Karlsruhe Institute of Technology

Numerical Results and Expansions for $gg \to HH$

3rd FCC Physics and Experiments Workshop | JHEP 1911 (2019) 024

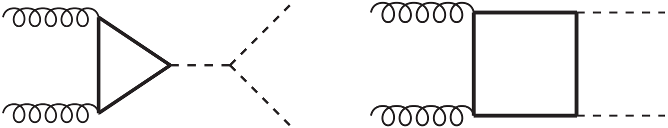
J. Davies, G. Heinrich, S. Jones, M. Kerner, G. Mishima, M. Steinhauser, D. Wellmann

| January 13, 2020

INSTITUTE FOR THEORETICAL PARTICLE PHYSICS (TTP)

Introduction

Dominant channel at a hadron collider: gluon fusion.



$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

Projectors: $\mathcal{F}_{tri} + \mathcal{F}_{box1} = P_{1\,\mu\nu}\mathcal{M}^{\mu\nu}$, $\mathcal{F}_{box2} = P_{2\,\mu\nu}\mathcal{M}^{\mu\nu}$.

Gives access to the Higgs self-coupling λ_{HHH} via $\mathcal{F}_{tri}$.

- experimentally challenging measurement (small cross-section)
- perhaps feasible with HL-LHC, more-so with FCC

NLO Theory Status

Karlsruhe Institute of Technology

LO

- full result [Glover,van der Bij ‘88][Plehn,Spira,Zerwas ‘98]

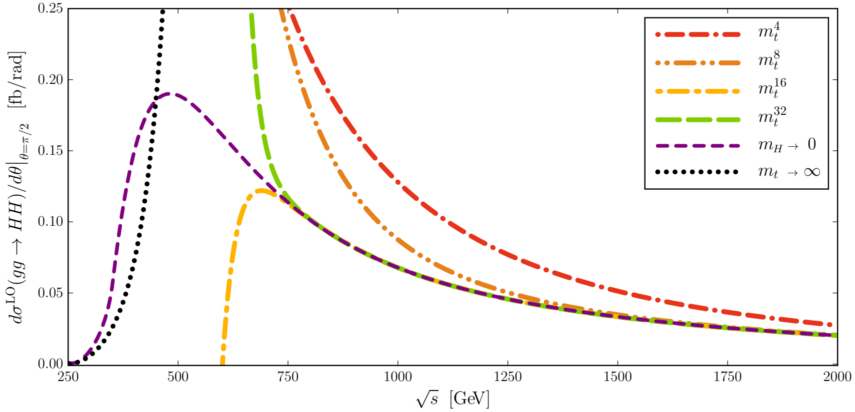
NLO

- numerical result [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke ‘16]
  [Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher ‘18]
- large-m_t limit [Dawson,Dittmaier,Spira ‘98] [Grigo,Hoff,Melnikov,Steinhauser ‘13]
  [Degrassi,Giardino,Gröber ‘16]
- Padé approx. (large-m_t + threshold) [Gröber,Maier,Rauh ‘17]
- small-p_T limit [Bonciani,Degrassi,Giardino,Gröber ‘16]
- high-energy expansion [Davies,Mishima,Steinhauser,Wellmann ‘18,‘19]

Also results at NNLO and $\mathrm{N^3LO}$.

Leading Order

- High-energy limit: $s, t \gg m_t^2 > m_H^2$
- Large-m_t: $m_t \to \infty$



High-Energy Limit

Procedure:

Amplitude in terms of Feynman integrals: $I(m_H^2, m_t^2)$

$\downarrow$

Expand around $m_H^2 = 0$: $I(0, m_t^2) + m_H^2 I'(0, m_t^2) + \cdots$

$\downarrow$

IBP reduce Feynman integrals to master integrals: $J(0, m_t^2)$

$\downarrow$

Determine master integrals around $m_t^2 = 0$:

$$J(0, m_t^2) = \sum_{m,n} C_{m,n} \, (m_t^2)^m \, \log{(m_t^2)}^n$$

$\downarrow$

Amplitude for $\mathbf{s, t \gg m_t^2 > m_H^2}$

Software

Diagram generation	`qgraf`	[Nogueira '93]
Topology mapping	`q2e/exp`	[Harlander,Seidelsticker,Steinhauser '97]
Physics, projection	`TFORM 4.2`	[Ruijl,Ueda,Vermaseren '17]
$m_H^2 = 0$ expansion	`LiteRed`	[Lee '13]
IBP Reduction	`FIRE 5.2`	[Smirnov '14]
	(`LiteRed`)	[Lee '13]

