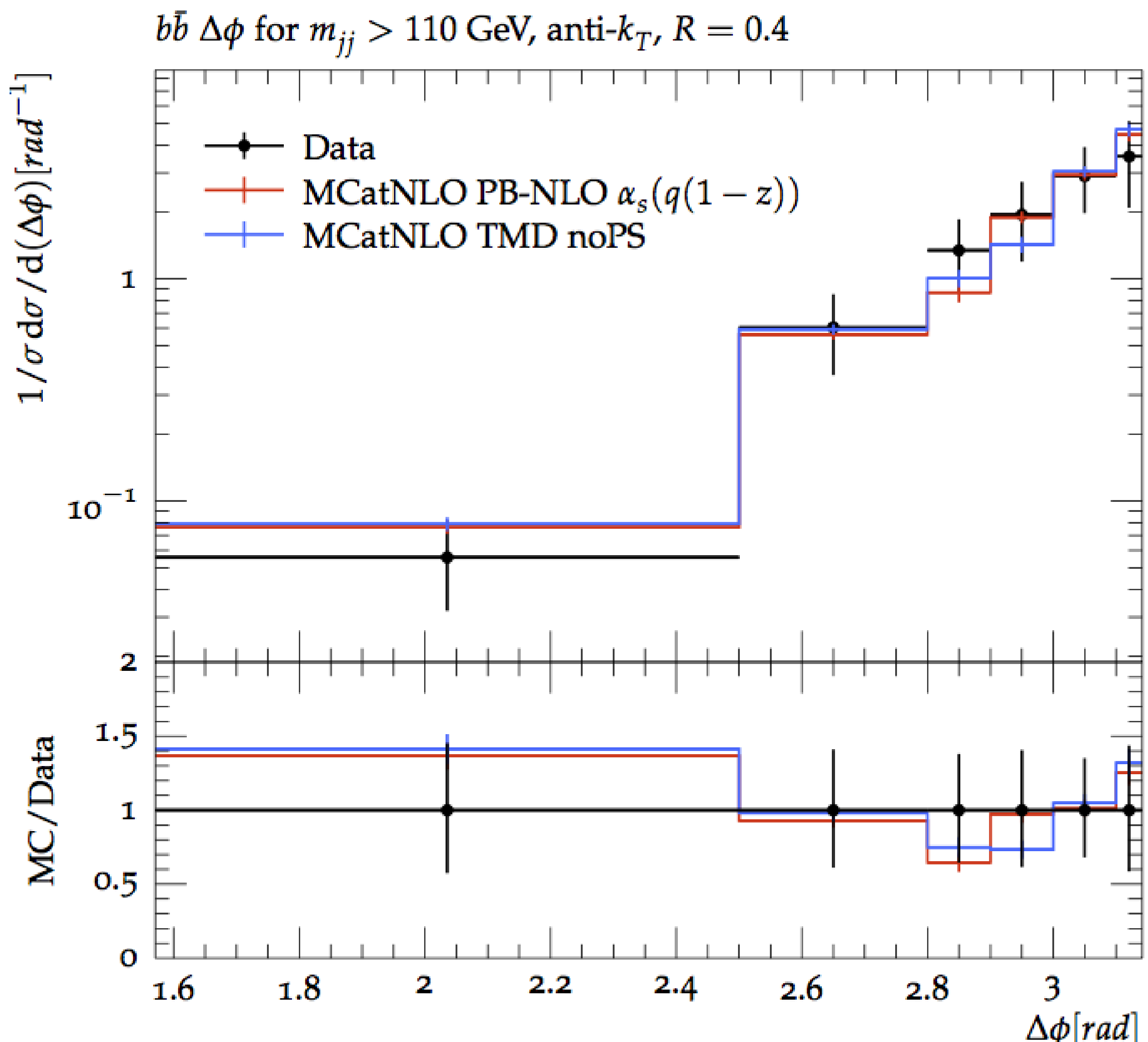


A Bermudez et al, in progress

- Events by NLO POWHEG 2 jets
- PB TMD (with angular ordering)
- TMD parton shower

The role of TMD densities and TMD showers: b-jets

A Bermudez et al, in progress



ATLAS 7 TeV data [E. Phys. J. C71 (2011) 1846]

TMD distributions (unpolarized and polarized)

TABLE I

(Colour on-line) Quark TMD pdfs: columns represent quark polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) *versus* medium gray (pink)) indicate structures that are T-even or T-odd, respectively. T-even and T-odd structures involve, respectively, an even or odd number of spin-flips.

