LFV and slepton mass splittings at the LHC

António J. R. Figueiredo¹

in collaboration with

A. Abada², J. C. Romão¹ and A. M. Teixeira³.

¹CFTP - Centro de Física Teórica de Partículas / Univ. Técnica de Lisboa,
 ²LPT - Laboratoire de Physique Theorique d'Orsay / Univ. Paris Sud XI,
 ³LPC - Laboratoire de Physique Corpusculaire de Clermont-Ferrand / Univ. Blaise Pascal.

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Introduction: Beyond the SM overview and Motivation

Beyond the SM: supersymmetry + seesaw type-I

- Eases the Higgs fine-tuning problem and offers a solution to the hierarchy problem;
- One possible explanation for small neutrino masses.

$$\mathcal{L}_{\mathsf{dim5}}^{\nu} = \frac{1}{2} \left(\frac{f}{\Lambda} \right)_{ij} (\phi \, i\sigma_2 L_i) \cdot \left(\phi \, i\sigma_2 L_j \right) \Rightarrow \mathcal{M}_{ij}^{\nu} = -\mathcal{v}_u^2 \left(\frac{f}{\Lambda} \right)$$
$$\left(\frac{f}{\Lambda} \right) = \mathcal{Y}^{\nu} \mathcal{M}_R^{-1} \mathcal{Y}^{\nu T} , \quad \mathcal{Y}^{\nu T} = \frac{1}{\mathcal{V}_u} \sqrt{\mathcal{M}_R} \, R \sqrt{\hat{M}^{\nu}} \mathcal{U}_{PMNS}^{\dagger}$$

 $(3 \times 2)_{R} + (3+3)_{PMNS} + 3_{m_{\nu}} + 3_{\mathcal{M}} = 18$ Unknown: 14_{continuous} + 1_{LH hierarchy type}

Motivation: (s)lepton sector

- Lepton flavour information;
- Involved in clean signals at colliders multi-lepton final states;
- Will carry some hint on the true mechanism for generating neutrino masses.

Slepton masses in the CMSSM (1/3)

Slepton masses after EWSB

$$(m_{\tilde{l},LL}^2)_{ij} = (m_{\tilde{L}}^2)_{ij} + \delta_{ij} \left[m_{l_j}^2 + M_Z^2 \left(-1/2 + s_w^2 \right) \cos 2\beta \right]$$

$$(m_{\tilde{l},RR}^2)_{ij} = (m_{\tilde{l}_R}^2)_{ij} + \delta_{ij} \left[m_{l_j}^2 + M_Z^2 s_w^2 \cos 2\beta \right]$$

$$(m_{\tilde{l},RL}^2)_{ij} = (m_{\tilde{l},LR}^2)_{ji}^* = v_d (A')_{ji} - \delta_{ij} \mu m_{l_j} \tan \beta$$

Universal LFC trilinear couplings $\Rightarrow v_d(A^l)_{ji} = \delta_{ij}m_{l_i}A_0$. Universal SU(2)_L \otimes U(1) gaugino masses $(m_{1/2})$ and $\alpha_2 > \alpha_1 = 5\alpha'/3$ $\Rightarrow m_{wino} > m_{bino}$

+ Universal slepton masses (m_0) \Rightarrow $m_{ ilde{L}}$ > $m_{ ilde{l}_R}$

$$\begin{split} m_{\tilde{L}}^2 &\simeq m_0^2 + 0.5 \, m_{1/2}^2 + 0.0375 \, m_{1/2}^2 + \delta m_{\tilde{L}}^2 \\ m_{\tilde{l}_R}^2 &\simeq m_0^2 + 0.15 \, m_{1/2}^2 + \delta m_{\tilde{l}_R}^2 \end{split}$$

where $\delta m_{\tilde{L},\tilde{l}_R}^2$ stands for flavour dependent RGE contributions: $(\delta m_{\tilde{L}}^2)_{ij} \simeq \frac{1}{2} (\delta m_{\tilde{l}_R}^2)_{ij} \simeq -\frac{1}{8\pi^2} \delta_{ij} |Y_i'|^2 \left(m_{H_d}^2 + (m_{\tilde{L}}^2)_{ii} + (m_{\tilde{l}_R}^2)_{ii} + |A_0|^2 \right) \ln \frac{M_{GUT}}{M_{SUSY}}$

