Prospects for the $\hat{H}$-parameter at future colliders

Admir Greljo

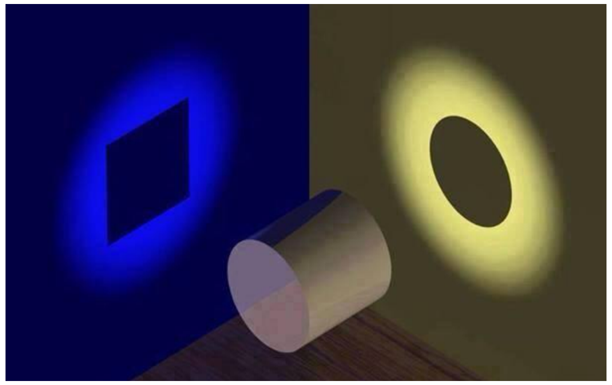
3rd FCC Physics and Experiments Workshop, CERN, 15.1.2020

The question

How does the Higgs propagate? (*)

[Englert, Giudice, Greljo, Mccullough] 1903.07725

(*) assuming the mass gap between NP and SM



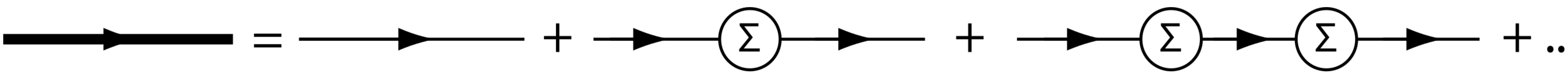
From several angles

- Oblique corrections
- Kallen-Lehmann spectral rep.
- EFT
- UV models
- Phenomenology

Aperitif

- Dyson equation

$$\Delta_h = \Delta_{\rm SM}(1 + \Sigma_h \Delta_h)$$



- Self-energy
  (Sum of all 1PI vacuum polarization diagrams)

$$\Sigma_h(p^2) = \Delta_{\rm SM}^{-1}(p^2) - \Delta_h^{-1}(p^2)$$

$$\Delta_{\rm SM}^{-1} = p^2 - m_h^2$$

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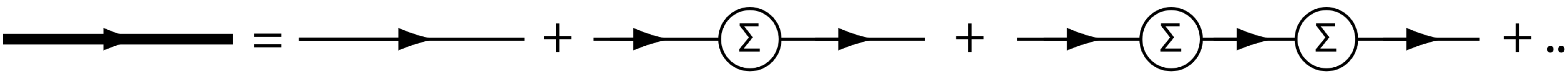

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- **Example:** Real scalar field h

$$\mathscr{L} \supset \frac{\hat{H}}{2\,m_h^2}\left(\partial_\mu \partial^\mu h\right)^2$$

$$-i\Sigma(p^2) = i\frac{\hat{H}}{m_h^2}p^4$$

$$\boxed{\hat{H} = -\frac{m_h^2}{2}\Sigma_h''(m_h^2)}$$

Aperitif

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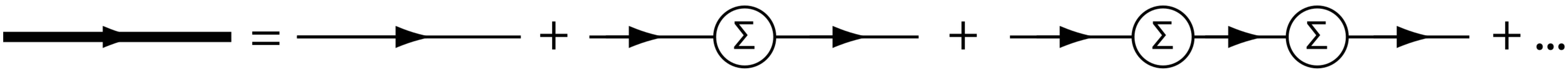

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$$-i\Sigma(p^2) = i\frac{\hat{H}}{m_h^2}p^4 + i(p^2\delta_Z - m_h^2\delta m)$$

On-shell renormalisation conditions:
(Pole with unit residue at the physical mass)

$$\Sigma(m_h^2) = 0 \;, \;\; \Sigma'(m_h^2) = 0$$

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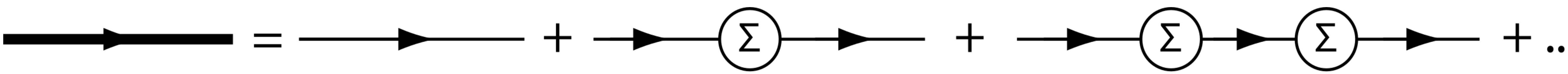

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- Renormalised propagator
(EFT expansion)

$$\Delta_h(p^2) = \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2}$$

Electroweak oblique corrections

- **S, T, W, Y** parameters
  - new physics in the self-energy of EW gauge bosons

[Peskin, Takeuchi] '90
[Altarelli, Barbieri] '91

- Oblique corrections **play a key role** in BSM theory:
  - SUSY, composite Higgs, extra dimension, etc…

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$$\hat{T} = \mathcal{O}(q^0) \qquad \hat{S} = \mathcal{O}(q^2)$$

$$\Pi_V(q^2) = \Pi_V(0) + q^2\Pi'_V(0) + \frac{q^4}{2}\Pi''_V(0) + \ldots$$

$$\hat{W}, \hat{Y} = \mathcal{O}(q^4)$$

$V = \{W^+W^-, W_3W_3, BB, W_3B\}$

There are 4 such two-point amplitudes for a total of 12 parameters, when the expansions are truncated at order q^4. However, after imposing normalisation and symmetry constraints, and by examining the EFT expansion, only 4 parameters can be matched to dim-6 operators.

[Barbieri, Pomarol, Rattazzi, Strumia] hep-ph/0405040

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- **S, T** @ LEP-1
- **W, Y** @ LEP-2, renewed interest in the LHC era

[Wulzer et al] 1609.08157

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Oblique Higgs parameter

[Englert, Giudice, Greljo, Mccullough]
1903.07725

- How does the Higgs boson propagate? (*)

- What is the analogue of **W,Y** in Higgs physics?

$$\mathcal{L}_{\hat{H}} = \frac{\hat{H}}{m_h^2} |\Box H|^2 \qquad\qquad \Box \equiv D^\mu D_\mu$$

- ***$\hat{H}$: the hallmark of off-shell Higgs physics***

(*) Framed within a general EFT context the answer to this question is unphysical and basis-dependent. However there is a broad class of microscopic theories (called Universal theories) which single out a specific EFT basis in which this question not only becomes well-defined, but also plays a key role in mapping out the boundaries of the UV.

Universal EFT

The complete set of CP-even operators in the Universal basis.

'Higgs-only'

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2 \qquad \mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2 \qquad \mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D^\mu H|^2$$

'Gauge-only'

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2 \qquad \mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2 \qquad \mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

'Mixed gauge-Higgs'

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu} \qquad \mathcal{O}_{GG} = \frac{g_s^2\, c_{GG}}{M^2}|H|^2 G^{a,\mu\nu}G^a_{\mu\nu}$$

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$$\mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu}W^a_{\mu\nu}$$

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(*) There exist a field basis in which all leading-order effects are captured by dim-6 operators built from SM bosonic fields only.

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Relations between oblique parameters and Wilson coefficients

$$\hat{S} = 4\left(c_{WB} + \frac{c_W + c_B}{4}\right)\frac{m_W^2}{M^2} \qquad \hat{T} = c_T\,\frac{v^2}{M^2}$$

$$\hat{W} = c_{2W}\,\frac{m_W^2}{M^2} \qquad \hat{Y} = c_{2B}\,\frac{m_W^2}{M^2}$$

$$\hat{Z} = c_{2G}\,\frac{m_W^2}{M^2} \qquad \hat{H} = c_\Box\,\frac{m_h^2}{M^2}$$

Universal EFT

- How?
  - NP interacts primarily with the SM bosons, or
  - NP couples to the conserved currents

$$\Box H = J_H$$

Field redefinitions by equation of motion

'Boson-only' basis

$$|\Box H|^2$$

'Conventional' basis

$$|J_H|^2$$

$$J_H = \mu^2 H - 2\lambda |H|^2 H - \bar{q} i \sigma_2 Y_u^\dagger u - \bar{d} Y_d q - \bar{e} Y_e \ell$$

(*) e.g. 4 top

(*) $\hat{H}$-parameter unjustly neglected in the electroweak fits.

Universal EFT

Present constraints

$\hat{H} < 0.12$ [CMS] 1908.06463 Vs $\hat{W}, \hat{Y} \lesssim \mathcal{O}(10^{-3})$ [LEP2]

Is $\hat{H}$ useful?

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$[g_*^0]$	$[g_*^2]$	$[g_*^4]$
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$$\hat{Z} = c_{2G}\frac{m_W^2}{M^2} \qquad \hat{H} = c_\Box \frac{m_h^2}{M^2}$$

- The 'Higgs-only' operators ordered by the power of the NP coupling.
- $\hat{\mathrm{H}}$-parameter uniquely charting the space of UV completions.

UV model examples

- Example 1: **New scalar matter**

$$\mathcal{L} = \mathcal{L}_{\rm SM} + |D_\mu \widetilde{H}|^2 + |\partial_\mu \widetilde{S}|^2 + \kappa \left(D_\mu \widetilde{H} D^\mu H + \text{h.c.} \right) - V(H, \widetilde{H}, \widetilde{S})$$

g* - a coupling in the scalar potential

In the $g^* \to 0$ limit, only $\mathcal{O}_\Box$ generated.

