Electroweak calculations for $\sin^2\theta_W^{eff}$ measurements, from LEP to LHC and FCC, ambiguities of definitions and systematics

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Aim: for inventory ($\longrightarrow$ archivization) of programs and results

Massive LPEP phenomenological efforts around intertwined topics:

- Precision measurements at LEP 1 including definition of *observable quantities*
- How does it fit to general picture of electroweak calculations
- Electroweak programs and libraries, versions and variants.
- How does it translates into ambiguities for $\sin^2\theta_W^{eff}$: (i) definition (ii) ambiguities (iii) parametric ambiguities.
- Numerical results; useful for choices of future EW schemes too.

LEP times legacy for electroweak physics: meaning of $\sin^2\Theta_W^{eff}$

- **Precision measurements at LEP 1:** 100 kevt samples: M_Z, N_ν, $\sin^2\Theta_W$, consistency checks of SM as a field theory. 1989-1995 $\rightarrow$ QED, line-shape corrections, genuine weak, semi-analytic and MC predictions. $\sin^2\Theta_W^{eff} = 0.23153$ measured with $\pm 16 \cdot 10^{-5}$ precision, Phys.Rept.427:257-454,2006 Note the year!
- At Tevatron the same as at LEP, so called $\alpha(0)$, scheme for $\sin^2\Theta_W^{eff}$.
- **Electroweak measurements at LEP 2:** 1-10 kevt samples: M_W, triple gauge couplings, New Physics. 1995-2000 ,$\rightarrow$ **s- t- channel gauge cancellation importan, precision requirements eased.**
- **Expected precision measurement at HL LHC** ATL-PHYS-PUB-2018-037: $\sin^2\Theta_W^{eff}$ to be measured with $\pm 15 \cdot 10^{-5}$ uncertainty.

FCC will do better, but backward compatibility important.

KKMC

KKMC is presented already, features important for EW-corrections.

Full and exact phase-space generator for $e^+e^- \to f\bar{f}(n\gamma), n = 0, 1, 2, 3, \ldots.$ This was possible because of additional (conformal) symmetry for eikonal soft photon amplitudes and phase space.

QED matrix elements were introduced up to second order in the scheme of Yennie Frautchi Suura.

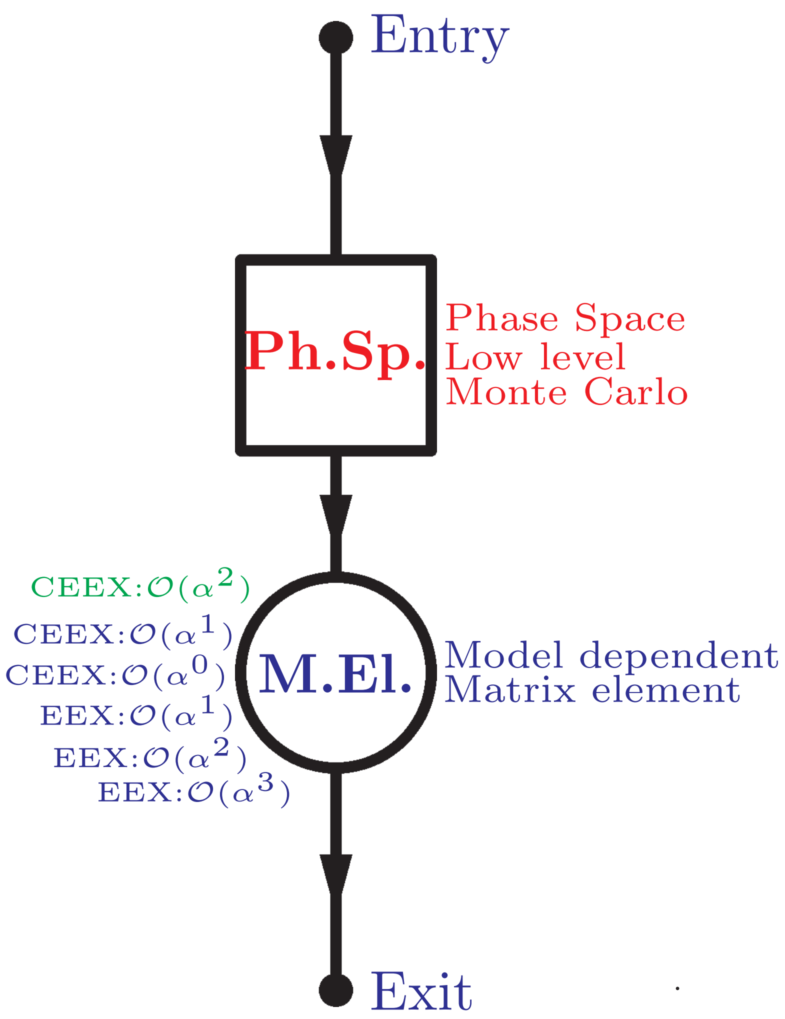
KKMC is the Monte Carlo, where *matrix element* $\times$ *phase space* paradigm was applied for the *case of re-summation*.