Slepton masses in the CMSSM (2/3)

Low energy slepton masses non-universality caused by one unique source, Y', which is communicated through:

- F-type 4-scalar interactions after EWSB $\propto m_{l_i}^2 \leftarrow$ negligible;
- 2 Left-Right (LR) mixing $\propto m_{l_i} (A_0 \mu \tan \beta);$
- Solution Sector Flavoured RGE induced contribution $\propto Y_{l_i} m_0^2$.

Effects 2. and 3. mainly relevant for the stau sector as $m_{\tau} \gg m_{\mu} \gg m_{e}$.

Slepton mass splittings (SMS) $\frac{\Delta m}{m}(x, y) \equiv 2|x - y|/(x + y)$

$$\begin{aligned} \frac{\Delta m}{m}(\tilde{\mu}_L, \tilde{\tau}_2) &\approx \quad \frac{1}{16\pi^2} |Y_{\tau}|^2 \left[3\left(\frac{m_0}{m_{\tilde{L}}}\right)^2 + \left(\frac{A_0}{m_{\tilde{L}}}\right)^2 \right] + \frac{1}{2} \left(\frac{m_{\tau}^2}{m_{\tilde{L}}^2 - m_{\tilde{l}_R}^2}\right) \left(\frac{A_0 - \mu \tan\beta}{m_{\tilde{L}}}\right)^2 \\ &\approx \quad (0.07\% - 2.32\%) \, 10^{-2} c_{\beta}^{-2} + (0.07 - 0.7) \, 10^{-2} t_{\beta}^2 \\ \frac{\Delta m}{m} (\tilde{e}_L, \tilde{\mu}_L) &\approx \quad \left(\frac{m_{\mu}}{m_{\tau}}\right)^2 \frac{\Delta m}{m} (\tilde{\mu}_L, \tilde{\tau}_2) \approx 3.6 \times 10^{-3} \frac{\Delta m}{m} (\tilde{\mu}_L, \tilde{\tau}_2) \end{aligned}$$

assumptions^a: typical $m_{1/2} \approx 400 \text{ GeV}$, $m_0 \approx 100 \text{ GeV}$, $|\mu| \approx \left(200 \text{ GeV} + \sqrt{m_0^2 + 0.5m_{1/2}^2}\right)$ and maximum $|A_0| \approx 1 \text{ TeV}$.

^aStandard window: $m_{\tilde{\chi}_{0}^{0}} - m_{\tilde{l}_{l}} \geq 10$ GeV and $\tilde{\chi}_{1}^{0}$ is the LSP.

Slepton masses in the CMSSM (3/3)



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Mass measurement strategies at the LHC (1/3)

Two standard methods

- Reconstruct entire decay chains by measuring all final state momenta;
- Construct invariant kinematical quantities which are "easy" to measure. Their distribution edges provide information on mass relations between decay chain's intermediate states.

R-parity conserving model \Rightarrow each SUSY event has two stable WIMPs \Rightarrow large amount of missing energy.

\Rightarrow **Conclusion:** 1st method cannot be used.

[B. C. Allanach and C. G. Lester and M. A. Parker and B. R. Webber, **Measuring sparticle masses** in non-universal string inspired models at the LHC, hep-ph/0007009v2]

[Henri Bachacou and Ian Hinchliffe and Frank E. Paige, Measurements of Masses in SUGRA Models at LHC, hep-ph/9907518v1]

SUSY @ LHC occurs primarily by gluon-gluon and quark-gluon fusion and quark-quark scattering $p p \rightarrow \tilde{q} \tilde{q}, \rightarrow \tilde{q} \tilde{q}^{\dagger}$ and $\rightarrow \tilde{q} \tilde{q}$.

