Loop corrections to Higgs mass spectra and decay BRs in BSM

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Outline

Introduction

Higher-order corrections for the NMSSM Higgs masses

Higher-order corrections for the NMSSM Higgs decays

Outlook
From the SM to the (N)MSSM

- **SM Hierarchy problem**: Quantum corrections to Higgs mass $m_H$ quadratically divergent in the SM ⇒ $m_H$ stabilized by SUSY, MSSM minimal SM extension, 2 Higgs doublets

- **But $\mu$–problem**: $\mu \hat{H}_u \cdot \hat{H}_d$ term in the superpotential, $\mu$ SUSY-invariant but of the order of EW scale ⇒ naturalness problem!

- **A solution**: effective $\mu$–term generated dynamically ⇔ NMSSM

**NMSSM Superpotential**

$$\mathcal{W} = \hat{u}_R^* y_u \left( \hat{Q} \cdot \hat{H}_u \right) - \hat{d}_R^* y_d \left( \hat{Q} \cdot \hat{H}_d \right) - \hat{e}_R^* y_e \left( \hat{L} \cdot \hat{H}_d \right) + \lambda \hat{S} \left( \hat{H}_u \cdot \hat{H}_d \right) + \frac{1}{3} \kappa \hat{S}^3$$

- **3+2 neutral Higgs** bosons, **2 charged Higgs** bosons, **4+1 neutralinos**

- Singlet field $S$ gets a vacuum expectation value ⇒ $\mu_{\text{eff}} = \lambda v_s / \sqrt{2}$

- Higgs sector less restricted in NMSSM compared to MSSM:
  
  upper bound on tree-level in the MSSM: $M_H \leq M_Z$
  ⇒ large quantum corrections to get $M_H \approx 125$ GeV

  Fine-tuning less important in the NMSSM, higher tree-level bound

  $$M^2_H \leq M^2_Z \left( \cos(2\beta)^2 + \frac{\lambda^2}{g^2} \sin(2\beta)^2 \right), \quad g^2 = (g_1^2 + g_2^2)/2 \quad \text{[see Ellwanger, Hugonie, Teixeira, (2010)]}$$

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Loop corrections to Higgs mass spectra and decay BRs in BSM

FCC Workshop, 15/01/2020
From the real NMSSM to the complex NMSSM

- **CP violation:** CKM matrix can explain $CP$ violation in $K - \bar{K}$ system but not the baryon asymmetry in the Universe: amount of $CP$ violation (from CKM) in the SM too small

  One solution

  $CP$ violation enhanced with complex parameters

- **Richer Higgs spectrum:** complex parameters in the NMSSM

  $\Rightarrow$ $CP$ violation at tree–level, all the Higgs bosons mix:

\[
H_i = \sum_k S_{ik}\phi_k, \quad \phi = (h_d, h_u, s, a, a_s)
\]

\[
(S_{ik}) \ 5 \times 5 \ matrix
\]
From the superpotential to the scalar potential:

\[
V = \left| \lambda \right|^2 \left| S \right|^2 \left( H_u^\dagger H_u + H_d^\dagger H_d \right) + \left| \lambda \left( H_u^\dagger \epsilon H_d \right) + \kappa S \right|^2 + \frac{1}{2} g_2^2 \left| H_u^\dagger H_d \right|^2 \\
+ \frac{1}{8} \left( g_1^2 + g_2^2 \right) \left( H_u^\dagger H_u - H_d^\dagger H_d \right)^2 + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_S^2 \left| S \right|^2 \\
+ \left( \lambda A_\lambda \left( H_u^\dagger \epsilon H_d \right) S + \frac{1}{3} \kappa A_\kappa S^3 + \text{c.c} \right)
\]

Higgs fields with phases:

\[
H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d + h_d + i a_d \\ \sqrt{2} H_d^- \end{pmatrix} \quad H_u = \frac{e^{i \phi_u}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H_u^+ \\ v_u + h_u + i a_u \end{pmatrix} \\
S = \frac{e^{i \phi_s}}{\sqrt{2}} \left( v_s + h_s + i a_s \right) \quad \tan \beta = \frac{v_u}{v_d}
\]
FCC target precision for Higgs decays

Exp error on $\Gamma(H \rightarrow xx) \Leftrightarrow$ expected precision on the (effective) SM-like coupling $g_{Hxx}^2$. [FCC CDC Volume 1, EPJC 70 (2019) 474]

