

Loop corrections to Higgs mass spectra and decay BRs in BSM

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Julien Baglio





Outline

Introduction

Higher-order corrections for the NMSSM Higgs masses

Higher-order corrections for the NMSSM Higgs decays

Outlook



From the SM to the (N)MSSM

- **SM Hierarchy problem:** Quantum corrections to Higgs mass m_H quadratically divergent in the SM $\Rightarrow m_H$ stabilized by SUSY, MSSM minimal SM extension, **2 Higgs doublets**
- **But μ -problem:** $\mu \hat{H}_u \cdot \hat{H}_d$ term in the superpotential, μ SUSY-invariant but of the order of EW scale \Rightarrow naturalness problem!
- **A solution:** effective μ -term generated dynamically \Leftrightarrow **NMSSM**

NMSSM Superpotential

$$\mathcal{W} = \hat{u}_{RY}^* y_u (\hat{Q} \cdot \hat{H}_u) - \hat{d}_{RY}^* y_d (\hat{Q} \cdot \hat{H}_d) - \hat{e}_R^* y_e (\hat{L} \cdot \hat{H}_d) + \lambda \hat{S} (\hat{H}_u \cdot \hat{H}_d) + \frac{1}{3} \kappa \hat{S}^3$$

- ▶ **3+2 neutral Higgs bosons, 2 charged Higgs bosons, 4+1 neutralinos**
- ▶ Singlet field S gets a vacuum expectation value $\Rightarrow \mu_{\text{eff}} = \lambda v_s / \sqrt{2}$
- ▶ Higgs sector less restricted in NMSSM compared to MSSM:
upper bound on tree-level in the MSSM: $M_H \leq M_Z$
 \Rightarrow large quantum corrections to get $M_H \simeq 125$ GeV

Fine-tuning less important in the NMSSM, higher tree-level bound

$$M_H^2 \leq M_Z^2 \left(\cos(2\beta)^2 + \frac{\lambda^2}{g^2} \sin(2\beta)^2 \right), \quad g^2 = (g_1^2 + g_2^2)/2 \quad [\text{see Ellwanger, Hugonie, Teixeira, (2010)}]$$



From the real NMSSM to the complex NMSSM

- **CP violation:** CKM matrix can explain CP violation in $K - \bar{K}$ system but not the baryon asymmetry in the Universe:
amount of CP violation (from CKM) in the SM too small

One solution

CP violation enhanced with complex parameters

- **Richer Higgs spectrum:** complex parameters in the NMSSM
 $\Rightarrow CP$ violation at tree-level, all the Higgs bosons mix:

$$H_i = \sum_k S_{ik} \phi_k, \quad \phi = (h_d, h_u, s, a, a_s)$$

(S_{ik}) 5×5 matrix



The scalar potential in the NMSSM

- From the superpotential to the scalar potential:

$$\begin{aligned} V = & |\lambda|^2 |S|^2 \left(H_u^\dagger H_u + H_d^\dagger H_d \right) + |\lambda (H_u^T \epsilon H_d) + \kappa S^2|^2 + \frac{1}{2} g_2^2 |H_u^\dagger H_d|^2 \\ & + \frac{1}{8} (g_1^2 + g_2^2) \left(H_u^\dagger H_u - H_d^\dagger H_d \right)^2 + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_S^2 |S|^2 \\ & + \left(\lambda A_\lambda (H_u^T \epsilon H_d) S + \frac{1}{3} \kappa A_\kappa S^3 + \text{c.c} \right) \end{aligned}$$

- Higgs fields with phases:

$$\begin{aligned} H_d &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_d + h_d + i a_d \\ \sqrt{2} H_d^- \end{pmatrix} & H_u &= \frac{e^{i\phi_u}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H_u^+ \\ v_u + h_u + i a_u \end{pmatrix} \\ S &= \frac{e^{i\phi_s}}{\sqrt{2}} (v_s + h_s + i a_s) \\ \tan \beta &= \frac{v_u}{v_d} \end{aligned}$$



FCC target precision for Higgs decays

Exp error on $\Gamma(H \rightarrow xx) \Leftrightarrow$ expected precision on the (effective) SM-like coupling g_{Hxx}^2 [FCC CDC Volume 1, EPJC 70 (2019) 474]

Partial width	FCC-ee target
$H \rightarrow b\bar{b}$	$\sim 0.6\%$
$H \rightarrow c\bar{c}$	$\sim 1.2\%$
$H \rightarrow \tau^+\tau^-$	$\sim 0.7\%$
$H \rightarrow \mu^+\mu^-$	$\sim 9.0\%$
$H \rightarrow gg$	$\sim 1.0\%$
$H \rightarrow \gamma\gamma$	$\sim 3.9\%$
$H \rightarrow WW \rightarrow 4f$	$\sim 0.4\%$
$H \rightarrow ZZ \rightarrow 4f$	$\sim 0.2\%$

\Rightarrow Higher-order corrections mandatory! Define target for theory uncertainties



Higher-order corrections for the Higgs masses in the NMSSM

1-loop mass calculations: Ender, Graf, Mühlleitner, Rzehak, PRD 85 (2012) 075024; Graf, Gröber, Mühlleitner, Rzehak, Walz, JHEP 1210 (2012) 122

2-loop mass calculations: Mühlleitner, Nhung, Rzehak, Walz, JHEP 1505 (2015) 128; Dao, Gröber, Krause, Mühlleitner, Rzehak, JHEP 08 (2019) 114



Disclaimer of this talk

- I will only present results from my group there, in the framework of NMSSMCALC
- My apologies for the other calculators on the market, namely FeynHiggs, FlexibleSUSY, FlexibleEFTHiggs, NMSSMTools, Sphenox, and SOFTSUSY
- See also [Drechsel, Gröber, Heinemeyer, Mühlleitner, Rzehak, Weiglein, EPJC 77 (2017) 366] for a comparison between some of them
- For a parallel approach to loop corrections to Higgs decays, see e.g. [Domingo, Heinemeyer, Paßer, Weiglein, EPJC 78 (2018) 942; Domingo, Paßer, EPJC 79 (2019) 905] ⇒ see also Sven's talk!