Strong and electroweak sectors mass hierarchy:

 $m_{\tilde{g}} > m_{wino} > m_{bino}$, $m_{\tilde{q}} > m_{\tilde{l}}$

Mass measurement strategies at the LHC (2/3)

Therefore, two main squark decay modes:

$$\ \, {\tilde q}_L \rightarrow {\tilde \chi}^0_2 q \Leftarrow {\it BR} ({\tilde q}_L \rightarrow {\tilde \chi}^0_2 \, q) \approx 0.31;$$

 $\ 2 \ \ \tilde{q}_R \to \tilde{\chi}_1^0 q;$

followed by $\tilde{\chi}_2^0$ decay to final state leptons or $b\bar{b}$ hadronization:

•
$$\tilde{\chi}_{2}^{0} \rightarrow \bar{\nu} \, \tilde{\nu}_{L} \rightarrow \bar{\nu} \nu \tilde{\chi}_{1}^{0} \Leftarrow$$
 unobservable but $\sum_{\nu} BR_{\nu} \approx 50\%$ in our scenario;
• $\tilde{\chi}_{2}^{0} \rightarrow \tilde{\chi}_{1}^{0} h \rightarrow \tilde{\chi}_{1}^{0} X (b \bar{b} ...) \Leftrightarrow BR \approx 1 - 3\%;$
• $\tilde{\chi}_{2}^{0} \rightarrow \tilde{\chi}_{1}^{0} Z \rightarrow \tilde{\chi}_{1}^{0} I \bar{I} \Leftrightarrow BR \lesssim 0.2\%.$
• $\tilde{\chi}_{2}^{0} \rightarrow \tilde{I}_{L,R} \bar{I} \rightarrow \tilde{\chi}_{1}^{0} I \bar{I};$

From the chain $\tilde{q}_L \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{l}_{L,R} \rightarrow \tilde{\chi}_1^0$ we can construct 3 observable di-particle invariant masses whose end-points have a common structure:

$$\begin{split} m_{ll}^{(max)} &= \textit{M}(m_{\tilde{\chi}_{2}^{0}}, \textit{m}_{\tilde{l}_{L,R}}, m_{\tilde{\chi}_{1}^{0}}), \ m_{l(near)q}^{(max)} &= \textit{M}(m_{\tilde{q}_{L}}, m_{\tilde{\chi}_{2}^{0}}, m_{\tilde{l}_{L,R}}), \ m_{l(tar)q}^{(max)} &= \textit{M}'(m_{\tilde{q}_{L}}, m_{\tilde{\chi}_{2}^{0}}, m_{\tilde{l}_{L,R}}, m_{\tilde{\chi}_{1}^{0}}) \\ \text{and one tri-particle invariant mass} \end{split}$$

$$m_{llq}^{(max)} = M(m_{\tilde{q}_L}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_1^0})$$

neglecting the jet mass.

$$M(x, y, z) = \frac{1}{y} \sqrt{(x^2 - y^2)(y^2 - z^2)}, \quad M'(x, y, w, z) = \frac{1}{w} \sqrt{(x^2 - y^2)(w^2 - z^2)}$$

Mass measurement strategies at the LHC (3/3)

Standard window representative points

Point	<i>m</i> ₀ [GeV]	m _{1/2} [GeV]	<i>A</i> ₀ [GeV]	$\tan \beta$	$sign(\mu)$	$\Omega_{CDM} h^2$
A	80	380	0	10	1	0.109
В	200	944	900	10	1	0.115
С	78.9	386	-1000	3	1	0.103

(SPHENO 3.0 + MICROMEGAS v2.2)

Approximate mass spectrum

Point	$m_{\tilde{\chi}^0_2}$ [GeV]	$m_{\tilde{\chi}_1^0}$ [GeV]	$m_{\tilde{l}_{X(L)}}$ [GeV]	$m_{\tilde{l}_{X(R)}}$ [GeV]	m _{q̃} [GeV]
A	285	152	~ 272	\sim 163	740 – 800
В	744	396	~ 638	\sim 399	1700 – 1820
С	296	155	~ 273	~ 165	700 – 820
(SPHENO 3.0)					

