3rd FCC Physics and Experiments Workshop, 13-17 January 2020, CERN
### Tau Physics topics

- **tau properties** (mass, lifetime, $g-2$, EDM, CPV ...)
- **tau decays** measurements and searches for
  - EW precision measurements
  - $|V_{us}|$ determination, low energy QCD
  - LFV
- treated the ones I consider most interesting
- consider only **Fcc-ee at Z peak**, which is the most interesting context

### I prefer to consider the following as not specifically tau topics

- processes where the tau is a decay product
  - $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$ process (EW)
  - $H, Z \rightarrow \tau^+\tau^-$ decays (EW)
  - $H, Z \rightarrow \mu^+\tau^-$ (LFV in EW heavy boson decays)
  - $W^- \rightarrow \tau^-\bar{\nu}_\tau$ (EW)
  - $B \rightarrow D^(*)\tau\nu$ (B Physics anomalies)
  - heavy flavour hadrons decays with LFV and that involve also tau leptons
Past work

- Mogens Dam presented on Tau Physics at Tau 2018 and at Fcc-ee Workshop of Jan 2019
- Tau Physics is also covered on the Fcc CDR
- sensitivity estimates in above documents are fair

Here

- some personal sensitivity estimations
  - using my experience in ALEPH, $\text{BaBar}$, SuperB, HFLAV
  - preferentially extrapolating from LEP rather than $B$-factories
  - some more detail on non-luminosity-scaling systematic contributions
- comparison with other future facilities sensitivity estimates
- recycled but updated from my Granada 2019 ESG presentation
# Hot / interesting Physics topics involving tau measurements

## Lepton Flavour Violation
- Natural extension of Standard Model, often present in New Physics models
- Clean and unambiguous signal of New Physics
- Large improvements coming on muon LFV (DeeMe, Mu2e, COMET, Project X/Mu2e, PRISM/PRIME)
- Muon LFV present and future limits typically more constraining than tau LFV ones
- Tau LFV limits are however prevailing for some specific models
- Tau LFV measurements decisive to investigate LFV possibly discovered on muons

## Heavy Flavour anomalies $R_{D^{(*)}}$ and $R_{K^{(*)}}$
- Lepton universality tests based on tau measurements impose significant constraints

## Non-standard neutrino interactions (NSI)
- Tau lepton universality tests and tau lifetime can constrain model parameters that determine $\epsilon_{\tau\tau}$
### unitarity discrepancy on the first row of the CKM matrix (recent issue)

- inclusive $\tau \rightarrow X_s \nu_\tau$ methods to measure $|V_{us}|$ (independent of Lattice QCD)
- depends on Cabibbo-suppressed tau branching fractions and spectral functions

### $\alpha_s$ from tau measurements

- spread of different determinations exceeds individual uncertainties
- different opinions on importance and treatment of duality violations
- tau spectral functions required to improve understanding and precision

### $\sim \sigma$ discrepancy on muon gyromagnetic anomaly $a_\mu$

- tau spectral functions might complement $e^+ e^-$ data to compute HVP contribution
 Tau Lepton Flavour Violation
Tau LFV searches probe & constrain New Physics models

MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006

- \( \frac{\text{BR}(\mu \rightarrow e\gamma)}{\text{BR}(\tau \rightarrow \mu\gamma)} \leq \frac{\theta_1}{\pi/4} \)
- \( \frac{\text{BR}(\tau \rightarrow \mu\gamma)}{\text{BR}(\mu \rightarrow e\gamma)} \leq \frac{\theta_2}{\pi/4} \)
- \( \theta_3 = 0 \)

- \( m_{N3} = 10^{10} \text{ GeV} \)
- \( m_{N2} = 10^{11} \text{ GeV} \)
- \( m_{\nu_1} = 10^{-12} \text{ eV} \)

- excluded by MEG 2016
- excluded by BaBar 2010

- typical NP models
  - \( B(\tau \rightarrow \mu\gamma) \sim 10^{-1000} \times B(\mu \rightarrow e\gamma) \)
  - muon LFV searches more effective

- doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019

- Yukawa-inspired scenario \((d \rightarrow 4)\)

