“Tell me that you have found no sign of New Physics again, I dare you. I double dare you. Tell me one more goddamn time!”
Higgs Couplings in the NMSSM
Determining the Nature of BSM Physics

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1. Motivation
2. (N)MSSM Higgs decays
3. BSM Higgs: what can we learn?
4. Conclusions
1. Motivation

Two Facts:

- We have an SM-like Higgs discovery!
- The SM cannot be the ultimate theory!

Conclusion: It cannot be “the SM Higgs”!
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We have an SM-like Higgs discovery!

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Conclusion: It cannot be “the SM Higgs”!

Q: Does the BSM physics have any (relevant) impact on the Higgs?
Q’: Which model?
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Two Facts:

- We have an SM-like Higgs discovery!
- The SM cannot be the ultimate theory!

Conclusion: It cannot be “the SM Higgs”!

Q: Does the BSM physics have any (relevant) impact on the Higgs?

Q’: Which model?

A1: check changed properties

A2: check for additional Higgs bosons

A2’: check for additional Higgs bosons above and below 125 GeV
**Higgs coupling measurements at $e^+e^-$ colliders**

Initial measurement: $\sigma \times \text{BR}$

**recoil method**: $e^+e^- \rightarrow ZH, Z \rightarrow e^+e^-, \mu^+\mu^-$

⇒ measurement of the Higgs production cross section

⇒ **NO** additional theoretical assumptions needed for absolute determination of partial widths

⇒ indirect measurement of total width

⇒ direct extraction of partial widths (couplings)

⇒ search for deviations from the SM

⇒ distinction between different models
Future expectations for $\kappa$ (kappa-3 framework)

⇒ very roughly similar results
⇒ FCC-hh/-he/-ee appears better
⇒ FCC-hh uses different theory assumptions, uncertainties $\lesssim 1\%$
⇒ also remember different time scales!
Intrinsic uncertainties for decay widths:  

"ILC/CEPC/FCC-ee" = expected precision on $g_{Hxx}^2$ (incl. HL-LHC meas.)

<table>
<thead>
<tr>
<th>Partial width</th>
<th>QCD</th>
<th>electroweak</th>
<th>total</th>
<th>future</th>
<th>ILC/CEPC/FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \rightarrow WW \rightarrow 4f$</td>
<td>$&lt; 0.5%$</td>
<td>$&lt; 0.3%$</td>
<td>$\sim 0.5%$</td>
<td>$\lesssim 0.4%$</td>
<td>$0.6/1.9/0.8%$</td>
</tr>
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<td>$&lt; 0.3%$</td>
<td>$\sim 0.5%$</td>
<td>$\lesssim 0.3%$</td>
<td>$0.4/0.4/0.3%$</td>
</tr>
<tr>
<td>$H \rightarrow gg$</td>
<td>$\sim 3%$</td>
<td>$\sim 1%$</td>
<td>$\sim 3.2%$</td>
<td>$\sim 1%$</td>
<td>$1.7/2.2/1.8%$</td>
</tr>
<tr>
<td>$H \rightarrow \gamma\gamma$</td>
<td>$&lt; 0.1%$</td>
<td>$&lt; 1%$</td>
<td>$&lt; 1%$</td>
<td>$&lt; 1%$</td>
<td>$2.4/2.4/2.4%$</td>
</tr>
<tr>
<td>$H \rightarrow Z\gamma$</td>
<td>$\lesssim 0.1%$</td>
<td>$\sim 5%$</td>
<td>$\sim 5%$</td>
<td>$\sim 1%$</td>
<td>$22/13/20%$</td>
</tr>
<tr>
<td>$H \rightarrow b\bar{b}$</td>
<td>$\sim 0.2%$</td>
<td>$&lt; 0.3%$</td>
<td>$&lt; 0.4%$</td>
<td>$\sim 0.2%$</td>
<td>$1.2/1.8/1.3%$</td>
</tr>
<tr>
<td>$H \rightarrow c\bar{c}$</td>
<td>$\sim 0.2%$</td>
<td>$&lt; 0.3%$</td>
<td>$&lt; 0.4%$</td>
<td>$\sim 0.2%$</td>
<td>$2.4/4.0/2.6%$</td>
</tr>
<tr>
<td>$H \rightarrow \tau^+\tau^-$</td>
<td>$-$</td>
<td>$&lt; 0.3%$</td>
<td>$&lt; 0.4%$</td>
<td>$&lt; 0.1%$</td>
<td>$1.3/1.9/1.3%$</td>
</tr>
<tr>
<td>$H \rightarrow \mu^+\mu^-$</td>
<td>$-$</td>
<td>$&lt; 0.3%$</td>
<td>$&lt; 0.3%$</td>
<td>$&lt; 0.1%$</td>
<td>$7.8/7.8/7.8%$</td>
</tr>
<tr>
<td>$\Gamma_{\text{tot}}$</td>
<td></td>
<td></td>
<td>$\sim 0.3%$</td>
<td></td>
<td>$1.1/1.8/1.2%$</td>
</tr>
</tbody>
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⇒ talk by S.H.
Needed for LHC/CLIC/ILC/CEPC/FCC-ee... physics:

