Numerical Results and Expansions for $gg \rightarrow HH$

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J. Davies, G. Heinrich, S. Jones, M. Kerner, G. Mishima, M. Steinhauser, D. Wellmann
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Introduction

Dominant channel at a hadron collider: gluon fusion.

\[ \mathcal{M}^{\mu\nu} \sim A_1^{\mu\nu} (F_{\text{tri}} + F_{\text{box1}}) + A_2^{\mu\nu} (F_{\text{box2}}) \]

Projectors: \( F_{\text{tri}} + F_{\text{box1}} = P_{1\mu\nu} \mathcal{M}^{\mu\nu} \), \( F_{\text{box2}} = P_{2\mu\nu} \mathcal{M}^{\mu\nu} \).

Gives access to the Higgs self-coupling \( \lambda_{HHH} \) via \( F_{\text{tri}} \).

- experimentally challenging measurement (small cross-section)
- perhaps feasible with HL-LHC, more-so with FCC
NLO Theory Status

LO

- full result

NLO

- numerical result
  
  [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zicke '16]
  
  [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '18]

- large-\(m_t\) limit
  
  [Dawson, Dittmaier, Spira '98]
  
  [Grigo, Hoff, Melnikov, Steinhauser '13]
  
  [Degrassi, Giardino, Gröber '16]

- Padé approx. (large-\(m_t\) + threshold)
  
  [Gröber, Maier, Rauh '17]

- small-\(p_T\) limit
  
  [Bonciani, Degrassi, Giardino, Gröber '16]

- high-energy expansion
  
  [Davies, Mishima, Steinhauser, Wellmann '18, '19]

Also results at NNLO and \(N^3\)LO.
Leading Order

- **High-energy limit:** $s, t \gg m_t^2 > m_H^2$
- **Large-$m_t$:** $m_t \to \infty$

![Graph](image-url)

**Legend:**
- $m_t^4$
- $m_t^8$
- $m_t^{16}$
- $m_t^{32}$
- $m_H \to 0$
- $m_t \to \infty$

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**Introduction**
- High-Energy Limit
- Padé Approximation
- Results
- Conclusion

High-Energy Limit

Procedure:

Amplitude in terms of Feynman integrals: \( I(m_H^2, m_t^2) \)

\[ \downarrow \]

Expand around \( m_H^2 = 0 \): \( I(0, m_t^2) + m_H^2 I'(0, m_t^2) + \cdots \)

\[ \downarrow \]

IBP reduce Feynman integrals to master integrals: \( J(0, m_t^2) \)

\[ \downarrow \]

Determine master integrals around \( m_t^2 = 0 \):

\[ J(0, m_t^2) = \sum_{m,n} C_{m,n} (m_t^2)^m \log (m_t^2)^n \]

\[ \downarrow \]

Amplitude for \( s, t \gg m_t^2 > m_H^2 \)
## Software

<table>
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<tr>
<th>Diagram generation</th>
<th>qgraf</th>
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<td>Topology mapping</td>
<td>q2e/exp</td>
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<td>$m_H^2 = 0$ expansion</td>
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<td>IBP Reduction</td>
<td>FIRE 5.2</td>
<td>[Smirnov ‘14]</td>
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<td>(LiteRed)</td>
<td>[Lee ‘13]</td>
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</table>

Feynman Diagrams: $8^{LO} + 118^{NLO}$

↓

Feynman Integrals: $26K$ (+$120K (m_H^2 \text{ exp}))$

↓

Masters Integrals: $10^{LO} + 161^{NLO}$
Compute masters: differential equations

Differentiate master integrals wrt \(X \in \{s, t, m_t^2\}\). IBP reduce result:

\[
\frac{d}{dX} \vec{J} = M(s, t, m_t^2, \epsilon) \cdot \vec{J}.
\]

\(m_t^2\) equation: substitute high-energy ansatz for each master integral,

\[
J = \sum_i \sum_j \sum_k C_{ijk}(s, t) \epsilon^i (m_t^2)^j \log (m_t^2)^k.
\]

Obtain a system of linear equations for coefficients \(C_{ijk}(s, t)\). Solve!