\tilde{q}_L production cross section

	NLO Cross section [pb]			
Point	$\tilde{q}_L \tilde{g}$	$\tilde{q}_L \tilde{q}$	$ ilde{q}_L ilde{q}^\dagger+ ilde{q} ilde{q}_L^\dagger$	total
A	1.56	0.92	0.90	3.39
В	3.88×10^{-3}	$7.79 imes 10^{-3}$	2.84×10^{-3}	1.45×10^{-2}
С	1.41	0.85	0.82	3.08

(PROSPINO2.1)

RGE induced LFV & mass splittings in type I SUSY seesaw

Low energy LH charged slepton masses and trilinear couplings

$$\begin{split} m_{\tilde{L}_{ij}}^{2} &\simeq \delta_{ij} m_{0}^{2} + \frac{1}{8\pi^{2}} \left(3m_{0}^{2} + A_{0}^{2} \right) \frac{Y_{jk}^{\nu} t_{k} Y_{ki}^{\nu\dagger}}{Y_{ki}^{\nu}}, \quad t_{k} \equiv \ln \left(\frac{(M_{R})_{k}}{M_{GUT}} \right) \\ A_{ij}^{\prime} &\simeq \delta_{ij} A_{0} Y_{ii}^{\prime} + \frac{3}{16\pi^{2}} A_{0} Y_{ii}^{\prime} Y_{ik}^{\nu} t_{k} Y_{kj}^{\nu\dagger} \end{split}$$

Small angle approximation

Obminant
$$(m_{\bar{l},LL}^2)_{ii}$$
, driven by SU(2)_L gaugino:
 $m_{\bar{l},LL}^2 \simeq m_0^2 + 0.5 m_{1/2}^2 + 0.0375 m_{1/2}^2$

 $\textcircled{0} Mixing LL \gg RL, LR$

$$R^{ ilde{l}} \simeq egin{pmatrix} 1 & \delta_{12} & \delta_{13} \ -\delta_{12} & 1 & \delta_{23} \ -\delta_{13} & -\delta_{23} & 1 \ \end{pmatrix}$$
, $\delta_{ij} = rac{\Delta m^2_{ ilde{L}(ij)}}{m^2_{ ilde{L}(ii)} - m^2_{ ilde{L}(jj)}}$

Assumptions on unknown seesaw parameters

Recall Casas-Ibarra parametrization:

$$Y^{\nu T} = \frac{1}{v_u} \sqrt{M_R} R \sqrt{\hat{M}^{\nu}} U^{\dagger}_{PMNS}$$

Assumptions:

• $(M_R)_3 \gg (M_R)_{1,2}$ (hierarchical RH neutrinos);

2 R = 1;

- **IBM** mixing angles except for the Chooz (θ_{13}) ;
- Inormal ordered light neutrinos.

The RH neutrinos induced flavour violation is thus proportional to

$$\delta'_{ji} \equiv v_u^2 \begin{bmatrix} Y^{\nu} t Y^{\nu\dagger} \end{bmatrix}_{ij} \Rightarrow \begin{cases} (M_R)_3^{-1} t_3^{-1} \delta'_{21} \simeq m_3 \frac{c_{13} s_{13}}{\sqrt{2}} e^{i\delta} \\ (M_R)_3^{-1} t_3^{-1} \delta'_{31} \simeq m_3 \frac{c_{13} s_{13}}{\sqrt{2}} e^{i\delta} \\ (M_R)_3^{-1} t_3^{-1} \delta'_{32} \simeq m_3 \frac{c_{13}^2}{2} \end{cases}$$

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Experimental signals

At colliders:

Flavoured slepton mass splittings – expected slepton mass splittings sensitivity @ LHC:

 $\sim \mathcal{O}(0.1)\%;$

- Sizable LFV decay width $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 I_i I_j (i \neq j);$
- Multiple edges in LFC di-lepton invariant mass distribution $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 / \bar{I}$.