Processes $e^+e^- \to f\bar{f}(n\gamma), n = 0, 1, 2, 3, 4, ...$ were covered.

Amount of documentation and tests was extensive, main publications were about 100 pages each: S. Jadach et al. Comput.Phys.Commun. 130 (2000) 260 Phys.Rev. D63 (2001) 113009,

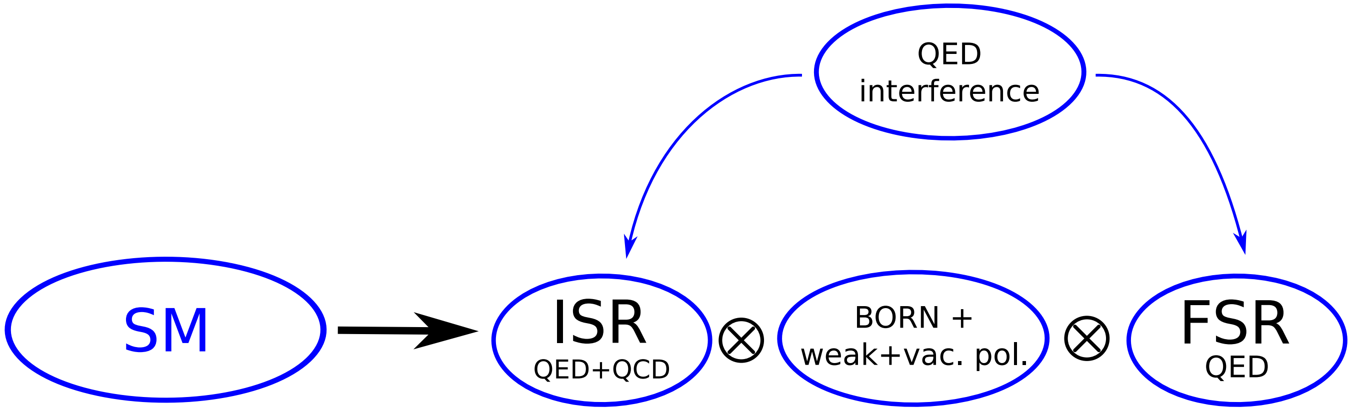
KKMC does not directly exploit *Improved Born approximation* only its form-factors.

Textbook principle "matrix element × full phase space" for technical and physics uncertainties



- Phase-space Monte Carlo simulator is a module producing "raw events" (including importance sampling for possible intermediate resonances/singularities)
- Library of Matrix Elements; input for "model weight"; independent module
- "model weight"-s may differ by implemented variant of electroweak form-factors (see later).
- Correlated samples techniques. Variants used for the difference semi-analytical vs. Monte Carlo simulation. Beware: relation of crude level phase-space and semianalytical integration variables, Phys. Rev. D41 (1990) 1425.
- Useful for fit arrangements!

Production and decay of Z/γ^* and EW effects in KKMC



- That is the picture which emerged **after lot of pain.**
- Genuine weak corrections were calculated at one loop level, but:
- Separated-out QED corrections at the second order with exclusive exponentiation. Elecroweak at one loop level, but with important partial resummations.
- QED ISR with vacuum polarization corrections were called **'line-shape corrections'**. Up to **3 loop** QCD contributions for quark loops used. For low energies vacuum polarization was obtained from dispersion relations and $e^+e^- \to hadrons$ data.

