Groomed heavy jet mass at NNLO +NNNLL accuracy in lepton collisions

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based on arXiv:1603.08927, 1606.03453, 1807.11472 and work in progress

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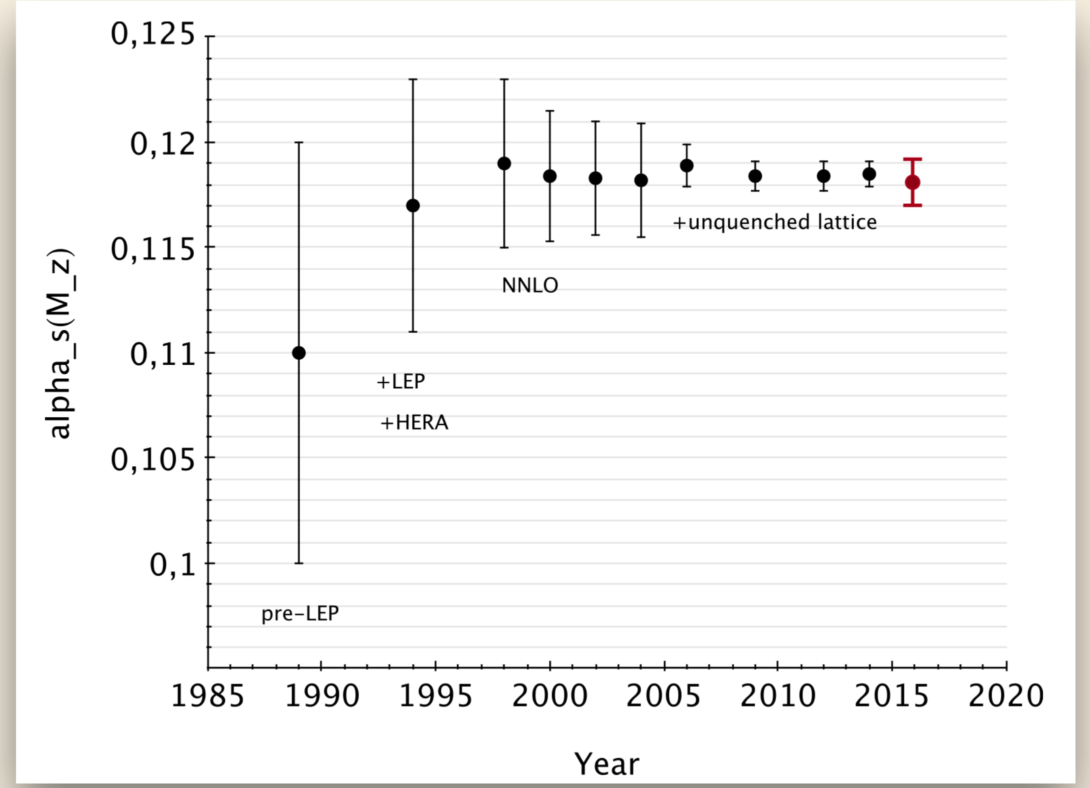
Outline

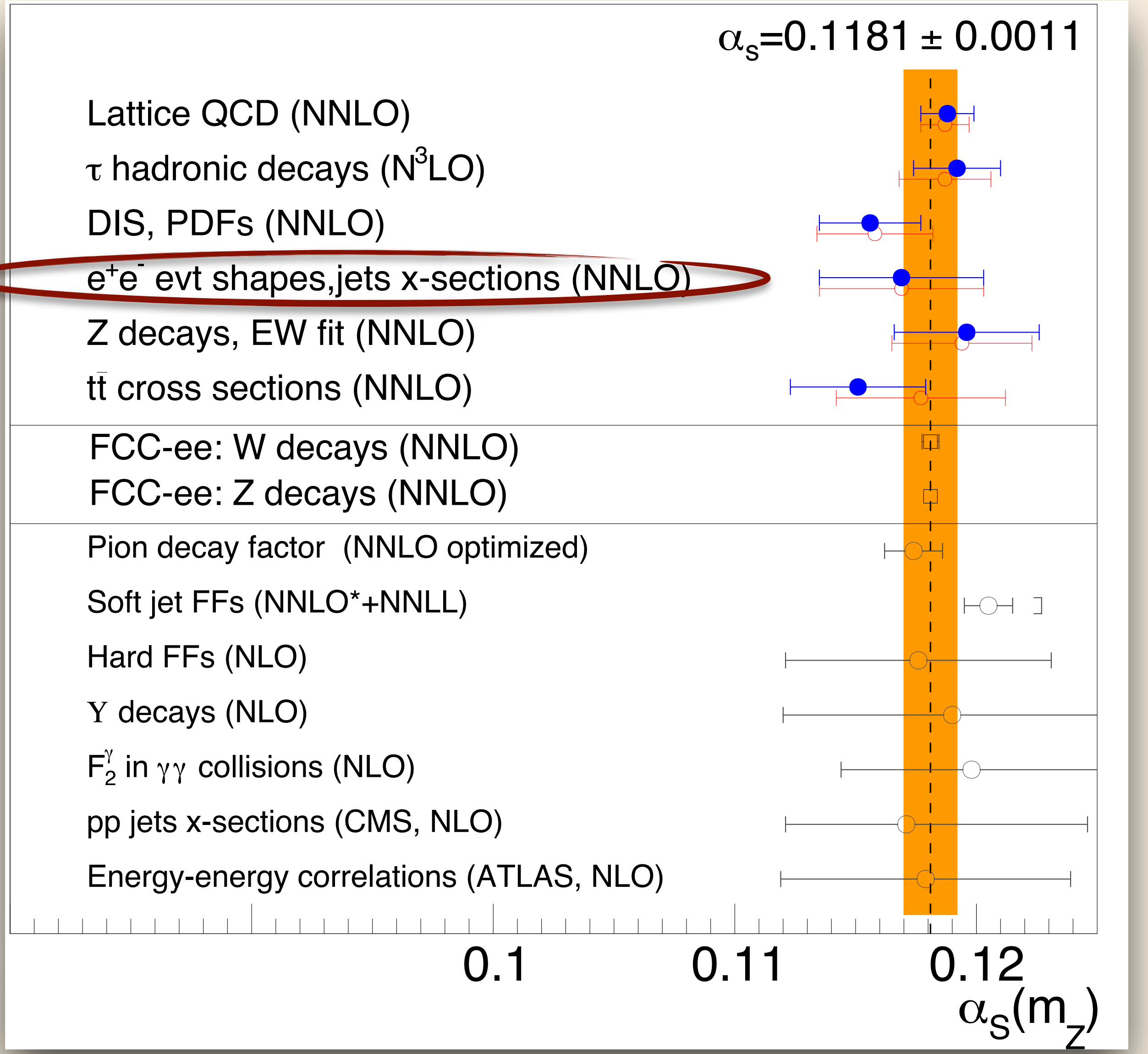
- Why event shapes in lepton collisions?
- New developments since LEP
- New prospects: groomed event shapes
- Conclusions

Why event shapes in lepton collisions?

- e^+e^- event shapes, jets have long been considered ideal for measuring α_s



summary of α_s determinations:



D. d'Enterria, arXiv: 1806.06156

D. d'Enterria and S. Kluth (eds), arXiv: 1907.01435

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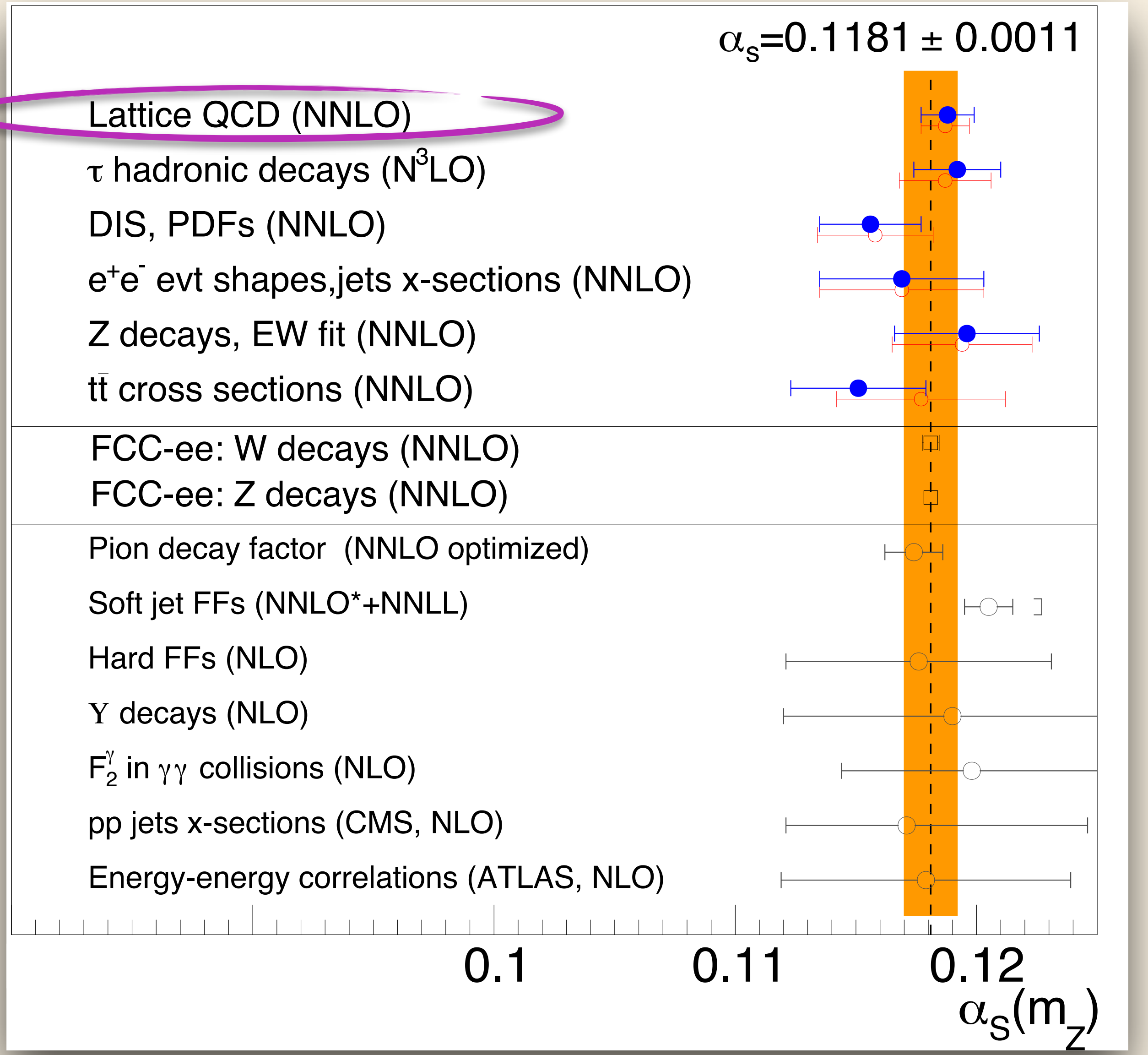

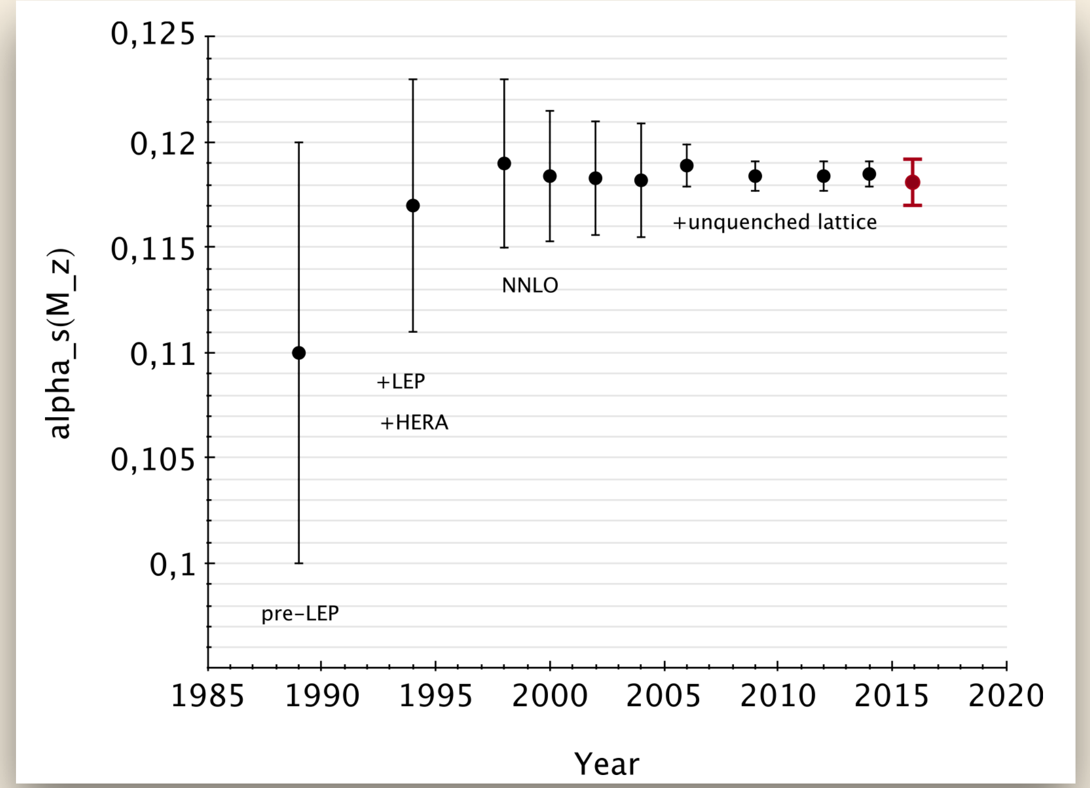

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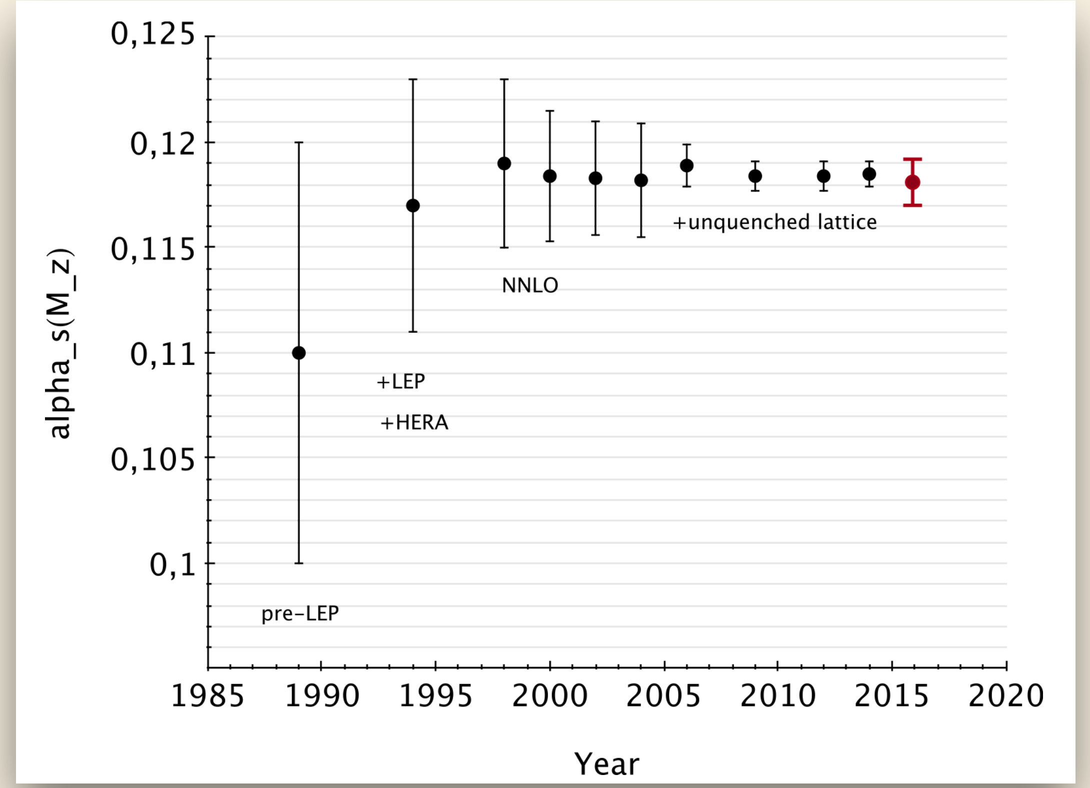
Why event shapes in lepton collisions?