- Turning on g* (scalar interactions) generates other Higgs-only operators.

- Operators involving gauge bosons generated at one-loop.

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- Example 2: Flat extra dimension

(*) We take the Higgs and gauge bosons to propagate in the bulk, with the fermions localised at one end of the extra dimension. For the Higgs we allow a bulk mass $M_{\rm Bulk}$ and boundary conditions allowing for a massless zero mode localised away from the matter brane.

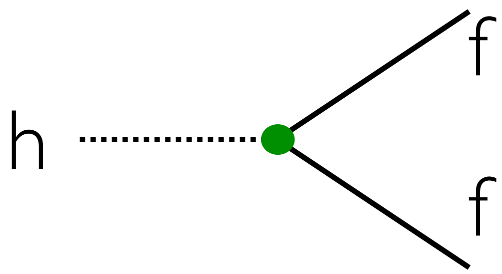
$$M_n^2 = M_{\rm Bulk}^2 + n^2/R^2$$

$$\frac{\hat{H}}{\hat{W}} = c_1(\alpha) \left(1 - \alpha^2\right) \frac{3}{\pi^2} \frac{m_h^2}{m_W^2}$$

- can be arbitrarily large

Physical effects

- Higgs couplings to fermions get universal modification



$$\frac{y_f}{y_f^{\rm SM}} = 1 - \hat{H} - c_H \frac{v^2}{2M^2}$$

Assuming only one operator (*) very strong assumption

$\hat{H} < 0.16$ **LHC today**

$\hat{H} < 0.04$ **HL-LHC projection**

$\hat{H} < 0.005$ **FCC-ee projection**

Physical effects

- Higgs couplings to massive gauge bosons by the 'Higgs-only' set

$$\mathcal{L} = \left(g_{hWW}^{\rm SM}\, W^{+\mu} W_\mu^-\, \mathcal{D}_W + g_{hZZ}^{\rm SM}\, Z^\mu Z_\mu\, \mathcal{D}_Z\right) h\,,$$

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(*) off-shell measurements needed

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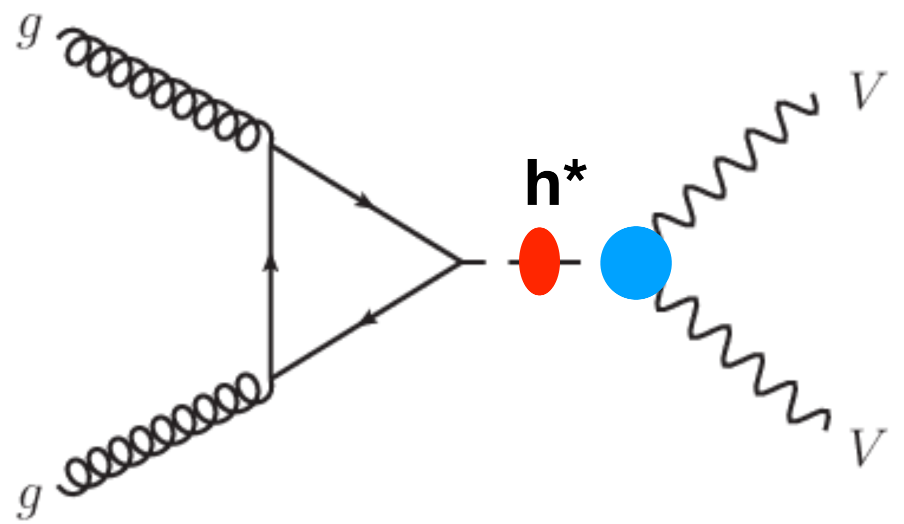
$gg \to ZZ$ cancelation!

$$\left(\frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2}\right)\left[1 - \hat{H}\left(1 - \frac{p^2}{m_h^2}\right)\right] = \frac{1}{p^2 - m_h^2}$$



$gg \to ZZ$ is not a probe of off-shell Higgs physics in the EFT regime

(*) go to other basis by $\Box H = J_H$

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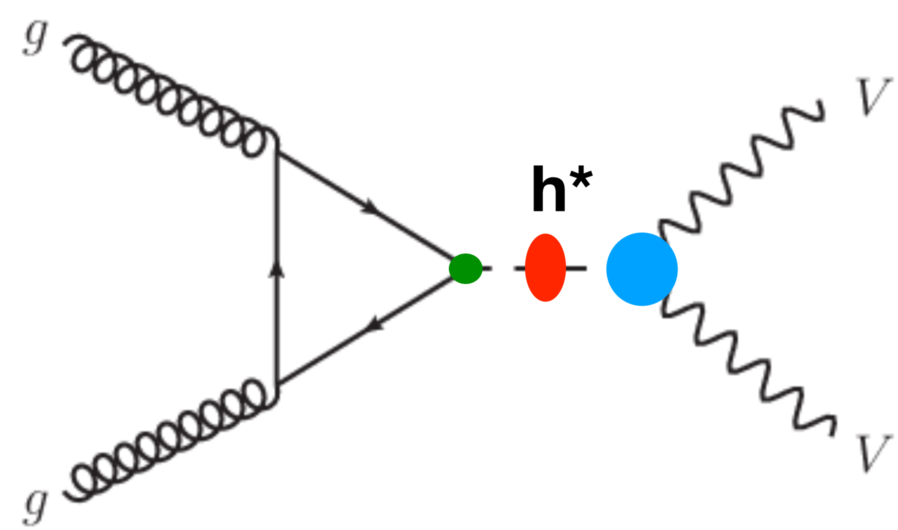

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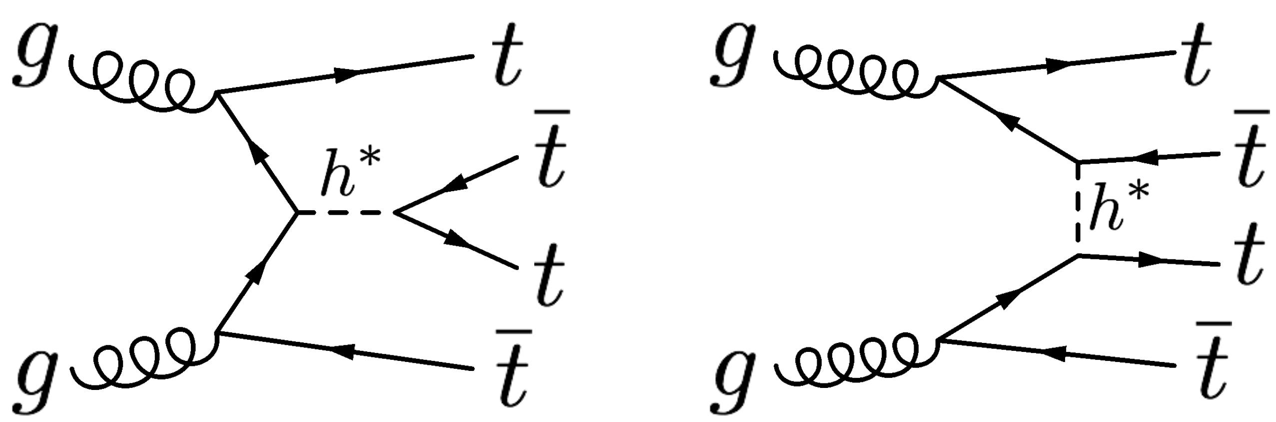
- Up to the top Yukawa correction

$$\frac{y_f}{y_f^{\rm SM}} = 1 - \hat{H} - c_H \frac{v^2}{2M^2}$$

Physical effects

- ***Four-top production for*** $\mathcal{L}_{\hat{H}} = \frac{\hat{H}}{m_h^2} |\Box H|^2$



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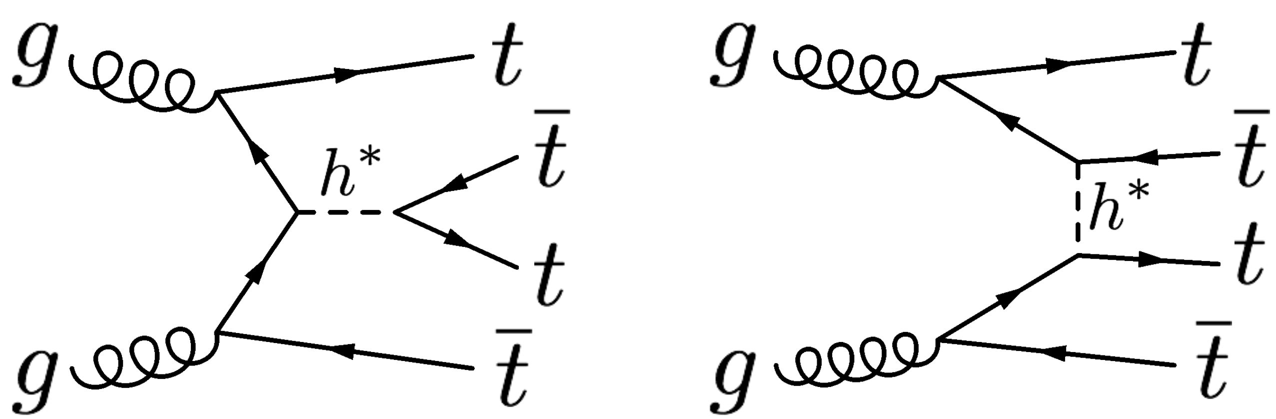