At low energy experiments:

$$BR(I_i \to I_j \gamma) \approx \left(rac{1}{c_{eta}^2 s_{eta}^4}
ight) \left(rac{3m_0^2 + A_0^2}{\overline{m}_{\tilde{L}}^4}
ight)^2 |\delta'_{ji}|^2 imes \left\{egin{array}{c} 6.36 imes 10^{-10} \ 3.58 imes 10^{-9} \ 3.58 imes 10^{-9} \ , \ ext{for } i = \mu \end{array}
ight.$$

[PDG 2008]

Mode	BR (at 90% CL)	Mode	BR (at 90% CL)
$\mu \rightarrow \boldsymbol{e} \gamma$	$< 1.2 \times 10^{-11}$	$\mu ightarrow e ar{e} e$	$< 1.0 imes 10^{-12}$
$\tau \rightarrow \boldsymbol{e} \gamma$	$< 1.1 imes 10^{-7}$	$ au ightarrow oldsymbol{e} oldsymbol{ au} oldsymbol{ au} ightarrow oldsymbol{ heta} oldsymbol{ heta}$	$< 3.6 imes 10^{-8}$
$\tau \to \mu \gamma$	$<4.5 imes10^{-8}$	$\tau \to \mu \bar{\mu} \mu$	$< 3.2 imes 10^{-8}$

Future sensitiveness $BR(\mu \rightarrow e\gamma) \gtrsim 10^{-13}$ [MEG experiment].

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Slepton mass splittings as a $I_i \rightarrow I_j \gamma$ indicator



Low $\theta_{13} \Rightarrow \tau - \mu$ mixing is $\sim 1/s_{13}^2$ enhanced, dominating the slepton flavour mixing¹:

$$rac{\Delta m}{m}(ilde{\mu}_L, ilde{ au}_2)\simeq 2rac{\Delta m}{m}(ilde{m{e}}_L, ilde{\mu}_L)pprox \left|rac{(m_{ ilde{L}}^2)_{23}}{(m_{ ilde{l}}^2)_{33}}
ight|$$

¹Not true in any of these situations: (i) non-hierarchical RH neutrinos, (ii) hierarchical RH neutrinos with a non-trivial *R*-matrix, (iii) light neutrinos with an inverted mass spectrum.

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$m_{ ilde{\chi}^0_2}$ + slepton mass splittings as a tan eta vs $|m{A}_0|$ probe

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$m_{\tilde{\chi}^0_2}$ + slepton mass splittings as a tan β vs $|A_0|$ probe Remarks

Perturbative bounds on Y^{ν} :

 Robust upper bound (independent of LFV decay rates) on slepton mass splittings for a fixed |A₀|;

 $\ \ \, \mathbf{2} \ \ \, |A_0|\sim 0 \ \, \mathrm{GeV} \Rightarrow \mathrm{Max}\big[\tfrac{\Delta m}{m}(\tilde{e}_L,\tilde{\mu}_L)\big]\lesssim 0.8\%.$

Current upper bounds on LFV decay rates imply

$$\begin{split} & \mathsf{Max}\!\left[\frac{\Delta m}{m}(\tilde{e}_L,\tilde{\mu}_L)\right] \approx 15\% \\ & \Rightarrow \tan\beta \lesssim 3 \text{ and } |A_0| \approx 1 \text{ TeV while } m_{\tilde{\chi}_2^0} \approx 350 - 450 \text{ GeV} \end{split}$$

for $\theta_{13} \lesssim 0.3^{o}$.