- e^+e^- event shapes, jets have long been considered ideal for measuring α_s
- recent prevailing view: lattice is unbeatable
- yet determination of α_s from experiments remains desirable



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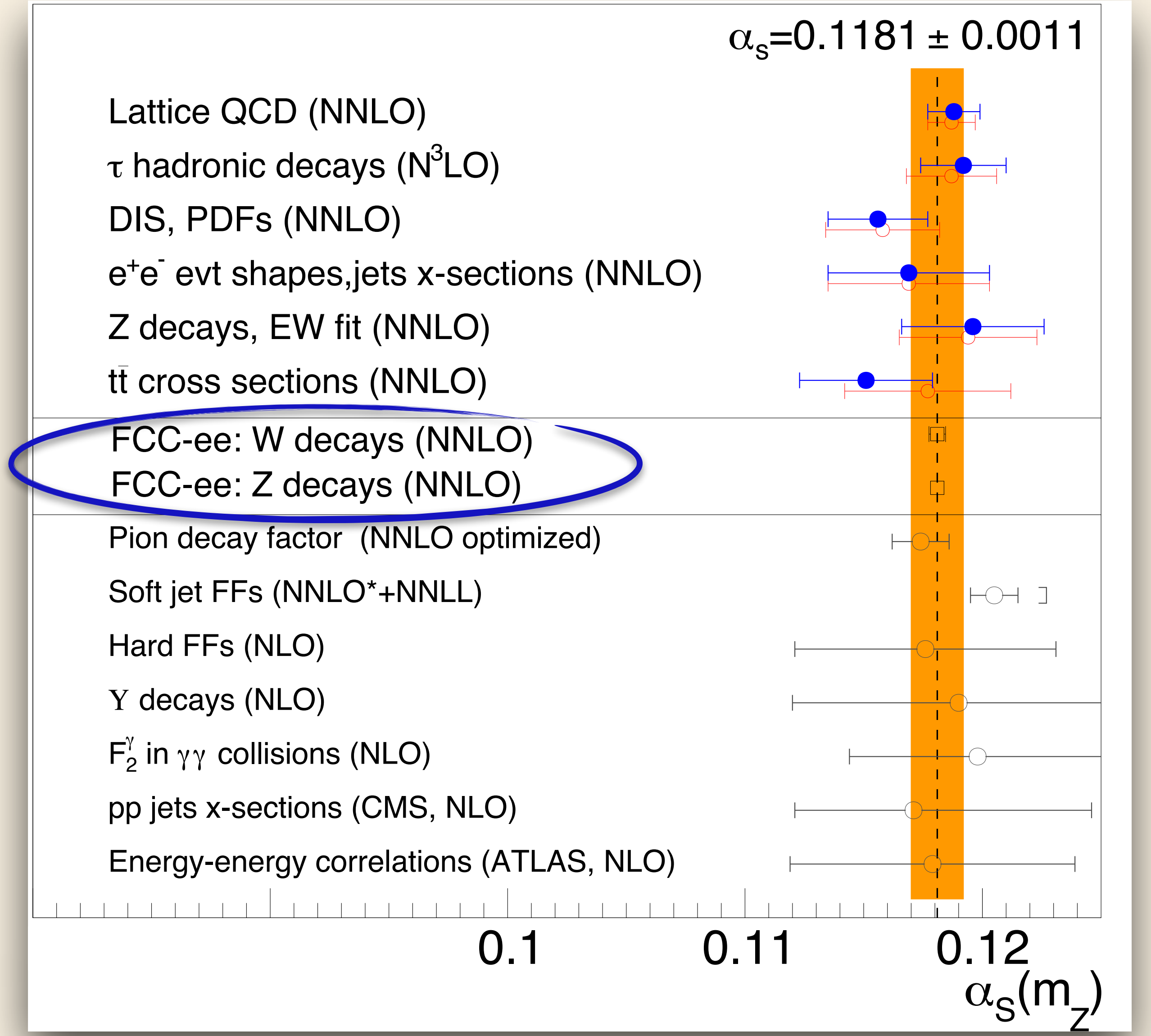

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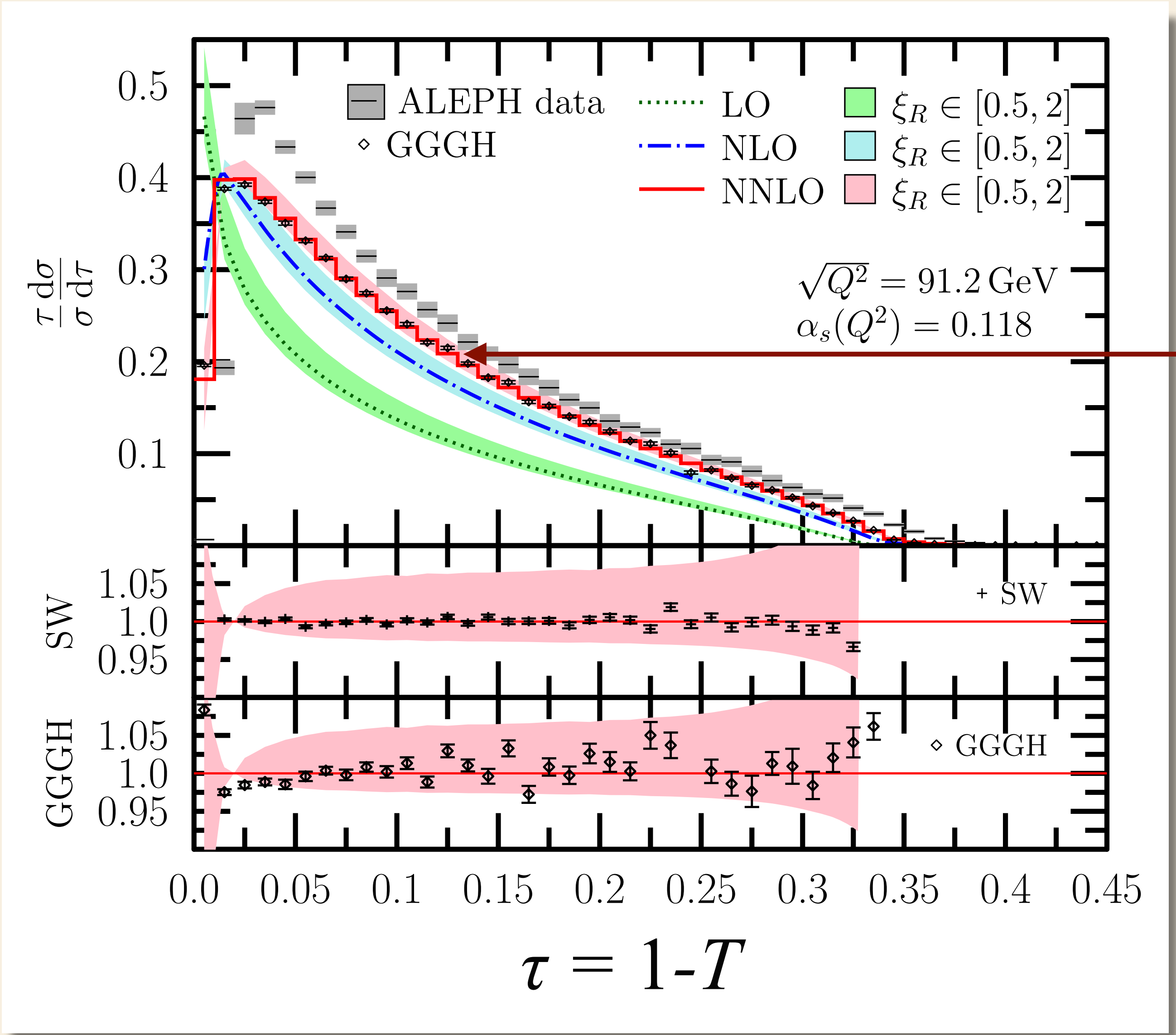
New since LEP

Impact of corrections at NNLO



fixed-order PT is insufficient to describe data

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$$

$$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau} = \left(\frac{\alpha_s}{2\pi}\right) A(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(\tau)$$

A, B and C computed with MCCSM (=Monte Carlo for the CoLoRFulNNLO Subtraction Method)

V. Del Duca et al, arXiv:1603.08927

Causes of failure

1. QCD radiative corrections are large
2. fixed-order perturbation theory fails when logarithms become large → we need
   A. resummation of such logarithmic terms at all orders
   B. matching of fixed order and resummed predictions