NMSSM Higgs masses at the 1-loop order

- **LO Higgs masses:** diagonalizing the 3×3 CP-even \mathcal{M}_S and the 3×3 CP-odd (+ Goldstone) \mathcal{M}_A mass matrices give the LO masses

$$m_{H_1} < m_{H_2} < m_{H_3} \text{ and } m_{A_1} < m_{A_2}$$

sizable higher order corrections compulsory to obtain one SM-like H_i boson

- **1-loop corrections:** define input parameters, calculate renormalized self-energies in a mixed renormalization scheme

- 7 parameters renormalized on-shell: $e, M_W, M_Z, M_{H^\pm}, t_{h_u}, t_{h_d}, t_s$

- 5 parameters renormalized $\overline{\text{DR}}$: $\tan \beta, \lambda, \kappa, A_k, v_s$

- ▶ $\overline{\text{DR}}$ field renormalization constants for the Higgs fields

$$H_d = \left(1 + \frac{1}{2}\delta Z_d\right) \hat{H}_d \quad H_u = \left(1 + \frac{1}{2}\delta Z_u\right) \hat{H}_u \quad S = \left(1 + \frac{1}{2}\delta Z_s\right) \hat{S}$$

- ▶ Tadpole condition: $\hat{T}_{h_u/d/s} = 0 \Rightarrow \delta t_{h_u/d/s} = T_{h_u/d/s}$



NMSSM Higgs masses at the 1-loop order

- **1-loop corrections:** with the renormalized self-energies $\hat{\Sigma}_{H_i H_j}^{(1)}(p^2)$ calculate the zeros of the **1-loop 2-point vertex matrix**

$$\Gamma^{11}(p^2) = i \begin{pmatrix} p^2 - m_{H_1}^2 + \hat{\Sigma}_{H_1 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_3}^{(1)}(p^2) \\ \hat{\Sigma}_{H_2 H_1}^{(1)}(p^2) & p^2 - m_{H_2}^2 + \hat{\Sigma}_{H_2 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_2 H_3}^{(1)}(p^2) \\ \hat{\Sigma}_{H_3 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_3 H_2}^{(1)}(p^2) & p^2 - m_{H_3}^2 + \hat{\Sigma}_{H_3 H_3}^{(1)}(p^2) \end{pmatrix}$$

Algorithm to calculate the masses

1. Set $p^2 = m_{H_1}^2$ in the self-energies $\hat{\Sigma}$
2. Solve $\det \hat{\Gamma}(p^2) = 0$
3. Extract the smallest root obtained m_{tmp}^2
4. Set $p^2 = m_{\text{tmp}}^2$ in the self-energies and back to 2. n times as long as $|m_{\text{tmp},n}^2 - m_{\text{tmp},n-1}^2| > 10^{-9}$ (desired accuracy) \Rightarrow get the 1-loop mass M_{H_1}
5. Start again at 1. with $m_{H_2}^2$, extract the next-to-smallest root, and so on



Complex parameters and renormalization

- **Many new phases in the complex NMSSM**, for the parameters M_1 , M_2 , A_t , etc

In the Higgs sector: 6 phases ϕ_u , ϕ_s , ϕ_λ , ϕ_κ , ϕ_{A_λ} , and ϕ_{A_κ}

At tree-level, only one relevant phase: only 3 combinations in the mass matrix

$$\Psi = \phi_\lambda - \phi_\kappa + \phi_u - 2\phi_s$$

$$\Psi_\lambda = \phi_\lambda + \phi_{A_\lambda} + \phi_u + \phi_s$$

$$\Psi_\kappa = \phi_\kappa + \phi_{A_\kappa} + 3\phi_s$$

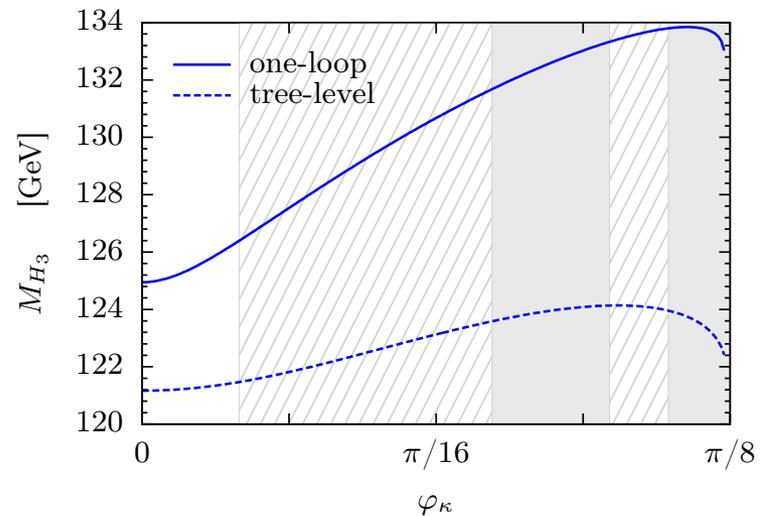
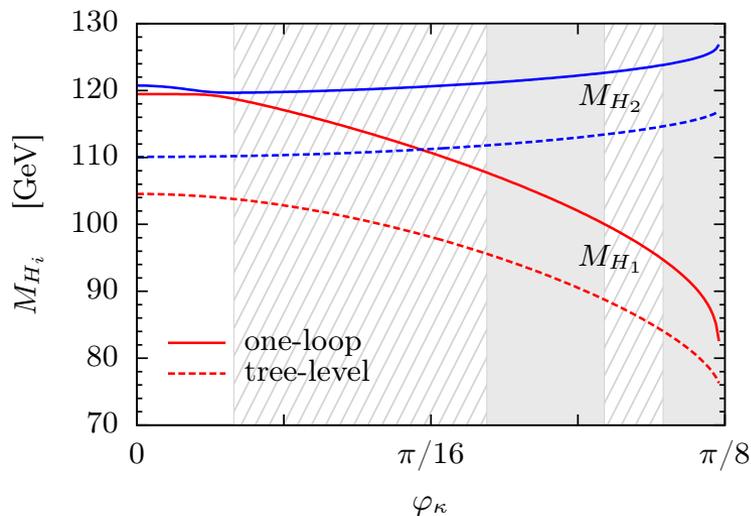
Ψ_λ , Ψ_κ fixed by two tadpole conditions

- **Renormalization in the Higgs sector:**

- ▶ on-shell renormalization for e , M_W , M_Z , M_{H^\pm}
- ▶ $\overline{\text{DR}}$ renormalization for $\tan\beta$, v_s , $|\lambda|$, $|\kappa|$, $|A_\kappa|$ and the phases
- ▶ **vanishing phases counter-terms** but for two tadpole conditions t_a and t_{a_s}

Complex phases have sizable effects:

■ **Tree-level CP violation ($\Psi \neq 0$):**



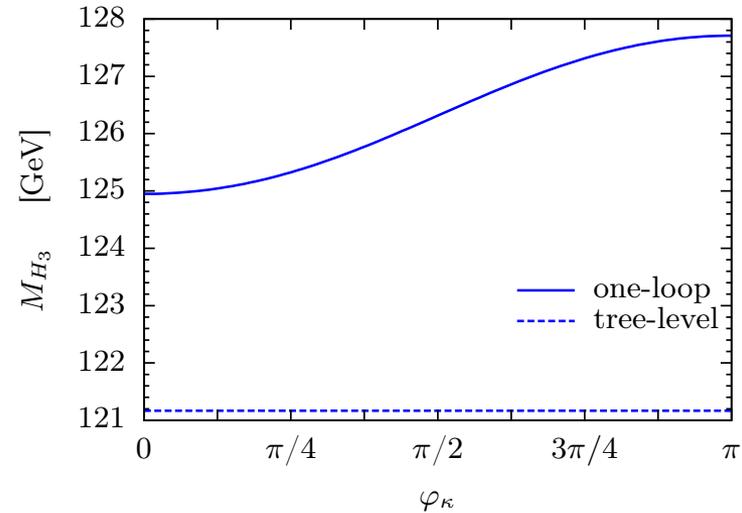
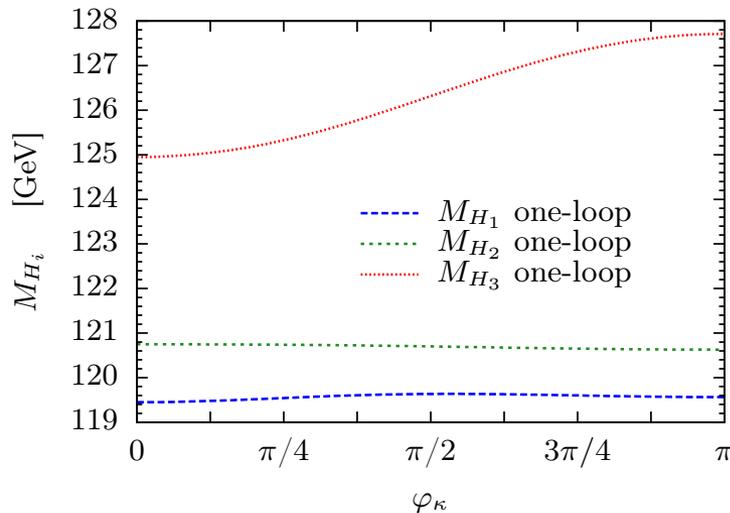
$$|\lambda| = 0.72, \quad |\kappa| = 0.2, \quad \tan \beta = 3, \quad v_s = 389 \text{ GeV}, \quad |A_\kappa| = 27 \text{ GeV},$$

$$|A_\lambda| = 928 \text{ GeV}, \quad A_t = -875 \text{ GeV}, \quad A_b = A_l = -963 \text{ GeV}, \quad M_{\text{SUSY}} = 1 \text{ TeV}$$

$$M_1 = 145 \text{ GeV}, \quad M_2 = 200 \text{ GeV}, \quad M_3 = 600 \text{ GeV}, \quad Q = 300 \text{ GeV}$$

Complex phases have sizable effects:

- **No tree-level CP violation ($\Psi = 0$), $\phi_\lambda = \phi_\kappa \neq 0$:**

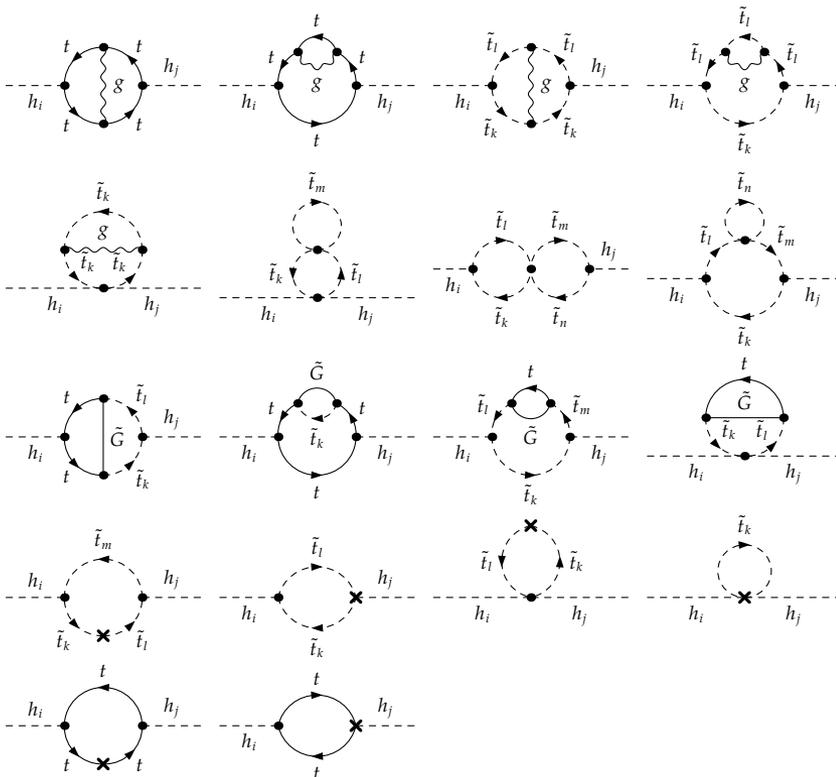


- ▶ One loop corrections have a **sizable impact** (restore some scenarios)
- ▶ No tree-level CP violation scenario: still slight impact of ϕ_κ over H_3 through its coupling to stops

2-loop effects in Higgs mass calculation

2-loop order $\mathcal{O}(\alpha_t \alpha_s + \alpha_t^2)$ in the gaugeless limit and in the approximation of vanishing external momentum:

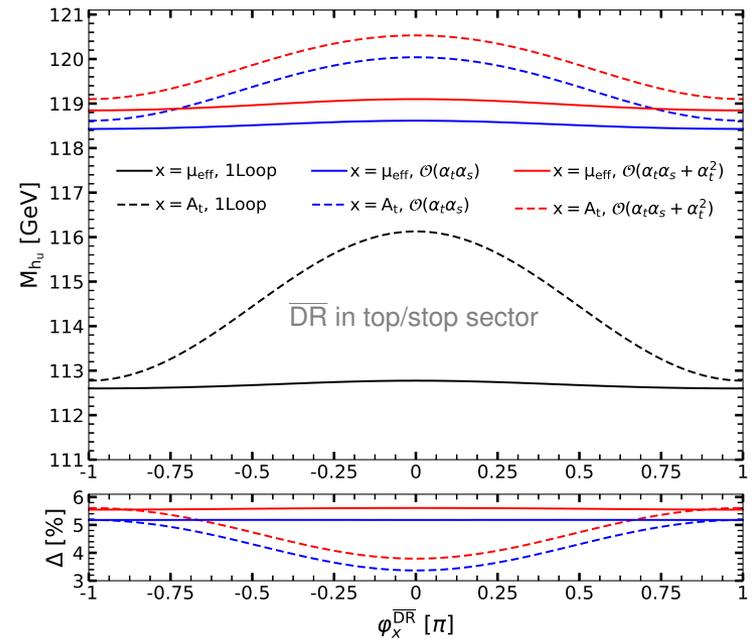
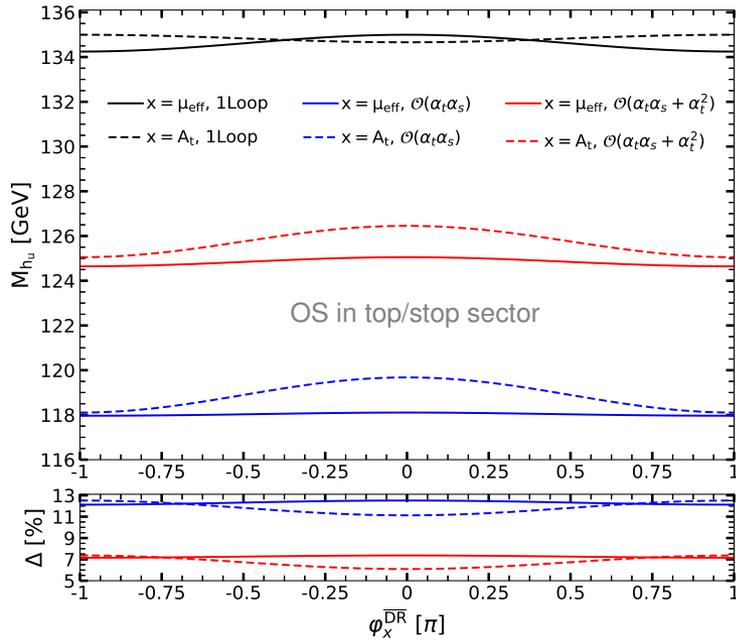
$$\hat{\Sigma}_{H_i H_j}(p^2) = \hat{\Sigma}_{H_i H_j}^{(1)}(p^2) + \hat{\Sigma}_{H_i H_j}^{(2), \alpha_t \alpha_s}(0) + \hat{\Sigma}_{H_i H_j}^{(2), \alpha_t^2}(0)$$