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T}, $h_{1T}^\perp$

TABLE II

(Colour on-line) Gluon TMD pdfs: columns represent gluon polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) *versus* medium gray (pink)) indicate structures that are T-even or T-odd, respectively. T-even and T-odd structures involve, respectively, an even or odd number of spin-flips. Linearly polarized gluons represent a double spin-flip structure.

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

R. Angeles-Martinez et al., “Transverse momentum dependent (TMD) parton distribution functions: status and prospects”, Acta Phys. Polon. B46 (2015) 2501

Non-resolvable emissions and unitarity method

- Introduce resolution scale z_M, where $1 - z_M \sim \mathcal{O}(\Lambda_{\mathrm{QCD}}/\mu)$.

- Classify singular behavior of splitting kernels $P_{ab}(z, \alpha_s)$ in non-resolvable region $1 > z > z_M$:

$$P_{ab}(\alpha_{\mathrm{S}}, z) = D_{ab}(\alpha_{\mathrm{S}})\delta(1-z) + K_{ab}(\alpha_{\mathrm{S}}) \frac{1}{(1-z)_+} + R_{ab}(\alpha_{\mathrm{S}}, z)$$

$$\text{where} \quad \int_0^1 \frac{1}{(1-z)_+} \varphi(z)\, dz = \int_0^1 \frac{1}{1-z} [\varphi(z) - \varphi(1)]\, dz$$

and $R_{ab}(\alpha_{\mathrm{S}}, z)$ contains logarithmic and analytic contributions for $z \to 1$

- Expand plus-distributions in non-resolvable region and use sum rule $\sum_c \int_0^1 z\, P_{ca}(\alpha_{\mathrm{S}}, z)\, dz = 0 \;\; (\text{for any } a)$ to eliminate D-terms in favor of K- and R-terms

$\Rightarrow$ real-emission probabilities exponentiate into Sudakov form factors

- angular ordering: q_t = (1 - z) q'

$$k_\perp = -\sum_i q_{\perp,i}$$

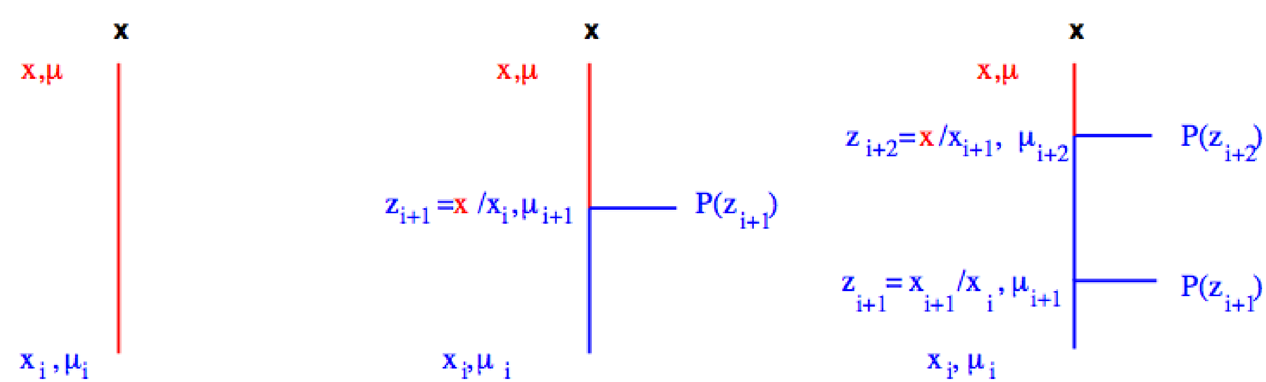
Integrated PB-TMD with $z_M \to 1$ and $\alpha_s \to \alpha_s(\mu'^2)$ ---> collinear PDFs

QCD evolution and soft-gluon resolution scale

[Jung, Lelek, Radescu, Zlebcik & H, PLB772 (2017) 446 + in progress]

$$\widetilde{f}_a(x,\mu^2) = \Delta_a(\mu^2)\, \widetilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz\, P_{ab}^{(R)}(\alpha_{\rm S}(\mu'^2), z)\, \widetilde{f}_b(x/z, \mu'^2)$$

$$\text{where} \quad \Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\; z\, P_{ba}^{(R)}(\alpha_{\rm S}(\mu'^2), z) \right)$$