An example of analytic structure of the perturbative expansion

$$\frac{\tau}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{2\pi}\right) A(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(\tau)$$

$$L = -\ln\tau$$

$$A(\tau) = A_1 L + A_0,$$

$$B(\tau) = B_3 L^3 + B_2 L^2 + B_1 L + B_0,$$

$$C(\tau) = C_5 L^5 + C_4 L^4 + C_3 L^3 + C_2 L^2 + C_1 L + C_0$$

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&\quad\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots
\end{aligned}$$

LL NLL N^2LL N^3LL …

for $L \sim 1/\alpha_s$ we need resummation of logarithmic terms at all orders

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LL NLL N^2LL N^3LL …

matching predictions at fixed order with resummed has to avoid double counting — achieved by removing coefficients known analytically (precisely)

→ need coefficients in fixed order also precisely

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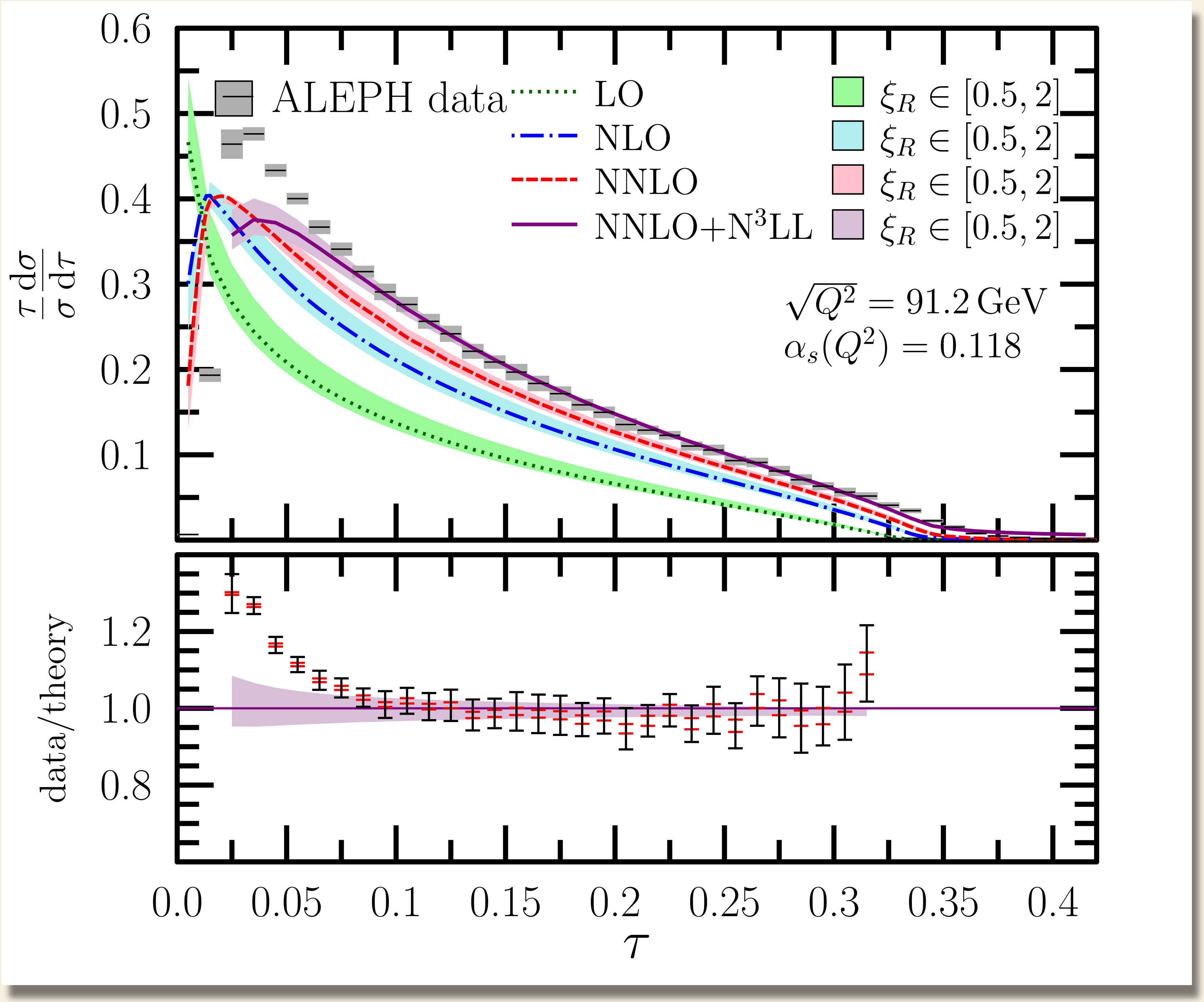
some precise predictions are available:

- NNLO+N^{3}LL for 1-T, C-parameter & heavy jet mass (ρ)
- NNLO+N^{2}LL for broadenings and EEC

Matching NNLO with N^3LL



Works for $\tau > 0.1$, fails in peak regions

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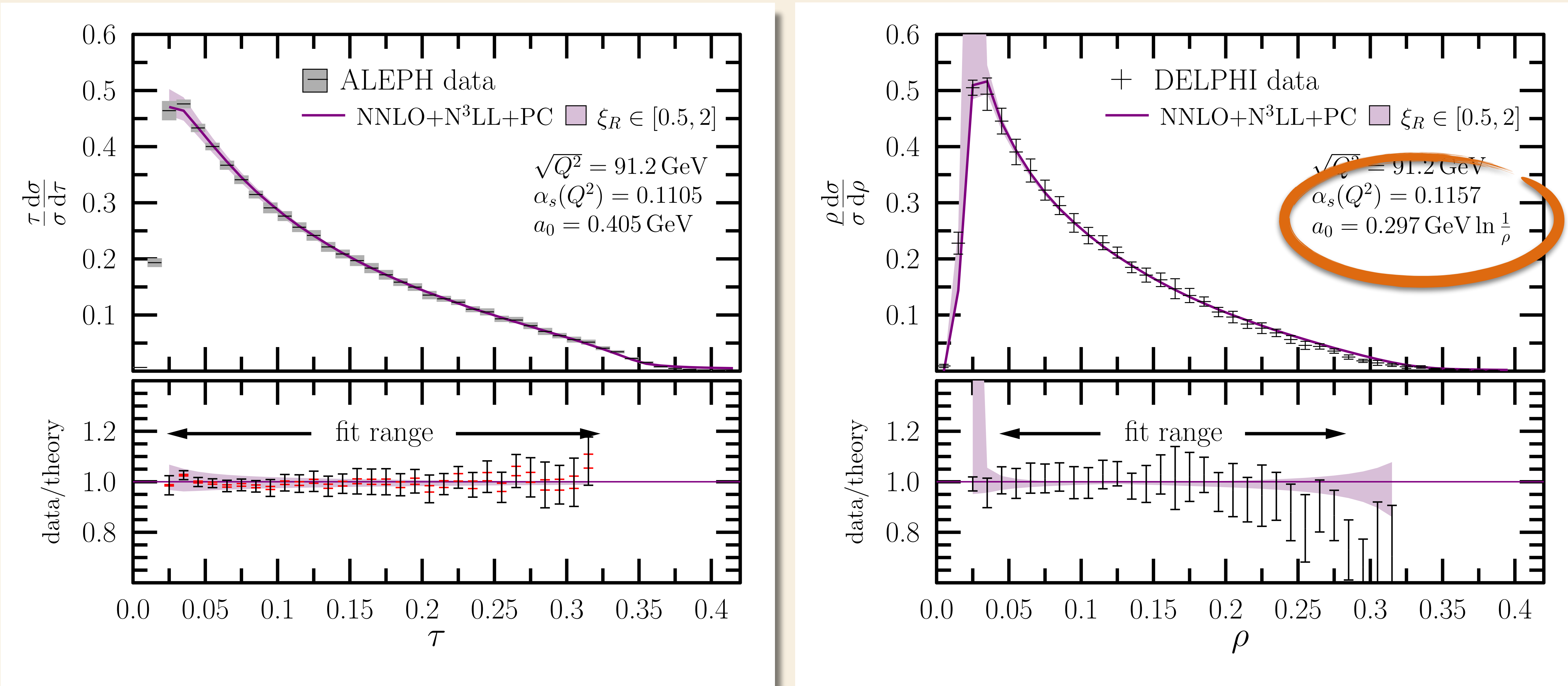
   B. matching of fixed order and resummed predictions
3. hadronization corrections are
   A. large, especially for small values of the event shape, i.e near the peak
   B. not well understood from first principles

two options:

- estimate of hadronisation using modern MC tools
- use analytic model for power corrections

both have their caveats

Fit data on thrust and heavy jet mass with NNLO+N^{3}LL+PC



… does not look universal

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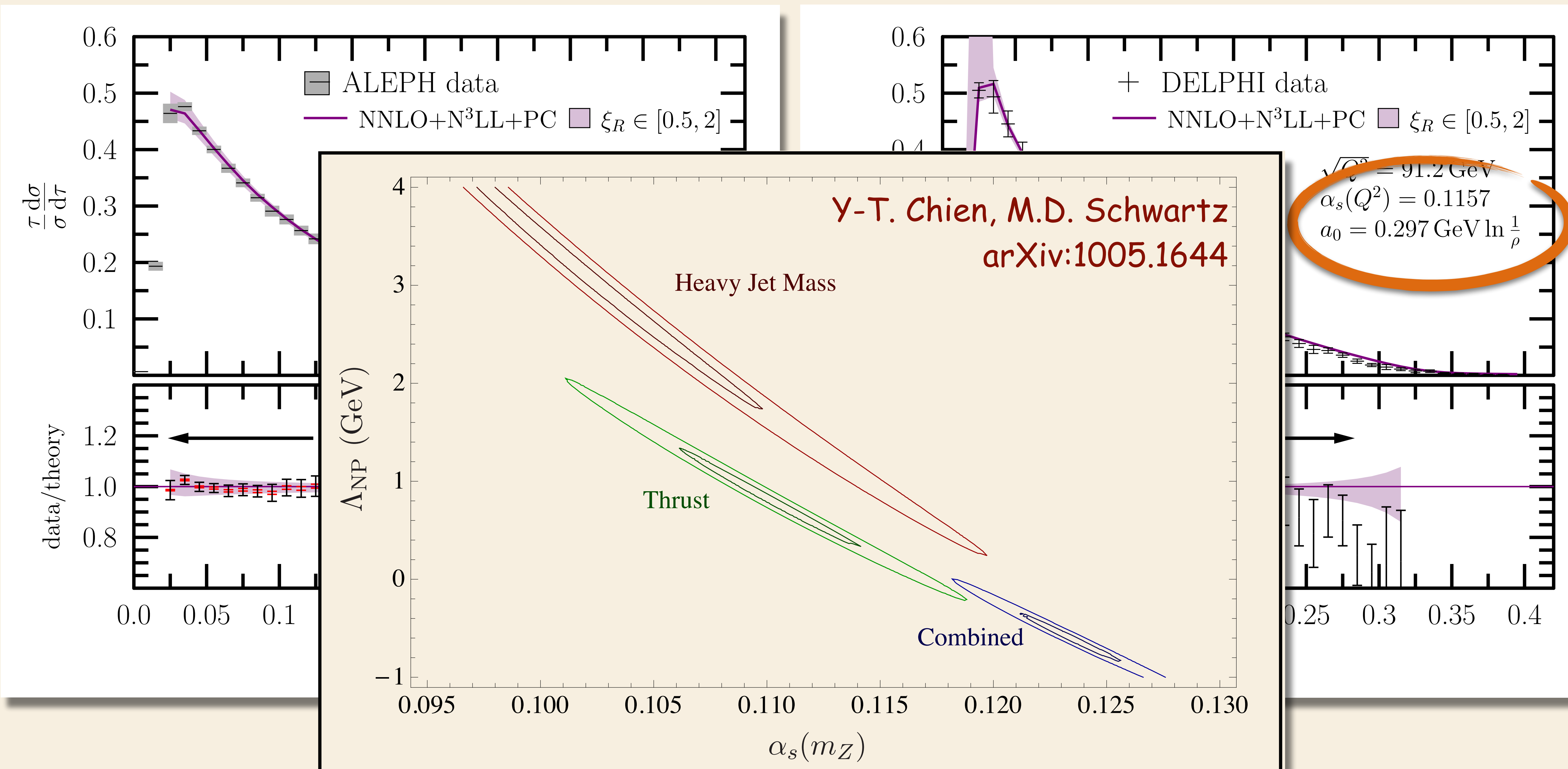

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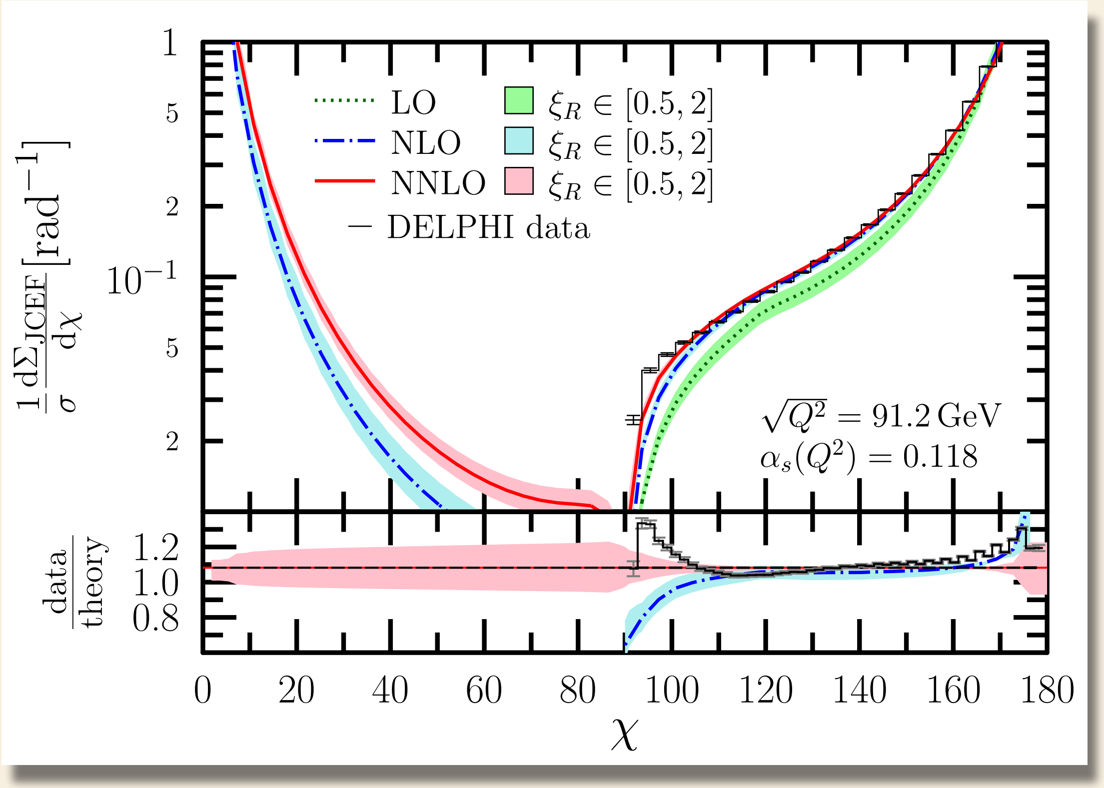
5. Monte Carlo estimates are model dependent

How to improve?

✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: "large uncertainty in small quantity is small uncertainty"

jet cone energy fraction: $$\frac{d\Sigma_{\rm JCEF}}{d\cos\chi} = \sum_i \int \frac{E_i}{Q} d\sigma_{e^+e^-\to i+X}\,\delta\left(\cos\chi - \frac{\vec{p}_i\cdot\vec{n}_T}{|\vec{p}_i|}\right)$$



V. Del Duca et al, arXiv:1606.03453

How to improve?