- specific models / parameter space regions
  - part of plot only constrained by tau LFV limits
Present Tau LFV limits, present and future Muon LFV limits


incoming years will provide remarkable progress on Muon LFV searches

Tau LFV limits: present and future with Belle II and LHCb-HL

\begin{tabular}{|c|c|c|c|c|}
\hline
\( \gamma \) & IP\(^0\) & IS\(^0\) & IV\(^0\) & III & Ihh & \( \Lambda h \) \\
\hline
10\(^{-5}\) & 10\(^{-6}\) & 10\(^{-7}\) & 10\(^{-8}\) & 10\(^{-9}\) & 10\(^{-10}\) & \\
\hline
\end{tabular}

90\% C.L. upper limits for LFV \( \tau \) decays

Fcc-ee Tau LFV sensitivity for $\tau \to \mu\gamma$, $\tau \to 3\mu$ (Granada 2019 ESG)

- **Fcc-ee estimate for $\tau \to \mu\gamma$**
  - M. Dam simulation with 2% of full Fcc-ee statistics

- **Fcc-ee estimate for $\tau \to \mu\mu\mu$**
  - My guestimate extrapolating BABAR limit assuming background free (optimistic)

- **Red** more solid estimates
- **Orange** less solid estimates
- Dates of future results are arbitrary, for plotting convenience

As reported at Granada 2019 ESG
Revised guestimate for Fcc-ee limit on $\tau \rightarrow \mu\mu\mu$

Belle II sensitivity (Physics Book)
- extrapolated from Belle limit at $0.782\,\text{ab}^{-1}$ to $50\,\text{ab}^{-1}$
- assumption background-free efficient selection with $\sim 60\times$ luminosity ($\sim$ fair)

Fcc-ee sensitivity, my guestimate
- tau pairs at Fcc-ee: $5\times 10^{12} \times 3.3\% = 1.65\times 10^{11}$, $3.5\times$ than Belle II
- assume selection efficiency $4\times$ better from comparison of DELPHI and $\text{BABAR} \tau \rightarrow \mu\gamma$ searches
- $m_\tau$ resolution comparable with $B$-factories
- $E$ resolution worse (850 MeV if M. Dam $\tau \rightarrow \mu\gamma$ study vs. 50-100 MeV in $\text{BABAR}$
- assumption background-free efficient more stressed than at Belle II
- revise my Granada Fcc-ee guestimate to same as Belle II, to account for worse $E$ resolution
- some simulation could produce a better assessment
Fcc-ee Tau LFV sensitivity for $\tau \to \mu \gamma$, $\tau \to 3\mu$

**Fcc-ee estimate for $\tau \to \mu \gamma$**

Fcc CDR, M. Dam simulation with 2% of full Fcc-ee statistics

**Fcc-ee estimate for $\tau \to \mu \mu \mu$**

my guestimate extrapolating from Belle and BelleII
Lepton Universality Tests
HFLAV Tau 2018 report

\[
\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{B_{\tau e} \tau_\mu m_\tau^5 f_{\mu e} R_{\mu} R_{\mu}^\mu}{B_{\mu e} \tau_\tau m_\tau^5 f_{\tau e} R_{\gamma} R_{\gamma}^\tau R_{\gamma}^\tau}} = 1.0010 \pm 0.0014 = \sqrt{\frac{B_{\tau e}}{B_{\tau e}^{SM}}} \\
\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{B_{\tau \mu} \tau_\mu m_\tau^5 f_{\mu e} R_{\mu} R_{\mu}^\mu}{B_{\mu e} \tau_\tau m_\tau^5 f_{\tau e} R_{\tau} R_{\tau}^\tau}} = 1.0029 \pm 0.0014 = \sqrt{\frac{B_{\tau \mu}}{B_{\tau \mu}^{SM}}} \\
\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{B_{\tau \mu} f_{\tau e}}{B_{\tau e} f_{\tau \mu}}} = 1.0018 \pm 0.0014
\]

using Standard Model predictions for leptons \(\lambda, \rho = e, \mu, \tau\) (Marciano 1988)

\[
\Gamma[\lambda \to \nu_\lambda \rho \bar{\nu}_\rho(\gamma)] = \Gamma_{\lambda \rho} = \Gamma_{\lambda \beta} \mathcal{B}_{\lambda \rho} = \frac{B_{\lambda \rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192 \pi^3} f \left(\frac{m_\rho^2}{m_\lambda^2}\right) R_{\tau \gamma}^\lambda R_{\gamma}^\lambda
\]