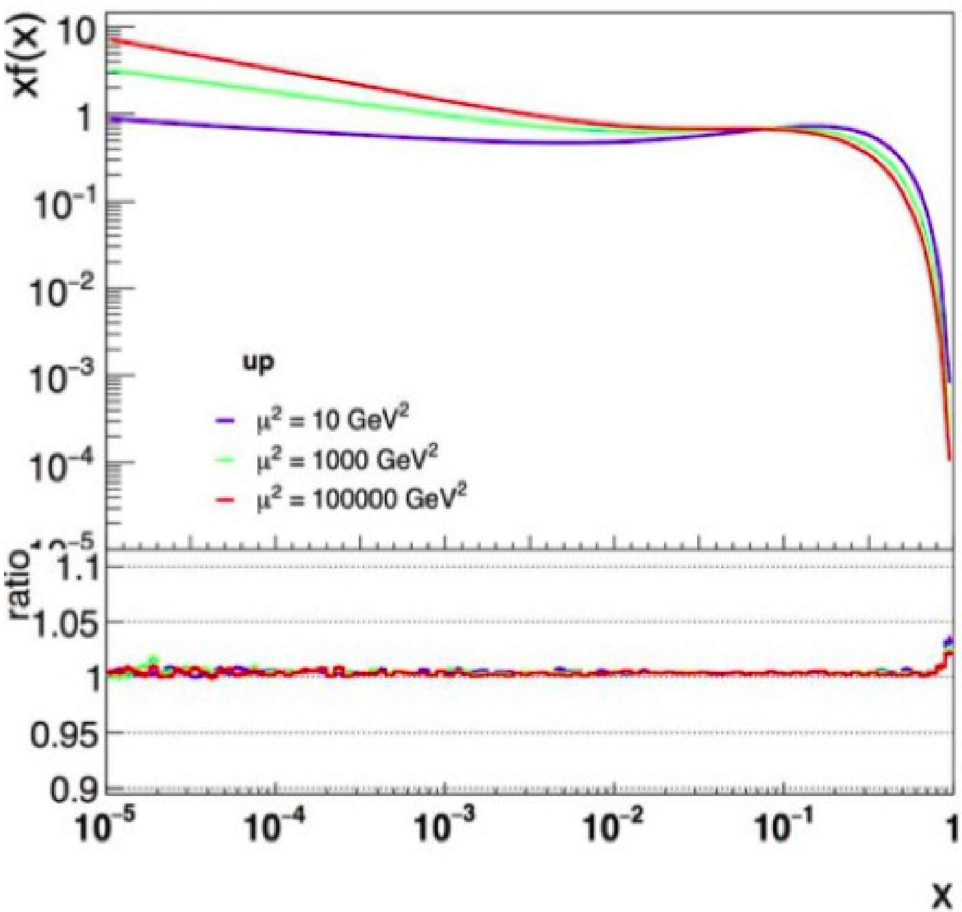


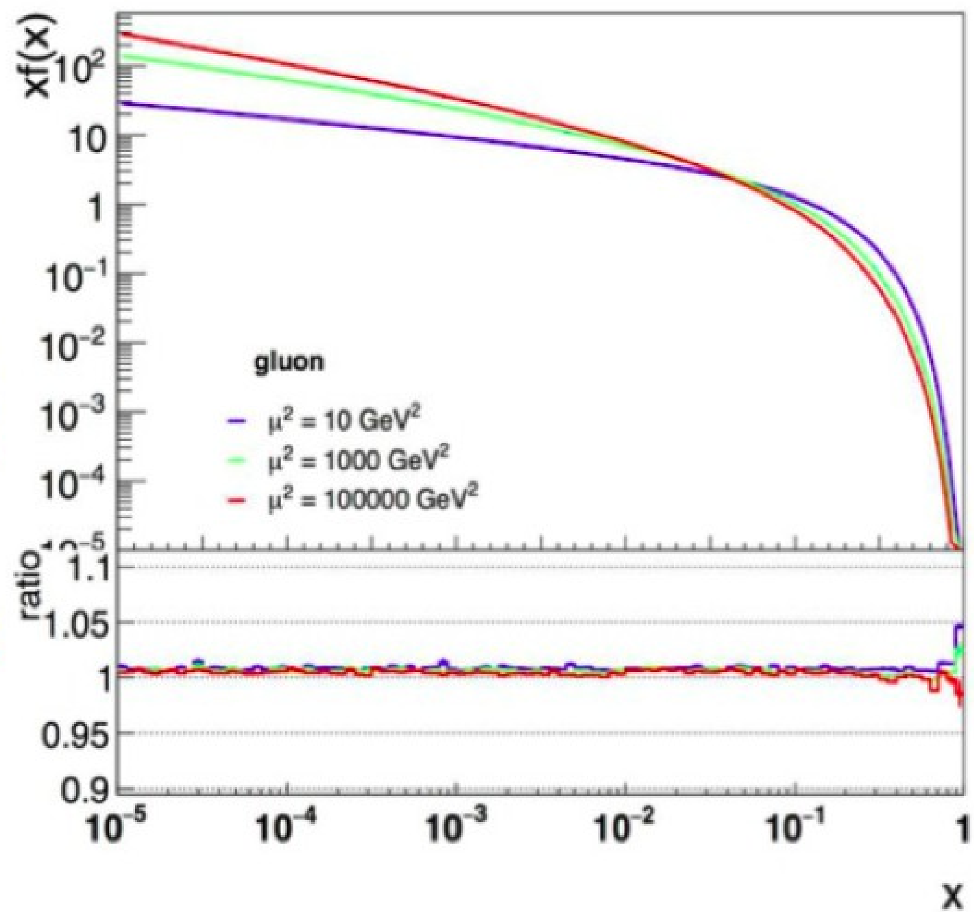
▷ soft-gluon resolution parameter z_M separates resolvable and nonresolvable branchings

