Perspective of the $\alpha_s(M_Z)$ extraction from the FCC data

Andrii Verbytskyi$^1$ in collaboration with
Andrea Banfi$^3$, Adam Kardos$^2$, Pier Francesco Monni$^4$, Stefan Kluth$^1$, Gábor Somogyi$^2$, Zoltán Ször$^5$, Zoltán Trocsányi$^6$, Zoltán Tulipánt$^2$ and Giulia Zanderighi$^1$

The 3rd FCC Physics and Experiments Workshop, Geneve, Switzerland, 13-18 January 2020
\( \alpha_s: \) motivation in the past and in the future

- As of 2020 \( \alpha_s \) is known with precision of 1\% if calculated from measurements with at least NNLO precision.
- However, there is a large spread between measurements which should be explained.

How to use the data from the future colliders to get the explanation?

Source: arXiv:1806.06156v1
The traditional methods to extract $\alpha_s$ from $e^+e^-$ data

- $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$ (high energy) or $R_{\text{had}}$
  - + Theory precision: $\alpha_s^4$
  - + Direct measurement
  - + Check running
  - + Good understanding of theory: EW, quark masses
  - - Large experimental uncertainties
  - Theory from Ref. [1]

- Electroweak fits
  - + High final precision?
  - + No nonperturbative corrections
  - - Indirect measurement
  - - Some model dependence, e.g. running
  - - Dep. on many SM parameters
  - * Actually include non-$e^+e^-$ data
  - Full analysis with FCC/ILC in Ref. [2]

- $\tau$ decays
  - + High final precision?
  - - Some model dependence
  - - Dep. on many SM parameters
  - * Can use any source of $\tau$, e.g. $B$-factories
  - To be studied

- Observables in $e^+e^- \rightarrow \text{hadrons}$
  - + Theory precision: $\alpha_s^3 + \text{resummation}$
  - + Direct measurement
  - + Check running
  - + High final precision?
  - - Model dependence: hadronisation

No perfect method, all of them should be combined.
Data needed for the methods

Which data would be perfect for these methods? How much data would be sufficient to suppress statistical uncertainties?

- "Mini QCD $\times$ EW fits", e.g. $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$ (high energy), $R_{\text{had}}$ for $Z$ or $W$
  - Widest $\sqrt{s}$ range
  - Preferably w/o or with a separated $e^+e^- \rightarrow VV$ background

- Electroweak/SM fits
  - Widest $\sqrt{s}$ range
  - $\sqrt{s} = M_Z$
  - Separate measurements of $e^+e^- \rightarrow VV$

- $\tau$ decays
  - Preferably w/o $e^+e^- \rightarrow VV$ background

- Observables in $e^+e^- \rightarrow \text{hadrons}$
  - Widest $\sqrt{s}$ range
  - Higher $\sqrt{s}$ is preferable
  - Preferably w/o $e^+e^- \rightarrow VV$ background
  - Preferably with $e^+e^- \rightarrow \gamma \text{ hadrons}$

- Limited by lumi detector precision, unknown

- Systematic uncertainties dominate, unknown

- At least $\mathcal{O}(10^8 - 10^9)$ $\tau$ events

- At least $\mathcal{O}(10^8 - 10^9)$ $e^+e^- \rightarrow \text{hadrons}$ events, depending on $\sqrt{s}$ range

Data with $\sqrt{s} < M_Z$ and $\sqrt{s} \approx M_Z$ is strongly preferable from the first principles for most methods.
Mini QCD × EW fits

Idea: some quantity (number) can be predicted in QCD × QED as

\[
\text{Observable} = C(1 + \sum_{k=0}^{N} c_k \alpha_s^k(M_Z) + \delta_{\text{massive}} + \delta_{\text{EW}} + \delta_{\text{nonperturbative}})
\]

For some the \(C\) and/or \(\delta_{\text{massive}}, \delta_{\text{EW}}, \delta_{\text{nonperturbative}}\) can be calculated with so high precision that the uncertainties on the input EW parameters can be ignored. As of 2020 the following quantities are known to \(N^3\text{LO}\) in pQCD: \(R_{\text{had},Z}, R_{\text{had},W}, \sigma_{\text{tot}}(e^+e^- \to \text{hadrons})\).

Recent example: \(\alpha_s(M_Z)\) at \(N^3\text{LO}\) from \(R_{\text{had},W}\) [3] delivers \(\alpha_s(M_Z) = 0.117 \pm 0.040\).