Precise and consistent prediction of

- Higgs boson masses
- Higgs boson mixings
- Higgs boson couplings
- Higgs boson production cross sections
- Higgs boson decay widths/branching ratios

⇒ (partially) provided by FeynHiggs
⇒ Please: repeat this exercise in your favorite model! Not EFT ...
The MSSM:

⇒ Superpartners for Standard Model particles
Enlarged Higgs sector: Two Higgs doublets

\[ H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \]

\[ H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \]

\[ V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \]

\[ + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \]

gauge couplings, in contrast to SM

physical states: \( h^0, H^0, A^0, H^\pm \)

Goldstone bosons: \( G^0, G^\pm \)

Input parameters: (to be determined experimentally)

\[ \tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta) \]
Enlarged Higgs sector: Two Higgs doublets with $\mathcal{CP}$ violation

\[
H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}
\]

\[
H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} e^{i\xi}
\]

\[
V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a \bar{H}_2^b + \text{h.c.}) 
\]

\[
\quad + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2
\]

Gauge couplings, in contrast to SM

Physical states: $h^0, H^0, A^0, H^\pm$ ⇒ re-enters via loop corrections!

2 $\mathcal{CP}$-violating phases: $\xi, \arg(m_{12})$ ⇒ can be set/rotated to zero

Input parameters: (to be determined experimentally)

\[
\tan \beta = \frac{v_2}{v_1}, \quad M_{H^\pm}^2
\]
Z$_3$ invariant NMSSM

MSSM Higgs sector: Two Higgs doublets

\[ H_1 = \begin{pmatrix} H^1_1 \\ H^2_1 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \]

\[ H_2 = \begin{pmatrix} H^1_2 \\ H^2_2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \]

\[ V = (\tilde{m}_1^2 + |\mu|^2)H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\mu|^2)H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab}H^a_1H^b_2 + \text{h.c.}) \]

\[ + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \]
\textbf{Z}_3\text{ invariant NMSSM}

\textbf{NMSSM Higgs sector: Two Higgs doublets + one Higgs singlet}

\begin{align*}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \\
S &= v_s + S_R + IS_I
\end{align*}

\begin{align*}
V &= (\tilde{m}_1^2 + |\mu\lambda S|^2)H_1\bar{H}_1 + (\tilde{m}_2^2 + |\mu\lambda S|^2)H_2\bar{H}_2 - m_{12}^2(\epsilon_{ab}H_1^aH_2^b + \text{h.c.}) \\
&\quad + \frac{g^2 + g'^2}{8} (H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \frac{g^2}{2} |H_1\bar{H}_2|^2 \\
&\quad + |\lambda(\epsilon_{ab}H_1^aH_2^b) + \kappa S^2|^2 + m_S^2|S|^2 + (\lambda A\lambda(\epsilon_{ab}H_1^aH_2^b)S + \frac{\kappa}{3}A\kappa S^3 + \text{h.c.})
\end{align*}

Free parameters:

\[\lambda, \kappa, A_\kappa, M_{H\pm}, \tan \beta, \mu_{\text{eff}} = \lambda v_s\]
Higgs spectrum:

$\mathcal{CP}$–even: $h_1, h_2, h_3$

$\mathcal{CP}$–odd: $a_1, a_2$

charged: $H^+, H^-$

Goldstones: $G^0, G^+, G^-$

Neutralinos:

$\mu \rightarrow \mu_{\text{eff}}$

compared to the MSSM: one singlino more

$\rightarrow \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0$
Mass of the lightest \( \mathcal{CP} \)-even Higgs:

\[
m_{h,\text{tree,NMSSM}}^2 = m_{h,\text{tree,MSSM}}^2 + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta
\]

Mass of the \( \mathcal{CP} \)-odd Higgs:

\[
\text{MSSM: } M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta) = \mu B (\tan \beta + \cot \beta)
\]

\[
\text{NMSSM: } "M_A^2" = \mu_{\text{eff}} B_{\text{eff}} (\tan \beta + \cot \beta)
\]

with \( B_{\text{eff}} = A_\lambda + \kappa s, \mu_{\text{eff}} = \lambda_s \quad \Rightarrow \quad \text{one very light } a_1
\]

Mass of the charged Higgs:

\[
\text{MSSM: } M_{H^\pm}^2 = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2
\]

\[
\text{NMSSM: } M_{H^\pm}^2 = M_A^2 + v^2 \left( \frac{g^2}{2} - \lambda^2 \right)
\]
Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m_{h,\text{tree,NMSSM}}^2 = m_{h,\text{tree,MSSM}}^2 + \frac{M_Z^2 \lambda^2}{g^2} \sin^2 2\beta$$

Mass of the $\mathcal{CP}$-odd Higgs:

MSSM: \[ M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta) = \mu B (\tan \beta + \cot \beta) \]

NMSSM: \[ "M_A'^2" = \mu_{\text{eff}} B_{\text{eff}} (\tan \beta + \cot \beta) \]

with \( B_{\text{eff}} = A_\lambda + \kappa s \), \( \mu_{\text{eff}} = \lambda s \) \( \Rightarrow \) one very light \( a_1 \)

Mass of the charged Higgs:

MSSM: \[ M_{H^\pm}^2 = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2 \]

NMSSM: \[ M_{H^\pm}^2 = M_A^2 + v^2 \left( \frac{g^2}{2} - \lambda^2 \right) \]

\( \Rightarrow \) \( M_{h_1,\text{MSSM,tree}} \leq M_{h_1,\text{NMSSM,tree}} \), one light \( a_1 \), \( M_{H^\pm,\text{MSSM,tree}} \geq M_{H^\pm,\text{NMSSM,tree}} \)
The FeynHiggs Ansatz for masses
(taken from talk by [P. Drechsel])

General idea: treat the MSSM part exactly as in the MSSM

- full inverse propagator in CP-even sector for mass determination

\[
\Delta^{-1}(k^2) = i \left[ k^2 \mathbb{1} - M_{\phi\phi} + \hat{\Sigma}^{(1L)}(k^2) + \hat{\Sigma}^{(2L)}(k^2 = 0) \right]
\]

- included corrections from FEYNHIGGS at 2-loop order:
  - orders \( \mathcal{O}(\alpha_s \alpha_t, \alpha_s \alpha_b, \alpha_t^2, \alpha_t \alpha_b) \)
  - resummed large logarithms

⇒ any deviation from the MSSM can directly attributed to the extended model!
⇒ kind of obvious, but only FeynHiggs does it . . .
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⇒ kind of obvious, but only FeynHiggs does it . . .
⇒ same Ansatz for Higgs decays!
FeynHiggs “workflow”:

- **MSSM modelfile**
  - defines matter content, Feynman rules, renormalization conditions

- **FeynArts**
  - generates Feynman diagrams

- **FormCalc**
  - computes Feynman diagrams

- **Numerical evaluation**
  - replace parameters, Looptools

- **FeynHiggs**
  - precision observables for the Higgs sector of the MSSM

- **Higgs self-energies, vertex functions, etc. in analytical form**

- **Observables**
  - Higgs masses, decay widths, etc.
Idea for this work:

- Renormalization of the CP-conserving Higgs-sector
  [Drechsel, Galeta, Heinemeyer, Weiglein, (2016)];
- CP-violating NMSSM, on-shell neutral Higgs
  [Drechsel, F.D., Paßehr (2017)];
- Neutral Higgs decays into SM particles at full one-loop order
  [F.D., Heinemeyer, Paßehr, Weiglein, (2018)];
- Higgs-to-Higgs + Higgs-to-SUSY on-going...
- Inclusion within FeynHiggs in an unforeseeable future...