Feynman Diagrams: $\mathbf{8}^{LO} + \mathbf{118}^{NLO}$

$\downarrow$

Feynman Integrals: **26K** (**+120K** (m_H^2 exp))

$\downarrow$

Masters Integrals: $\mathbf{10}^{LO} + \mathbf{161}^{NLO}$

Compute masters: differential equations

Differentiate master integrals wrt $X \in \{s,t,m_t^2\}$. IBP reduce result:

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}.$$

m_t^2 **equation:** substitute high-energy ansatz for each master integral,

$$J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log\left(m_t^2\right)^k.$$

Obtain a system of linear equations for coefficients $C_{ijk}(s,t)$. **Solve!**

... we require **Boundary Conditions**

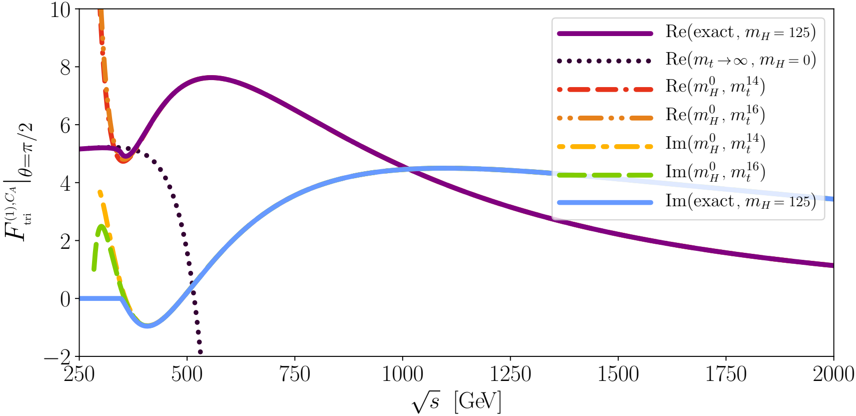
- determine leading powers in $m_t^2 \to$ fixes some $C_{ijk}(s,t)$

Here we determine the amplitude to m_t^{32}.

Results: Form Factors

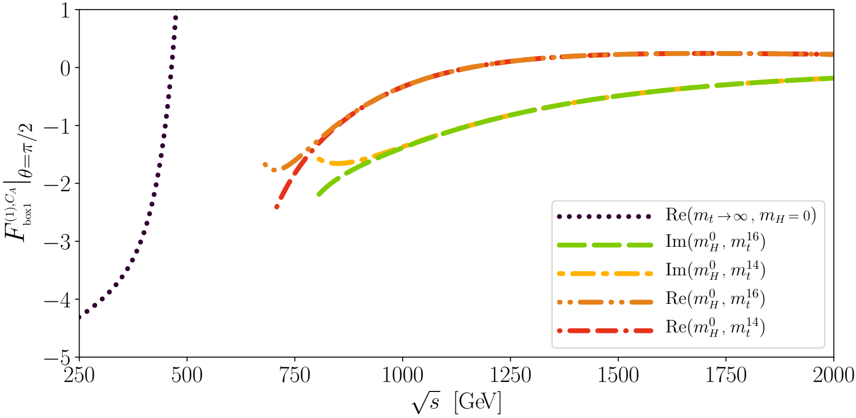
(Renorm. and IR subtraction: $\mathcal{F}_X^{(1)} = \mathcal{F}_X^{(1),IR-div.} - K_g^{(1)}\mathcal{F}_X^{(0)}$)

$\mathcal{F}_{tri}$ known analytically at NLO: $gg \to H$.