- ✓ Correct for hadronisation, 2nd option:
  - – estimate of hadronisation using modern MC tools
- ✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: “large uncertainty in small quantity is small uncertainty”

- – precluster hadrons and compute shapes from jets

Decamp et al [ALEPH], Phys.Lett. B257 (1991) 479-491

- – groomed event shapes, designed to reduce contamination from non-perturbative effects

Groomed heavy jet mass

mMDT grooming algorithm

1. Divide the final state of an e+e− → hadrons event into two hemispheres in any infrared and collinear safe way.
2. In each hemisphere, run the Cambridge/Aachen jet algorithm to produce an angular-ordered pairwise clustering history of particles.
3. Undo the last step of the clustering for the one hemisphere, and split it into two particles; check if these particles pass the mass drop condition, which is defined for e^+e^- collisions as:

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\mathrm{cut}}$$

   where E_i and E_j are the energies of the two particles

3. If the splitting fails this condition, the softer particle is dropped and the groomer continues to the next step in the clustering at smaller angle.
4. If the splitting passes this condition the procedure ends and any observable can be measured in the remaining hemispheres

mMDT groomed heavy jet mass

Factorization formula for $\tau_\mathrm{L}, \tau_\mathrm{R} \ll z_\mathrm{cut} \ll 1$ $\quad \tau_i = \frac{m_i^2}{E_i^2}$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau_\mathrm{L}\,\mathrm{d}\tau_\mathrm{R}} = H(Q^2)S(z_\mathrm{cut})\left[J(\tau_\mathrm{L})\otimes S_c(\tau_\mathrm{L}, z_\mathrm{cut})\right]\left[J(\tau_\mathrm{R})\otimes S_c(\tau_\mathrm{R}, z_\mathrm{cut})\right]$$

C. Frye et al, arXiv: 1603.09338

Convolutions —

true product for Laplace transforms:

$$\frac{\sigma(\nu_\mathrm{L}, \nu_\mathrm{R})}{\sigma_0} = H(Q^2)S(z_\mathrm{cut})\tilde{J}(\nu_\mathrm{L})\tilde{S}_c(\nu_\mathrm{L}, z_\mathrm{cut})\tilde{J}(\nu_\mathrm{R})\tilde{S}_c(\nu_\mathrm{R}, z_\mathrm{cut})$$

Modified mass drop tagger groomed heavy jet mass:

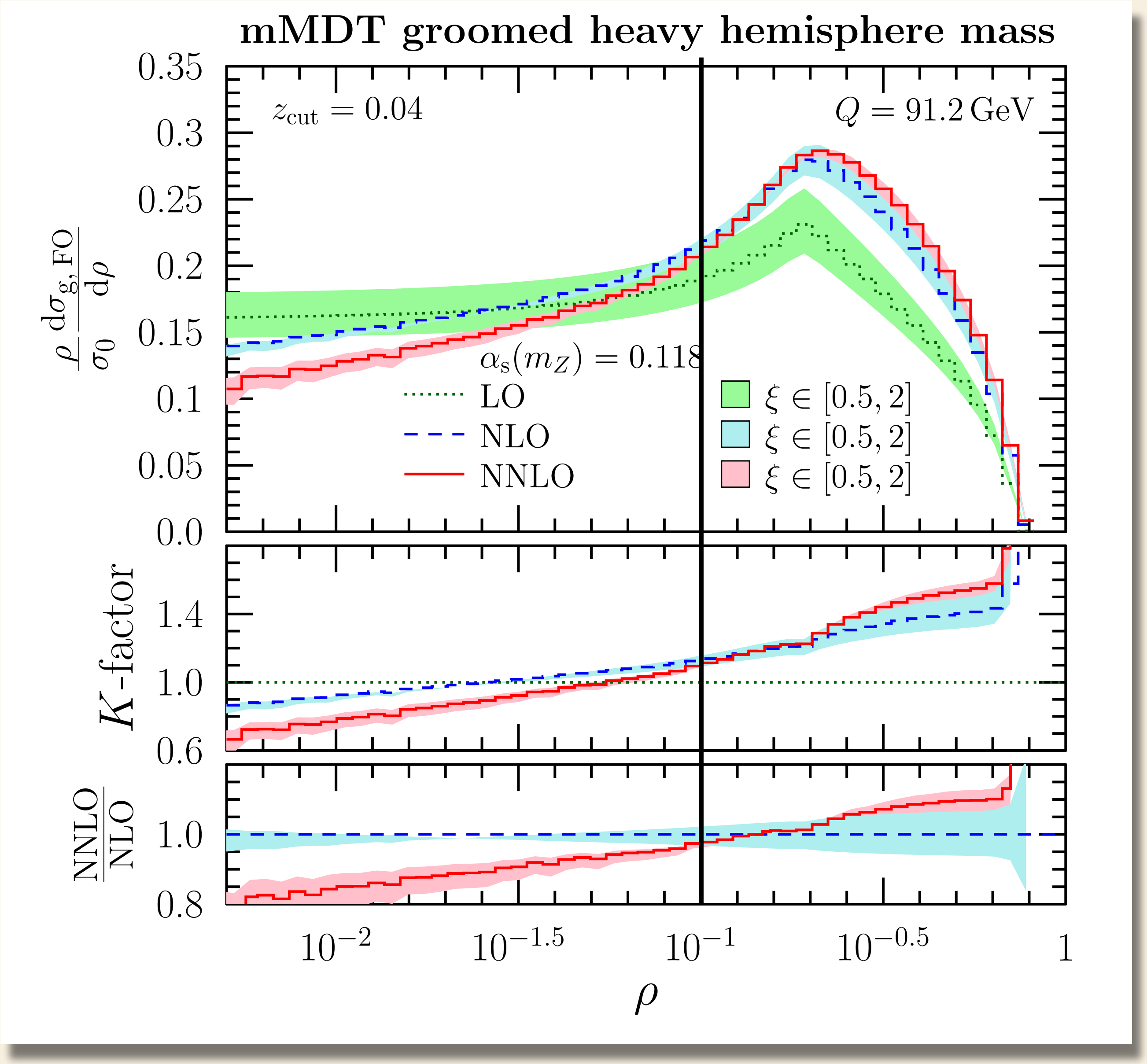
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_\mathrm{g}}{\mathrm{d}\rho} \equiv \int \mathrm{d}\tau_\mathrm{L}\,\mathrm{d}\tau_\mathrm{R}\,\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau_\mathrm{L}\,\mathrm{d}\tau_\mathrm{R}}\left[\Theta(\tau_\mathrm{L}-\tau_\mathrm{R})\,\delta(\rho-\tau_\mathrm{L}) + \Theta(\tau_\mathrm{R}-\tau_\mathrm{L})\,\delta(\rho-\tau_\mathrm{R})\right]$$

mMDT groomed heavy jet mass

$$\rho\frac{\mathrm{d}\sigma_{\mathrm{g,NNLO}}}{\mathrm{d}\rho} = \frac{\alpha_s}{2\pi}A_{\mathrm{g}} + \left(\frac{\alpha_s}{2\pi}\right)^2\left[B_{\mathrm{g}} + A_{\mathrm{g}}\beta_0\log\frac{\mu}{Q}\right]$$
$$+\left(\frac{\alpha_s}{2\pi}\right)^3\left[C_{\mathrm{g}} + 2B_{\mathrm{g}}\beta_0\log\frac{\mu}{Q} + A_{\mathrm{g}}\left(\frac{\beta_1}{2}\log\frac{\mu}{Q} + \beta_0^2\log^2\frac{\mu}{Q}\right)\right]$$

A, B and C are computed with MCCSM (=Monte Carlo for the CoLoRFulNNLO Subtraction Method)

Converges for ρ > 0.1, cannot be trusted for ρ < 0.1



A. Kardos et al, arXiv: 1807.11472

mMDT groomed heavy jet mass

Resummation is made possible by the RGEs:

$$\mu\frac{\partial\tilde{F}}{\partial\mu} = \left(d_F\Gamma_{\mathrm{cusp}}\log\frac{\mu^2}{\mu_F^2} + \gamma_F\right)\tilde{F}\,, \quad (\tilde{F} = H\,,\ S\,,\ \tilde{J}\,,\ \tilde{S}_c)$$

order of ingredients needed for N^nLL resummation

	Γ_{cusp}	γ_F	β	c_F
LL	α_s	-	α_s	-
NLL	α_s^2	α_s	α_s^2	-
NNLL	α_s^3	α_s^2	α_s^3	α_s
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2