\[
G_\lambda = \frac{g_\lambda^2}{4 \sqrt{2} M_W^2} ; \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x ; \quad f_{\lambda \rho} = f \left(\frac{m_\rho^2}{m_\lambda^2}\right)
\]

\[
R_{\tau \gamma}^\lambda = 1 + \frac{3}{5} \frac{m_\lambda^2}{M_W^2} + \frac{9}{5} \frac{m_\rho^2}{M_W^2} ; \quad R_{\gamma}^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi} \left(\frac{25}{4} - \pi^2\right) ; \quad \text{all statistical correlations included}
\]
Lepton universality tests with hadronic decays

<table>
<thead>
<tr>
<th>HFLAV Tau 2018 report</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\frac{g_\tau}{g_\mu})<em>\pi = 0.9958 \pm 0.0026$ , \quad $(\frac{g</em>\tau}{g_\mu})_K = 0.9879 \pm 0.0063$ .</td>
</tr>
</tbody>
</table>

Averaging the three $g_\tau/g_\mu$ ratios:

$$(\frac{g_\tau}{g_\mu})_{\tau+\pi+K} = 0.9999 \pm 0.0014 .$$

using Standard Model predictions

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \rightarrow h\nu_\tau)}{\mathcal{B}(h \rightarrow \mu\nu_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta_h)m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2$$

rad. corr. $\delta_\pi = (0.16 \pm 0.14)\%$, \quad $\delta_K = (0.90 \pm 0.22)\%$ (Decker 1994)

note: electron tests less precise because $h \rightarrow e\nu$ decays are helicity-suppressed
**Canonical tau lepton universality test plot**

\[
B'(\tau \rightarrow e\bar{\nu}) = \text{average of} \left\{ \frac{B(\tau \rightarrow e\bar{\nu})}{B(\tau \rightarrow \mu\bar{\nu})} \cdot \frac{f_{\tau e}}{f_{\tau \mu}} \right\}
\]

\[
\frac{B'(\tau \rightarrow e\bar{\nu})_{\tau \mu}}{B(\mu \rightarrow e\bar{\nu})_{\tau \mu}} = \frac{g_T^2}{g_{e\mu}} \frac{m^5_{\tau}}{m^5_{\mu}} \frac{f_{\tau e} R_{\gamma}^{T \tau}}{f_{\mu e} R_{\gamma}^{\mu \tau}} \frac{R_{W}^{T \tau}}{R_{W}^{\mu \tau}}
\]

\[
\left( \frac{g_T}{g_{e\mu}} \right)^2 = \frac{B'(\tau \rightarrow e\bar{\nu})_{\tau \mu}}{B(\mu \rightarrow e\bar{\nu})_{\tau \mu}} \frac{m^5_{\tau}}{m^5_{\mu}} \frac{f_{\mu e} R_{\gamma \mu}^{\tau \mu}}{f_{\tau e} R_{\gamma \mu}^{T \tau}} \frac{R_{W}^{T \tau}}{R_{W}^{\mu \tau}}
\]

\[
(g_T/g_{e\mu}) = 1.0020 \pm 0.0013
\]

\([g_{e\mu} = g_e = g_\mu \text{ assuming } g_e = g_\mu]\)

**\(\Delta(g_T/g_{e\mu})\) contributions**

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<tr>
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<th>(\Delta\text{input})</th>
<th>(\Delta(g_T/g_{e\mu}))</th>
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<tr>
<td>(B'_{\tau \rightarrow e})</td>
<td>0.178%</td>
<td>0.089%</td>
</tr>
<tr>
<td>(\tau_T)</td>
<td>0.172%</td>
<td>0.086%</td>
</tr>
<tr>
<td>(m_\tau)</td>
<td>0.007%</td>
<td>0.017%</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>0.125%</td>
</tr>
</tbody>
</table>

**best measurements**

| \(B'_{\tau \rightarrow e}\) | ALEPH |
| \(\tau_T\) | Belle |
| \(m_\tau\) | BES III |
Tau Lepton universality constrains models for $B R_D^{\tau/\ell} - R_K^{\mu/e}$ anomalies