Essential building block

- One of the important segment for numerical calculation was library to calculate improved Born amplitude (and improved effective couplings) and help to establish what $\sin^2\Theta_W^{eff}$ actually mean.
- Since LEP time and until now family of DIZET library versions was used.
- Before I will compare numerical results of these versions, let me recall some features, which enable separating out such electroweak sector calculations. For that purpose numerical results from DIZET version 6.21 will be used.
- Then I will monitor results from 6.21 to 6.45 (2019) version of the program
- **This may be used to clarify what need to be included for FCC data analysis and also point to archivization efforts available with KKMC efforts.**

properties of Improved Born amplitude

Let us start with the lowest order coupling constants (without EW corrections) of the Z boson to fermions, $\sin\theta_W^2 = s_W^2 = 1 - m_W^2/m_Z^2$ (on-shell scheme) and T_3^f denotes third component of the isospin.

The vector v_e, v_f and axial a_e, a_f couplings for leptons and quarks are defined with the formulas below:

$$\begin{aligned} v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)/\Delta \\ v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2)/\Delta \\ a_e &= (2 \cdot T_3^e)/\Delta \\ a_f &= (2 \cdot T_3^f)/\Delta \end{aligned} \tag{1}$$

where

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \tag{2}$$

properties of Improved Born amplitude

With this notation, matrix element for the $e^+e^- \to Z/\gamma^* \to f\bar{f}$ (or $q\bar{q} \to Z/\gamma^* \to l^+l^-$), ME_{Born}, can be written as:

$$
\begin{aligned}
ME_{Born} &= [\bar{u}\gamma^\mu v g_{\mu\nu}\bar{v}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \frac{\chi_\gamma(s)}{s} \\
&+ [\bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu u \cdot (v_e \cdot v_f) + \bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u \cdot (v_e \cdot a_f) \\
&+ \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u \cdot (a_e \cdot a_f)] \cdot \frac{\chi_Z(s)}{s}
\end{aligned} \quad (3)
$$

Z-boson and photon propagators read respectively as

$$\chi_\gamma(s) = 1 \quad (4)$$

$$\chi_Z(s) = \frac{G_\mu \dot{M}_z^2}{\sqrt{2} \cdot 8\pi \cdot \alpha_{QED}(0)} \cdot \Delta^2 \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z} \quad (5)$$

At the peak of resonance $|\chi_Z(s)| \times (v_e \cdot v_f) > (q_e \cdot q_f)$ and as a consequence, angular distribution asymmetries of leptons are proportional to $v_e = (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)$. This gives good sensitivity for s_W^2 measurement.

Above and below resonance we are sensitive to lepton and quark charge instead ...

properties of Improved Born amplitude

Born cross-section, for $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$ (for $e^+e^- \to Z/\gamma^* \to \ell^+\ell^-$ too):

$$\frac{d\sigma^{q\bar{q}}_{Born}}{d\cos\theta}(s,\cos\theta,p) = (1+\cos^2\theta)\,F_0(s)+2\cos\theta\,F_1(s)-p[(1+\cos^2\theta)\,F_2(s)+2\cos\theta\,F_3(s)] \quad (6)$$

p polarization of the outgoing leptons. The $\cos\theta$ of angle between incoming quark and outgoing lepton in the rest frame of outgoing leptons. All rely on second order spherical harmonics. Also with transverse spin. Form-factors read:

$$\begin{aligned}
F_0(s) &= \frac{\pi\alpha^2}{2s}[q_f^2 q_\ell^2 \cdot \chi^2_\gamma(s) + 2\cdot\chi_\gamma(s) Re\chi_Z(s)\, q_f q_\ell v_f v_\ell + |\chi^2_Z(s)|^2 (v_f^2+a_f^2)(v_\ell^2+a_\ell^2)], \\
F_1(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, 2v_f a_f 2v_\ell a_\ell], \quad (7 \\
F_2(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2+a_f^2) 2v_\ell a_\ell], \\
F_3(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2+a_f^2) 2v_\ell a_\ell],
\end{aligned}$$

properties of Improved Born amplitude

Why is it of interest?