<table>
<thead>
<tr>
<th>Partial width</th>
<th>FCC-ee target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \rightarrow b\bar{b}$</td>
<td>$\sim 0.6%$</td>
</tr>
<tr>
<td>$H \rightarrow c\bar{c}$</td>
<td>$\sim 1.2%$</td>
</tr>
<tr>
<td>$H \rightarrow \tau^+\tau^-$</td>
<td>$\sim 0.7%$</td>
</tr>
<tr>
<td>$H \rightarrow \mu^+\mu^-$</td>
<td>$\sim 9.0%$</td>
</tr>
<tr>
<td>$H \rightarrow gg$</td>
<td>$\sim 1.0%$</td>
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<tr>
<td>$H \rightarrow \gamma\gamma$</td>
<td>$\sim 3.9%$</td>
</tr>
<tr>
<td>$H \rightarrow WW \rightarrow 4f$</td>
<td>$\sim 0.4%$</td>
</tr>
<tr>
<td>$H \rightarrow ZZ \rightarrow 4f$</td>
<td>$\sim 0.2%$</td>
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</tbody>
</table>

$\Rightarrow$ Higher-order corrections mandatory! Define target for theory uncertainties
Higher-order corrections for the Higgs masses in the NMSSM


Disclaimer of this talk

- I will only present results from my group there, in the framework of **NMSSMCALC**

- My apologies for the other calculators on the market, namely **FeynHiggs, FlexibleSUSY**, **FlexibleEFTHiggs, NMSSMTools, Spheno, and SOFTSUSY**

- See also [Drechsel, Gröber, Heinemeyer, Mühlleitner, Rzehak, Weiglein, EPJC 77 (2017) 366] for a comparison between some of them

- For a parallel approach to loop corrections to Higgs decays, see e.g. [Domingo, Heinemeyer, Paßer, Weiglein, EPJC 78 (2018) 942; Domingo, Paßer, EPJC 79 (2019) 905] ⇒ see also Sven’s talk!
NMSSM Higgs masses at the 1-loop order

- **LO Higgs masses**: diagonalizing the $3 \times 3$ CP-even $M_S$ and the $3 \times 3$ CP-odd (+ Goldstone) $M_A$ mass matrices give the LO masses $m_{H_1} < m_{H_2} < m_{H_3}$ and $m_{A_1} < m_{A_2}$

  sizable higher order corrections compulsory to obtain one SM-like $H_i$ boson

- **1-loop corrections**: define input parameters, calculate renormalized self-energies in a mixed renormalization scheme

  - 7 parameters renormalized on-shell: $e$, $M_W$, $M_Z$, $M_{H^\pm}$, $t_{hu}$, $t_{hd}$, $t_s$
  - 5 parameters renormalized $\overline{DR}$: $\tan \beta$, $\lambda$, $\kappa$, $A_k$, $v_s$

  $\overline{DR}$ field renormalization constants for the Higgs fields

  $$H_d = \left(1 + \frac{1}{2} \delta Z_d \right) \hat{H}_d \quad H_u = \left(1 + \frac{1}{2} \delta Z_u \right) \hat{H}_u \quad S = \left(1 + \frac{1}{2} \delta Z_s \right) \hat{S}$$

  $\overline{DR}$ Tadpole condition: $\hat{T}_{h_{u/d/s}} = 0 \Rightarrow \delta t_{h_{u/d/s}} = T_{h_{u/d/s}}$

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NMSSM Higgs masses at the 1-loop order

1-loop corrections: with the renormalized self-energies $\hat{\Sigma}_{H_i H_j}^{(1)}(p^2)$ calculate the zeros of the 1-loop 2-point vertex matrix

$$\Gamma^{11}(p^2) = i \begin{pmatrix} p^2 - m_{H_1}^2 + \hat{\Sigma}_{H_1 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_3}^{(1)}(p^2) \\ \hat{\Sigma}_{H_2 H_1}^{(1)}(p^2) & p^2 - m_{H_2}^2 + \hat{\Sigma}_{H_2 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_2 H_3}^{(1)}(p^2) \\ \hat{\Sigma}_{H_3 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_3 H_2}^{(1)}(p^2) & p^2 - m_{H_3}^2 + \hat{\Sigma}_{H_3 H_3}^{(1)}(p^2) \end{pmatrix}$$

Algorithm to calculate the masses

1. Set $p^2 = m_{H_1}^2$ in the self-energies $\hat{\Sigma}$
2. Solve $\text{det}(\Gamma(p^2)) = 0$
3. Extract the smallest root obtained $m_{\text{tmp}}^2$
4. Set $p^2 = m_{\text{tmp}}^2$ in the self-energies and back to 2. $n$ times as long as $|m_{\text{tmp},n}^2 - m_{\text{tmp},n-1}^2| > 10^{-9}$ (desired accuracy) $\Rightarrow$ get the 1-loop mass $M_{H_1}$
5. Start again at 1. with $m_{H_2}^2$, extract the next-to-smallest root, and so on
Complex parameters and renormalization

