Monochromatization Under Beamstrahlung in Circular e^+e^- Colliders

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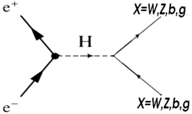
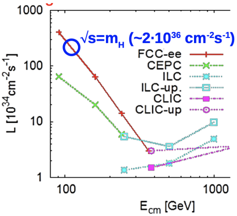
e Yukawa via s-channel $e^+e^- \to$ H production [3] I

- Higgs decay to e^+e^- is unobservable:
  $BR(\boldsymbol{H \to e^+e^-}) \ \propto \ m_e^2 \approx 5 \times 10 - 9$
- Resonant Higgs production considered so far only for muon collider: $s(\mu\mu H) \approx 70pb$. **Tiny κ_e Yukawa coupling** → Tiny $\sigma(e^+e^- \to H)$:

$$\sigma\left(e^+e^- \to H\right) = \frac{4\pi\Gamma_H^2 Br\left(H \to e^+e^-\right)}{\left(\hat{s} - M_H^2\right)^2 + M_H^2}$$
$$= \mathbf{1.64}\ \boldsymbol{fb(m_H = 125\ GeV, \Gamma_H = 4.2\ MeV)}$$



e Yukawa via s-channel $e^+e^- \to$ H production [4] II



Huge luminosities available at FCC-ee:
In theory, FCC-ee running at H pole-mass $\mathrm{L_{int}} \approx 20\ \mathrm{ab}^{-1}/\mathrm{yr}$ **would produce O(30.000) H's**

If we can control: (i) beam-energy spread, (ii) ISR, and (iii) huge backgrounds, then:

- **Electron Yukawa coupling** measurable.
- **Higgs width** measurable (threshold scan)?
- Separation of possible **nearly-degen.** H's?

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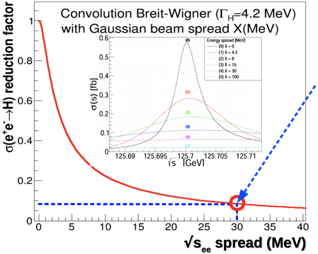
[3] See D. d'Enterria's slides

[4] See D. d'Enterria's slides

“Actual” s-channel $e^+e^- \to$ H cross section [5] I

- $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{H})$ for Breit-Wigner with natural $\Gamma_H = 4.2\ MeV$ width. But Higgs production is **greatly suppressed off resonant peak**

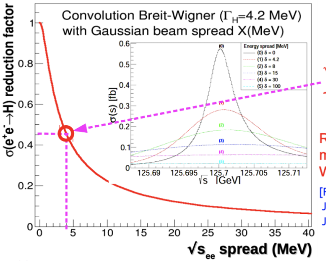
- **Convolution** of **Gaussian energy spread** of each $e^\pm$ beam with Higgs Breit-Wigner results on a (Voigtian) **effective cross-section decrease**:



For $\sqrt{s}_{spread} \approx$ 30 MeV:
Reduction factor: ×1/12

“Actual” s-channel $e^+e^- \rightarrow$ H cross section [6] II

- $\sigma(\mathrm{e^+e^-} \rightarrow \mathrm{H})$ for Breit-Wigner with natural $\Gamma_H = 4.2\ MeV$ width. But Higgs production is **greatly suppressed off resonant peak**
- **Convolution** of **Gaussian energy spread** of each $e^\pm$ beam with Higgs Breit-Wigner results on a (Voigtian) **effective cross-section decrease**:



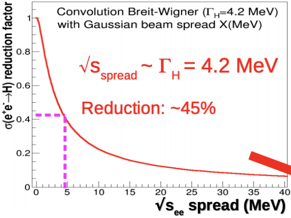
$\sqrt{s}_{spread} = \Gamma_H = 4.2$ MeV

~45% x-section reduction

Reachable with beams monochromatization?
What luminosity loss price?

[F.Zimmermann, A.Valdivia:
JACoW-IPAC2017-WEPIK015
JACoW-IPAC2019-MOPMP035]

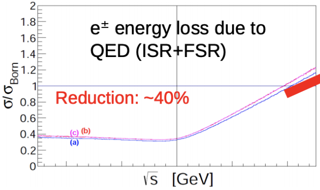
"Actual" s-channel $e^+e^- \rightarrow$ H cross section [7] III



$\sqrt{s}_{spread} \sim \Gamma_H = 4.2$ MeV

Reduction: ~45%

- Extra ~40% reduction due to QED radiation:



$e^\pm$ energy loss due to QED (ISR+FSR)