Reliability: Even w/o progress in theory these analysis can deleiver \(N^3\text{LO}\) results just with more data available.
\[ \Gamma(\tau^- \rightarrow \text{hadrons}\nu_{\tau}) \approx \text{C}(1 + \sum_{k=0}^{N} c_k \alpha_s^k(M_Z) + \delta_{\text{nonperturbative}}) \]

The main effort: estimate the \(\delta_{\text{nonperturbative}} \approx \mathcal{O}(1\%)\) precisely relying on data from OPAL and ALEPH on the hadronic final state mass in \(\tau\) decays.

The two used approaches, fixed-order and countour improved perturbation theories (FOPT [4] and CIPT [5]) give slightly different results[6].

Getting more data is crucial.
Would be interesting to know about perspectives of measurements in modern \(B\) factories.
## Results from $e^+e^-$

<table>
<thead>
<tr>
<th>Determination</th>
<th>Type</th>
<th>Data and procedure</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.110$^{+0.014}_{-0.017}$</td>
<td>$\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})$</td>
<td>CLEO, below 10 GeV</td>
<td>[7]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})$</td>
<td>LEP, SLC, included in EW fit</td>
<td>Also [8]</td>
</tr>
<tr>
<td>0.1194 ± 0.0029</td>
<td>EW fit</td>
<td>LHC/TVT/LEP/SLC</td>
<td>[9]</td>
</tr>
<tr>
<td>0.1165 ± 0.0012</td>
<td>$\tau$ decays</td>
<td>OPAL/ALEPH FOPT N$^3$LO</td>
<td>[6]</td>
</tr>
<tr>
<td>0.1185 ± 0.0015</td>
<td></td>
<td>OPAL/ALEPH CIPT N$^3$LO</td>
<td>[6]</td>
</tr>
<tr>
<td>0.1187 ± 0.0018</td>
<td></td>
<td>Jan 2020 PDG average</td>
<td>[10]</td>
</tr>
<tr>
<td>0.1171 ± 0.0031</td>
<td>$e^+e^- \rightarrow \text{hadrons}$</td>
<td>01.2020 PDG average</td>
<td>[10]</td>
</tr>
</tbody>
</table>

The PDG (world) “averages” have larger uncertainty due to the spread/disagreement between results.

*Would be nice to have better precision ... and better agreement between results.*
Results from $e^+e^- \rightarrow \text{hadrons}$

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<td>$0.1175 \pm 0.0025$</td>
<td>Non-global</td>
<td>ALEPH 3-jet rate (NNLO+MChad)</td>
<td>[12]</td>
</tr>
<tr>
<td>$0.1199 \pm 0.0059$</td>
<td>fit</td>
<td>JADE 3-jet rate (NNLO+NLL+MChad)</td>
<td>[13]</td>
</tr>
<tr>
<td>$0.1224 \pm 0.0039$</td>
<td>+MChad</td>
<td>ALEPH event shapes (NNLO+NLL+MChad)</td>
<td>[14]</td>
</tr>
<tr>
<td>$0.1172 \pm 0.0051$</td>
<td></td>
<td>JADE event shapes (NNLO+NLL+MChad)</td>
<td>[15]</td>
</tr>
<tr>
<td>$0.1189 \pm 0.0041$</td>
<td></td>
<td>OPAL event shapes (NNLO+NLL+MChad)</td>
<td>[16]</td>
</tr>
<tr>
<td>$0.1164^{+0.0028}_{-0.0026}$</td>
<td>Global fit</td>
<td>Thrust (NNLO+NLL+anlhad)</td>
<td>[17]</td>
</tr>
<tr>
<td>$0.1134^{+0.0031}_{-0.0025}$</td>
<td>+anlhad</td>
<td>Thrust (NNLO+NNLL+anlhad)</td>
<td>[18]</td>
</tr>
<tr>
<td>$0.1135 \pm 0.0011$</td>
<td></td>
<td>Thrust (SCET NNLO+N^3LL+anlhad)</td>
<td>[19]</td>
</tr>
<tr>
<td>$0.1123 \pm 0.0015$</td>
<td></td>
<td>C-parameter (SCET NNLO+N^3LL+anlhad)</td>
<td>[20]</td>
</tr>
<tr>
<td>$0.11750 \pm 0.00287$</td>
<td>Global fit</td>
<td>EEC (NNLO+N^2LL+MChad+NLO_{mb})</td>
<td>[21]</td>
</tr>
<tr>
<td>$0.11881 \pm 0.00131$</td>
<td>+MChad</td>
<td>2-jet rate (N^3LO+N^3LL+MChad+N^2LO_{mb})</td>
<td>[22]</td>
</tr>
</tbody>
</table>

Global fits and wide $\sqrt{s}$ range $\rightarrow$ best precision.