1. Condition: $s_W^2 = 1 - m_W^2/m_Z^2$ is important for some gauge cancellations, in case of multi-leg processes, but at the same time bring inconsistencies with measurements:
2. either m_W must be off by many experimental errors
3. or electroweak observables such as A_{FB} or P_τ by 50 % of their measurable values.
4. Nonetheless such on mass shell scheme is used by many programs of importance for QCD phenomenology.
5. Technical solutions using calculation of correcting weights are of interest.
6. BY-PRODUCT: separate leptonic degrees of freedom from the hadronic ones.

Improved Born survives ISR/FSR :

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_-$

$s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$

Mustraal: Monte Carlo for $e^+ e^- \to \mu^+ \mu^-$ (γ)

$$\sigma_{\mathrm{hard}} = \int \mathrm{d}\tau \left(X_{\mathrm{i}} + X_{\mathrm{f}} + X_{\mathrm{int}} \right),$$

The explicit forms of the three terms in σ_{hard} read:

$$X_{\mathrm{i}} = \frac{Q^2\alpha}{4\pi^2 s} \frac{1-\Delta}{k_+ k_-} s'^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t, u) + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t', u') \right], \tag{3.4}$$

$$X_{\mathrm{f}} = \frac{Q'^2\alpha}{4\pi^2 s} \frac{1-\Delta'}{k'_+ k'_-} s^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t', u) \right], \tag{3.5}$$

$$X_{\mathrm{int}} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \tfrac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right]$$
$$+ \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s-s') M\Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \tag{3.6}$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \to Z/\gamma^* \to \mu\ \mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

2

Improved Born survive electroweak loops:

EW corr. can be added with the help of form-factors, these form-factors are used in KKMC amplitudes

$$
\begin{aligned}
ME_{Born+EW} = \quad & [\bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{v}\gamma^{\nu} u] \cdot (q_e \cdot q_f) \cdot \Gamma_{V_{\Pi}} \cdot \frac{\chi_{\gamma}(s)}{s} \\
& +[\bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{\nu}\gamma^{\nu} u \cdot (v_e \cdot v_f \cdot vv_{ef}) + \bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{\nu}\gamma^{\nu}\gamma^{5} u \cdot (v_e \cdot a_f) \\
& +\bar{u}\gamma^{\mu}\gamma^{5} v g_{\mu\nu} \bar{\nu}\gamma^{\nu} u \cdot (a_e \cdot v_f) + \bar{u}\gamma^{\mu}\gamma^{5} v g_{\mu\nu} \bar{\nu}\gamma^{\nu}\gamma^{5} u \cdot (a_e \cdot a_f)] \cdot Z_{V_{\Pi}} \cdot \frac{\chi_Z(s)}{s}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2 \cdot K_e(s,t))/\Delta \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s,t))/\Delta \\
a_e &= (2 \cdot T_3^e)/\Delta \\
a_f &= (2 \cdot T_3^f)/\Delta, \qquad \chi_Z(s) = \frac{G_{\mu} \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}, \\
\Gamma_{V_{\Pi}} &= \frac{1}{2 - (1 + \Pi_{\gamma\gamma})}, \qquad Z_{V_{\Pi}} = \rho_{\ell f}(s,t), \qquad \chi_{\gamma}(s) = 1,
\end{aligned}
$$

$$
\begin{aligned}
vv_{ef} = \quad & \frac{1}{v_e \cdot v_f}[(2 \cdot T_3^e)(2 \cdot T_3^f) - 4 \cdot q_e \cdot s_W^2 \cdot K_f(s,t) - 4 \cdot q_f \cdot s_W^2 \cdot K_e(s,t) \\
& +(4 \cdot q_e \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{ef}(s,t)] \frac{1}{\Delta^2}
\end{aligned}
$$

Improved Born survive electroweak loops:

- **Electroweak corrections could kill the picture:**
  - Energy and angle dependent Form-factors multiply coupling constants
  - That could kill gauge dependence cancellations
  - Fortunately for the region close to the Z peak this dependence is weak, and that is why it can be partly ignored and dominant part hidden in $\sin^2\Theta_W$ redefinition into $\sin^2\Theta_W^{eff}$.
  - Exclusive exponentiation help to complete part of cancellations at the amplitude level. QED gauge invariant parts using form-factors of the same energy angular arguments, remain gauge invariant in their presence too.
  - These observations required elaborated efforts on Yennie-Frautchi-Suura formulation and analytic forms of its β_0, β_1 and β_2 functions to preserve QED gauge invariance.
  - I will not recall it here, but at some precision EW-ff may affect QED IFI interferences.
- Let me show some plots of form-factors calculated with DIZET 6.21 that is electroweak library used since decades by $\mathtt{KKMC}$ Monte Carlo.