... we require **Boundary Conditions**
- determine leading powers in \(m_t^2 \rightarrow \) fixes some \(C_{ijk}(s, t)\)

Here we determine the amplitude to \(m_t^{32}\).
Results: Form Factors

(Renorm. and IR subtraction: $\mathcal{F}_X^{(1)} = \mathcal{F}_X^{(1),\text{IR-div.}} - K_g^{(1)} \mathcal{F}_X^{(0)}$)

$\mathcal{F}_{\text{tri}}$ known analytically at NLO: \( gg \to H \).
Results: Form Factors

\( F_{box1}, F_{box2} \): no analytic result for comparison.
Results: $V_{\text{fin}}$

$V_{\text{fin}}$: IR finite (subtracted) virtual cross-section. Here, $m_t^{30}$, $m_t^{32}$ terms.

- We can compare with $\text{hhgrid}$ [Heinrich, Jones, Kerner, Luisoni, Vryonidou ‘17]
Padé Improved $V_{\text{fin}}$

Padé Approximant:

$$\frac{n}{m}(m_t^2) = \frac{a_0 + a_1 m_t^2 + a_2(m_t^2)^2 + \cdots + a_n(m_t^2)^n}{1 + b_1 m_t^2 + b_2(m_t^2)^2 + \cdots + b_m(m_t^2)^m}$$

- use high-energy expansion to fix coefficients $a_i, b_i$.
- evaluate for $m_t = 173\text{ GeV}$

Compute Padé approximants with $n + m = 16, |n - m| \leq 2$:

- $[8/8], [7/8], [8/7], [7/9], [9/7]$

Take (weighted) mean value, (weighted) stdev for error estimate.
Check: Master Integrals

Check the Padé approximation procedure at the level of Master Integrals:

- **Dashed**: high-energy expansion, **Dots**: PySecDec, **Solid**: [8/8] Padé

\[ -(l_1 + q_4)^2 \]

- **\( p_T = 350 \text{ GeV}, m_t^{30} \)**
- **\( p_T = 350 \text{ GeV} \)**
- **\( p_T = 250 \text{ GeV} \)**
- **\( p_T = 150 \text{ GeV} \)**
- **\( p_T = 350 \text{ GeV}, m_t^{32} \)**
- **\( p_T = 300 \text{ GeV} \)**
- **\( p_T = 200 \text{ GeV} \)**
- **\( p_T = 100 \text{ GeV} \)**

**Introduction**

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- Padé Approximation

**Results**

- Conclusion

Obvious next step: augment hhgrid with high-energy input points.
Results: Improved hhgrid

6320 points + 5157 high-energy points

Available from https://github.com/mppmu/hhgrid
Effect on distributions: $m_{hh}$

Real corrections from GoSam

![Graphs showing the effect on distributions for $m_{hh}$ on LHC 14 TeV and FCC 100 TeV.](image)

- LO
- NLO FTapprox
- NLO Grid
- NLO Grid + Padé

$LHC$ 14 TeV
PDF4LHC15 NLO
$\mu = m_{hh}/2$

$FCC$ 100 TeV
PDF4LHC15 NLO
$\mu = m_{hh}/2$
Effect on distributions: $p_{T,h}$

- LHC 14 TeV
  - PDF4LHC15 NLO
  - $\mu = m_{hh}/2$
- FCC 100 TeV
  - PDF4LHC15 NLO
  - $\mu = m_{hh}/2$

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- $K$ factor
Conclusions

Exact numerics are expensive, increasingly so at high energies.

- Augment with results of high-energy expansions
- Improved grid available: https://github.com/mppmu/hhgrid

Method can be applied to other processes

- \( gg \rightarrow ZZ, \; gg \rightarrow ZH, \; gg \rightarrow Hj, \; gg \rightarrow \gamma\gamma \)