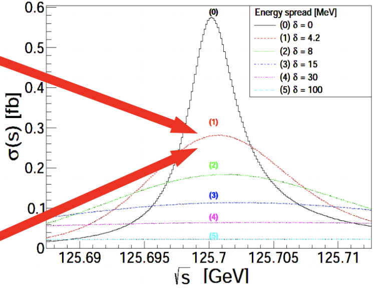
Reduction: ~40%

Note: Higgs pole known to within ±5MeV

Monochrom. goal: $\sqrt{s}_{spread} \approx \Gamma_H = 4.2$ MeV

- Full convolution of both effects:

[S.Jadach, R. Kycia, PLB755 (2016) 58]



$\sigma_{spread+ISR}(e^+e^-\rightarrow H) = 0.17\times\sigma(e^+e^-\rightarrow H) = 290$ ab

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[5] See D. d'Enterria's slides

[6] See D. d'Enterria's slides

[7] See D. d'Enterria's slides

FCCee Design

- Baseline beam relative energy spread σ_ϵ insufficient to produce a significant amount of Higgs

$$\sigma_\epsilon^2 \propto \frac{55\hbar c E_0^2}{32\sqrt{3\left(mc^2\right)^3}}$$

- Collision energy spread σ_w would require **reduction** of σ_ϵ

$$\sigma_w = \sqrt{2} E_0 \sigma_\epsilon$$

- A double ring system enables an alternative

Monochromatization Principle

- Baseline scheme:

$$\begin{aligned} E+\Delta E \rightarrow & \quad \leftarrow E+\Delta E \\ E \rightarrow & \quad \leftarrow E \\ E-\Delta E \rightarrow & \quad \leftarrow E-\Delta E \end{aligned}$$

- Monochromatization scheme:

$$\begin{aligned} E+\Delta E \rightarrow & \quad \leftarrow E-\Delta E \\ E \rightarrow & \quad \leftarrow E \\ E-\Delta E \rightarrow & \quad \leftarrow E+\Delta E \end{aligned}$$

Beamstrahlung

- Quantum Nonlinearity Parameter

$$\Upsilon \equiv \frac{|e|\hbar}{m_e^3 c^4}\sqrt{\left|\left(F_{\mu\nu}p^{\nu}\right)^2\right|} = \gamma\frac{B}{B_c}, \quad B_c = m^2/e \approx 4.4 \text{ GT}$$

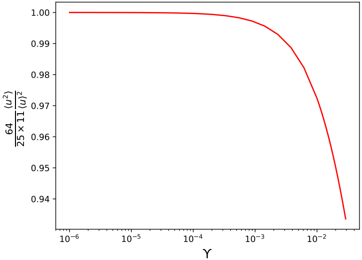
$$\Upsilon_{\text{max}} = 2\frac{r_e^2\gamma N_b}{\alpha\sigma_z\left(\sigma_x^* + \sigma_y^*\right)}$$

- Simulations are the typical approach for lattice design
- An analytical model is preferable for parameters optimization

Classical Description

- Standard Description

$$\left\langle u^2 \right\rangle \approx \frac{25 \times 11}{64} \langle u \rangle^2 \quad (\text{ constant } \rho)$$



- Validation of standard description for beamstrahlung

$$\langle \Upsilon \rangle \approx \frac{5}{6} \frac{r_e^2 \gamma N_b}{\alpha \sigma_z^* \left(\sigma_x^* + \sigma_y^* \right)} \ll 1$$

Expectation Value of Energy for Beamstrahlung Photons

- Average number of photons per collision

$$n_\gamma \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x^*} \frac{1}{\sqrt{1+\Phi_{\mathrm{piw}}^2}}$$

- Average relative energy loss

$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 \gamma N_b^2}{\sigma_z \sigma_x^{*2}} \frac{1}{\sqrt{1+\Phi_{\mathrm{piw}}^2}}$$

- Average photon energy normalized to the beam energy

$$\langle u \rangle = \frac{\delta_B}{n_\gamma} \approx \frac{2\sqrt{3}}{9} \frac{r_e^2 N_b \gamma}{\alpha \sigma_z \sigma_x^*}$$

Beamstrahlung

- Bremstrahlung increases energy spread, bunch length, and horizontal emittance

$$\sigma^2_{\epsilon,BS}(s) = \frac{1}{4}\tau_\epsilon \langle n_\gamma \langle u^2 \rangle_u \rangle_s$$

$$\sigma^2_{\mathbf{x}\beta,BS}(s) = \frac{1}{4}\tau_x \langle n_\gamma \langle u^2 \rangle_u \mathrm{H} \rangle_s \beta(s)$$

$$\mathrm{H}^*_x = \frac{1}{\beta^*_x}\left\{ D^{*2}_x + \left(\beta^*_x D'^*_x - \frac{1}{2}\beta'^*_x D^*_x \right)^2 \right\}$$