Nearly no dependence on $\cos\theta$ for $m_{ee} \simeq M_Z$

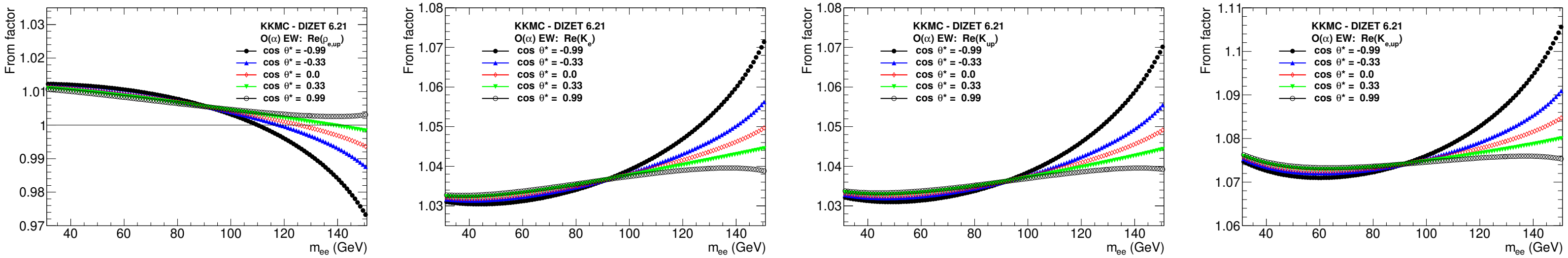


Figure 1: From Eur.Phys.J. C79 (2019) no.6, 480: Real parts of the $\rho_{e,up}$, $\mathscr{K}_e$, $\mathscr{K}_{up}$ and $\mathscr{K}_{e,up}$ EW form-factors of $ee \to Z \to u\bar{u}$ process, as a function of $\sqrt{s}$ and for the few values of $\cos\theta$. Note, that $\mathscr{K}_e$ depends on the flavour of outgoing quarks. KKMC **LEP time Default version of** DIZET **library was used. Note that in data analysis, experiments used updates for hadronic vacuum polarizations.**

Nearly no dependence on flavour for $m_{ee} \simeq M_Z$

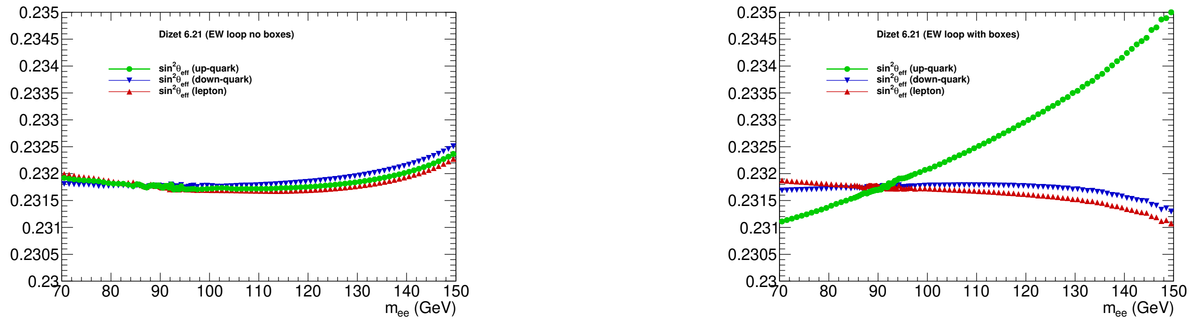


Figure 2: From Eur.Phys.J. C79 (2019) no.6, 480: Effective weak mixing angles $\sin^2\theta^f_{eff}(s,t)$ as a function of m_{ee} and $\cos\theta$ = 0, without (left-hand plot) and with (right-hand plot) box corrections. The $\mathscr{K}^f(s,t)$ form-factor calculated using `Dizet` library and on-mass-shell $s_W^2 = 0.22352$ were used. Real part is shown, imaginary part of $I_f^2(s,t)$ is only about 10^{-4}. `KKMC` **LEP time Default version of** `DIZET` **library was used. Note that in data analysis, experiments used updates for hadronic vacuum polarizations.**