The discrepancy between the analytic and MC hadronisation should be clarified.

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1 Credits to Ref. [11]
Observations from previous analyses of $e^+e^- \rightarrow$ hadrons

Experiment:
- PETRA, TRISTAN, PEP, LEP

Theory:
- $\alpha_s^3$ massless pQCD
- $+N^3LL$ for many diff. observables
- $+\alpha_s^2$ pQCD for massive quarks

Phenomenology:
- MC hadronisation
- Analytic hadronisation

Experimental problems:
- Only $\approx 50\%$ of data used for measurements, e.g. $\sqrt{s} \leq M_Z$ for EEC
- Many numerical values of the measurements are lost or not published
- Low statistics far from $M_Z$
- Some problems with exp. uncertainties

Theory problems:
- For most data either $R_b$ or $m_b^2/s$ is high. Not taken into account.
- Mixed $QED \times QCD$ corrections are not taken into account.

Phenomenology problems:
- MC models are ad-hoc
- Analytic hadronisation is not realistic

- **Experiment:** Produce better data and take care about it
- **Phenomenology:** Do better models
- **Theory:** Do $QED \times QCD$ and massive predictions. Maybe also with $e^+e^-$ in FS.
The FCC-$e^+e^-$ physics program extension

**FCC-$e^+e^-$ CDR is great ... and can be extended**

FCC–$e^+e^-$ data in range $\sqrt{s} = 20 – 91$ GeV$^2$ can help to solve exp./pheno problems simultaneously and will have side benefits.

- Fast to collect – $10^7 – 10^9$ events/day – supersedes all collected data in one day.
- Background free – perfect for most $\alpha_s$ analyses.

\[ \sqrt{s} \text{ (GeV)} \]
\[ \text{Cross-section (pb)} \]

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2 The lower bound depends on the actual capabilities of the machine.
The FCC-\(e^+e^-\) physics program extension

**FCC-\(e^+e^-\) CDR is great . . . and can be extended**

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- Fast to collect – \(10^7 – 10^9\) events/day – supersedes all collected data in one day.
- Background free – perfect for most \(\alpha_s\) analyses.

**A perfect scenario would be \(\approx 10\) equidistant energy points in range 20 – 90 GeV with \(10^7 – 10^8\) events each.**

- Perfect data for hadronisation studies.
- Additional data for electroweak fits, quark masses extraction and other analyses.
- Perfect data for detector calibration, e.g. \(e^+e^- \rightarrow 2\text{jets}, e^+e^- \rightarrow \mu^+\mu^-\), etc.

\(^3\)The lower bound depends on the actual capabilities of the machine.
It will take time to change the beam energy.

True. But it is acceptable to sacrifice a tiny fraction of running time to take a better data and better physics.

Need input from accelerator physicists and engineers.
The $\sqrt{s} = 20 \,–\, 91$ GeV data can be taken during high energy runs using ISR.

- The sys. uncertainties of such data will be much higher.
- Will take much more time to collect.
- Adjusting detector/reconstruction for such data could take even more time.
- Potential problems with acceptance of highly boosted events.
- Such data are not suitable for many analyses and calibration.
### L3 [23] LEPI:

<table>
<thead>
<tr>
<th>Type</th>
<th>$\sqrt{s}$, GeV</th>
<th>$\langle \sqrt{s} \rangle$, GeV</th>
<th>Int. Lumi (pb)</th>
<th>Selection Eff. (%)</th>
<th>Purity (%)</th>
<th>Sel. Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced</td>
<td>30–50</td>
<td>41.4</td>
<td>142.4</td>
<td>48.3</td>
<td>68.4</td>
<td>1247</td>
</tr>
<tr>
<td>Centre- of- Mass</td>
<td>50–60</td>
<td>55.3</td>
<td>142.4</td>
<td>41.0</td>
<td>78.0</td>
<td>1047</td>
</tr>
<tr>
<td>Energy</td>
<td>60–70</td>
<td>65.4</td>
<td>142.4</td>
<td>35.2</td>
<td>86.0</td>
<td>1575</td>
</tr>
<tr>
<td></td>
<td>70–80</td>
<td>75.7</td>
<td>142.4</td>
<td>29.9</td>
<td>89.0</td>
<td>2938</td>
</tr>
<tr>
<td></td>
<td>80–84</td>
<td>82.3</td>
<td>142.4</td>
<td>27.4</td>
<td>90.5</td>
<td>2091</td>
</tr>
<tr>
<td></td>
<td>84–86</td>
<td>85.1</td>
<td>142.4</td>
<td>27.5</td>
<td>87.0</td>
<td>1607</td>
</tr>
<tr>
<td>Z pole</td>
<td>91.2</td>
<td>91.2</td>
<td>8.3</td>
<td>98.5</td>
<td>99.8</td>
<td>248100</td>
</tr>
</tbody>
</table>