- **Many new phases in the complex NMSSM**, for the parameters $M_1$, $M_2$, $A_t$, etc

  In the Higgs sector: 6 phases $\phi_u$, $\phi_S$, $\phi_\lambda$, $\phi_\kappa$, $\phi_{A_\lambda}$, and $\phi_{A_\kappa}$

  **At tree–level, only one relevant phase**: only 3 combinations in the mass matrix

  \[
  \Psi = \phi_\lambda - \phi_\kappa + \phi_u - 2\phi_s \\
  \Psi_\lambda = \phi_\lambda + \phi_{A_\lambda} + \phi_u + \phi_s \\
  \Psi_\kappa = \phi_\kappa + \phi_{A_\kappa} + 3\phi_s
  \]

  $\Psi_\lambda$, $\Psi_\kappa$ fixed by two tadpole conditions

- **Renormalization in the Higgs sector**:
  - on–shell renormalization for $e$, $M_W$, $M_Z$, $M_{H^\pm}$
  - DR renormalization for $\tan \beta$, $v_s$, $|\lambda|$, $|\kappa|$, $|A_\kappa|$ and the phases
  - **vanishing phases counter–terms** but for two tadpole conditions $t_a$ and $t_{a_s}$
Exemplary results at the 1-loop order

Complex phases have sizable effects:
- **Tree–level CP violation ($\psi \neq 0$):**

\[
|\lambda| = 0.72, \quad |\kappa| = 0.2, \quad \tan \beta = 3, \quad \nu_s = 389 \text{ GeV}, \quad |A_\kappa| = 27 \text{ GeV},
|A_\lambda| = 928 \text{ GeV}, \quad A_t = -875 \text{ GeV}, \quad A_b = A_l = -963 \text{ GeV}, \quad M_{\text{SUSY}} = 1 \text{ TeV}
M_1 = 145 \text{ GeV}, \quad M_2 = 200 \text{ GeV}, \quad M_3 = 600 \text{ GeV}, \quad Q = 300 \text{ GeV}
\]
Complex phases have sizable effects:

- **No tree–level CP violation** ($\Psi = 0$), $\phi_\lambda = \phi_\kappa \neq 0$:
  - One loop corrections have a **sizable impact** (restore some scenarios)
  - No tree-level $CP$ violation scenario: still slight impact of $\phi_\kappa$ over $H_3$ through its coupling to stops
2-loop effects in Higgs mass calculation

2-loop order $\mathcal{O}(\alpha_t \alpha_s + \alpha_t^2)$ in the gaugeless limit and in the approximation of vanishing external momentum:

$$\hat{\Sigma}_{H_i H_j}(p^2) = \hat{\Sigma}_{H_i H_j}^{(1)}(p^2) + \hat{\Sigma}_{H_i H_j}^{(2), \alpha_t \alpha_s}(0) + \hat{\Sigma}_{H_i H_j}^{(2), \alpha_t^2}(0)$$

Set of parameters at 2-loop:

- OS renormalization for $M_{H^\pm}$, $\tan \beta$, $\nu$ (Higgs sector)
- $\overline{\text{DR}}$ renormalization for $\tan \beta$, $\nu_s$, $|\lambda|$, $|\kappa|$, $\text{Re } A_\kappa$, $\phi_\lambda$, $\phi_\kappa$, $\phi_u$, $\phi_s$ (Higgs sector)
- OS or $\overline{\text{DR}}$ renormalization for $m_t$, $m_{\tilde{Q}_3}$, $m_{\tilde{t}_R}$, $A_t$ (top/stop sector)
Examply results at the 2-loop order

2-loop $O(\alpha_s \alpha_t + \alpha_t^2)$ corrections of $O(6\%)$ on top of 1-loop result:

$|\lambda| = 0.301, \ |\kappa| = 0.299, \ \tan\beta = 4.44, \ \nu_s = 981.962 \text{ GeV}, \ \text{Re} A_{\tilde{\kappa}} = -791 \text{ GeV}, \ m_{t_R} = 881 \text{ GeV}, \ m_{\tilde{Q}_3} = 1226 \text{ GeV}, \ \phi_{A_q} = \pi, \ \phi_{A_A} = \phi_{\lambda, \kappa} = \phi_u = 0,$

$|M_1| = 644 \text{ GeV}, \ |M_2| = 585 \text{ GeV}, \ |M_3| = 1850 \text{ GeV}, \ M_{H^\pm} = 898 \text{ GeV}$