Set of parameters at 2-loop:

- OS renormalization for M_{H^\pm} , t_{h_j} , v (Higgs sector)
- $\overline{\text{DR}}$ renormalization for $\tan \beta$, v_s , $|\lambda|$, $|\kappa|$, $\text{Re } A_\kappa$, ϕ_λ , ϕ_κ , ϕ_U , ϕ_S (Higgs sector)
- OS or $\overline{\text{DR}}$ renormalization for m_t , $m_{\tilde{Q}_3}$, $m_{\tilde{t}_R}$, A_t (top/stop sector)

2-loop $\mathcal{O}(\alpha_s \alpha_t + \alpha_t^2)$ corrections of $\mathcal{O}(6\%)$ on top of 1-loop result:

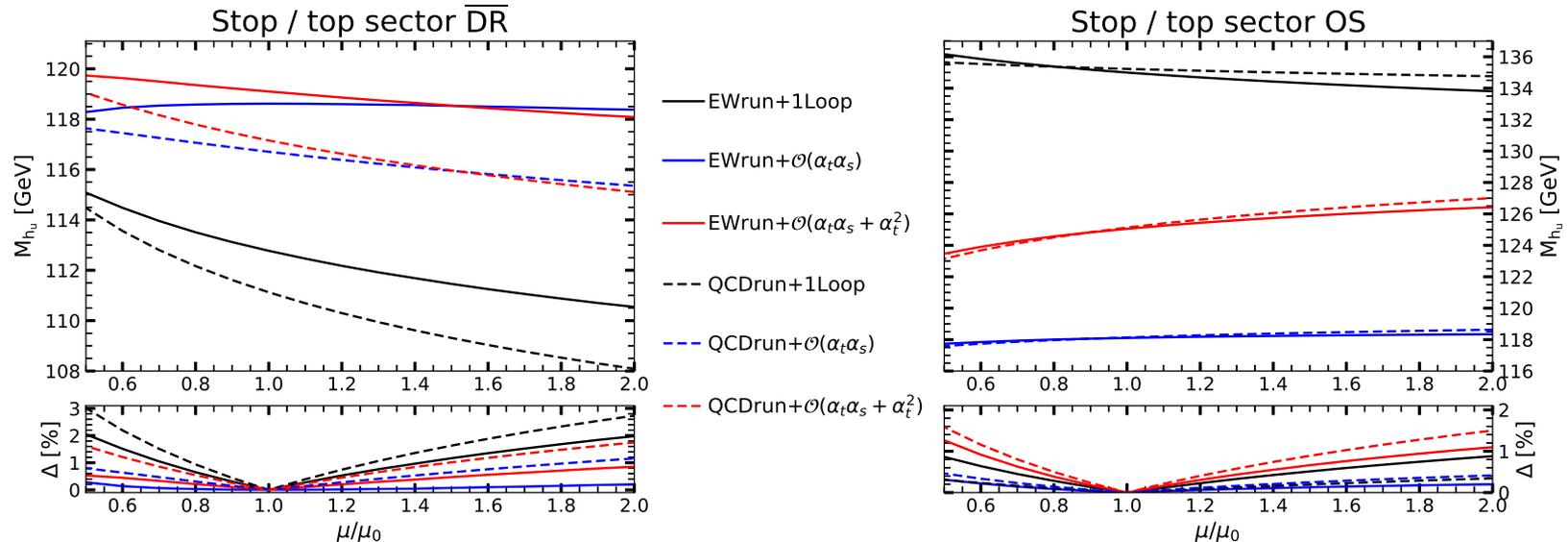


$$\begin{aligned}
 |\lambda| &= 0.301, \quad |\kappa| = 0.299, \quad \tan \beta = 4.44, \quad v_s = 981.962 \text{ GeV}, \quad \text{Re } A_{\kappa} = -791 \text{ GeV}, \\
 m_{\tilde{t}_R} &= 881 \text{ GeV}, \quad m_{\tilde{Q}_3} = 1226 \text{ GeV}, \quad \phi_{A_q} = \pi, \quad \phi_{A_l} = \phi_{M_j} = \phi_{\lambda, \kappa} = \phi_u = 0, \\
 |M_1| &= 644 \text{ GeV}, \quad |M_2| = 585 \text{ GeV}, \quad |M_3| = 1850 \text{ GeV}, \quad M_{H^\pm} = 898 \text{ GeV}
 \end{aligned}$$

Discussion of the theoretical uncertainty

Intrinsic theoretical uncertainty due to

1. renormalization scheme choice; 2. renormalization scheme variation



- QCDrun \equiv (OS \rightarrow $\overline{\text{DR}}$ for $m_t(\mu_R = M_Z)$) & RGEs for ($\mu_R = M_Z \rightarrow \mu_R = \mu$) at order $\mathcal{O}(\alpha_s + \alpha_s^2)$
- EWrun \equiv (OS \rightarrow $\overline{\text{DR}}$ for $m_t(\mu_R = M_Z)$) & RGEs for ($\mu_R = M_Z \rightarrow \mu_R = \mu$) at order $\mathcal{O}(\alpha_s + \alpha_t + (\alpha_s + \alpha_t)^2)$
- Uncertainty of $\mathcal{O}(1\%)$ in each scheme choice, $\mathcal{O}(3 - 4\%)$ between \neq schemes



Higher-order corrections for the Higgs decays in the NMSSM

Higgs decays and NMSSMCALC: **J.B.**, Gröber, Mühlleitner, Nhung, Rzehak, Spira, Streicher, Walz, CPC 185 (2014) 12

NLO SUSY-QCD+SUSY-EW corrections: **J.B.**, Krauss, Mühlleitner, Walz, JHEP 1510 (2015) 024; **J.B.**, Dao, Mühlleitner, arXiv:1907.12060

Trilinear Higgs couplings: Nhung, Mühlleitner, Streicher, Walz, JHEP 1311 (2013) 181; Mühlleitner, Nhung, Ziesche, JHEP 1512 (2015) 034