▷ no-branching probability Δ; real-emission probability $P^{(R)}$

- Equivalent to DGLAP evolution equation for zM → 1

Validation at LO
against semi-analytic result from QCDNUM





Agreement to better than 1 % over several orders of magnitude in x and mu

See also S. Jadach et al, 2004 – 2010
H. Tanaka et al, 2001 - 2005

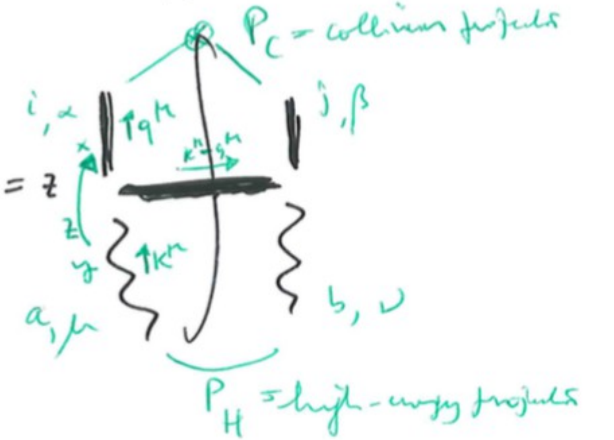
How to extend PB to small-x evolution?

$$\phi_{qg}(x) = \int_x^1 \frac{dy}{y}\, d^{2+2\varepsilon}\underline{k}_\perp \; \mathcal{H}_{qg}\left(\frac{x}{y}, \underline{k}_\perp\right) \mathcal{A}_g(y, \underline{k}_\perp)$$

- Promote splitting functions to TMD splitting functions:

$$K_{qg}(z, \underline{k}_\perp) = z\, \mathcal{H}_{qg}(z, \underline{k}_\perp) = \qquad (3)$$

$$= z$$



$$= \int \frac{d\tilde{q}^2}{\tilde{q}^2} \left(\frac{\tilde{q}^2}{\mu^2}\right)^{\varepsilon} z \frac{1}{(4\pi)^{\varepsilon}} \frac{1}{\Gamma(1+\varepsilon)} \Theta\left(Q^2 - \frac{\tilde{q}^2}{1-z} - z\underline{k}^2\right)$$

$$\cdot \underbrace{\frac{\alpha_s}{2\pi} T_R \left[\frac{\tilde{q}^2}{\tilde{q}^2 + z(1-z)\underline{k}^2}\right]^2 \left\{1 - \frac{2z(1-z)}{1+\varepsilon} + 4z^2(1-z)^2 \frac{\underline{k}^2}{\tilde{q}^2}\right\}}_{\text{TMD splitting function } P_{qg}(z,\, \underline{k}_\perp/\tilde{q}^2)}$$

$$\frac{\alpha_s}{2\pi} T_R \left[\frac{\tilde{q}^2}{\tilde{q}^2 + z(1-z)\underline{k}^2}\right]^2 \left\{\frac{(1-z)^2 + z^2 + \varepsilon}{1+\varepsilon} + 4z^2(1-z)^2 \frac{\underline{k}^2}{\tilde{q}^2}\right\}$$