known | known partially

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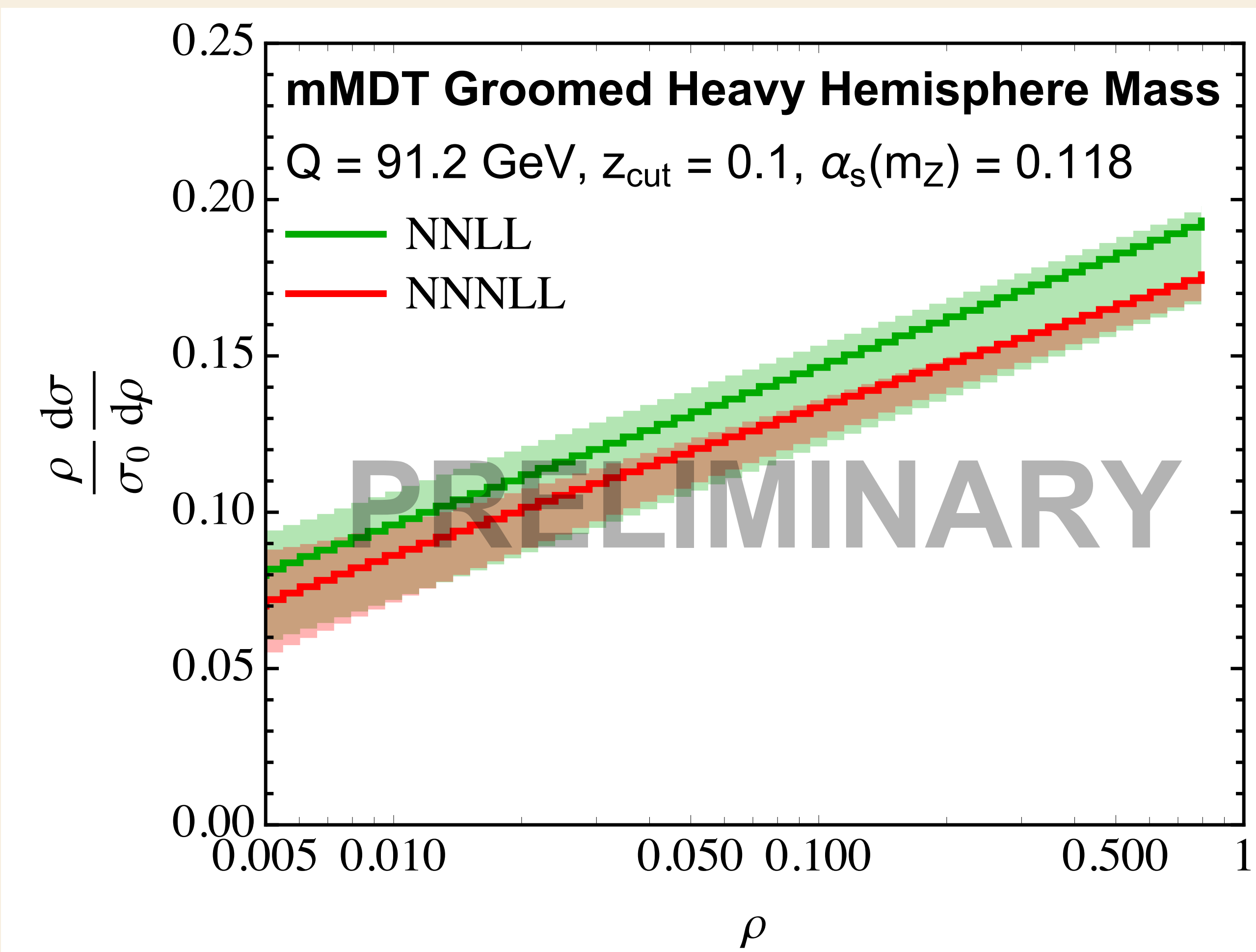

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NNLL	α_s^3	α_s^2	α_s^3	α_s
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2