$\mathcal{H}$ow important are higher orders

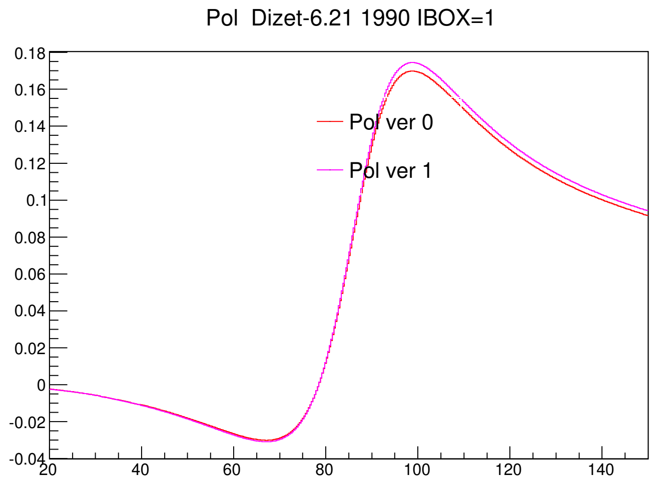
- Thanks to modular code organization it is easy to switch `KKMC` between `DIZET` EW library versions. Graphic programs can be used to monitor importance of introduced electroweak form-factors updates.
- For numerical results I have chosen τ-lepton polarization $P_\tau(m_{\tau\tau})$because $P_\tau(M_Z) \simeq 2(1 - 4\sin^2\theta_W^{eff})$.
- The following (next slide explain why these) versions of DIZET are compared: (i) 6.21 (ii) 6.42 (iii) 6.42 with updated vac-pol (iv) 6.45
- I explore resulting variations of $\Pi_{\gamma\gamma}$ vacuum polarization because it is numerically important.
- Environment to test variants of other contributions is prepared. To study graphical and in language of $\sin^2\theta_W^{eff}$.

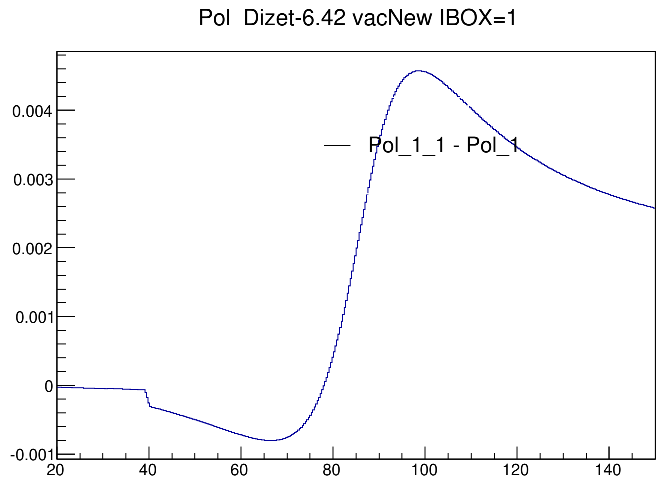
Why these DIZET *versions?*

- DIZET 6.21 is distributed with KKMC through CPC electroweak corrections and in particular vacuum polarization was not updated for backup compatibility. Note: upgrades of EW corrections within LEP experiments; not always documented well.
- DIZET 6.42 as in published ZFITTER Computer Physics Communications 174 (2006) 728 *ZFITTER: a semi-analytical program for fermion pair production in e^+e^- annihilation, from version 6.21 to version 6.42.* the last published/archived DIZET code version . Note: default $\Pi_{\gamma\gamma}$ from S. Eidelman, F. Jegerlehner, Z. Phys. C67(1995)585.
- DIZET 6.42 with $\Pi_{\gamma\gamma}$ updated by me to hadr5n17_compact.f parametrization from F. Jegerlehner, his web page, version of Oct 8 02:19:56 2017
- DIZET 6.45 VERSION 6.45 (30 Aug. 2019) with author added JEGERLEHNER (2017) vacuum polarization code and fermionic two loops corrections, AMT4 flag.
- There are good reasons to measure $\alpha_{QED}(M_Z)$ at FCC directly as you will see. Nonetheless backward compatibility to LEP and to LHC 2020-ties of a value.
- Many things become forgotten, important experts assistance not anymore available.