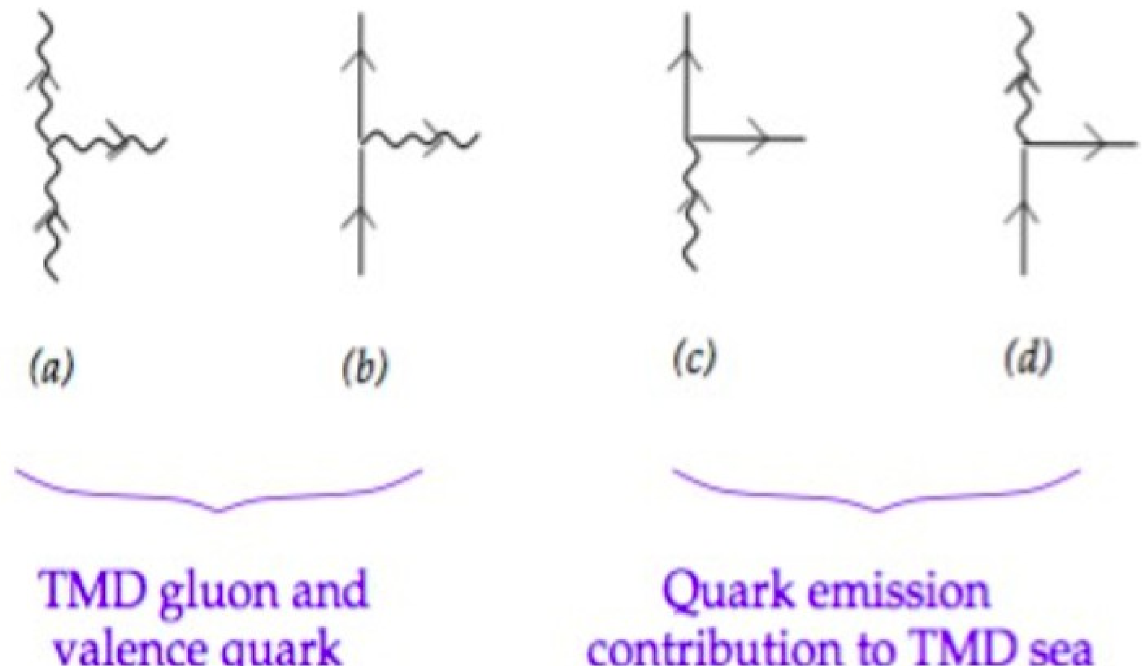
- controls summation of small-x logarithms in gluon-to-quark processes

How to extend PB to small-x evolution?

- The TMD gluon-to-quark splitting function has the schematic structure

$$\mathcal{P}_{g\to q}(z;q_\perp,k_\perp) = P^{(0)}_{qg}(z)\ \left(1+\sum_{n=1}^{\infty} b_n(z)(k_\perp^2/q_\perp^2)^n\right)$$

- Work is underway to extend this to splitting processes in all partonic channels:



Predictions for 13 TeV

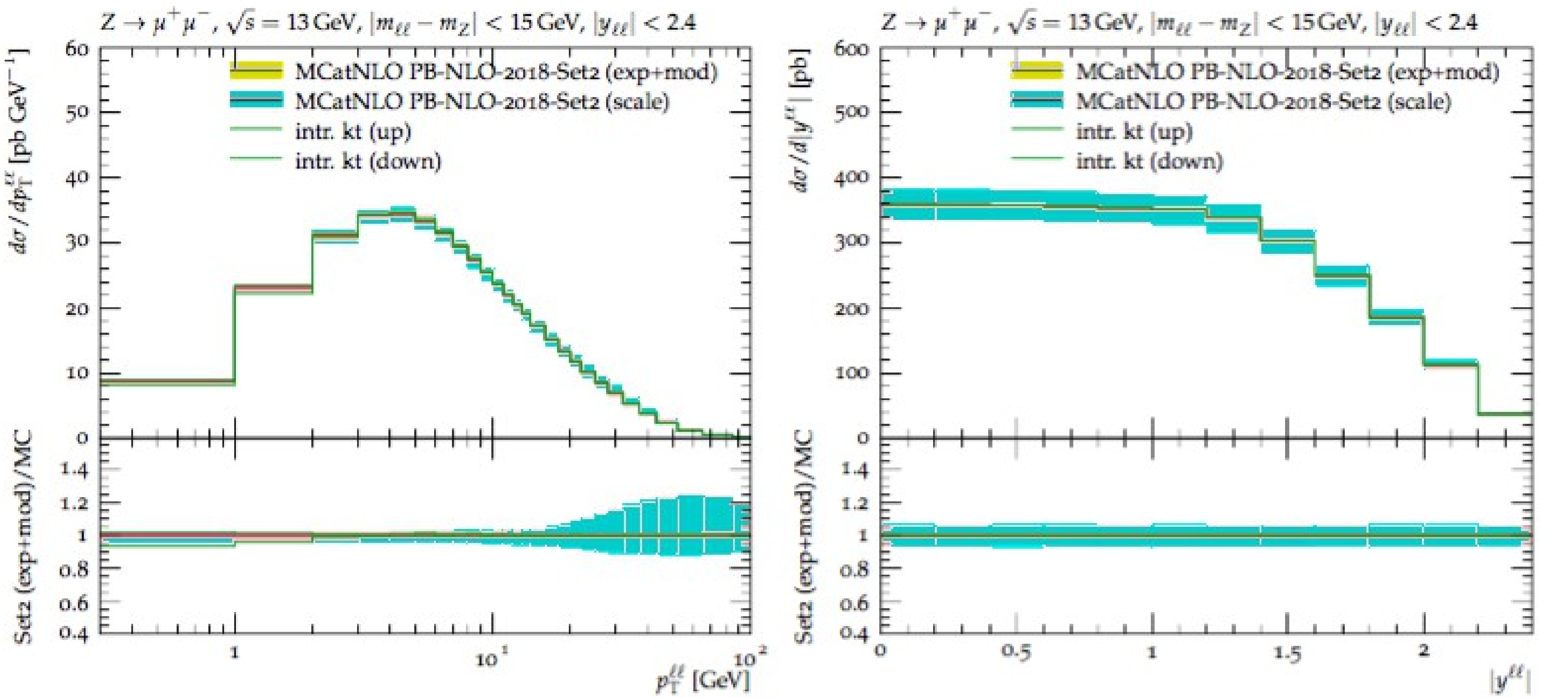
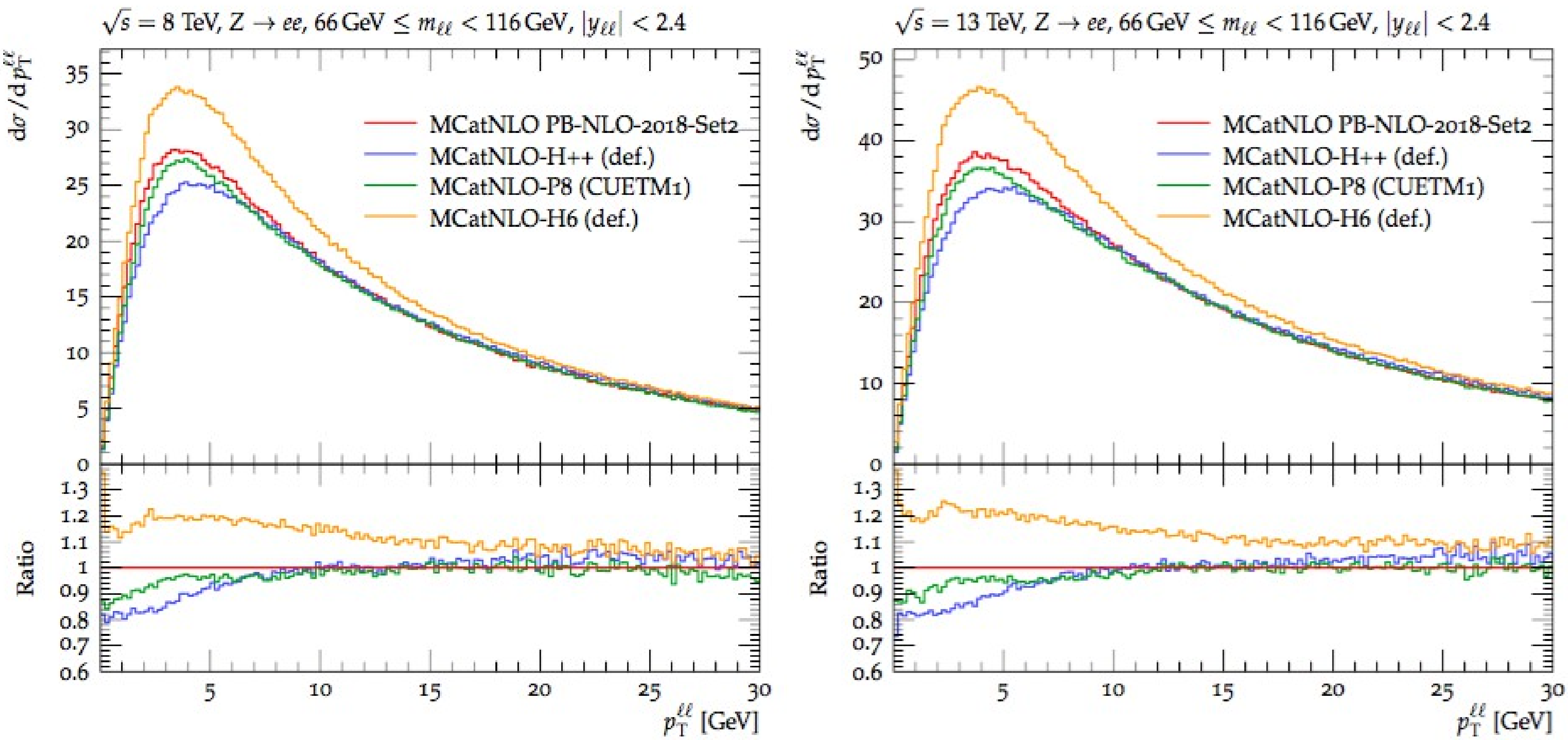