Feruglio, Paradisi, Pattori JHEP 09 (2017) 061
blue points correspond to parameter space region allowed by tau lepton universality
Tau Lepton universality constrains models for $B \frac{R_{\tau/\ell}^{T}}{R_{D(\ast)}^{\mu/e}}$ anomalies

Feruglio, Paradisi, Pattori JHEP 09 (2017) 061

blue points correspond to parameter space region allowed by tau lepton universality
Non standard neutrino interactions

blue region = model parameters’ \((Y_{\tau e}, \sin \phi, m_{h^+})\) consistent with tau LU & lifetime, given \(\epsilon_{\tau \tau}\)

\[
L_{NC} = -2\sqrt{2} G_F \sum_{f,P,\alpha,\beta} \epsilon_{f,P}^{\alpha,\beta} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P f)
\]

\[
L_{CC} = -2\sqrt{2} G_F \sum_{f,P,\alpha,\beta} \epsilon_{f,P}^{\alpha,\beta} (\bar{\nu}_\alpha \gamma^\mu P_L \ell_\beta)(\bar{f} \gamma_\mu P f')
\]

where \(G_F\) is Fermi’s constant and the \(\epsilon\) terms quantify the size of the new interaction relative to the weak scale. The sum is over matter fermions, typically \(f, f' \in \{e, u, d\}\) and \(P \in \{P_L, P_R\}\) are the chirality projection operators. These projection operators can also be reparameterized.

NSI model opens new tau decays

\[
\tau \nu_\alpha \ell \gamma \nu_\beta \rightarrow \tau \nu_\alpha \ell \gamma \nu_\beta
\]
# Required measurements for Tau Lepton Universality

**$\mathcal{B}_{\tau \rightarrow \ell \bar{\nu} \nu}$, $\mathcal{B}_{\tau \rightarrow h \nu}$**

- best existing experimental inputs: ALEPH, then other LEP experiments
- valuable experimental assets
  - tau statistics, PID and photon systematics
  - $e^+ e^-$ at $Z$-peak significantly better than $B$-factories energies

**$\tau_{\tau}$**

- best existing experimental inputs: Belle, then LEP experiments
- valuable experimental assets
  - tau statistics, vertexing
  - $e^+ e^-$ at $Z$-peak better than $B$-factories energies

**$m_{\tau}$**

- best existing experimental inputs: BES III then KEDR i.e. $e^+ e^-$ at $\tau^+ \tau^-$ threshold, then $B$-factories
- valuable experimental assets
  - $e^+ e^-$ at tau production threshold, small uncertainty on beam energy
  - tau-charm factories at threshold are best
  - Fcc-ee can provide interesting measurement, limited by systematics understanding
Precision Standard Model Studies
Unitarity violation in first row of CKM matrix

New radiative correction changes $|V_{ud}|$ and spoils unitarity (from E. Passemard, Kaon 2019)

$$|V_{ud}|^2 = \frac{2984.432(3)\,s}{ft\Delta_R^V}$$

used so far up to CKM 2018, PDG 2019

$\Delta_R^V = 0.02361(38)$

Marciano et al., PRL 96, 032002 (2006)

$|V_{ud}| = 0.97418(10)_{ft(18)}\Delta_R^V$

new dispersive calculation

$\Delta_R^V = 0.02467(22)$

Seng et al., PRL 121, 241804 (2018)

$|V_{ud}| = 0.97379(10)_{ft(11)}\Delta_R^V$

1.8 $\sigma$ smaller and more precise

$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00062(45)$

$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00154(32)$

$-1.4\,\sigma$

$-4.8\,\sigma$
**|V_{us}| determinations (non-exhaustive)**

**Conventional, using kaon measurements and lattice QCD, most precise**

\[ \Gamma(K \rightarrow \pi\ell\bar{\nu}_\ell) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{EW}^K \left( |V_{us}| f_{\pi K}^K(0) \right)^2 I_\ell^K \left( 1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi} \right)^2 \]

\[ \frac{\Gamma(K^{-} \rightarrow \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \rightarrow \ell^- \bar{\nu}_\ell)} = \left( \frac{f_{K^\pm}}{f_{\pi^\pm}} \right)^2 \frac{m_K (1 - m_\ell^2/m_K^2)^2}{m_\pi (1 - m_\ell^2/m_\pi^2)^2} \left( 1 + \delta_{EM} \right) \]