Learn from MSSM and adapt to the NMSSM. + Re-juvenate MSSM from NMSSM.

In the future: **FeynHiggs 3.0**

⇒ few numerical examples for the Higgs decays

⇒ Please: repeat this excercise in your favorite model! Not EFT . . .
SM-like Higgs state

- Radiative corrections dominated by QCD-corrections;
- Unitary tree-level approximation works well;
- Difference wrt. FH (small): $h \rightarrow g(g^* \rightarrow b \bar{b})$
  (whether $h \rightarrow b \bar{b}$ or $h \rightarrow gg$ is an experimental question).
Bringing the NMSSM to the same level: NMSSM in the MSSM limit

Heavy doublet Higgs states at ~ 1 TeV

- Sizable EW corrections due to Sudakov logarithms;
- Unitary tree-level approximation ‘fails’ ~ 10% off;
- Difference wrt. FH (minor): UV scale in $\Delta_b$ (higher-order).
Bringing the NMSSM to the same level: NMSSM in the MSSM limit

Heavy CP-even doublet Higgs state

- black curve: SM 1L prediction of Prophecy4f rescaled (as FH);
- Red curve: full one-loop (on-shell);
- Rescaling procedure fails for a decoupling state $g_{HHVV} / g_{HSMVV} \approx 0$. 
Overall (N)MSSM Higgs decay uncertainty estimates

- $h_i \rightarrow q\bar{q}$: SM-like: SM NNLO QCD, EW NNLO, SUSY 2L: $\sim 5\%$
  heavy: as SM-like, Sudakov logs: $\sim 5 - 10\%$

- $h_i \rightarrow \ell\bar{\ell}$: SM-like: $\lesssim 1\%$
  heavy: Sudakov logs for very heavy Higgses $\lesssim 10\%$

- $h_i \rightarrow WW^{(*)}, ZZ^{(*)}$: SM-like: $\lesssim 1\%$
  heavy: missing 2L (very small width): $\lesssim 50\%$

- $h_i \rightarrow \gamma\gamma, gg, \gamma Z$: $\gamma\gamma$: NLO QCD, LO EW: $\lesssim 4\%$
  $gg$: NLO QCD, EW: $\lesssim 4\%$
  $\gamma Z$: LO: $\sim 5\%$

- $h_i \rightarrow$ SUSY SUSY: [S.H., C. Schappacher ’14-’16]
  1L effects 10 – 20%, 2L?

- all decays: $U_{ij}, Z_{ij}$: few %, effects close to threshold?

$\Rightarrow$ approaching $e^+e^-$ prec. for SM-like Higgs (not for heavy Higgses yet)
3. BSM Higgs: what can we learn?

– let’s assume that we do see a deviation

– What do we learn from that?
Required precision for Higgs couplings?

MSSM example:

\[
\kappa_V \approx 1 - 0.5\% \left( \frac{400 \text{ GeV}}{M_A} \right)^4
\]

\[
\kappa_t = \kappa_c \approx 1 - \mathcal{O}(10\%) \left( \frac{400 \text{ GeV}}{M_A} \right)^2 \cot^2 \beta
\]

\[
\kappa_b = \kappa_\tau \approx 1 + \mathcal{O}(10\%) \left( \frac{400 \text{ GeV}}{M_A} \right)^2
\]

Composite Higgs example:

\[
\kappa_V \approx 1 - 3\% \left( \frac{1 \text{ TeV}}{f} \right)^2
\]

\[
\kappa_F \approx 1 - (3 - 9)\% \left( \frac{1 \text{ TeV}}{f} \right)^2
\]

⇒ couplings to bosons in the per mille range

⇒ couplings to fermions in the per cent range

⇒ theory/experimental match?
Let us assume that we do see a deviation

What do we learn from that?
How do we learn something from that?