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(*) Modify the numerator of the scalar propagator in **propagators.py** file of the SM UFO

$$(1-\hat{H})^2\left(1-\frac{\hat{H}}{m_h^2}(p^2-m_h^2)\right)$$

"Energy help accuracy"

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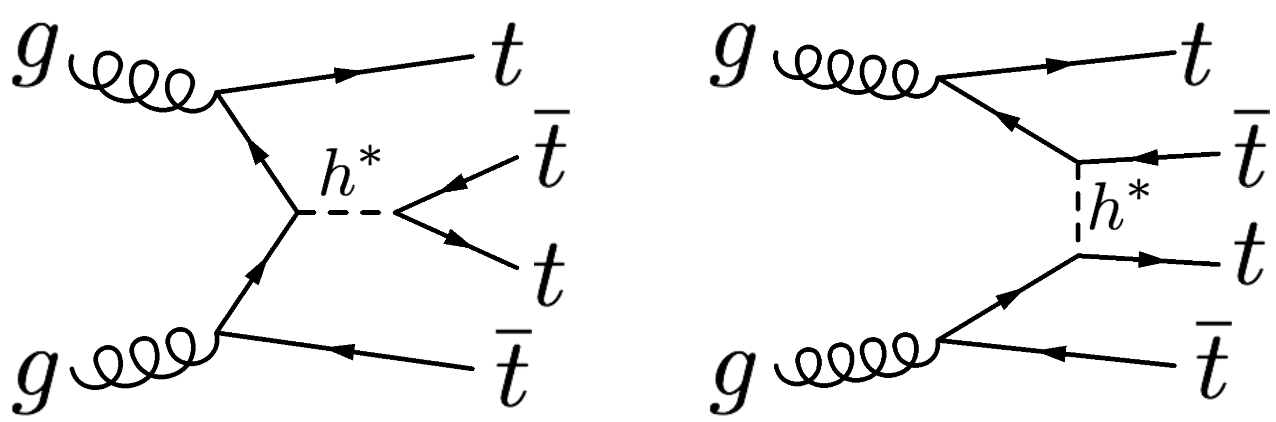

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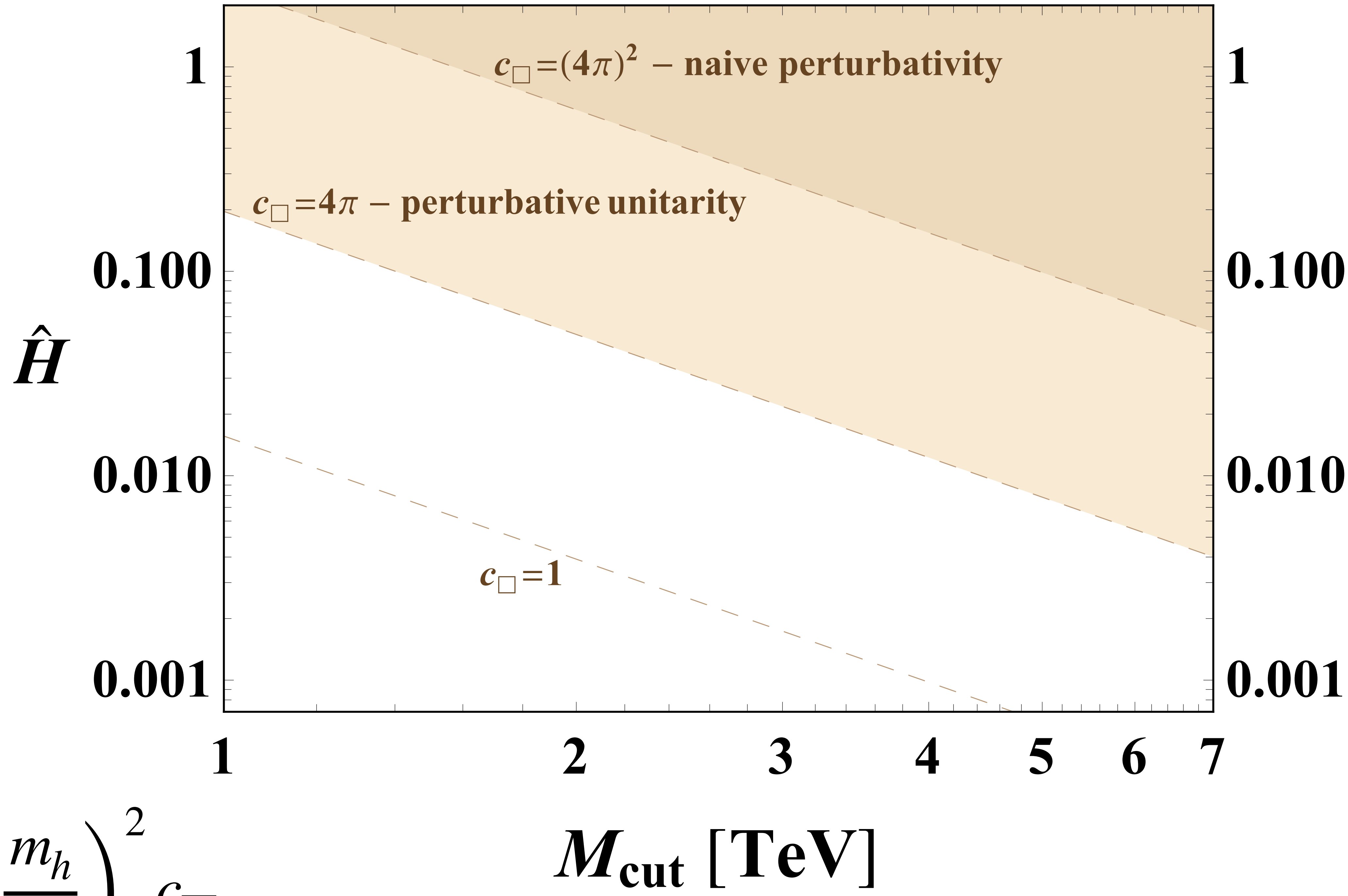
"Energy help accuracy"

- We use the HL-LHC ATLAS projection analysis for inclusive ***pp > 4t*** [ATL-PHYS-PUB-2018-047]
- Same-sign dilepton and trilepton search is particularly clean $S/\sqrt{B} \sim 10$ and S/B in the range of 2.3 to 5.5
- 9% statistical (inclusive, HL-LHC) and 16% systematic uncertainty (mainly theory prediction on ***4t***, ***ttV***, ***ttH***)
- We perform kinematical cuts in H_T and m_{4t} to optimise for $\mathcal{L}_{\hat{H}} = \frac{\hat{H}}{m_h^2}|\Box H|^2$

Projections

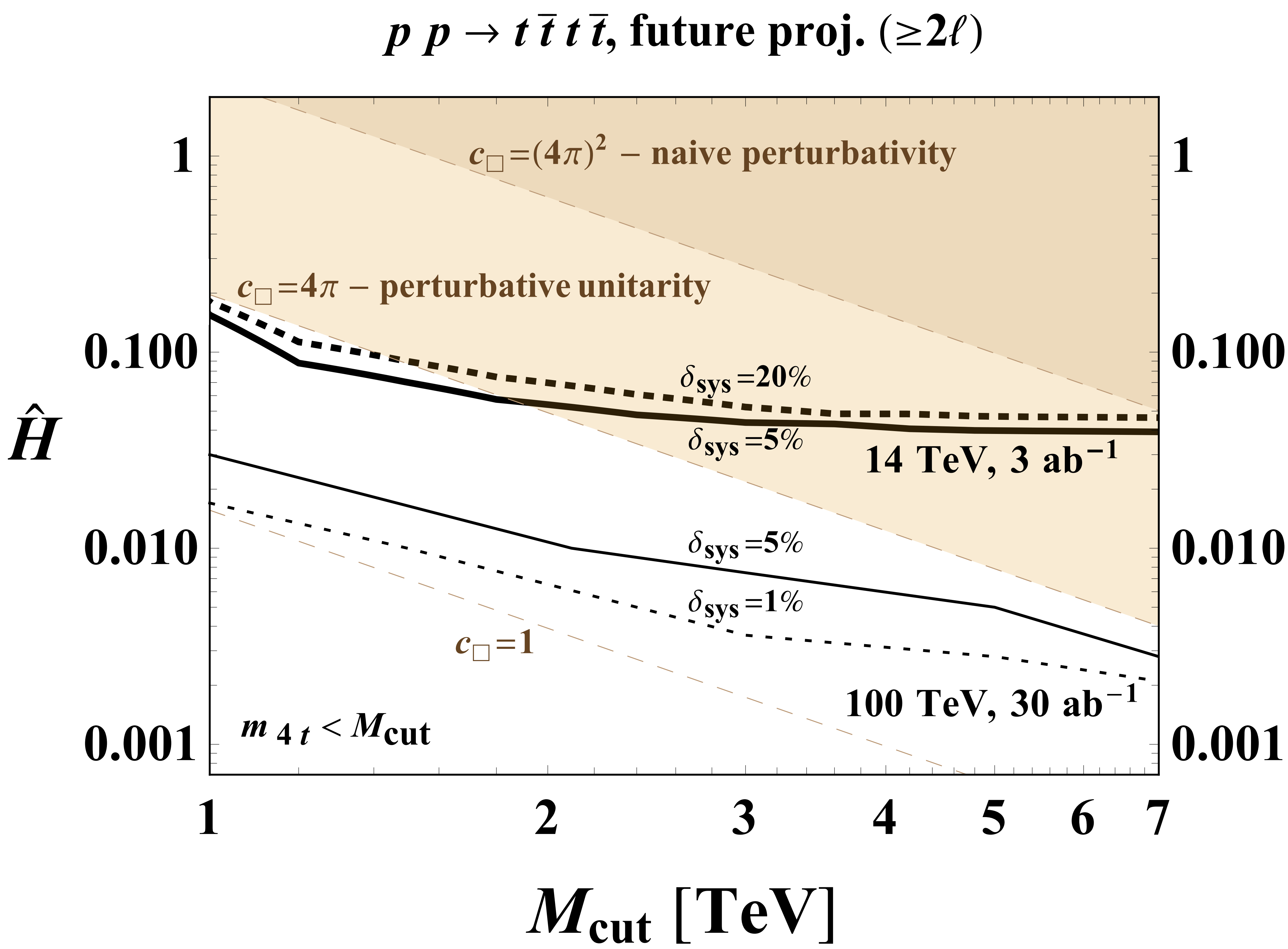
$$\mathcal{L}_{\hat{H}} = \frac{\hat{H}}{m_h^2}|\Box H|^2$$