**Using tau measurements and OPE, no lattice QCD**

\[ R(\tau \rightarrow X_{\text{strange}} \nu) = \frac{R(\tau \rightarrow X_{\text{non-strange}} \nu)}{|V_{us}|^2} - \delta R_{\tau,\text{SU3 breaking}}, \]

\[ R(\tau \rightarrow X_{\text{strange}} \nu) = \frac{R(\tau \rightarrow X_{\text{non-strange}} \nu)}{|V_{us}|^2} - \delta R_{\tau,\text{SU3 breaking}}, \]

**Using tau measurements and lattice QCD**

\[ \frac{\Gamma(\tau^- \rightarrow K^- \nu_\tau)}{\Gamma(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{|V_{us}|^2 \left( \frac{f_{K^\pm}}{f_{\pi^\pm}} \right)^2 (1 - m_\ell^2/m_K^2)^2 R_{\tau/K}}{(1 - m_\ell^2/m_\pi^2)^2 R_{\tau/\pi}} R_{K/\pi} \]

\[ \frac{\Gamma(\tau^- \rightarrow K^- \nu_\tau)}{\Gamma(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{G_F^2 |V_{us}|^2 m_\tau^3 \left( 1 - \frac{m_K^2}{m_\tau^2} \right)^2 R_{\tau/K} R_{K\mu_2}}{16\pi \hbar} \]
“$\tau \rightarrow X_s \nu$ inclusive” $|V_{us}|$ determination

Gamiz et al. 2003 method

$$|V_{us}|_{\tau s} = \sqrt{\frac{R_s}{|V_{ud}|^2} \left( \frac{R_{VA}}{R_{theory}} - 1 \right)}$$

where

$$R_s = \frac{B(\tau \rightarrow X_s \nu)}{B(\tau \rightarrow e\bar{\nu})}$$

$$R_{VA} = \frac{B(\tau \rightarrow X_d \nu)}{B(\tau \rightarrow e\bar{\nu})}$$

$$\delta R_{theory} = SU(3)$$-breaking correction

Gamiz, Jamin, Pich, Prades, Schwab 2003/2005

$\delta R_{theory}$ from OPE calculation + tau spectral functions does not require lattice QCD inputs

other methods

- Hudspith 2018, uses also tau spectral functions
- Boyle 2018, tau spectral function and lattice QCD
- reliability of Gamiz method has been questioned

Required tau measurements

- Cabibbo-suppressed tau BRs
- tau spectral functions

comparison with CKM unitarity determination of $|V_{us}|$ is equivalent to testing the unitarity of the first row of the CKM matrix
\( \alpha_s(m_\tau) \) from

- \( R_{VA} = B(\tau \rightarrow X_d\nu)/B(\tau \rightarrow e\bar{\nu}) \)
- tau spectral functions
- extrapolation to \( M_Z \) competitive with other methods
- \( \alpha_s(m_\tau) \) confirms running of \( \alpha_s \)

Recent discussions on tau determinations

- FOPT and CIPT extractions get significantly different results
- different groups get significantly different results
- disagreement on treatment of duality violations
- Pich 2019
  - Boito, Golterman, Maltman, Peris 2019
  - Pich, Rojo, Sommer, Vairo 2018
  - Boito, Golterman, Maltman, Peris 2017
  - Pich, Rodríguez-Sánchez 2016

Required tau measurements

- tau spectral functions
- tau branching fractions

\( \alpha_s(M_Z^2) \) from tau decay measurements

- \( \alpha_s(m_\tau) \) from
- \( R_{VA} = B(\tau \rightarrow X_d\nu)/B(\tau \rightarrow e\bar{\nu}) \)
- tau spectral functions
- extrapolation to \( M_Z \) competitive with other methods
- \( \alpha_s(m_\tau) \) confirms running of \( \alpha_s \)
Muon $g-2$ hadronic contribution from tau

- $\alpha_{\mu}^{2\pi,LO}$ from
- $\tau \rightarrow \pi\pi^0\nu$ spectral function
- normalization could come from $B(\tau \rightarrow \pi\pi^0\nu)$, $\tau_\tau$
- isospin rotation (associated theory systematics)
- tau data $\Rightarrow$ reduced discrepancy with exp.
- presently $e^+e^-$ data more precise and complete