Results: Form Factors

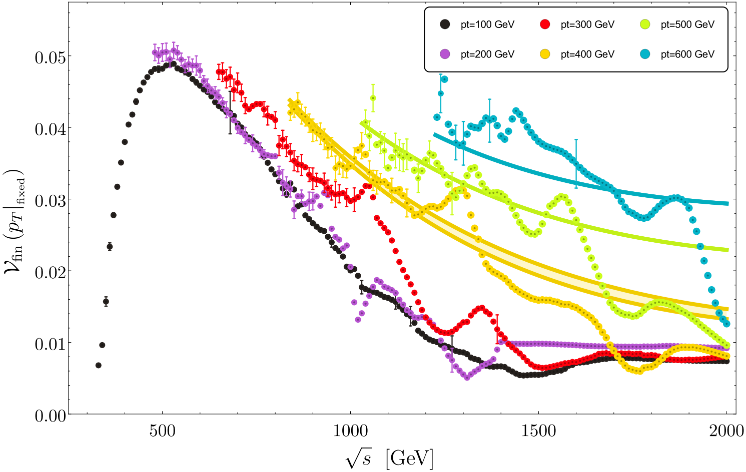
$\mathcal{F}_{box1}$, $\mathcal{F}_{box2}$: no analytic result for comparison.



Results: V_{fin}

V_{fin}: IR finite (subtracted) virtual cross-section. Here, m_t^{30}, m_t^{32} terms.

- We can compare with `hhgrid` [Heinrich,Jones,Kerner,Luisoni,Vryonidou '17]



Padé Improved V_{fin}

Padé Approximant:

$$[n/m](m_t^2) = \frac{a_0 + a_1 m_t^2 + a_2 (m_t^2)^2 + \cdots + a_n (m_t^2)^n}{1 + b_1 m_t^2 + b_2 (m_t^2)^2 + \cdots + b_m (m_t^2)^m}$$

- use high-energy expansion to fix coefficients a_i, b_i.
- evaluate for $m_t = 173$ GeV

Compute Padé approximants with $n + m = 16$, $|n - m| \leq 2$:

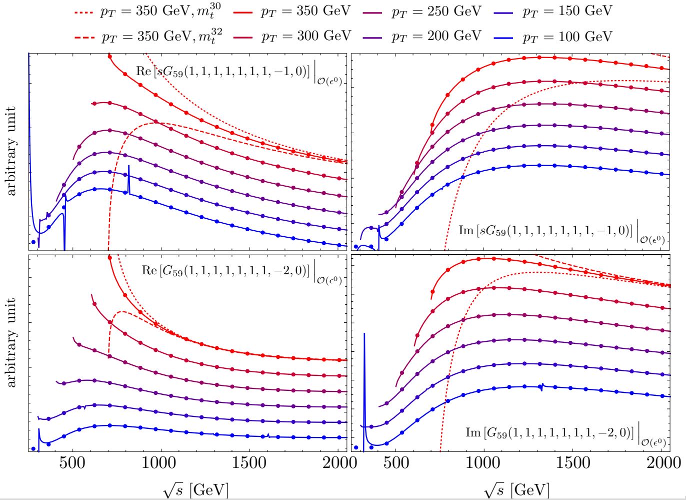
- $[8/8], [7/8], [8/7], [7/9], [9/7]$

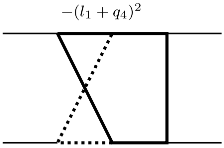
Take (weighted) mean value, (weighted) stdev for error estimate.

Check: Master Integrals

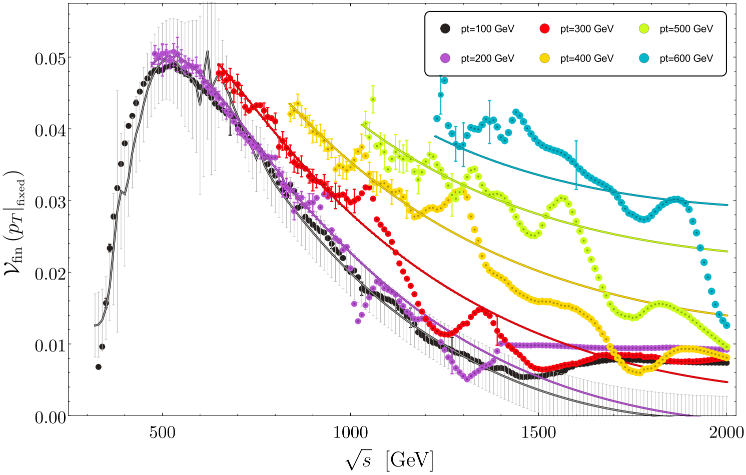
Check the Padé approximation procedure at the level of Master Integrals:

- Dashed: high-energy expansion, Dots: `PySecDec`, Solid: [8/8] Padé





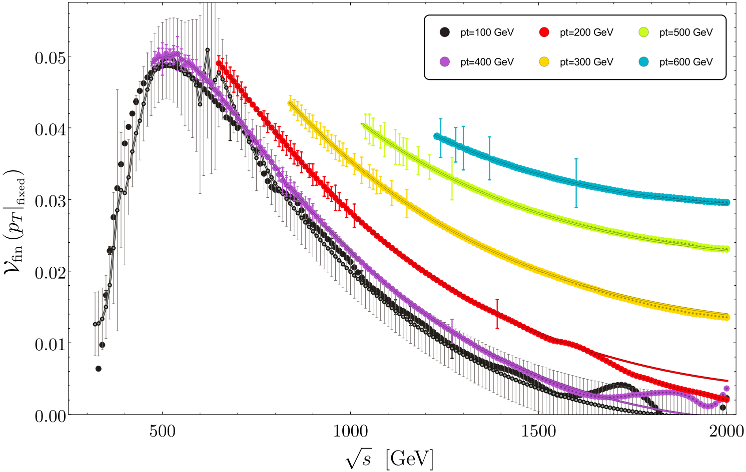
Results: Padé Improved V_{fin}



Obvious next step: augment `hhgrid` with high-energy input points.

Results: Improved `hhgrid`

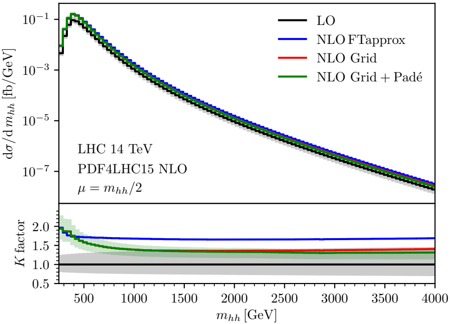
6320 points + 5157 high-energy points

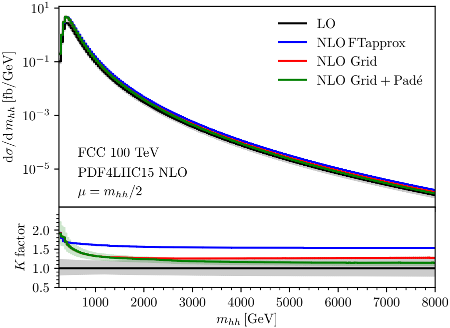


Available from `https://github.com/mppmu/hhgrid`

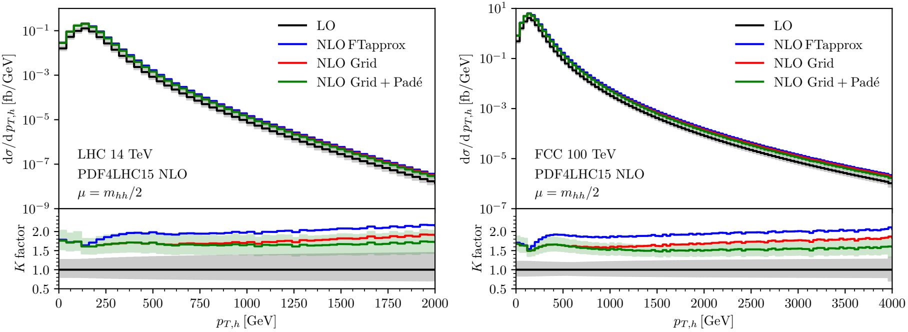
Effect on distributions: m_{hh}

Real corrections from `GoSam`





Effect on distributions: $p_{T,h}$



Conclusions

Exact numerics are expensive, increasingly so at high energies.

- Augment with results of high-energy expansions
- Improved grid available: `https://github.com/mppmu/hhgrid`

Method can be applied to other processes

- $gg \to ZZ$, $gg \to ZH$, $gg \to Hj$, $gg \to \gamma\gamma$