Electron dipole moments: King, Mühlleitner, Nevzorov, Walz, Nucl.Phys.B 901 (2015) 526



2-loop Higgs boson masses in the NMSSM, allowing for complex parameters
+
calculation of the Higgs boson decay widths in the NMSSM
based on the latest version of **HDECAY**

[Djouadi, Kalinowski, Spira, CPC 108 (1998) 56; Djouadi, Mühlleitner, Spira, Acta Phys.Polon. B38 (2007) 635; Djouadi, Kalinowski, Mühlleitner, Spira, CPC 238 (2019) 214]

- Real or complex parameters
- $\mathcal{O}(\alpha_t \alpha_s + \alpha_t^2)$ 2-loop masses, electric dipole moments
- Decay widths and branching ratios including dominant higher order QCD corrections
- Off-shell decays into VV^* , $H_i V^*$, tt^* , bt^*
- Dominant SUSY corrections in the decays into quarks and leptons: implemented with **effective Δ_b -Yukawa couplings calculated in the NMSSM**



NMSSMCALC 3.0: The Higgs sector in the NMSSM

2-loop Higgs boson masses in the NMSSM, allowing for complex parameters

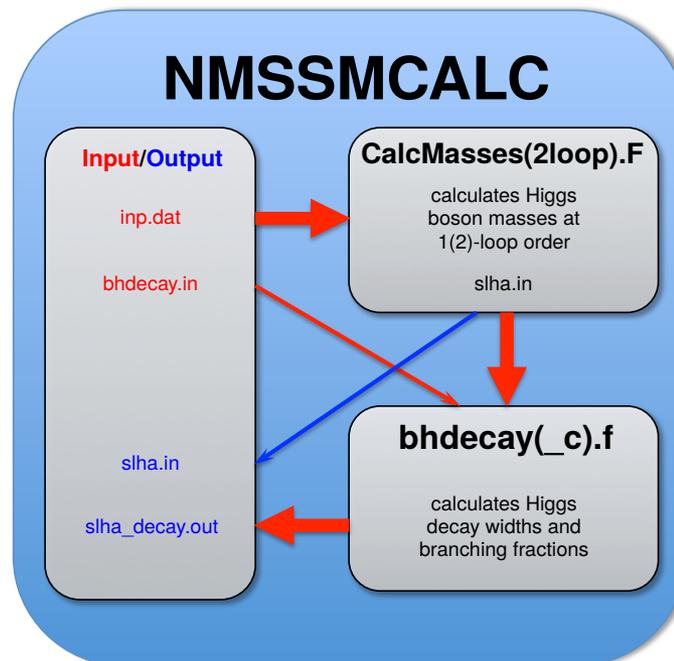
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Current status of mass calculation in NMSSMCALC:

- ▶ 2-point 1-loop integrals with complex momentum
⇒ no instabilities due to threshold singularities
- ▶ approximate 2-loop order for the calculation of the Higgs spectrum with $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$ corrections

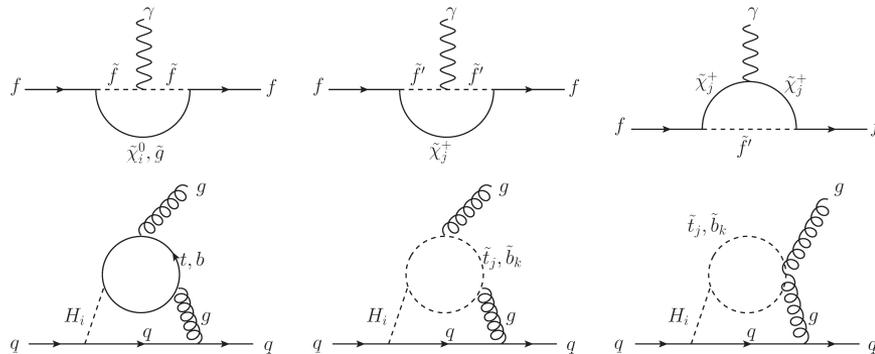
To download NMSSMCALC:
<https://www.itp.kit.edu/~maggie/NMSSMCALC>



Inclusion of electric dipole moments

CP-violating effects enhance the (chromo-)electric dipole moments (EDM), predicted to be very small in the SM, and not yet observed

Exp upper bounds:



■ Electron EDM $\lesssim 10^{-29} \text{e.cm}$
[ACME, Nature 562 (2018) 355]

■ Thallium EDM $\lesssim 9 \times 10^{-25} \text{e.cm}$
[Regan et al, PRL 88 (2002) 071805]

■ Neutron EDM $\lesssim 3 \times 10^{-26} \text{e.cm}$
[Baker et al, PRL 97 (2006) 131801]

■ Mercury EDM $\lesssim 3.1 \times 10^{-29} \text{e.cm}$
[Griffith et al, PRL 102 (2009) 101601]

Implemented in NMSSMCALC at the 2-loop level, new switch in Block MODSEL

[King, Mühlleitner, Nevzorov, Walz, Nucl.Phys.B 901 (2015) 526]