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Intrinsic theoretical uncertainty due to
1. renormalization scheme choice; 2. renormalization scheme variation

- QCDrun ≡ (OS → DR for $m_t(\mu_R = M_Z)$) & RGEs for $(\mu_R = M_Z \rightarrow \mu_R = \mu)$ at order $O(\alpha_s + \alpha_s^2)$

- EWrun ≡ (OS → DR for $m_t(\mu_R = M_Z)$) & RGEs for $(\mu_R = M_Z \rightarrow \mu_R = \mu)$ at order $O(\alpha_s + \alpha_t + (\alpha_s + \alpha_t)^2)$

- Uncertainty of $O(1\%)$ in each scheme choice, $O(3 – 4\%)$ between $\neq$ schemes
Higher-order corrections for the Higgs decays in the NMSSM

*Higgs decays and NMSSMCALC:* J.B., Gröber, Mühlleitner, Nhung, Rzehak, Spira, Streicher, Walz, CPC 185 (2014) 12


2-loop Higgs boson masses in the NMSSM, allowing for complex parameters

+ calculation of the Higgs boson decay widths in the NMSSM

based on the latest version of HDECAY


- Real or complex parameters
- $\mathcal{O}(\alpha_t \alpha_s + \alpha_i^2)$ 2-loop masses, electric dipole moments
- Decay widths and branching ratios including dominant higher order QCD corrections
- Off-shell decays into $VV^*, H_i V^*, tt^*, bt^*$
- Dominant SUSY corrections in the decays into quarks and leptons: implemented with effective $\Delta_b$-Yukawa couplings calculated in the NMSSM
2-loop Higgs boson masses in the NMSSM, allowing for complex parameters

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NMSSMCALC 3.0: The Higgs sector in the NMSSM

2-loop Higgs boson masses in the NMSSM, allowing for complex parameters
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  based on the latest version of HDECAY


Current status of mass calculation in NMSSM CALC:

- 2-point 1-loop integrals with complex momentum
  ⇒ no instabilities due to threshold singularities

- approximate 2-loop order for the calculation of the Higgs spectrum with $O(\alpha_t \alpha_s + \alpha_t^2)$ corrections

To download NMSSM CALC:
https://www.itp.kit.edu/~maggie/NMSSM CALC
Inclusion of electric dipole moments

CP-violating effects enhance the (chromo-)electric dipole moments (EDM), predicted to be very small in the SM, and not yet observed

Exp upper bounds:

- Electron EDM $\lesssim 10^{-29} e\text{.cm}$

- Thallium EDM $\lesssim 9 \times 10^{-25} e\text{.cm}$
  [Regan et al, PRL 88 (2002) 071805]

- Neutron EDM $\lesssim 3 \times 10^{-26} e\text{.cm}$

- Mercury EDM $\lesssim 3.1 \times 10^{-29} e\text{.cm}$
  [Griffith et al, PRL 102 (2009) 101601]

Implemented in NMSSMCALC at the 2-loop level, new switch in Block MODSEL


Block MODSEL

3 1  # NMSSM
5 2  # CP violation
10 1  # EDMs: 0, no EDMs); 1, with EDMs); 2, EDMs individual contributions

Output with results rescaled to exp upper-bound
Impact of EDMs constraints

Still a sizable portion of NMSSM phases allowed by EDMs constraints

\[ \phi_2 = \phi_\kappa + 3\phi_s, \text{ specific NMSSM phase} \]

\[ \phi_1 = \phi_\lambda + \phi_s + \phi_u, \text{ MSSM-like phase} \]
New in 2019: Extension of **NMSSMCalc** to include the complete NLO SUSY-QCD and SUSY-EW corrections to neutral Higgs decay widths:

- \( \Gamma(H \rightarrow f\bar{f}) \): decay to fermions at 1-loop SUSY-QCD, SUSY-EW, running masses for the quarks
- \( \Gamma(H \rightarrow W^+W^-, ZZ) \): (improved) tree-level for off-shell decays to massive SM gauge bosons, 1-loop SUSY-EW for on-shell decays of heavy Higgs bosons
- \( \Gamma(H \rightarrow ZH_j, H_jH_k) \): Higgs-to-Higgs and Higgs+Z decays at 1-loop NLO SUSY-EW
- \( \Gamma(H \rightarrow \tilde{f}_i\tilde{f}_j) \): decay to sfermions at 1-loop SUSY-QCD, SUSY-EW
- \( \Gamma(H \rightarrow \tilde{\chi}_i\tilde{\chi}_j, \tilde{\chi}_i^+\tilde{\chi}_j^-) \): decay to neutralinos and charginos at 1-loop SUSY-EW