Preliminary EW-libs, tabulation and plotting tests.

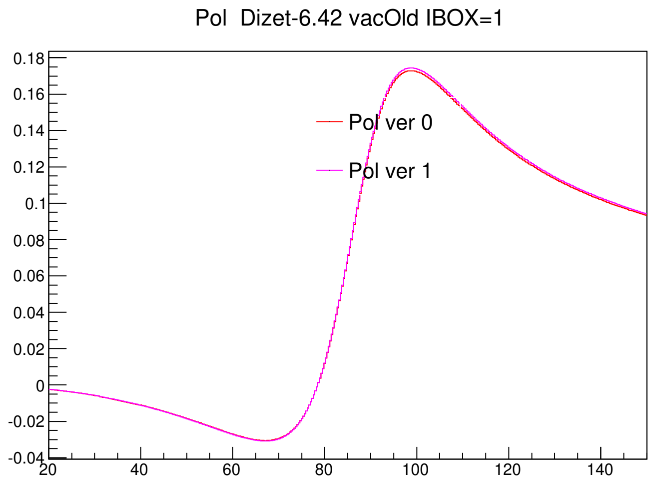
Figure 3: Comparison of $P_\tau(m_{\tau^+\tau^-})$ for calculation with two distinct EW-initialization. The first, is distribution initialization of `KKMC` as published. Thus with not final M_Z, M_H, m_t. Also hadronic vacuum polarization is not as used by LEP experiments. Initialization of `DIZET 6.42` is much closer to best of present days. Still difference correspond only to $\sim 37 \cdot 10^{-5}$ for $\sin^2\theta_W^{eff}$ ambiguity. Note bump at 40 GeV. It was numerically insignificant at low precision, will be investigated in the next days. `DIZET` variants are denoted respectively on left and right plot, ver 0, ver 1 correspond to these versions. APOLOGY: these plots are semi-automatically prroduced with testing package.

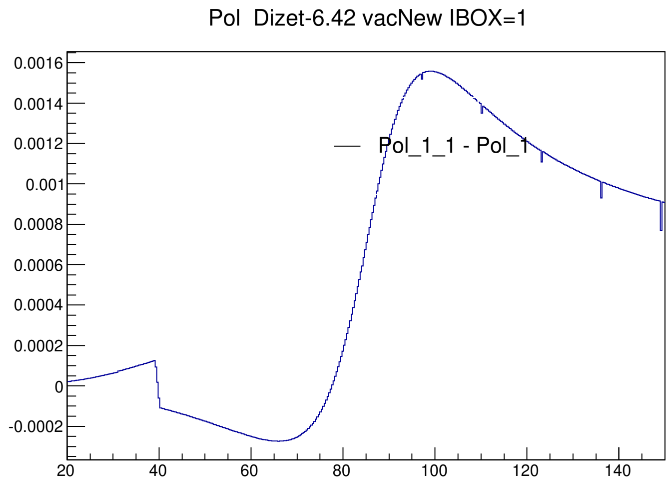




Preliminary EW-libs, tabulation and plotting tests.

Figure 4: Comparison of $P_\tau(m_{\tau^+\tau^-})$ for calculation with two distinct EW-initialization. The first, is initialization as of the shelf `DIZET 6.42`. For the second hadronic vacuum polarization is up to date (as on previous slide). difference correspond to $\sim 15 \cdot 10^{-5}$ for $\sin^2\theta_W^{eff}$ ambiguity. This is solely due to $\Pi_{\gamma\gamma}$. Further bumps appear, above Z peak another hint of technical issues.