⇒ We have to compare the observed deviation with predicted deviations

⇒ Preferrably with the predicted deviations in a concrete models
  (A comparison with an EFT result subsequently requires the mapping
to concrete models anyway . . . )

⇒ Needed: sufficiently precise predictions in BSM model
  close to ready: MSSM, NMSSM
  (I am not aware of uncertainty estimates in other models)

⇒ in the following:

\[
\text{model prediction (w/o TH unc.)} \Leftrightarrow e^+e^- \text{ precision}
\]

⇒ “Wäscheleinen-Plots”
  (concrete: ILC500 – FCC-ee similar!)
Wäscheleine I: $e^+e^-$ precision vs. 2HDM type II prediction:

[T. Barklow et al., '17]
Wäscheleine II: $e^+e^-$ precision vs. 2HDM type X prediction:

[T. Barklow et al., '17]
Wäscheleine III: $e^+e^-$ precision vs. 2HDM type Y prediction:

[T. Barklow et al., ’17]
Wäscheleine IV: $e^+e^-$ precision vs. Composite Higgs prediction:

[T. Barklow et al., '17]
Wäscheleine V: $e^+e^-$ precision vs. HxSM prediction:

[T. Barklow et al., '17]
Wäscheleine VI: $e^+e^-$ precision vs. Higgs-Radion prediction:
[T. Barklow et al., '17]
MSSM Wäscheleine I: $e^+e^-$ precision vs. $M_h^{125}$ ($M_A = 700$ GeV, $\tan\beta = 8$)

[H. Bahl et al – PRELIMINARY]
MSSM Wäscheleine II: $e^+e^-$ precision vs. $M_h^{125} (M_A = 1000 \text{ GeV}, \tan \beta = 8)$

\[ H. \ Bahl \ et \ al \ – \ PRELIMINARY \]

\[ \Rightarrow \text{only } e^+e^- \text{ measurements allows to set upper limit on } M_A \]
MSSM Wäscheleine III: $e^+e^- \text{ vs. } M_{h}^{125,EFT}(\tilde{\chi})$ ($M_A = 700$ GeV, $\tan \beta = 3$)

[H. Bahl et al – PRELIMINARY]
MSSM Wäscheleine IV: $e^+e^-$ vs. $M_{h^{125, \text{EFT}}} (\tilde{\chi}) (M_A = 1000 \text{ GeV}, \tan \beta = 3)$

[H. Bahl et al – PRELIMINARY]

$\Rightarrow$ only $e^+e^-$ measurements allows to set upper limit on $M_A$
MSSM Wäscheleine V: $e^+e^-$ vs. $M_h^{125}$ ($M_A = 1000$ GeV, $\tan \beta = 8$)

[H. Bahl et al – PRELIMINARY]

⇒ MSSM vs. 2HDM: very challenging!
4. Conclusions

- High precision prediction for cross sections and branching ratios are crucial for coupling constant determination

- Predictions (in the SM and BSM) needed at/below the per-cent level!

- (N)MSSM Higgs decays:
  - SM-like Higgs: approaching $e^+e^-$ precision
  - Heavy Higgses: much larger uncertainties (exp. situation unclear)

- Please: repeat this exercise in your favorite model! Not EFT . . .

- BSM Higgs: deviations in per-cent range ⇒ What can we learn?
  ⇒ Compare $e^+e^-$ precision with concrete BSM expectations
  ⇒ Wäscheleinen-Plots ($e^+e^-$ precision vs. BSM deviation)
  ⇒ clear distinction between (selection of) models possible
Higgs Days at Santander 2020
Theory meets Experiment
28 September - 02 October

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Gervasio.Gomez@cern.ch
http://hdays.csic.es
Further Questions?
HDECAY provides a QCD/large $\tan\beta$-corrected width (including SQCD) \approx our green line;

Full one-loop shows EW Sudakov logarithms for heavy states.

Here: $h_1$ SM-like; $h_2$ (640 GeV) and $a_1$ (320 GeV) singlet-like; $h_3$ and $a_2$ doublet-like (1 TeV).
• HDECAY performs at the same order as us with 5-flavor radiation;

• ~ 4% deviation due to normalization factor (difference of EW 2-loop and QCD 3-loop order).

Here: \( h_1 \) SM-like; \( h_2 \) (650 GeV) and \( a_1 \) (320 GeV) singlet-like; \( h_3 \) and \( a_2 \) doublet-like (1 TeV).