- Two different assumptions are made for the vertical emittance

From Instantaneous to average effects

- An additional geometrical factor is required

$$\left\langle u^2 \right\rangle \approx Z_{\mathrm{c}} \frac{25 \times 11}{64} \langle u \rangle^2$$

- Excitation contribution due to beamstrahlung is defined once geometrical factor is specified

$$\left\{ n_\gamma \left\langle u^2 \right\rangle \right\}_{\mathrm{BS}} \approx \frac{25 \times 11}{36 \pi^{3/2}} Z_c \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 \sigma_x^{*3}} \frac{1}{\sqrt{1 + \Phi_{\mathrm{piw}}^2}}$$

Model Benchmark I

$$Z_{\mathrm{c}} \equiv \frac{\langle 1/\rho^2 \rangle}{\langle 1/\rho \rangle^2}$$

$$\left\langle \frac{1}{\rho} \right\rangle = \int_{x,y,z,s} dxdydzds \frac{1}{\rho(x,y,s,z)} \frac{\exp\left(-\frac{\left(x+z\frac{\theta_c}{2}\right)^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y(s)^2} - \frac{z^2}{2\sigma_z^2} \right)}{(2\pi)^{3/2}}$$

$$\left\langle \frac{1}{\rho^2} \right\rangle = \int_{x,y,z,s} dxdydzds \frac{1}{\rho(x,y,s,z)^2} \frac{\exp\left(-\frac{\left(x+z\frac{\sigma}{2}\right)^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y(s)^2} - \frac{z^2}{2\sigma_z^2} \right)}{(2\pi)^{3/2}}$$

Model Benchmark II

- Instantaneous local bending radius is not uniform

$$\frac{1}{\rho(x,y,t)} = |\Theta(x,y)| \frac{1}{\sqrt{2\pi}\sigma_z/2} \exp\left(-\frac{(ct)^2}{2\left(\sigma_z/2\right)^2}\right)$$

$$\Theta(x,y) = \Delta y' + i\Delta x' = -\frac{2N_t r_e}{\gamma}\mathcal{F}\left(x, y, \sigma_y(s)\right)$$

$$\mathcal{F}\left(x,y,\sigma_y(s)\right) = \frac{\sqrt{\pi}}{\sqrt{2(\sigma_x^2 - \sigma_y(s)^2)}} \left(w\left[\frac{x+iy}{\sqrt{2(\sigma_x^2 - \sigma_y(s)^2)}}\right] - e^{-\frac{x^2}{2\sigma_x^2}} - \frac{y^2}{2\sigma_y(s)^2} w\left[\frac{\frac{x\sigma_y(s)}{\sigma_x} + i\frac{y\sigma_x}{\sigma_y(s)}}{\sqrt{2(\sigma_x 2 - \sigma_y(s)^2)}}\right]\right)$$

A set of couple equations

$$\varepsilon_{x,\mathrm{tot}} = \varepsilon_{x,\mathrm{SR}} + \frac{\tau_x n_{1P}}{4\,T_{\mathrm{rev}}} \left\{ n_\gamma < u^2 > \right\}_{\mathrm{BS}} \mathcal{H}_x^*$$

$$\sigma^2_{\delta,\mathrm{tot}} = \sigma^2_{\delta,\mathrm{SR}} + \frac{n_{\mathrm{IP}} \tau_E, \mathrm{SR}}{4\,T_{\mathrm{rev}}} \left\{ n_\gamma < u^2 > \right\}_{\mathrm{BS}}$$

$$\mathcal{H}_x^* \equiv \frac{\left(\beta_x^* D_x'^* + \alpha_x^* D_x^*\right)^2 + D_x^{*2}}{\beta_x^*}$$

$$\tau_x \approx 2\tau_\epsilon$$

$$\sigma_{z,\ \mathrm{tot}} \ = \frac{\alpha_{\mathrm{C}} C}{2\pi Q_s} \sigma_{\delta,\ \mathrm{tot}}$$

Explicit Coupled Equations

- Monochromatization limit

$$D_x^* \sigma_{\delta,\,\text{tot}} \gg \sqrt{\beta_x^* \varepsilon_x}$$

- Optical parameters at IP define the coupled equations

$$\varepsilon_{x,\,\text{tot}} \approx \varepsilon_{x,\mathrm{SR}} + \frac{2V\mathcal{H}_x^*}{D_x^{*3}\sigma_{\delta,\,\text{tot}}^5}$$

$$\sigma_{\delta,\,\text{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \frac{V}{D_x^{*3}\sigma_{\delta,\,\text{tot}}^5}$$

$$V \equiv \frac{25 \times 11}{4 \times 36\pi^{3/2}\alpha} Z_c \frac{n_{\mathrm{IP}}\tau_{E,\mathrm{SR}}}{T_{\mathrm{rev}}} \frac{r_e^5 N_b^3 \gamma^2 \left(2\pi Q_s\right)^2}{\left(\alpha_{\mathrm{C}} C\right)^2 \sqrt{1+\Phi_{\mathrm{piw}}^2}}$$