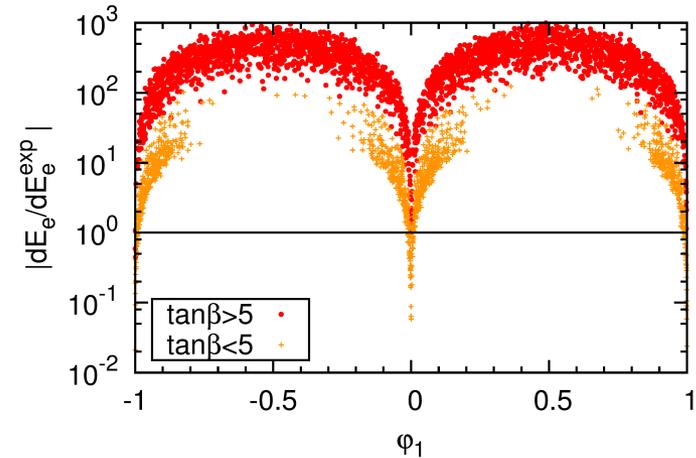
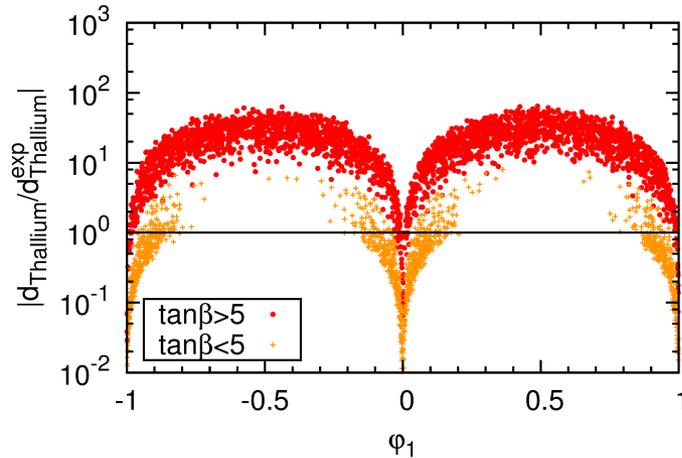
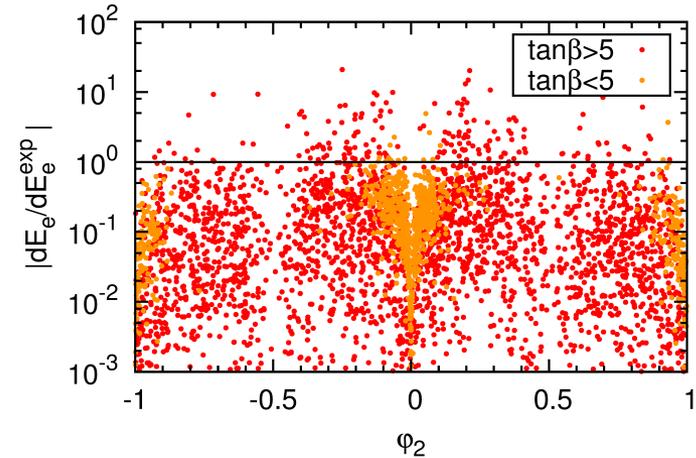
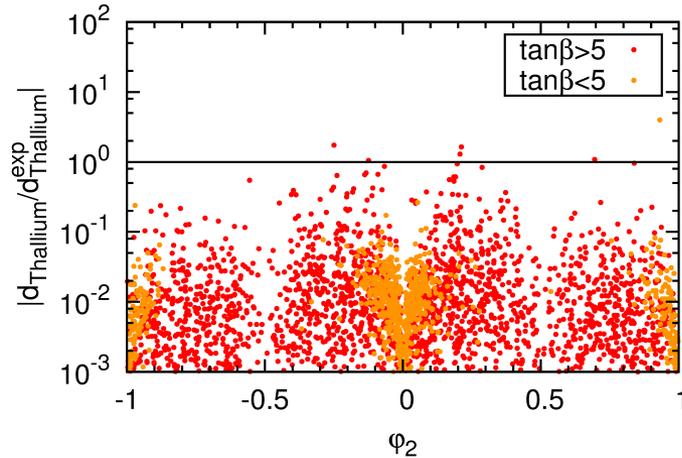
Block MODSEL

```
3 1 # NMSSM
5 2 # CP violation
10 1 # EDMs: 0, no EDMs); 1, with EDMs); 2, EDMs individual contributions
```

Output with results rescaled to exp upper-bound

Impact of EDMs constraints

Still a sizable portion of NMSSM phases allowed by EDMs constraints



$\phi_2 = \phi_\kappa + 3\phi_s$, specific NMSSM phase $\phi_1 = \phi_\lambda + \phi_s + \phi_U$, MSSM-like phase



NMSSMCALCEW: NLO Higgs decay widths

New in 2019: Extension of NMSSMCALC to include the complete NLO SUSY-QCD and SUSY-EW corrections to neutral Higgs decay widths:

- $\Gamma(H_i \rightarrow f\bar{f})$: decay to fermions at 1-loop SUSY-QCD, SUSY-EW, running masses for the quarks
- $\Gamma(H_i \rightarrow W^+W^-, ZZ)$: (improved) tree-level for off-shell decays to massive SM gauge bosons, 1-loop SUSY-EW for on-shell decays of heavy Higgs bosons
- $\Gamma(H_i \rightarrow ZH_j, H_jH_k)$: Higgs-to-Higgs and Higgs+Z decays at 1-loop NLO SUSY-EW
- $\Gamma(H_i \rightarrow \tilde{f}_i\tilde{f}_j)$: decay to sfermions at 1-loop SUSY-QCD, SUSY-EW
- $\Gamma(H_i \rightarrow \tilde{\chi}_i\tilde{\chi}_j, \tilde{\chi}_i^+\tilde{\chi}_j^-)$: decay to neutralinos and charginos at 1-loop SUSY-EW

To download NMSSMCALCEW:
<https://www.itp.kit.edu/~maggie/NMSSMCALCEW>



Some technical details

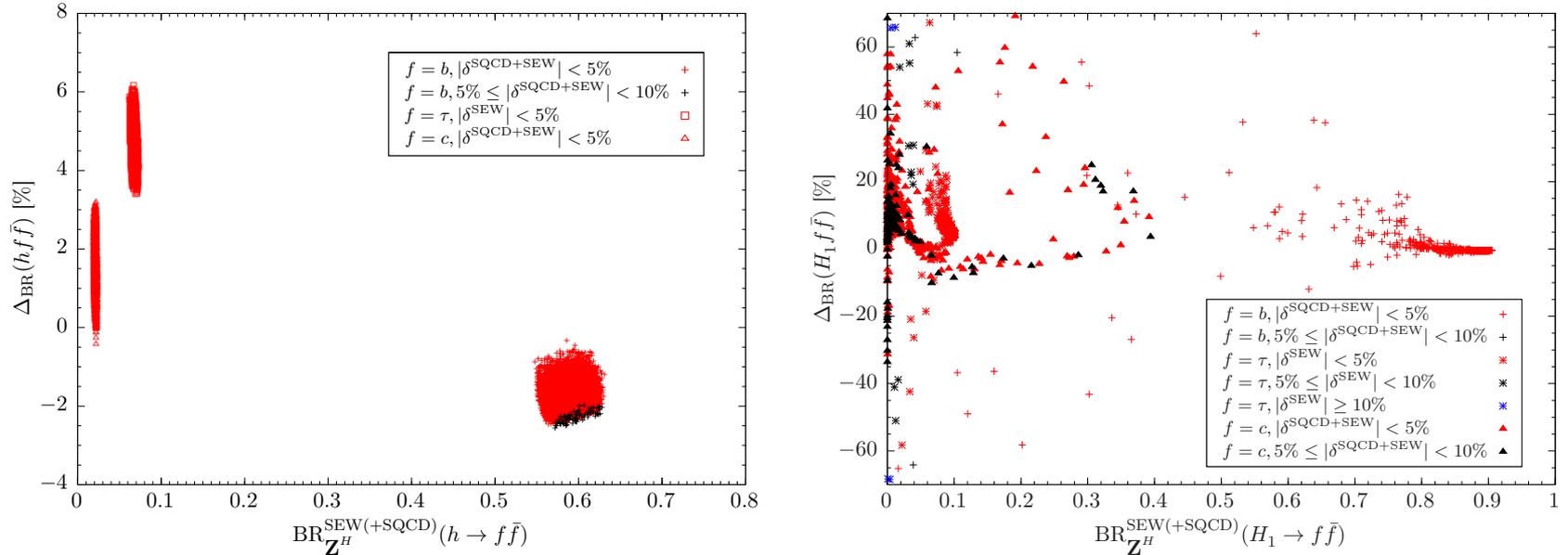
- **Loop corrections to squark, neutralinos, and weakino masses** needed, 2 descriptions for the wave-function renormalization (WFR):
 - ▶ **EMT description** [see e.g. Espriu, Manzano, Talavera, PRD 66 (2002) 07600]: 2 \neq WFR for ingoing and outgoing states
 - no mixing self-energies, absorptive part of loop functions included ✓
 - no hermiticity of the Lagrangian ✗
 - ▶ **Denner description** [Denner, Fort.Phys. 41 (1993) 307]: only 1 common WFR
 - hermiticity of the Lagrangian ✓
 - absorptive part of the loop functions must be removed ✗
- **Z-Goldstone boson mixing $\delta\mathcal{M}_{GZ,mix}$** dealt with **Slavnov-Taylor identities** adapted to loop-corrected masses [Nhung, Mühlleitner, Streicher, Walz, JHEP 11 (2013) 181; Domingo, Heinemeyer, Paßer, Weiglein, EPJC 78 (2018) 942]

$$M_Z \hat{\Sigma}_{h_i, G}(M_{h_i}^2) + i M_{h_i}^2 \hat{\Sigma}_{h_i, Z}(p^2) = 0$$