Required tau measurements

- tau spectral functions
- tau branching fractions
Tau Mass
Current world average: \( m_\tau = 1776.86 \pm 0.12 \text{ MeV} \)

Best in world: BES3 (threshold scan) \( m_\tau = 1776.91 \pm 0.12 \text{ (stat.)} ^{+0.10}_{-0.13} \text{ (syst.)} \text{ MeV} \)

Best at LEP: OPAL \( m_\tau = 1775.1 \pm 1.6 \text{ (stat.)} \pm 1.0 \text{ (syst.)} \text{ MeV} \)

- About factor 10 from world’s best
- Main result from endpoint of distribution of pseudo-mass in \( \tau \rightarrow 3\pi^\pm(n\pi^0)\nu_\tau \)
- Dominant systematics:
  - Momentum scale: 0.9 MeV
  - Energy scale: 0.25 MeV (including also \( \pi^0 \) modes)
  - Dynamics of \( \tau \) decay: 0.10 MeV

Same method from Belle

- Main systematics
  - Beam energy & tracking system calib.: 0.26 MeV
  - Parameterisation of the spectrum edge: 0.18 MeV

\[ m_\tau = 1776.61 \pm 0.13 \text{ (stat.)} \pm 0.35 \text{ (syst.) MeV} \]
Prospects for FCC-ee:

- 3 prong, 5 prongs, (perhaps even 7 prongs?)
- Statistics $10^5$ times OPAL: $\delta_{\text{stat}} = 0.004$ MeV

Systematics:

- At FCC-ee, $E_{\text{BEAM}}$ known to better than 0.1 MeV (~ 1 ppm) from resonant depolarisation
  - Negligible effect on $m_\tau$.
- Likely dominant experimental contribution comes from understanding of the mass scale
  - Use high stats $e^+e^- \rightarrow \mu^+\mu^-$ sample to fix momentum scale. Extrapolate down to momenta typical for $\tau \rightarrow 3\pi$.
  - Use $D^0 \rightarrow K^-\pi^+ / K^-\pi^+\pi^-\pi^-$ and $D^+ \rightarrow K^-\pi^+\pi^+$ to fix mass scale ($m_D$ known to 50 keV)
- Reduce uncertainty from parametrisation of spectrum edge by use of theoretical spectrum checked against high statistics data
- Cross checks using 5-prongs

Suggested overall systematics: $\delta_{\text{syst}} = 0.1$ MeV

- Could potentially touch current precision but probably no substantial improvement?
Fcc-ee sensitivity for $m_\tau$
Tau Lifetime
## Tau Lifetime

### τ MEAN LIFE

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<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
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#### PDG 2019

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#### tau lifetime precision

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<tr>
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- 260 Belle II guestimate, extrapolating from 0.711 ab$^{-1}$ to 50 ab$^{-1}$
- 5 Fcc-ee, stat. only extrapolation from ALEPH (1e5) to Fcc-ee (1.65e11) tau pairs

⇒ what are the limiting systematics?
## Tau Lifetime systematics at LEP


- IP impact parameter difference on 1-1-prong tau pairs
  - trimming, backgrounds, impact parameter resolution, alignment
- MD miss-distance on 1-1-prong tau pairs
  - resolution on MD, bias, selection
- DL transverse decay length on 3-1 and 3-3 prong tau pairs
  - alignment


- MIPS, momentum-weighted impact parameter sum
  - resolution on impact parameter sum, bias (from MC)
  - bias (from MC), vertex chisq cut
- IPD, impact parameter difference
  - resolution and trimming of outliers
- DL, decay length
  - vertex chisq cut

Expect that all these systematics scale with $\frac{1}{\sqrt{N_{\text{events}}}}$, although questionable if up to a factor $1/\sim 1300$. Including alignment systematics.
### Alignment systematic

- alignment calibration precision improves with statistics
- misalignment effects zero at first order for uniform azimuthal acceptance
  

- still, questionable how far this holds
- related systematic that does not scale

  absolute length scale of vertex detector average elements spacing, reliable to $10^{-4}$ or 100 ppm