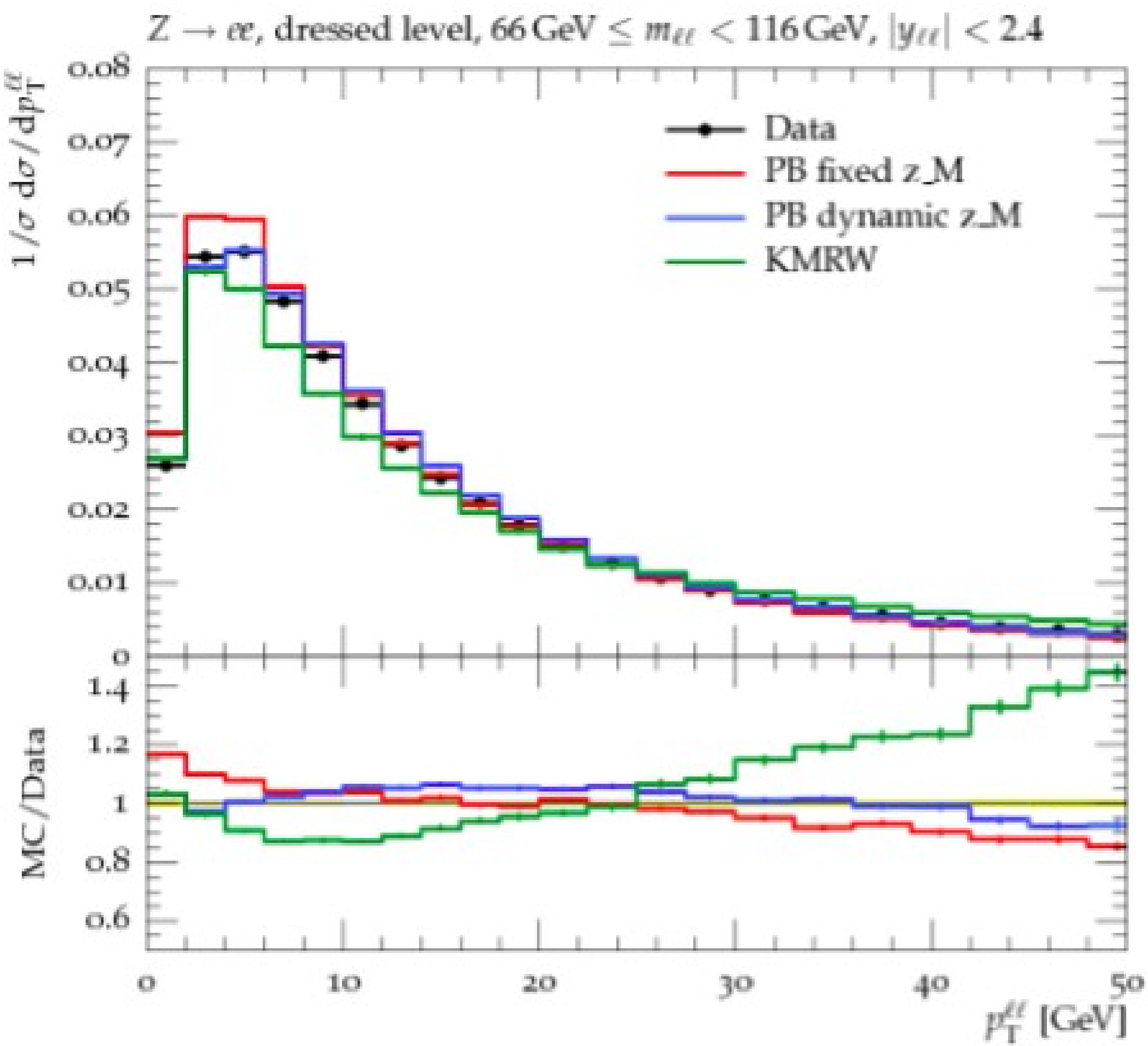
Figure 7: Transverse momentum p_T (left) and rapidity y spectra of Z-bosons at $\sqrt{s} = 13$ TeV from the prediction after including TMDs. The pdf (not visible) and the scale uncertainties are shown. In addition shown are predictions when the mean of the intrinsic gauss distribution is varied by a factor of 2 up and down.