- **Real corrections including $\Gamma(H_i \rightarrow W^+ W^- \gamma)$** to ensure IR-finiteness, but loop-corrected Higgs masses used in the amplitude \Rightarrow gauge-invariance not enforced, see [Dao, Fritz, Krause, Mühlleitner, Patel, arXiv:1911.07197] for a detailed discussion

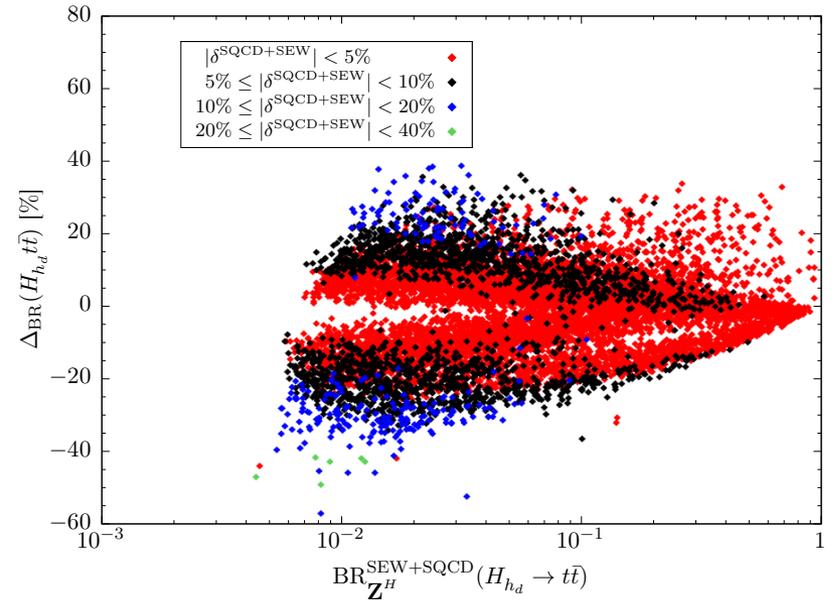
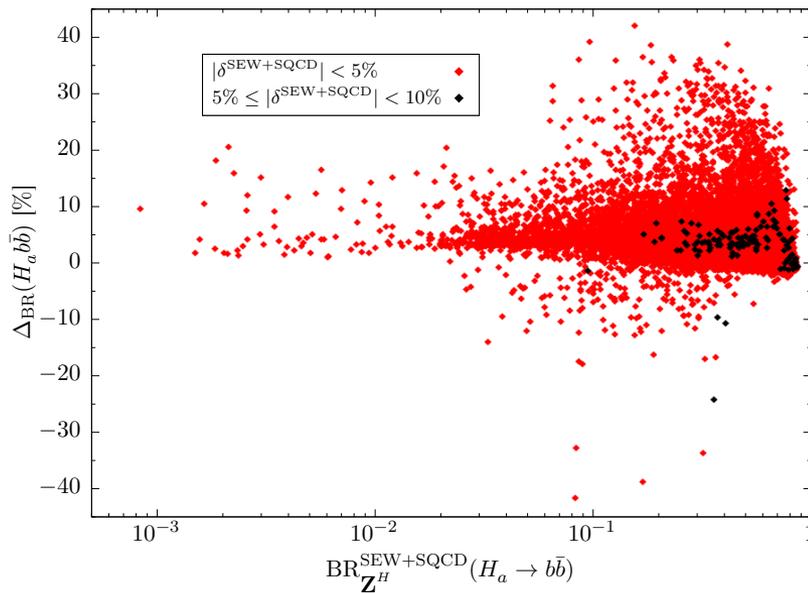
Decays into SM fermions

Random scan of the NMSSM parameter space, δ 1-loop vertex correction, $\Delta_{\text{BR}}(H_i XY)$ comparison to NMSSMCALC 3.0 for $\text{BR}(H_i \rightarrow XY)$

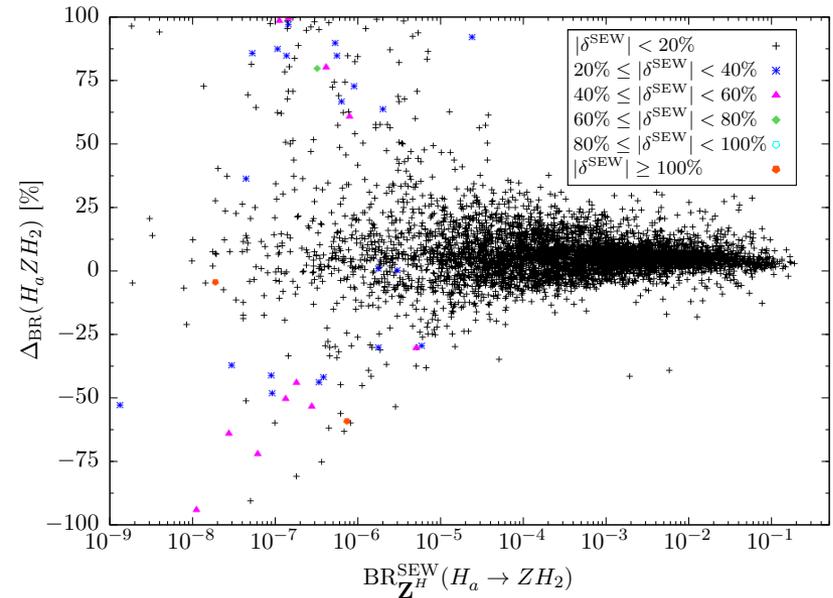
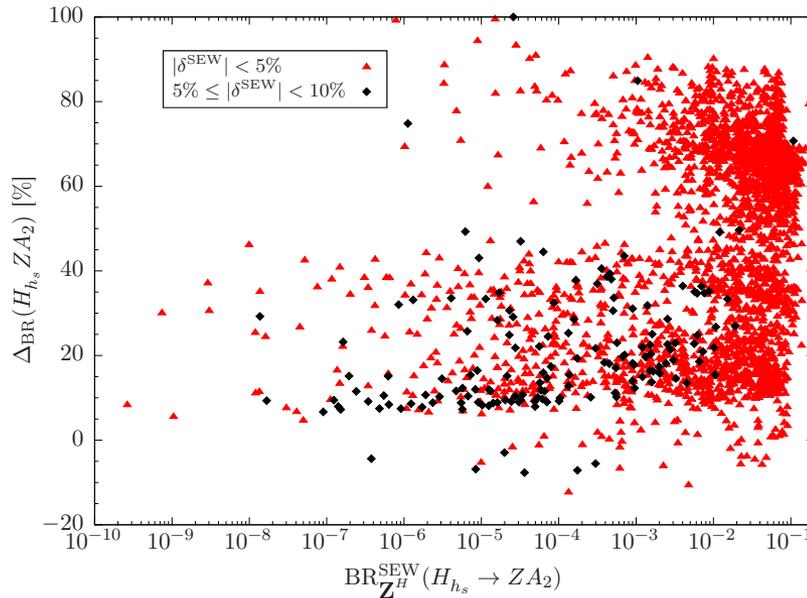