$\alpha_s(M_Z)_{41 \text{ GeV}} = 0.1418 \pm 0.0053(\text{stat.}) \pm 0.0030(\text{exp. syst.}) \pm 0.0055(\text{hadr.}) \pm 0.0085(\text{theory})(NLO)$

$\alpha_s(M_Z)_{55 \text{ GeV}} = 0.1260 \pm 0.0047(\text{stat.}) \pm 0.0056(\text{exp. syst.}) \pm 0.0066(\text{hadr.}) \pm 0.0062(\text{theory})(NLO)$

$\alpha_s(M_Z)_{91 \text{ GeV}} = 0.1210 \pm 0.0008(\text{stat.}) \pm 0.0017(\text{exp. syst.}) \pm 0.0040(\text{hadr.}) \pm 0.0052(\text{theory})(NLO)$

### OPAL [24] LEPI:

<table>
<thead>
<tr>
<th>$E_\gamma$ [GeV]</th>
<th>Events</th>
<th>$\sqrt{s'}_{\text{Mean}}$ [GeV]</th>
<th>$\sqrt{s'}_{\text{Mean}}$ [GeV]</th>
<th>Background [%]</th>
<th>$\tau\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Non-rad. MH</td>
<td>Isolated tracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10–15</td>
<td>1560</td>
<td>78.1 ± 1.7</td>
<td></td>
<td>6.0 ± 0.7</td>
<td>6.2 ± 0.9</td>
</tr>
<tr>
<td>15–20</td>
<td>954</td>
<td>71.8 ± 1.9</td>
<td></td>
<td>3.1 ± 0.5</td>
<td>4.9 ± 0.8</td>
</tr>
<tr>
<td>20–25</td>
<td>697</td>
<td>65.1 ± 2.0</td>
<td></td>
<td>2.6 ± 0.6</td>
<td>6.3 ± 1.1</td>
</tr>
<tr>
<td>25–30</td>
<td>513</td>
<td>57.6 ± 2.3</td>
<td></td>
<td>5.1 ± 1.1</td>
<td>7.9 ± 1.4</td>
</tr>
<tr>
<td>30–35</td>
<td>453</td>
<td>49.0 ± 2.6</td>
<td></td>
<td>4.5 ± 1.1</td>
<td>9.6 ± 1.6</td>
</tr>
<tr>
<td>35–40</td>
<td>376</td>
<td>38.5 ± 3.5</td>
<td></td>
<td>5.2 ± 1.2</td>
<td>13.1 ± 1.9</td>
</tr>
<tr>
<td>40–45</td>
<td>290</td>
<td>24.4 ± 5.3</td>
<td></td>
<td>10.4 ± 2.3</td>
<td>12.9 ± 1.7</td>
</tr>
</tbody>
</table>

$\alpha_s(M_Z)_{\text{comb}} = 0.1182 \pm 0.0015(\text{stat.}) \pm 0.0038(\text{exp. syst.}) \pm 0.0070(\text{hadr.}) \pm 0.0062(\text{theory})(NLO)$

$\dagger$ Specific problems: hadronisation, systematics, statistics.
The FCC-$e^+e^-$ physics program extension: contra IIb

Future $e^+e^-$ FACILITY1

- ... 
- 203Y-203Z Taking ISR data, need for dedicated PhDs, calibration.
- ... Complicated analysis, manpower/resources shortage.
- ... Difficulties with theory interpretation.

Results are irrelevant before analysis is completed.

The dedicated runs can hedge other risks.

Future $e^+e^-$ FACILITY2

- 202X Planning low $\sqrt{s}$ runs.
- ... Preparation for analysis with $\sqrt{s} = M_Z$ data.
- Jan 203Z Taking low energy data, getting results, publish.