### Systematics from kinematics of tau decay

- $\tau_\tau = \frac{\lambda_\tau}{\beta \gamma} = \frac{\lambda_\tau}{\sqrt{E_{\text{beam}}^2 - m_{\tau}^2}} \cdot \frac{\sqrt{(E_{\text{beam}} - E_{\text{rad}}^{MC})^2 - m_{\tau}^2}}{m_{\tau}}$

<table>
<thead>
<tr>
<th>systematic [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $E_{\text{beam}}$</td>
</tr>
<tr>
<td>68 $m_{\tau} \text{ PDG 2019}$</td>
</tr>
<tr>
<td>7 $m_{\tau} \text{ possible measurement at Super Charm-Tau Factories}$</td>
</tr>
<tr>
<td>? MC accuracy on average radiation energy loss (*) (estimated 100 ppm for $BABAR$)</td>
</tr>
</tbody>
</table>

(*) depends on
- accuracy of generator, can be checked measuring momentum distribution of di-muon events
- accuracy of simulation of efficiency of selection procedure vs. $E_\tau$ (scales with luminosity)
- replace M.Dam / CDR estimate of 0.004 fs / 140 ppm with 0.0029 fs / 100 ppm
- more precise measurement not excluded in principle
- to be investigated accuracy of simulation of average radiation energy loss (neglected here)
Tau branching fractions and spectral functions
world averages of large BRs still dominated by LEP
- background separation from dileptons and hadrons much better
- higher selection purity and efficiency
- possible to tag single tau with good efficiency and purity and observe the other one
  ⇒ wonderful base for reducing systematics using data, exploited in particular by ALEPH

*B*-factories improved on small branching fractions using statistics
  ⇒ Fcc-ee statistics $1300 \times$ ALEPH, $175 \times$ Belle, $3.5 \times$ BelleII (& better efficiency w.r.t. *B*-factories)

Fcc-ee is best imaginable context for tau BR measurements

*what are the limiting systematics?*
# Systematics of main ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

## Total systematic errors for branching ratios measured from the 1994–1995 data sample

### $\pi^0$ systematics

<table>
<thead>
<tr>
<th>Topology</th>
<th>$n^0$</th>
<th>sel</th>
<th>bkg</th>
<th>pid</th>
<th>int</th>
<th>trk</th>
<th>dyn</th>
<th>mcs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>0.011</td>
<td>0.021</td>
<td>0.029</td>
<td>0.019</td>
<td>0.009</td>
<td>0.000</td>
<td>0.000</td>
<td>0.015</td>
<td>0.045</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.004</td>
<td>0.020</td>
<td>0.020</td>
<td>0.021</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
<td>0.015</td>
<td>0.039</td>
</tr>
<tr>
<td>$h$</td>
<td>0.071</td>
<td>0.016</td>
<td>0.010</td>
<td>0.022</td>
<td>0.002</td>
<td>0.014</td>
<td>0.000</td>
<td>0.019</td>
<td>0.083</td>
</tr>
<tr>
<td>$h_1$</td>
<td>0.063</td>
<td>0.027</td>
<td>0.019</td>
<td>0.011</td>
<td>0.045</td>
<td>0.009</td>
<td>0.000</td>
<td>0.027</td>
<td>0.090</td>
</tr>
<tr>
<td>$h_2$</td>
<td>0.089</td>
<td>0.021</td>
<td>0.014</td>
<td>0.004</td>
<td>0.007</td>
<td>0.003</td>
<td>0.040</td>
<td>0.028</td>
<td>0.105</td>
</tr>
<tr>
<td>$h_3$</td>
<td>0.056</td>
<td>0.012</td>
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<td>0.000</td>
<td>0.008</td>
<td>0.001</td>
<td>0.008</td>
<td>0.030</td>
<td>0.068</td>
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<tr>
<td>$h_4$</td>
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<td>0.005</td>
<td>0.011</td>
<td>0.000</td>
<td>0.015</td>
<td>0.000</td>
<td>0.000</td>
<td>0.019</td>
<td>0.040</td>
</tr>
<tr>
<td>$3h$</td>
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<td>0.021</td>
<td>0.018</td>
<td>0.004</td>
<td>0.012</td>
<td>0.014</td>
<td>0.006</td>
<td>0.015</td>
<td>0.059</td>
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<tr>
<td>$3h_1$</td>
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<td>0.017</td>
<td>0.029</td>
<td>0.002</td>
<td>0.041</td>
<td>0.009</td>
<td>0.007</td>
<td>0.018</td>
<td>0.066</td>
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<tr>
<td>$3h_2$</td>
<td>0.027</td>
<td>0.008</td>
<td>0.015</td>
<td>0.000</td>
<td>0.009</td>
<td>0.003</td>
<td>0.012</td>
<td>0.014</td>
<td>0.038</td>
</tr>
<tr>
<td>$3h_3$</td>
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<td>0.012</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.010</td>
<td>0.006</td>
<td>0.019</td>
</tr>
<tr>
<td>$5h$</td>
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<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>$5h_1$</td>
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<td>0.000</td>
<td>0.006</td>
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<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>Class 14</td>
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<td>0.003</td>
<td>0.022</td>
<td>0.002</td>
<td>0.024</td>
<td>0.000</td>
<td>0.000</td>
<td>0.011</td>
<td>0.037</td>
</tr>
</tbody>
</table>