Parameters Optimization I

- Monochromatization Factor

$$\lambda = \sqrt{D_x^{*2}\sigma^2_{\delta,\ \mathrm{tot}} \,/\, (\varepsilon_{x,\ \mathrm{tot}}\ \beta_x^*) + 1}$$

- Luminosity

$$\mathcal{L} = f_r \frac{n_b N_b^2}{4\pi\sigma_x^*\sigma_y^*}$$

$$\sigma_x^* = \sqrt{\beta_x^*\epsilon_{x,tot} + D_x^{*2}\sigma^2_{\delta,tot}}$$

$$\sigma_y^* = \sqrt{\beta_y^*\epsilon_{y,tot} + D_y^{*2}\sigma^2_{\delta,tot}}$$

Parameters Optimization II

- Target Monochromatization parameter is kept constant to compare with standard solutions

$$D_x^* = S \times D_{x,0}^* \quad \beta_x^* = S^2 \times \beta_{x,0}^*$$

- Beam current is kept constant in order to match power constraints

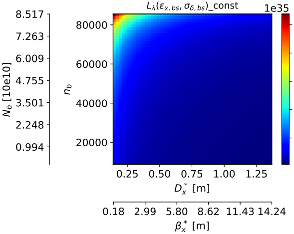
$$n_b = n_{b,0} \times T \quad N_b = N_{b,0}/T$$

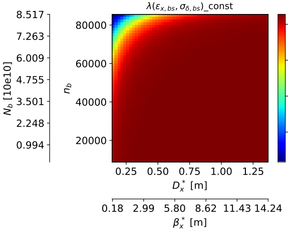
- Beamstrahlung may compromise performance

$$L\left(\varepsilon_{x,\text{ tot}}\left(T,S\right)\right) \text{ and } \lambda\left(\varepsilon_{x,\text{ tot}}\left(T,S\right)\right)$$

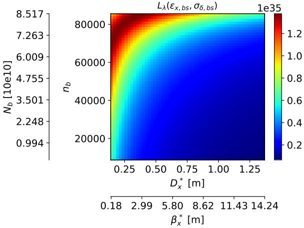
Map of Variables I

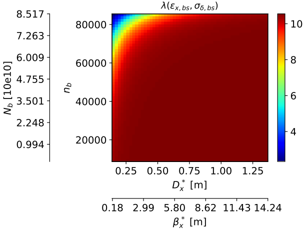
ϵ_y = **constant**





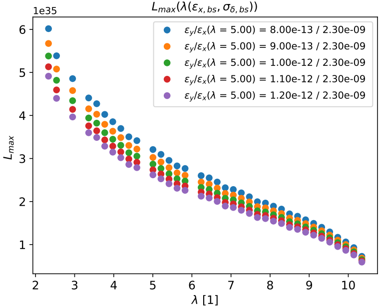
ϵ_y / ϵ_x = **constant**



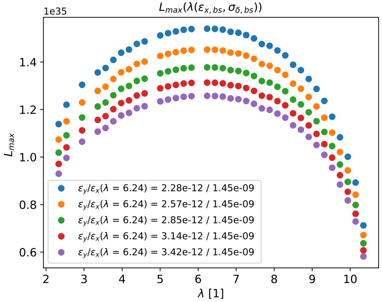


Optimized Luminosity

$\epsilon_y =$ **constant**



$\epsilon_y/\epsilon_x =$ **constant**



Conclusions

- An analytical model to analyse Beamstrahlung effects is proposed
- Agreement exist between standard simulation codes and the analytical model
- Monochromatization scheme is optimisable for direct Higgs production:

$$\epsilon_y = \textbf{constant} : \lambda \approx 6.03 \quad L_{opt}[10^{35} cm^{-2} s^{-1}] \approx 2.87$$

$$\epsilon_y/\epsilon_x = \textbf{constant} : \lambda \approx 6.00 \quad L_{opt}[10^{35} cm^{-2} s^{-1}] \approx 1.38$$

- Practical implementation depend on the future of future colliders

Future Work

- Modify Final Focus System
- Explore Short/Weak Regime Implications
- Coding Time Optimization

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