- New implementation (with full WFR factor \mathbf{Z}^H and NLO amplitudes) close to NMSSMCALC 3.0 (with unitary Higgs mixing matrix) for SM-like Higgs boson h
- Vertex corrections at most $\mathcal{O}(10\%)$ for SM-like state h
- Same mostly true for singlet-like H_1 boson, $\Delta_{\text{BR}}(H_1)$ mostly below 30%

Decays into SM fermions

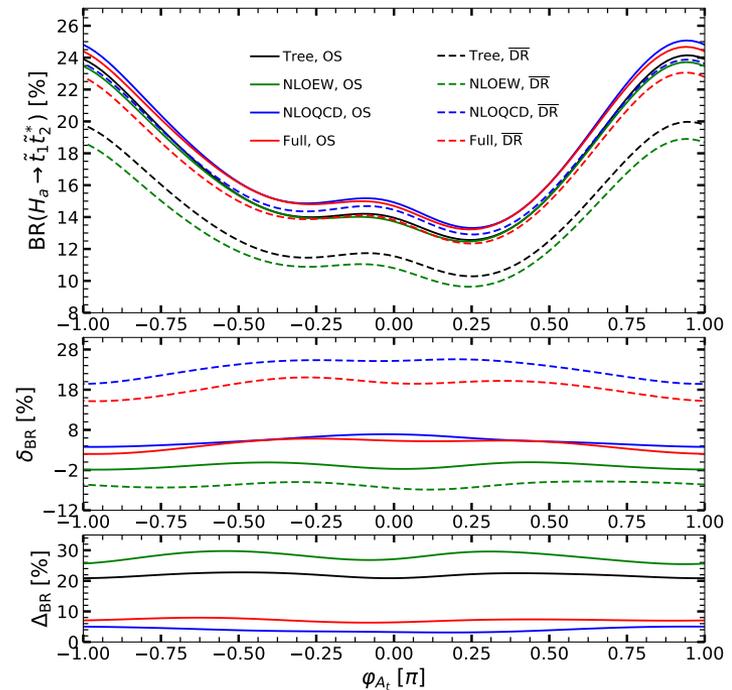
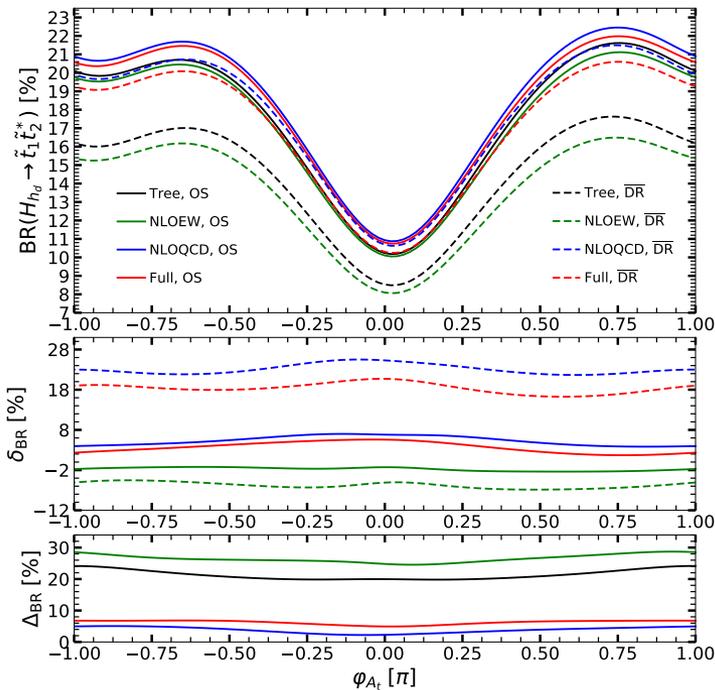


- $|\Delta_{\text{BR}}(H_a b \bar{b})|$ and $|\Delta_{\text{BR}}(H_{h_d} t \bar{t})|$ mostly below 40%
- Vertex corrections can reach 40% for the H_{h_d} state



- Many points with large $\text{BR}(H_{h_s} \rightarrow Z A_2)$, A_2 doublet-like
- $|\Delta(H_{h_s} Z A_2)|$ up to 80% while $|\delta| < 10\% \Rightarrow$ corrections here driven by Z^H factor
- $|\Delta(H_a Z H_2)|$ mostly below 25% (H_2 singlet-like), vertex corrections can reach 100% \Rightarrow large corrections for very small BRs where 1-loop squared are also taken into account

Decays into a squark pair



- $\delta_{\text{BR}} \equiv$ vertex correction, $\Delta_{\text{BR}} \equiv$ difference between OS and $\overline{\text{DR}}$
- SUSY-EW corrections negative, SUSY-QCD corrections positive, cancelation between the two of them
- vertex corrections larger in $\overline{\text{DR}}$ than in OS
- strong decrease of Δ_{BR} when both EW and QCD taken into account!



Conclusions and outlook

In view of the HL-LHC and FCC programs, plenty of room for improvement!

- **2-loop mass calculation:**

- ▶ **2-loop $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$ corrections in the gaugeless limit at zero momentum current state-of-the-art in the mixed renormalization scheme;**

agreement with other codes when taking into account \neq input setup

[see e.g. Staub et al, CPC 202 (2016) 113; Goodsell, Staub, EPJC 77 (2017) 46]

- ▶ **For the future: 2-loop terms $\mathcal{O}(\alpha_s\alpha_b + \alpha_t\alpha_b)$ to be done for the complex NMSSM** (known in the real NMSSM, see FeynHiggs 3)

- **NLO decay widths: Inclusion of NLO SUSY-(QCD+)EW corrections in the complex NMSSM for all neutral Higgs decays now known!**

→ For the future: corrections for charged Higgs decays

- Uncertainties on the decay BRs: reaching FCC precision for the SM-like Higgs boson at the percent level, $\mathcal{O}(10 - 30\%)$ for the BSM Higgs states
⇒ room for improvement in the precision of the Higgs decay widths