Fine binning at low pT?



- dedicated measurements in the region of Z-boson pT < 5 -10 GeV?

Sensitivity to branching-scale dependent soft-gluon resolution scales

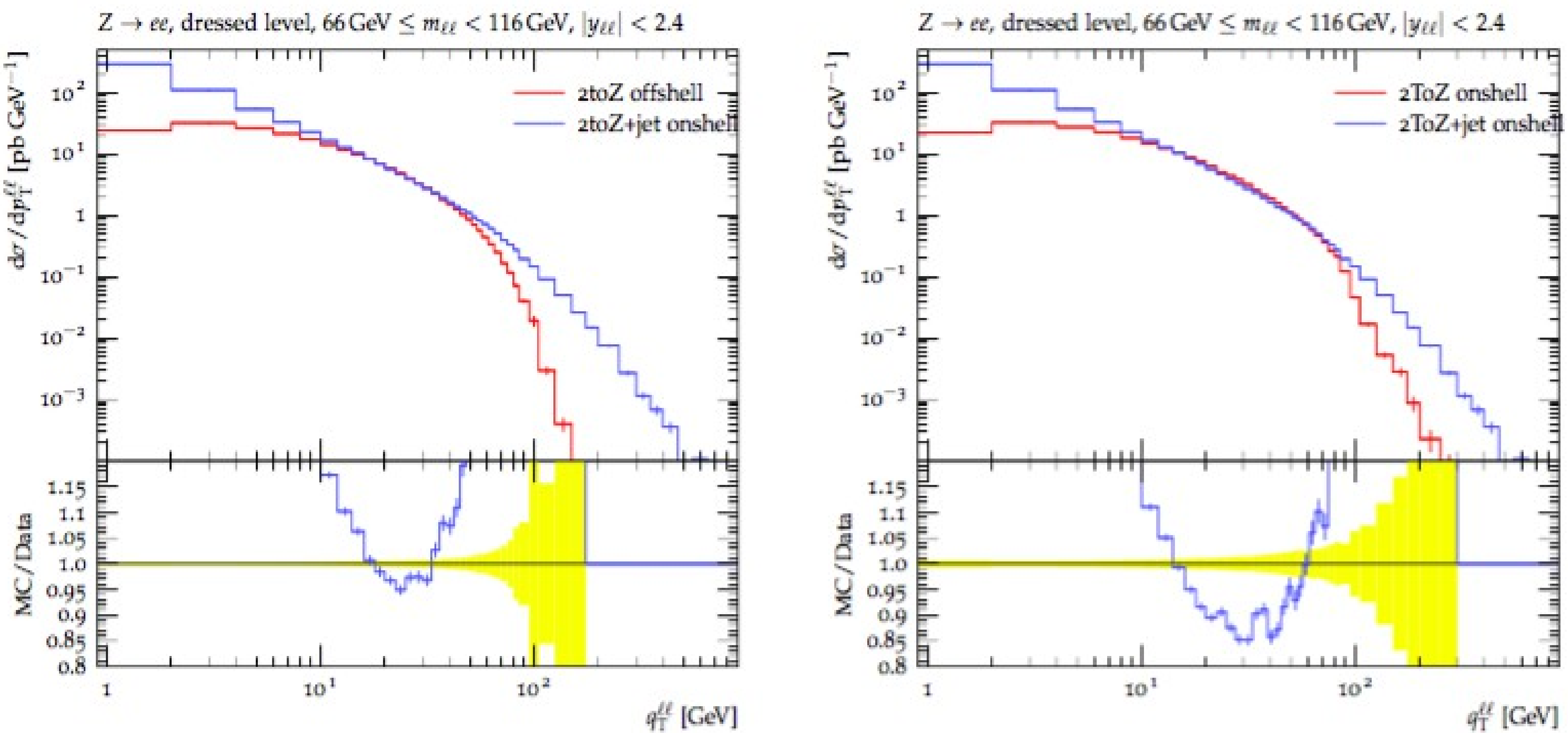


arXiv:1908.08524

Toward new approaches to matching/merging, locally in kT

Matching to hard process: off-shell ME with KaTie

van Hameren, A. CPC, 224, 371, 2018, arXiv 1611.00680



KaTie

[A. Van Hameren, talks at DESY MCEG Workshop, February 2019 and DIS2019 Workshop, April 2019]