New submissions for Thu, 9 Jan 3Z


The most precise determination of $\alpha_S(M_Z)$
FUTURE2 collaboration: A. Aname, B. Bname, C. Cname, D. Dname, et al. (1999 additional authors not shown)
Comments: 3 pages in LaTeX, to appear in JHEP next week

We present the most precise determination of the strong coupling constant based on the data taken by the FUTURE2 experiment at FACILITY2 over last weekend.
High energy data are more valuable, hadronisation effects (corrections) are larger at lower energies, e.g. as stated in Ref. [25]:

“Since the analysis in Ref. [21] only uses data at or below the Z pole, it is expected that future data from CEPC at 250 GeV can significantly reduce the hadronization uncertainty.”

More fundamentally: if we don’t understand hadronisation, we should not avoid it, but rather study it!

Higher hadronisation corrections are not equivalent to higher hadronisation modeling uncertainty on $\alpha_S$. There might be even anti-correlation between these quantities.
**FCC-$e^+e^-$ pseudo-data analysis**

The pseudo-data was generated according to the following rules:

- The central values were generated as (massless NNLO pQCD+small ad-hoc $\alpha_S^4$ term) $\times$ (MC hadronisation) for given energy.
- Statistical uncertainty is the best LEP statistical uncertainty at given energy scaled by the $\sqrt{\frac{\text{Number of LEP events}}{\text{Number of FCC events}}}$.
- Systematic uncertainty is the best LEP systematic uncertainty at given energy, or, for $\sqrt{s} < M_Z$ at Z pole.
- It was assumed that 50% of systematic uncertainty is correlated.
- All uncertainties were assumed to be normally distributed.

Two basic scenarios were compared:

- **FCC**: $3 \times 10^{12}$ ev @91 GeV, $3.6 \times 10^9$ ev @161 GeV, $8 \times 10^8$ ev @240 GeV
- **FCC+**: FCC + $10^7$ ev 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 GeV
The analysis was performed as a global fit with

- massless NNLO predictions from Ref. [26]
- MC hadronisation modelling similar to one from Ref. [22]

with multiple observables.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Data</th>
<th>Result</th>
<th>$\chi^2/ndof$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle C \rangle + \langle T \rangle$ simultaneously</td>
<td>FCC</td>
<td>$0.11680 \pm 0.00032(\text{exp.})$</td>
<td>$3.43/5$</td>
</tr>
<tr>
<td></td>
<td>FCC+</td>
<td>$0.11649 \pm 0.00009(\text{exp.})$</td>
<td>$57.61/27$</td>
</tr>
</tbody>
</table>

The FCC+ scenario provides visibly smaller uncertainty even taking into account quality of fit.
New ideas are needed in physics to get ultimate precision in measurements and get new knowledge. Otherwise we stuck with:

- Brute force number crunching with higher fixed&log orders.
- Need of more data.
- Miserable improvements in data analysis techniques, Neural Networks/Machine Learning “magic”, etc.

What it can be?

- Jet grooming [27], sub-jet substructure [28] and similar approaches?
- Now unprecise methods with more data, e.g. $\gamma$ structure functions [29]?
- Analytic calculations in theory [30]?
- Measurements with totally new observables?
- ...
Conclusions

- The data from FCC will play the most important role in the precise determination of $\alpha_S$.
- Taking low energy data at FCC is important for the QCD studies and will be beneficial for other studies.
- It is crucial to obtain the low energy data not only from the radiative events, but also from the dedicated low energy runs.
Backups
It is interesting to admit the differences between the hadronisation uncertainties of results from OPAL [24]

\[ 0.1182 \pm 0.0015(\text{stat.}) \pm 0.0038(\text{exp. syst.}) \pm 0.0070(\text{hadr.}) \pm 0.0062(\text{theory.})(NLO) \]

and JADE [15]:

\[ 0.1172 \pm 0.0006(\text{stat.}) \pm 0.0020(\text{exp. syst.}) \pm 0.0035(\text{hadr.}) \pm 0.0030(\text{theory.})(\text{NNLO + NLLA}) \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Type</th>
<th>( \sqrt{s} )</th>
<th>Hadr. unc.</th>
<th>Exp. syst. unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JADE</td>
<td>2008 Low energy</td>
<td>12-46</td>
<td>0.0035</td>
<td>0.0020</td>
</tr>
<tr>
<td>OPAL</td>
<td>2007 Radiative</td>
<td>10-45</td>
<td>0.0070</td>
<td>0.0038</td>
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Scaling analyses to FCC

L3 [23]:

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<td>98.5</td>
<td>99.8</td>
<td>248100</td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>

With a tighter selection from OPAL, the number of events would be order of magnitude smaller. The dedicated runs could obtain such amount of data in some days or even hours.
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