$$\hat{H} = \left(\frac{m_h}{M}\right)^2 c_\Box$$

cutoff mass M

Projections



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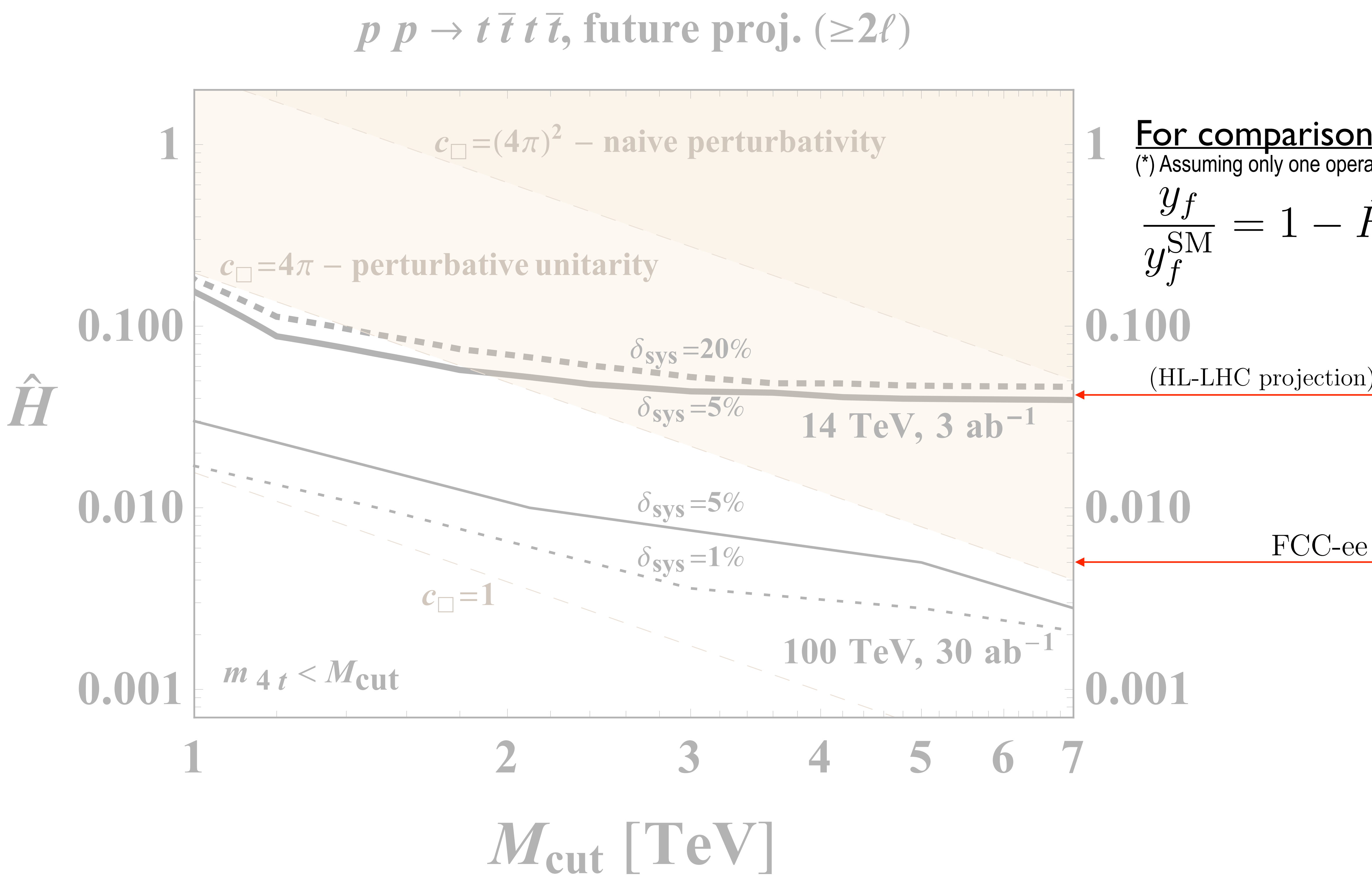


For comparison

(*) Assuming only one operator

$$\frac{y_f}{y_f^{\rm SM}} = 1 - \hat{H}$$

Kallen-Lehmann & EFT

- Expand the spectral representation in p^2/M^2

- Unitarity
- Positive norm
- Causality

$$\Delta_h(p^2) = \Delta_{\rm SM}(p^2) - \frac{1}{M^2}\sum_{n=1}^{\infty} c_n \left(\frac{p^2}{M^2}\right)^{n-1}$$

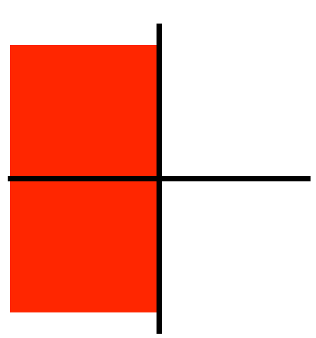
$$c_n = M^2 \int_0^1 dx\, \rho_X(M^2/x)\, x^{n-2}$$

$$p^2 \ll M^2$$

(i) Positivity

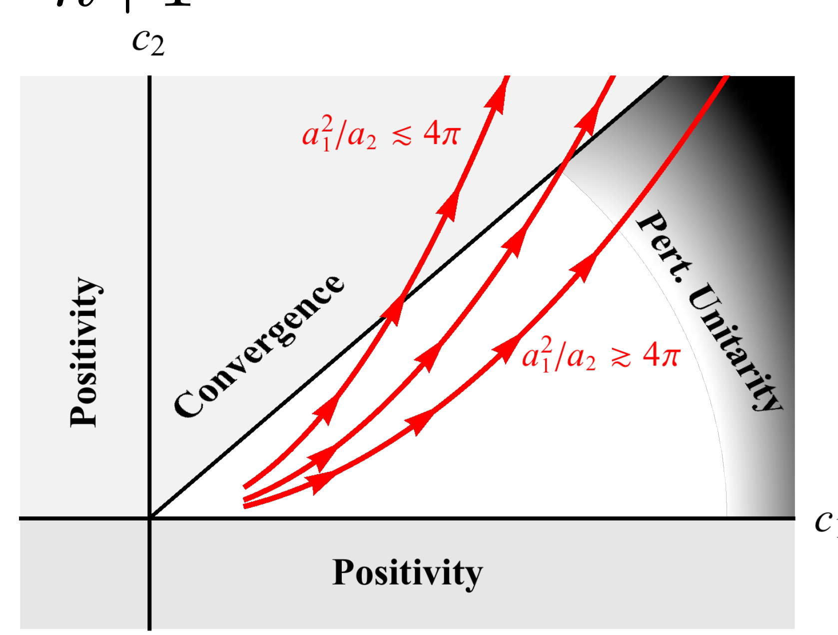
$$c_n \geqslant 0 \quad \forall n$$

$$\hat{H} \geqslant 0$$



(ii) Convergence

$$c_n \geqslant c_{n+1} \quad \forall n$$



Conclusions

- $\hat{H}$-parameter | *The* ***hallmark*** *of off-shell Higgs physics.*
- $|\Box H|^2$ | *Well-motivated operator within the Universal EFT. Uniquely charts an important class of microscopic theories.*
- *Four top production as the main experimental probe. The LHC reach is <u>sadly limited</u>.*
- *KL spectral rep. leads to* ***positivity & convergence*** *criteria.*