All numbers are absolute in per cent. The labels are defined as follows: photon and $n^0$ reconstruction ($n^0$), event selection efficiency (sel), non-$\tau$ background (bkg), charged particle identification (pid), secondary interactions (int), tracking (trk), Monte Carlo dynamics (dyn), Monte Carlo statistics (mcs), total systematic uncertainty (total).

- Many systematics but in general all limited only by data vs. MC comparisons
- Non-trivial to extrapolate to 1300$^2$ more data
Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

- non-tau backgrounds
  - estimated by varying MC estimate by 30%
  - does not trivially scale with luminosity, but can be improved
- tau pair selection
  - use break-mix method on data and MC, 0.1-0.2% uncertainties
  - dominant systematics from data statistics of tau vs. hadron cut separation
  - scales with luminosity, but correlations between hemispheres limit how much
- PID
  - uncertainties from control samples studies
  - partially scales with luminosity, but limited by achievable purity of control samples
- photon efficiency
  - uncertainties from control samples studies data-MC comparisons
    - fit data using predicted MC fake and genuine photon distributions and compare number of genuine photons
    - compare photons > 3 GeV as function of separation from tracks
    - compare converted photons
    - compare hadron to electron misidentification
    - compare photon identification efficiency
    - photon energy scale calibrated with momentum measurement on high-energy $e$ from tau decay
    - compare fake photons
Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

- $\pi^0$ efficiency
  - compare data and MC $D_{ij}$ distributions (probability $\gamma_i$, $\gamma_j$) of $\pi^0$ mass fit
- efficiency for $\pi^0$ with unresolved photons
  - compare data and MC 2nd moment of transverse energy in calorimeter cells
- radiative and bremsstrahlung photons
  - compare data and MC distributions
  - compare PHOTOS vs. exact calculation for $\tau \rightarrow \pi\pi^0\nu$ with radiative $E_\gamma > 12$ MeV
- tracking
  - compare data and MC on same sign events events (two tracks missing in one hemisphere)
- tau decay dynamic
  - reduced because acceptances are large and flat
  - will become important with higher statistics
  - can be partially addressed with iterative concurrent measurements where also invariant mass distributions are fitted on data (complicate)

Conclusion

- potential improvement w.r.t. ALEPH is $\sim 1300$ in precision
- only future actual analysis will be able to estimate limiting systematics in a reliable way
- guestimate: assume total uncertainty $10\times$ better that WA, which is about $20\times$ better than ALEPH
reasonably complete sets only measured at LEP (ALEPH, OPAL)
limited contributions from $B$-factories
studies at the $Z$ peak are by far the most favourable context
significant improvements are possible at Fcc-ee especially for the poorly measurer rare modes
analyses are complex and may be limited by manpower availability
Fcc-ee sensitivity for $\mathcal{B}(\tau \rightarrow \ell \bar{\nu} \nu)$
Fcc-ee (or comparable Z-peak $e^+e^-$ facilities) are the best for tau Physics

- there are several interesting measurements to be improved
- Fcc-ee permits impressive precision improvements
- fair share of systematics scale with luminosity if enough human effort is available
- identified possibly limiting systematics still allow large and interesting margins of improvement
- improvements on tau lifetime and leptonic BRs make desirable improvements on $m_\tau$ (SCTF?)

Thanks for your attention!