To download **NMSSMCalc**:
https://www.itp.kit.edu/~maggie/NMSSMCalcCEW
Some technical details

- Loop corrections to squark, neutralinos, and weakino masses needed, 2 descriptions for the wave-function renormalization (WFR):
  - **EMT description** [see e.g. Espriu, Manzano, Talavera, PRD 66 (2002) 07600]:  $2 \neq WFR$ for ingoing and outgoing states
    - no mixing self-energies, absorptive part of loop functions included ✓
    - no hermiticity of the Lagrangian ❌
  - **Denner description** [Denner, Fort.Phys. 41 (1993) 307]: only 1 common WFR
    - hermiticity of the Lagrangian ✓
    - absorptive part of the loop functions must be removed ❌

- **$Z$-Goldstone boson mixing $\delta M_{GZ,\text{mix}}$** dealt with Slavnov-Taylor identities adapted to loop-corrected masses [Nhung, Mühlleitner, Streicher, Walz, JHEP 11 (2013) 181; Domingo, Heinemeyer, Paßer, Weiglein, EPJC 78 (2018) 942]

\[
M_Z \hat{\Sigma}_{h_i} G(M_{h_i}^2) + \nu M_{h_i}^2 \hat{\Sigma}_{h_i} Z(p^2) = 0
\]

- **Real corrections including $\Gamma(H_i \to W^+ W^- \gamma)$** to ensure IR-finiteness, but loop-corrected Higgs masses used in the amplitude $\Rightarrow$ gauge-invariance not enforced, see [Dao, Fritz, Krause, Mühlleitner, Patel, arXiv:1911.07197] for a detailed discussion
Decays into SM fermions

Random scan of the NMSSM parameter space, $\delta$ 1-loop vertex correction, $\Delta_{BR}(H_iXY)$ comparison to $\text{NMSSMCALC}$ 3.0 for $\text{BR}(H_i \to XY)$

- New implementation (with full WFR factor $Z^H$ and NLO amplitudes) close to $\text{NMSSMCALC}$ 3.0 (with unitary Higgs mixing matrix) for SM-like Higgs boson $h$
- Vertex corrections at most $O(10\%)$ for SM-like state $h$
- Same mostly true for singlet-like $H_1$ boson, $\Delta_{BR}(H_1)$ mostly below 30%
Decays into SM fermions

- $|\Delta_{\text{BR}}(H_a b\bar{b})|$ and $|\Delta_{\text{BR}}(H_{h_d} t\bar{t})|$ mostly below 40%
- Vertex corrections can reach 40% for the $H_{h_d}$ state
Decays into Higgs+Z boson

- Many points with large BR(H_{h_s} \rightarrow ZA_2), A_2 doublet-like
- |Δ(H_{h_s}ZA2)| up to 80% while |δ| < 10% ⇒ corrections here driven by Z^H factor
- |Δ(H_aZH_2)| mostly below 25% (H_2 singlet-like), vertex corrections can reach 100% ⇒ large corrections for very small BRs where 1-loop squared are also taken into account
Decays into a squark pair

- $\delta_{BR} \equiv$ vertex correction, $\Delta_{BR} \equiv$ difference between OS and DR
- SUSY-EW corrections negative, SUSY-QCD corrections positive, cancelation between the two of them
- vertex corrections larger in DR than in OS
- strong decrease of $\Delta_{BR}$ when both EW and QCD taken into account!
Conclusions and outlook

In view of the HL-LHC and FCC programs, plenty of room for improvement!

■ **2-loop mass calculation:**
  
  ▶ **2-loop** $\mathcal{O}(\alpha_s \alpha_t + \alpha_t^2)$ corrections in the gaugeless limit at zero momentum current state-of-the-art in the mixed renormalization scheme;
  
  agreement with other codes when taking into account $\neq$ input setup

  [see e.g. Staub et al, CPC 202 (2016) 113; Goodsell, Staub, EPJC 77 (2017) 46]

  ▶ **For the future:** 2-loop terms $\mathcal{O}(\alpha_s \alpha_b + \alpha_t \alpha_b)$ to be done for the complex NMSSM (known in the real NMSSM, see FeynHiggs 3)

■ **NLO decay widths:** Inclusion of NLO SUSY-(QCD+)EW corrections in the complex NMSSM for all neutral Higgs decays now known!

  → For the future: corrections for charged Higgs decays

■ Uncertainties on the decay BRs: reaching FCC precision for the SM-like Higgs boson at the percent level, $\mathcal{O}(10 – 30\%)$ for the BSM Higgs states

  ⇒ room for improvement in the precision of the Higgs decay widths