Backup

Kallen-Lehmann Spectral Representation

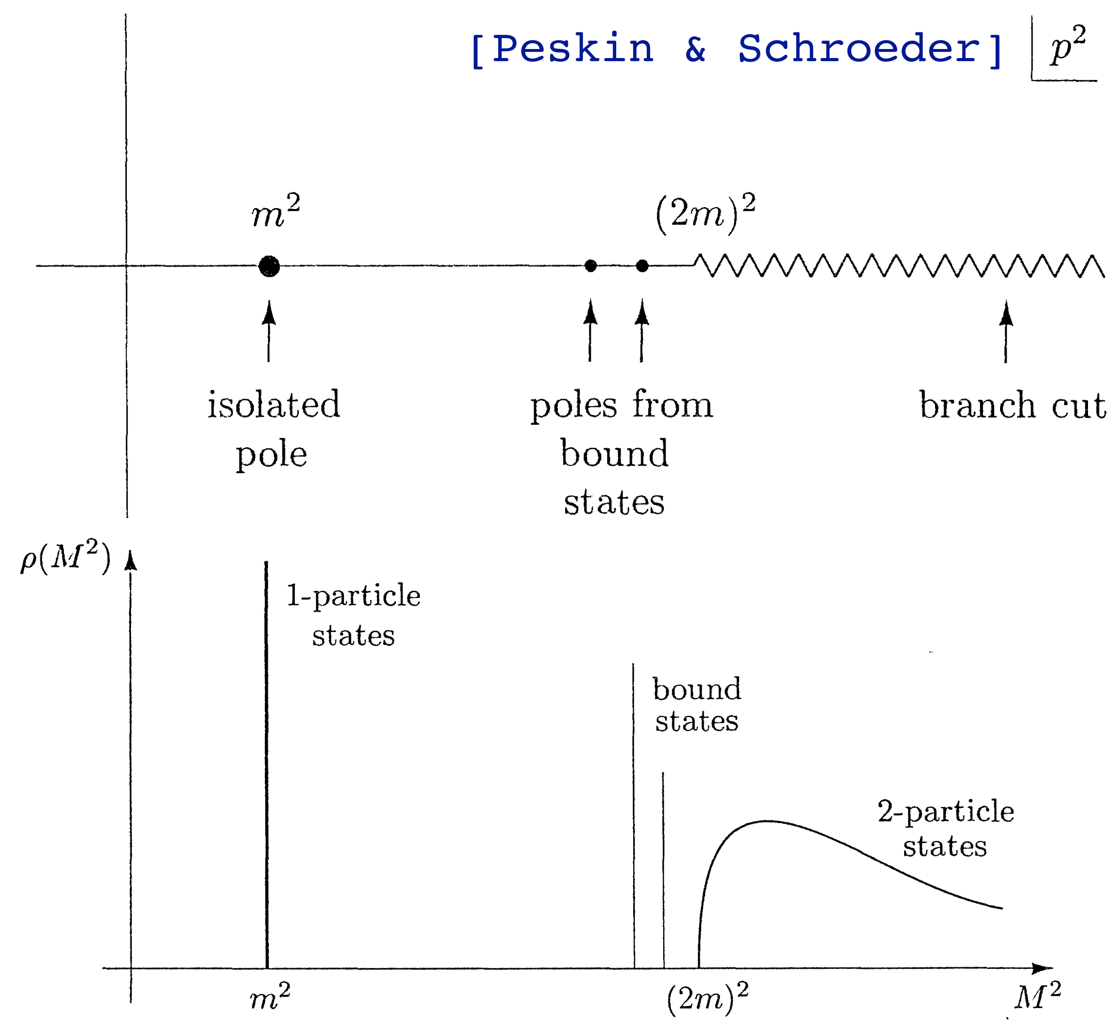
- General representation of the two-point correlation function

$$\int d^4x\, e^{ip\cdot x} \langle\Omega|\, T\phi(x)\phi(0)\,|\Omega\rangle = \int_0^\infty \frac{dM^2}{2\pi}\,\rho(M^2)\,\frac{i}{p^2-M^2+i\epsilon}$$

$$= \frac{iZ}{p^2-m^2+i\epsilon} + \int_{\sim 4m^2}^{\infty} \frac{dM^2}{2\pi}\,\rho(M^2)\,\frac{i}{p^2-M^2+i\epsilon}$$

Based on the basic QFT principles

- Unitarity
- Positive norm
- Causality



- **Positive spectral density function**

$$\rho(M^2) = \sum_\lambda (2\pi)\delta(M^2 - m_\lambda^2)\left|\langle\Omega|\,\phi(0)\,|\lambda_0\rangle\right|^2$$

Kallen-Lehmann & EFT

- Start with

$$\Delta_h(p^2) = -i \int d^4z \; e^{ipz} \langle 0|T\{h(z)h(0)\}|0\rangle \qquad \Delta_h(p^2) = \int_0^\infty dq^2 \frac{\rho_h(q^2)}{p^2 - q^2 + i\epsilon}$$

Kallen-Lehmann & EFT

- Start with

$$\Delta_h(p^2) = -i \int d^4z \; e^{ipz} \langle 0 | T\{h(z)h(0)\} | 0 \rangle \qquad \Delta_h(p^2) = \int_0^\infty dq^2 \frac{\rho_h(q^2)}{p^2 - q^2 + i\epsilon}$$

- Assume a heavy BSM state X in the Hilbert space is above a certain mass gap ***M***

$$\rho_X(q^2 < M^2) = 0 \qquad \rho_h(q^2) = \rho_{\rm SM}(q^2) + \rho_X(q^2)$$

$$\rho_X \propto |<0|h|X>|^2$$

Kallen-Lehmann & EFT

- Start with

$$\Delta_h(p^2) = -i \int d^4z \; e^{ipz} \langle 0 | T\{h(z)h(0)\} | 0 \rangle \qquad \Delta_h(p^2) = \int_0^\infty dq^2 \frac{\rho_h(q^2)}{p^2 - q^2 + i\epsilon}$$

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$$\rho_X \propto | < 0 | h | X > |^2$$

- Expand the spectral representation in p^2/M^2

$$\Delta_h(p^2) = \Delta_{\rm SM}(p^2) - \frac{1}{M^2} \sum_{n=1}^{\infty} c_n \left(\frac{p^2}{M^2} \right)^{n-1}$$

$$c_n = M^2 \int_0^1 dx \, \rho_X(M^2/x) \, x^{n-2}$$

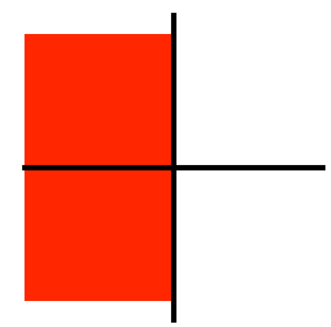
$$p^2 \ll M^2$$

Consistency condition

$$c_n = M^2 \int_0^1 dx \, \rho_X(M^2/x) \, x^{n-2}$$

(i) Positivity

$$\hat{H} \geqslant 0$$



$$c_n \geqslant 0 \quad \forall n$$

Consistency condition

$$c_n = M^2 \int_0^1 dx\, \rho_X(M^2/x)\, x^{n-2}$$

(ii) Convergence

$$c_n \geqslant c_{n+1} \quad \forall n$$

* This condition implies that higher orders in the EFT expansion are not only suppressed by additional powers of p^2/M^2 (which is smaller than one, whenever the EFT is valid), but their corresponding Wilson coefficients also become progressively smaller (or stay constant).

- Saturated for the single-state tree-level exchange
  $\rho_X(q^2) \propto \delta(q^2 - M^2)$

- D'Alembert's criterion - absolutely convergent series $\qquad \lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \frac{p^2}{M^2} \right| < 1$

Consistency condition

(ii) Convergence

$$\Delta_h(p^2) = \Delta_{\rm SM}(p^2) - \frac{1}{M^2}\sum_{n=1}^{\infty} c_n \left(\frac{p^2}{M^2}\right)^{n-1} \qquad\qquad c_n \geqslant c_{n+1} \quad \forall n$$

- In the low-energy experiment

  $$a_n \;\equiv\; c_n/M^{2n}$$

  are measurable

$$M^2 \leqslant \frac{a_n}{a_{n+1}} \quad \forall n \qquad\qquad \text{(convergence).}$$

- ***If consecutive powers in the EFT expansion were measured, one could place an upper bound on the value of the true cutoff.***