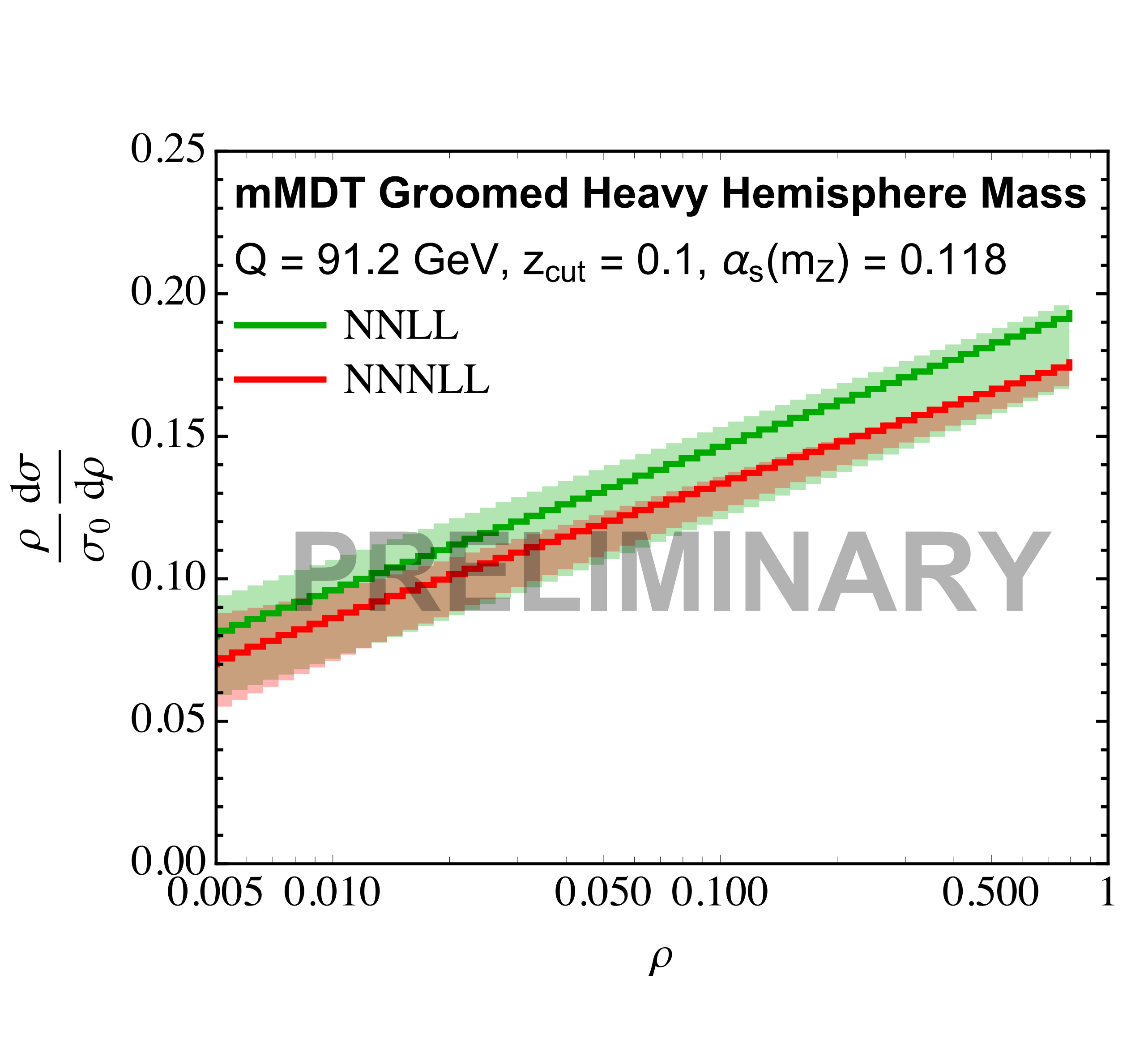
known | known partially

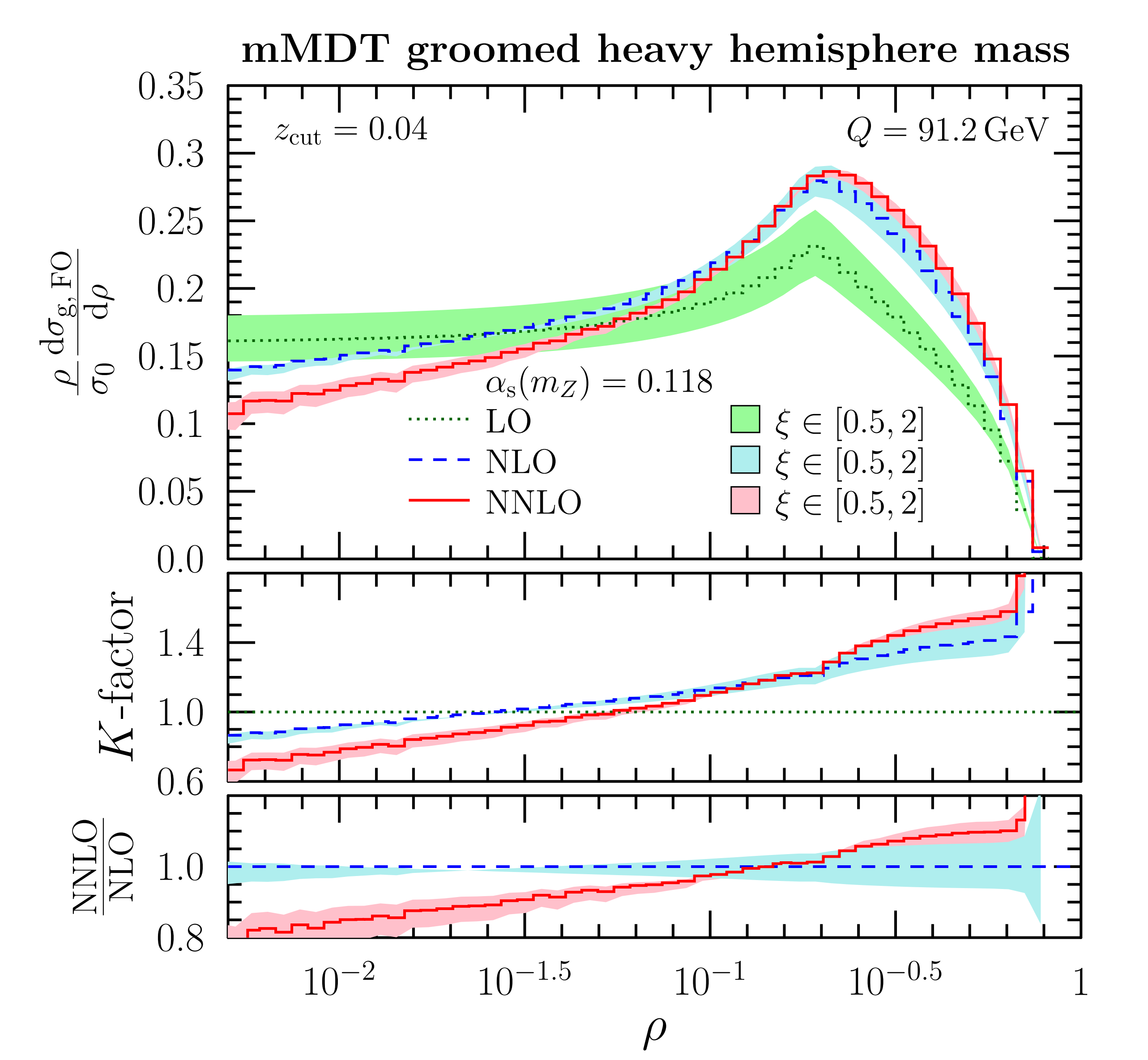
new in this plot: $c_{S_c}^{\text{mMDT}}$ and γ_S^{mMDT}

mMDT groomed heavy jet mass

N^3LL can be matched to N^2LO additively by subtracting the expansion of N^3LL through $O(\alpha_s^3)$

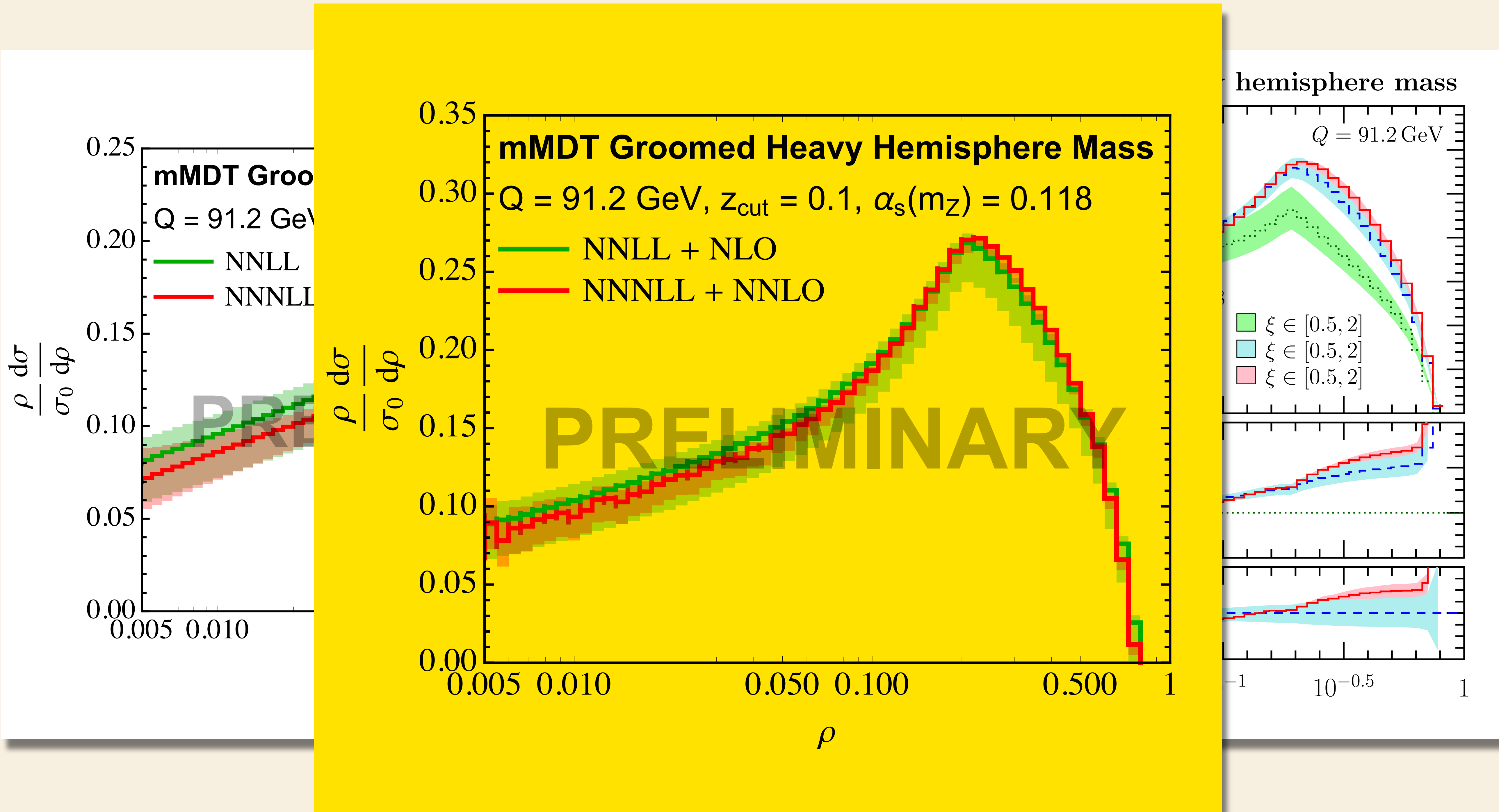
$$\frac{\rho}{\sigma_0}\frac{d\sigma_{g,\mathrm{FO+res}}}{d\rho} = \frac{\rho}{\sigma_0}\left(\frac{d\sigma_{g,\mathrm{N^3LL}}}{d\rho} + \frac{d\sigma_{g,\mathrm{N^2LO}}}{d\rho} - \frac{d\sigma_{g,\mathrm{LP}}}{d\rho}\right)$$





$$\frac{d\sigma_{g,\mathrm{LP}}}{d\rho} = \delta(\rho)D_{\delta,g} + \frac{\alpha_s}{2\pi}(D_{A,g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^2 (D_{B,g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^3 (D_{C,g}(\rho))_+$$

mMDT groomed heavy jet mass



Conclusions

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- ✓ MCCSM was used to compute differential distributions for groomed event shapes — mMDT groomed heavy jet mass among others

- ✓ Our predictions

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  careful selection of observables with small perturbative and non-perturbative corrections (and data — not discussed here)

- ✓ MCCSM was used to compute differential distributions for groomed event shapes — mMDT groomed heavy jet mass among others

- ✓ Our predictions
  - show good perturbative stability for $\rho > 10^{-1}$ (smaller scale dependence than un-groomed event shapes)
  - are stable numerically to $\rho \sim 10^{-4}$
  - were used to extract unknown constants needed for NNNLL resummation and matching

- ✓ NNLO+NNNLL additive matching is made possible the first time