Deviations from TBM ($\theta_{13} \gtrsim 1^{o}$) imply

for **current** LFV upper bounds:
$$Max\left[rac{\Delta m}{m}(ilde{e}_L, ilde{\mu}_L)
ight]\lesssim 7\%.$$

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$m_{\tilde{\chi}_{0}^{0}}$ + slepton mass splittings as a tan β vs $|A_{0}|$ probe



$m_{\tilde{\chi}^0_2} + { m slepton} \mbox{ mass splittings as a } { m tan} \, eta \, { m vs} \, |A_0| \mbox{ probe}$ Remarks

Future upper bound on $BR(\mu \rightarrow e \gamma)$ for $\theta_{13} \sim 0.1^{o}$ imply

$$\begin{aligned} \mathsf{Max} & \left[\frac{\Delta m}{m} (\tilde{e}_L, \tilde{\mu}_L) \right] \approx 6\% \\ \Rightarrow & \tan \beta \lesssim 3 \text{ and } |A_0| \approx 1 \text{ TeV while } m_{\tilde{\chi}_n^0} \approx 550 - 650 \text{ GeV} \end{aligned}$$

Deviations from TBM ($\theta_{13} \gtrsim 1^{o}$) imply

for **future**
$$BR(\mu \to e \gamma)$$
 upper bound: Max $\left[\frac{\Delta m}{m}(\tilde{e}_L, \tilde{\mu}_L)\right] \lesssim 2\%$
 $(m_{\tilde{\chi}_2^0} \approx 850-950 \text{ GeV}).$

Di-lepton invariant mass distributions



Point	$\frac{\Delta m}{m}(\tilde{\mu}_L, \tilde{\tau}_2)$	$\frac{\Delta m}{m}(\tilde{e}_L, \tilde{\mu}_L)$	(<i>M_R</i>) ₃ [GeV]	
Α	1.33%	0.73%	8.2×10^{14}	
В	6.40%	-	$1.9 imes 10^{15}$	
С	2.14%	-	$2.0 imes 10^{13}$	
$BR(\mu ightarrow e \gamma) \lessapprox 10^{-13}, heta_{13} = 0.1^{o} \Rightarrow BR(au ightarrow \mu \gamma) \lessapprox 3 imes 10^{-9}$				

 $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \, \bar{\mu} \, e) \approx 10^{-4} BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \, \bar{\tau} \, \mu) \ , \quad (\theta_{13} = 0.1^o)$

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Di-lepton invariant mass distributions

Remarks

Number of events expected for $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$

• Point A: \sim 24822 ($\tau\bar{\tau}$) and \sim 1554 ($\tau\bar{\mu}$);

2 Point B: \sim 34 ($\tau\bar{\tau}$) and \sim 34 ($\tau\bar{\mu}$);

• Point C: \sim 10461 ($\tau\bar{\tau}$) and \sim 5480 ($\tau\bar{\mu}$).

 $\tilde{\chi}^0_2 \rightarrow \tilde{\chi}^0_1 \, \bar{\mu} \, \boldsymbol{e} \Rightarrow$ rather unlikely to be observable.

 m_{\parallel} and LFC signals of LFV in the slepton sector:

Sufficient: 3 edges in LFC processes (point C);

- Necessary: 2 edges in LFC processes when
 [~]₂⁰ decay via [~]_B is highly suppressed;
- 2 edges in LFC processes with a heavy mass spectrum (point B) and proper dark matter relic density ($\Rightarrow m_{\tilde{\tau}_1} m_{\tilde{\chi}_1^0} \lesssim 0.1 \text{ GeV} \text{features a long lived } \tilde{\tau}_1$).

Flavour quasi-degenerated sleptons ($\tilde{\tau}_2$, $\tilde{\mu}_L$) give rise to:

- 3 edges in LFC processes;
- 2 Similar number of $\tau\mu$ and $\tau\tau$ events at the LHC.

Conclusion

A non-negligible $\tilde{e}_L - \tilde{\mu}_L$ mass splitting or a non-conventional $\frac{\Delta m}{m}(\tilde{e}_L, \tilde{\mu}_L)$ vs $\frac{\Delta m}{m}(\tilde{\mu}_L, \tilde{\tau}_2)$ correlation requires a proper explanation:

- Is slepton non-universality generated @ GUT (by SUSY-breaking)?
- or is it generated by the same mechanism responsible for neutrino masses? For example, a seesaw type-I

We have seen that the second answer implies

- Correlation between low energy LFV observables and slepton mass splittings;
- 2 Possible hints on $\tan \beta$ and $|A_0|$ for a given mass splitting;