Consistency condition

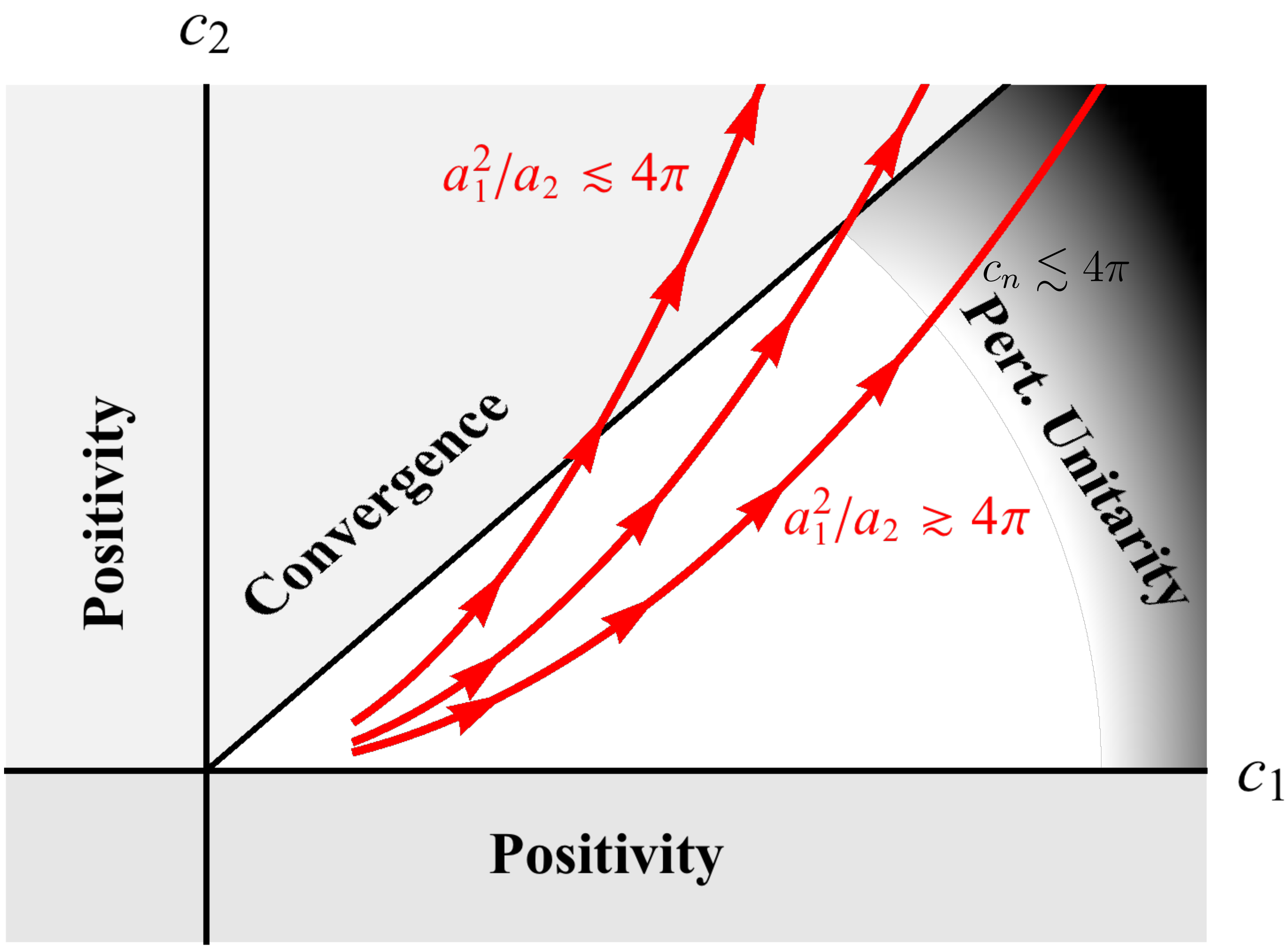


Figure 1. In the plane spanned by c_1 and c_2 (the two leading coefficients of the propagator derivative expansion) we show how the constraints from *(i) positivity, (ii) convergence, (iii) perturbative unitarity* single out a theoretically-allowed bounded region. An experimental measurement of a_1 and a_2 (the first two terms in a momentum expansion) selects the curve $c_2 = c_1^2\, a_2/a_1^2$. Examples of these curves (for different values of a_2/a_1^2) are shown by solid red lines, which are generated by varying the cutoff mass M. The value of M increases along the direction of the arrows. The stronger bound on M comes from *convergence* when $a_1^2/a_2 \lesssim 4\pi$ and from *perturbative unitarity* when $a_1^2/a_2 \gtrsim 4\pi$.

(*) A very short detour from the Higgs

Convergence in practice

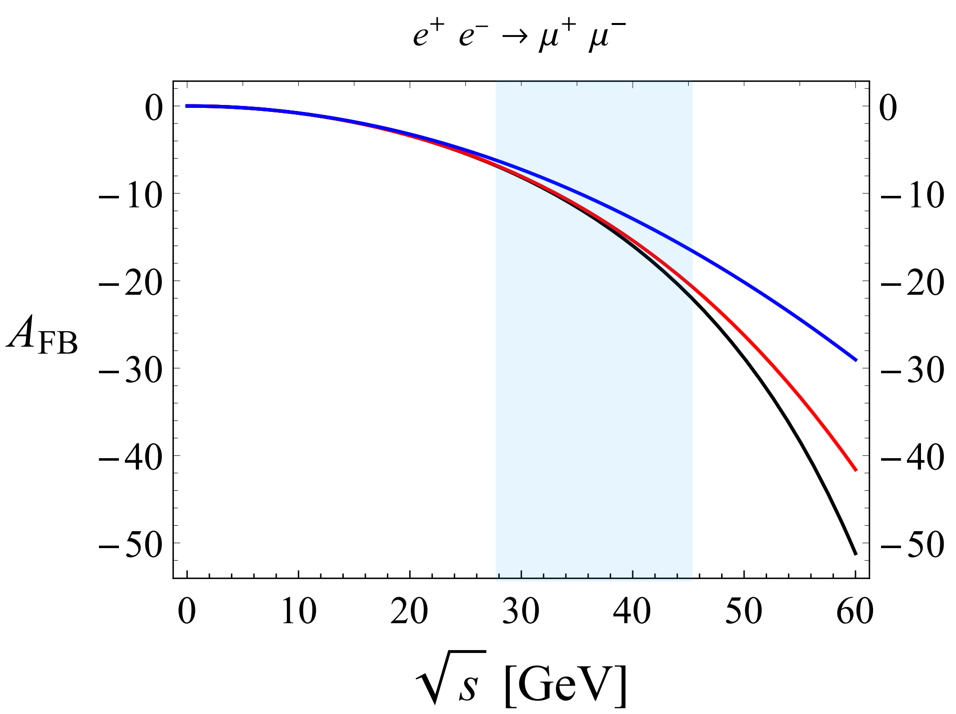
Lepton forward-backward asymmetry

$$e^+e^- \to \mu^+\mu^-$$

$$\mathcal{L} = (\bar{e}\gamma_\mu\gamma^5 e)\left(-a_1 + a_2\,\partial^2\right)(\bar{\mu}\gamma^\mu\gamma^5\mu)$$

* Positivity and convergence satisfied

$$A_{FB}(s) = -\frac{3\,a_1\,s\,\left(1+\frac{a_2}{a_1}\,s\right)}{8\,\pi\,\alpha^2}$$



Convergence in practice

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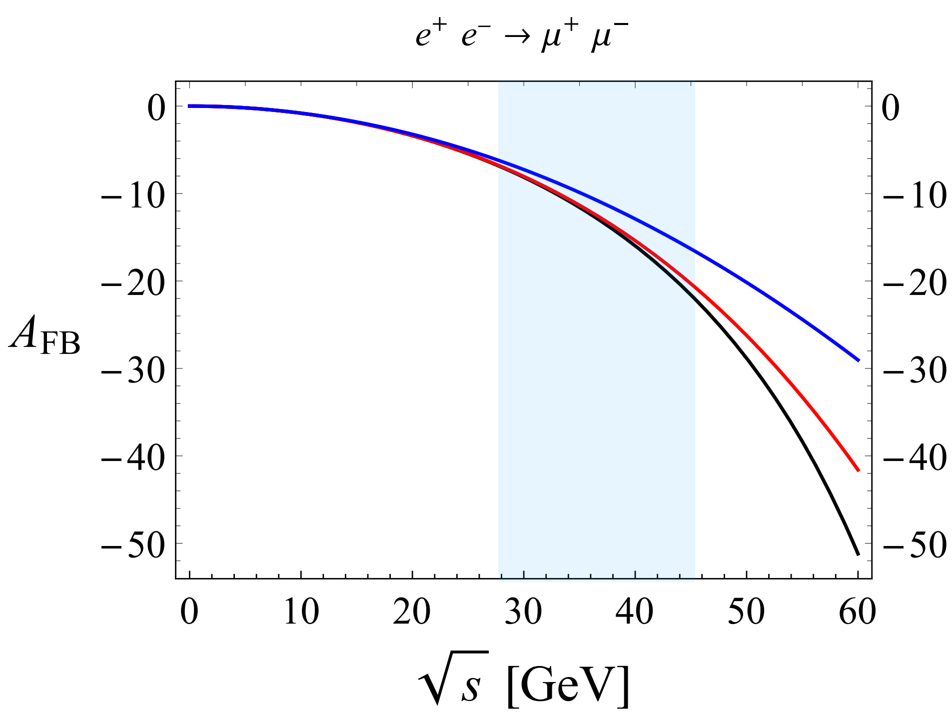


Fit to PDG data in the range 29–45 GeV

$$m_Z \lesssim 170\ \text{GeV}$$

2018 Review of Particle Physics.
M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

$A_{FB}^{(0,\mu)}$ CHARGE ASYMMETRY IN $e^+\,e^- \to \mu^+\mu^-$

INSPIRE search

OUR FIT is obtained using the fit procedure and correlations as determined by the LEP Electroweak Working Group (see the note ``The Z boson" and ref. LEP-SLC 2006). For the Z peak, we report the pole asymmetry defined by (3/4)A_eA_μ as determined by the nine-parameter fit to cross-section and lepton forward-backward asymmetry data.

ASYMMETRY (%)	STD. MODEL	$\sqrt{s}$ (GeV)	DOCUMENT ID		TECN
1.69 ± 0.13	**OUR FIT**				
1.59 ± 0.23	1.57	91.2	1 ABBIENDI	2001A	OPAL
1.65 ± 0.25	1.57	91.2	ABREU	2000F	DLPH
1.88 ± 0.33	1.57	91.2	ACCIARRI	2000C	L3
1.71 ± 0.24	1.57	91.2	2 BARATE	2000C	ALEP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
9 ± 30	−1.3	20	3 ABREU	1995M	DLPH
7 ± 26	−8.3	40	3 ABREU	1995M	DLPH
−11 ± 33	−24.1	57	3 ABREU	1995M	DLPH
−62 ± 17	−44.6	69	3 ABREU	1995M	DLPH
−56 ± 10	−63.5	79	3 ABREU	1995M	DLPH
−13 ± 5	−34.4	87.5	3 ABREU	1995M	DLPH
$-29.0^{+5.0}_{-4.8} \pm 0.5$	−32.1	56.9	4 ABE	1990I	VNS
−9.9 ± 1.5 ± 0.5	−9.2	35	HEGNER	1990	JADE
0.05 ± 0.22	0.026	91.14	5 ABRAMS	1989D	MRK2
−43.4 ± 17.0	−24.9	52.0	6 BACALA	1989	AMY
−11.0 ± 16.5	−29.4	55.0	6 BACALA	1989	AMY
−30.0 ± 12.4	−31.2	56.0	6 BACALA	1989	AMY
−46.2 ± 14.9	−33.0	57.0	6 BACALA	1989	AMY
−29 ± 13	−25.9	53.3	ADACHI	1988C	TOPZ
+5.3 ± 5.0 ± 0.5	−1.2	14.0	ADEVA	1988	MRKJ
−10.4 ± 1.3 ± 0.5	−8.6	34.8	ADEVA	1988	MRKJ
−12.3 ± 5.3 ± 0.5	−10.7	38.3	ADEVA	1988	MRKJ
−15.6 ± 3.0 ± 0.5	−14.9	43.8	ADEVA	1988	MRKJ
−1.0 ± 6.0	−1.2	13.9	BRAUNSCHWEIG	1988D	TASS
−9.1 ± 2.3 ± 0.5	−8.6	34.5	BRAUNSCHWEIG	1988D	TASS
$-10.6^{+2.2}_{-2.3} \pm 0.5$	−8.9	35.0	BRAUNSCHWEIG	1988D	TASS
$-17.6^{+4.4}_{-4.3} \pm 0.5$	−15.2	43.6	BRAUNSCHWEIG	1988D	TASS
−4.8 ± 6.5 ± 1.0	−11.5	39	BEHREND	1987C	CELL
−18.8 ± 4.5 ± 1.0	−15.5	44	BEHREND	1987C	CELL
+2.7 ± 4.9	−1.2	13.9	BARTEL	1986C	JADE
−11.1 ± 1.8 ± 1.0	−8.6	34.4	BARTEL	1986C	JADE
−17.3 ± 4.8 ± 1.0	−13.7	41.5	BARTEL	1986C	JADE
−22.8 ± 5.1 ± 1.0	−16.6	44.8	BARTEL	1986C	JADE
−6.3 ± 0.8 ± 0.2	−6.3	29	ASH	1985	MAC
−4.9 ± 1.5 ± 0.5	−5.9	29	DERRICK	1985	HRS
−7.1 ± 1.7	-5.7	29	LEVI	1983	MRK2
−16.1 ± 3.2	−9.2	34.2	BRANDELIK	1982C	TASS

Propagator versus Self-energy

The translation – at non-perturbative level

- Dyson equation

Self-energy

$$\Delta_h = \Delta_{\rm SM}(1 + \Sigma_h \Delta_h)$$

Propagator

$$\Delta_h(p^2) = \Delta_{\rm SM}(p^2) - \frac{1}{M^2}\sum_{n=1}^{\infty} c_n \left(\frac{p^2}{M^2}\right)^{n-1}$$

$$c_n \geqslant c_{n+1} \quad \forall n \qquad \text{(convergence)}$$

$$c_n \geqslant 0 \quad \forall n \qquad \text{(positivity)}$$

$$\mathscr{L} \supset -\frac{1}{2}h\left(\sum_{n=1}^{\infty}\hat{c}_n\left(\frac{-\partial^2}{M^2}\right)^n\right)(\partial^2 + m_h^2)h$$

$$\Sigma_h(p^2) = -(p^2 - m_h^2)\sum_{n=1}^{\infty}\hat{c}_n\left(\frac{p^2}{M^2}\right)^n$$

$$p^2 \gg m_h^2$$

$$\hat{c}_n = c_n + \sum_{j=1}^{n-1} c_j\,\hat{c}_{n-j}$$

$$\hat{c}_1 = c_1$$

$\hat{c}_n$ **positive, but can diverge with n**

For example, when $c_1 > 1$

Other criteria described in the paper

Propagator versus Self-energy

The translation – at non-perturbative level

- Dyson equation

Self-energy

$$\Delta_h = \Delta_{\mathrm{SM}}(1 + \Sigma_h \Delta_h)$$

Propagator

- Example: Real scalar field h

$$\Delta_h(p^2) = \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2}$$

- ***Correct EFT recipe***

- Example: Real scalar field h

$$\Sigma(p^2) = -\frac{\hat{H}}{m_h^2}\left(p^2 - m_h^2\right)^2$$

$$\Delta_h(p^2) = (p^2 - m_h^2 - \Sigma(p^2))^{-1}$$

- Ghost pole with mass

$$m_h/\sqrt{\hat{H}} \equiv M/\sqrt{c_1}$$

- Premature breakdown for $c_1 > 1$?

- Truncation of the self-energy wrong since no convergence

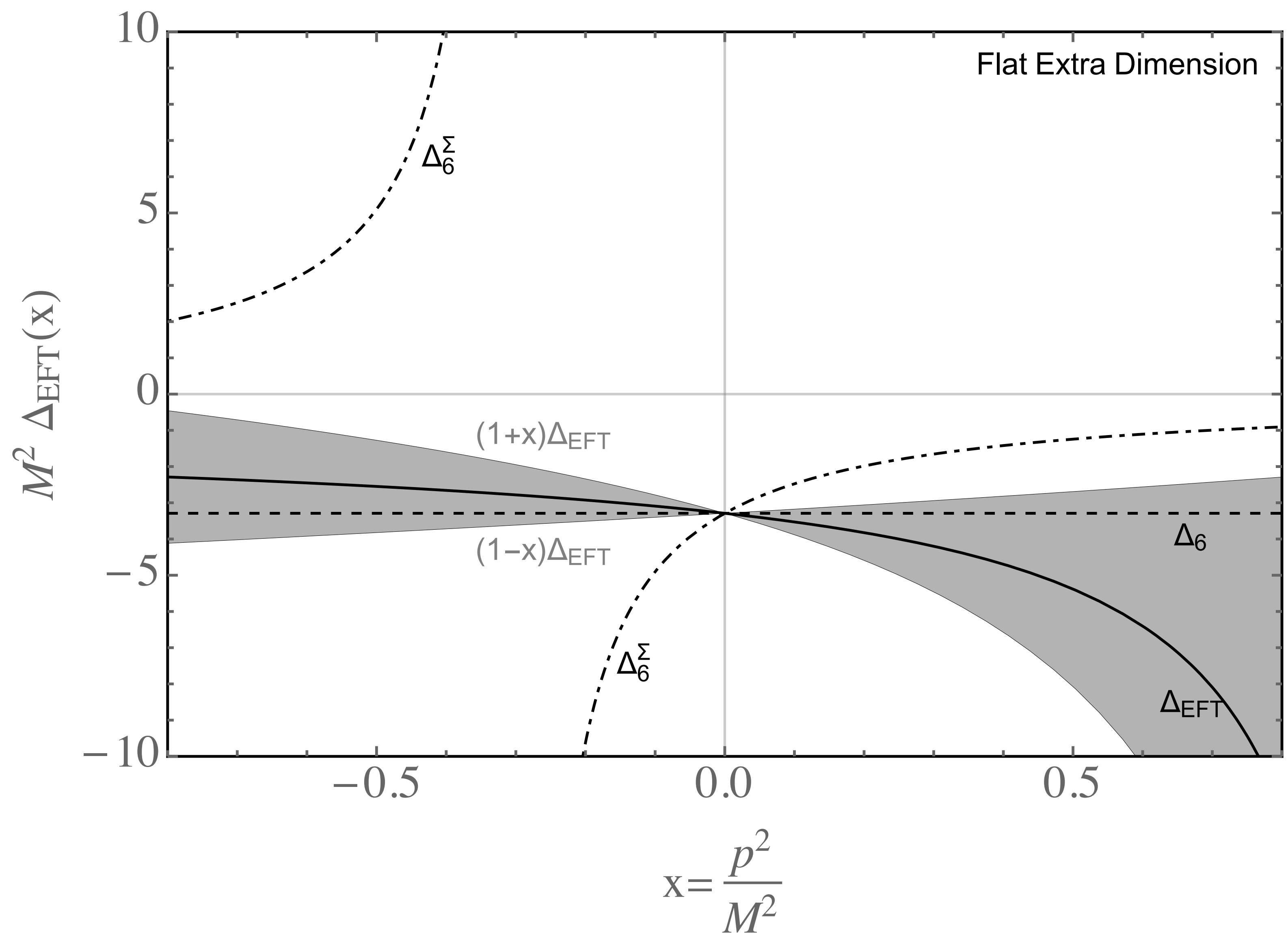
Convergence and the EFT validity

- A flat extra dimensional example for GB. Integrate out the KK excitations

$$\Delta(x) = \frac{\pi}{M^2}\frac{\cot\left(\pi\sqrt{x}\right)}{\sqrt{x}}$$

$$c_1 = \frac{\pi^2}{3} \quad , \quad c_2 = \frac{\pi^2}{15}c_1 \quad , \quad c_3 = \frac{2\pi^2}{21}c_2 \quad , \quad \ldots$$

$$\hat{c}_1 = \frac{\pi^2}{3} \quad , \quad \hat{c}_2 = \frac{2\pi^2}{5}\hat{c}_1 \quad , \quad \hat{c}_3 = \frac{17\pi^2}{42}\hat{c}_2 \quad , \quad \ldots$$



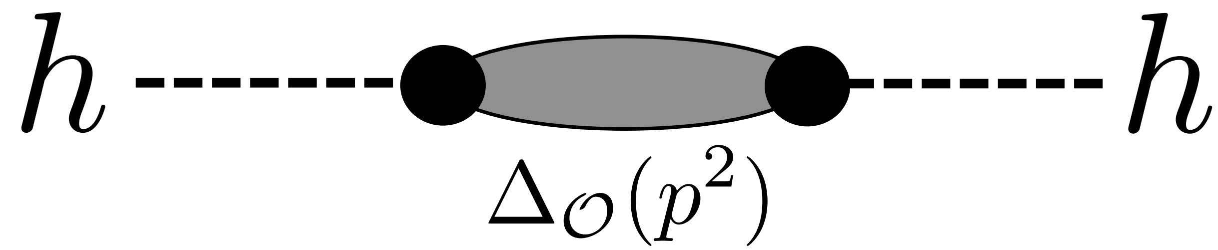
- ***Expand the propagator, not the self-energy***

$$\Delta_{EFT}(x) = \Delta(x) - \frac{1}{M^2 x} \quad , \quad \Delta_6(x) = \frac{c_1}{M^2} \quad , \quad \Delta_6^{\Sigma}(x) = \frac{1}{M^2}\left(\frac{1}{x - \hat{c}_1 x^2} - \frac{1}{x}\right)$$

Convergence in the self-energy

- For the class of BSM theories

$$\mathcal{L}_{\text{int}} = \kappa\, h^0 \mathcal{O}$$



$$\Sigma_h^0(p^2) = -\kappa^2 \Delta_{\mathcal{O}}(p^2) = i\kappa^2 \int d^4z\; e^{ipz} \langle 0 | T\{\mathcal{O}(z)\mathcal{O}(0)\} | 0 \rangle$$

the convergence of the self-energy expansion coefficients can be proved. **Both, c_n and $\hat{c}_n$ obey convergence.**

$$\Sigma_h(p^2) = \left(p^2 - m_h^2\right) \sum_{n=1}^{\infty} C_n \left(\frac{p^2}{M^2}\right)^n f_n\left(\frac{m_h^2}{p^2}\right)$$

$$C_n = \kappa^2 \int_0^1 dx\, \frac{\rho_{\mathcal{O}}(M^2/x)}{M^2}\, x^n \,,$$

$$f_n(y) = \frac{1 - (n+1) y^n + n y^{n+1}}{1 - y} \,.$$

Electroweak oblique corrections

- **S, T, W, Y** parameters
  - new physics in the self-energy of EW gauge bosons

- Oblique corrections <u>**play a key role**</u> in BSM theory:
  - SUSY, composite Higgs, extra dimension, etc…

$$\Pi_V(q^2) = \Pi_V(0) + q^2\Pi'_V(0) + \frac{q^4}{2}\Pi''_V(0) + \ldots$$

$$V = \{W^+W^-, W_3W_3, BB, W_3B\}$$

Adimensional form factors			operators			custodial
$g^{-2}\widehat{S}$	$=$	$\Pi'_{W_3B}(0)$	$\mathcal{O}_{WB}$	$=$	$(H^\dagger\tau^a H)W^a_{\mu\nu}B_{\mu\nu}/gg'$	$+$
$g^{-2}M_W^2\widehat{T}$	$=$	$\Pi_{W_3W_3}(0) - \Pi_{W^+W^-}(0)$	$\mathcal{O}_H$	$=$	$\lvert H^\dagger D_\mu H\rvert^2$	$-$
$-g^{-2}\widehat{U}$	$=$	$\Pi'_{W_3W_3}(0) - \Pi'_{W^+W^-}(0)$		$-$		$-$
$2g^{-2}M_W^{-2}V$	$=$	$\Pi''_{W_3W_3}(0) - \Pi''_{W^+W^-}(0)$		$-$		$-$
$2g^{-1}g'^{-1}M_W^{-2}X$	$=$	$\Pi''_{W_3B}(0)$		$-$		$+$
$2g'^{-2}M_W^{-2}Y$	$=$	$\Pi''_{BB}(0)$	$\mathcal{O}_{BB}$	$=$	$(\partial_\rho B_{\mu\nu})^2/2g'^2$	$+$
$2g^{-2}M_W^{-2}W$	$=$	$\Pi''_{W_3W_3}(0)$	$\mathcal{O}_{WW}$	$=$	$(D_\rho W^a_{\mu\nu})^2/2g^2$	$+$

[Barbieri, Pomarol, Rattazzi, Strumia] hep-ph/0405040

* Not invariant under redefinitions of the vector boson fields, universal theories to be defined rigorously in the EFT later.

*There are 4 such two-point amplitudes for a total of 12 parameters, when the expansions are truncated at order q^4. However, after imposing normalisation and symmetry constraints, and by examining the EFT expansion, only 4 parameters can be matched to dim-6 operators.

Electroweak oblique corrections

- **W,Y** energy enhanced
  "**Contact terms**" in the propagator

[Farina, Panico, Pappadopulo, Ruderman, Torre, Wulzer]
1609.08157

$$P_N = \begin{bmatrix} \frac{1}{q^2} - \frac{t^2\mathrm{W}+\mathrm{Y}}{m_Z^2} & \frac{t\left(\left(\mathrm{Y}+\hat{\mathrm{T}}\right)c^2+s^2\mathrm{W}-\hat{\mathrm{S}}\right)}{(c^2-s^2)\left(q^2-m_Z^2\right)} + \frac{t(\mathrm{Y}-\mathrm{W})}{m_Z^2} \\ \star & \frac{1+\hat{\mathrm{T}}-\mathrm{W}-t^2\mathrm{Y}}{q^2-m_Z^2} - \frac{t^2\mathrm{Y}+\mathrm{W}}{m_Z^2} \end{bmatrix}$$

$$P_C = \frac{1+\left(\left(\hat{\mathrm{T}}-\mathrm{W}-t^2\mathrm{Y}\right)-2t^2\left(\hat{\mathrm{S}}-\mathrm{W}-\mathrm{Y}\right)\right)/(1-t^2)}{(q^2-m_W^2)} - \frac{\mathrm{W}}{m_W^2}\,,$$

	universal form factor ($\mathcal{L}$)	contact operator ($\mathcal{L}'$)
W	$-\frac{\mathrm{W}}{4m_W^2}(D_\rho W^a_{\mu\nu})^2$	$-\frac{g_2^2\mathrm{W}}{2m_W^2} J_L{}^a_\mu J_L{}^\mu_a$
Y	$-\frac{\mathrm{Y}}{4m_W^2}(\partial_\rho B_{\mu\nu})^2$	$-\frac{g_1^2\mathrm{Y}}{2m_W^2} J_{Y\,\mu} J_Y{}^\mu$

TABLE I. *The parameters* W *and* Y *in their "universal" form (left), and as products of currents related by the equation of motion (right).*

Energy helps accuracy