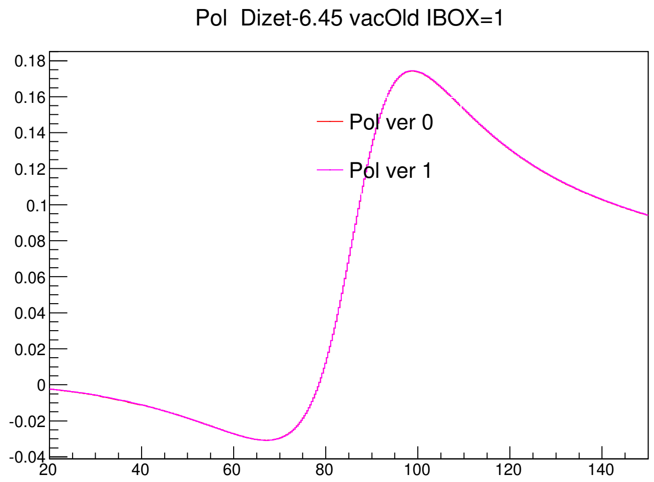


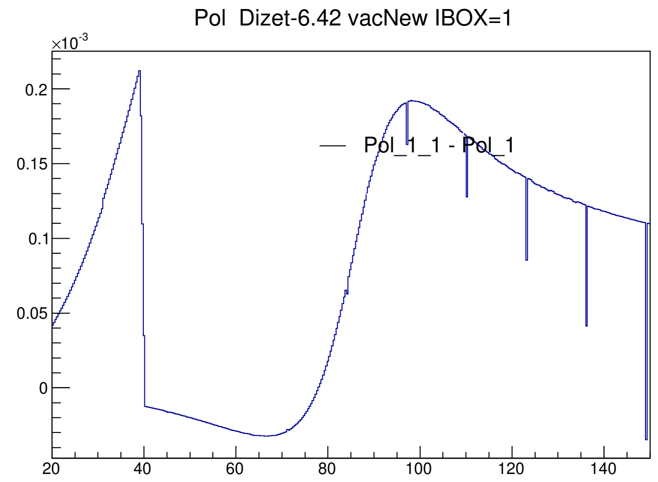


Preliminary EW-libs, tabulation and plotting tests.

Figure 5: Comparison of $P_\tau(m_{\tau^+\tau^-})$ for calculation with two distinct EW-initialization. The first, is initialization as of the shelf `DIZET 6.42` and `DIZET 6.45`. difference correspond to $\sim 2 \cdot 10^{-5}$ for $\sin^2\theta_W^{eff}$ ambiguity. This is much smaller than effect due to $\Pi_{\gamma\gamma}$. `DIZET` library is already updated for that, as a consequence of the discussions for LHC present day phenomenology. Bumps seem to become larger, because the differences scale is so much smaller now.

This is important for FCC to keep track **NOW** and archive too.





Summary.

- From the perspective of LEP1, differences between variants of EW-calculations are not large.
- Already for measurements at LHC $\Pi_{\gamma\gamma}$ variants are numerically important.
- LHC measurements $\sin^2\theta_W^{eff}$ precision is not expected to be better than $\sim 15 \cdot 10^{-5}$.
- Archivization of solutions used at: LEP1, Tevatron and LHC, require attention.

WARNING: some updates were completed by experiments and such changes are not always easy to retrieve.

- **This may become a challenge at FCC operation times, if no attention for proper archivization is paid now.**
- `DIZET` variants, will need to be revisited for FCC even if direct FCC $\alpha_{QED}(M_Z)$ measurements and different electroweak schemes will be used.
- **(i) Consistency with previous measurements (ii) compatible $\sin^2\theta_W^{eff}$ (iii) what new EW schemes must assure (iv) fixed or running Z-width? (v) parametric uncertainties**
- **Extra slides on Dizet initialization (from on-going work; E. Richter-Was et.al.) follow**

Table 1: `Dizet` initialisation flags: different versions defaults.

Input NPAR()	Internal flag	Dizet 6.21 Defaults in *Bardin:1989tq*	Dizet v6.42 Defaults in *Arbuzov:2005ma*	Dizet v6.45	Comments
NPAR(1)	IHVP	1	1	5	$\Delta\alpha^{(5)}_{had}$ param. from *Jegerlehner:2017zsb* in v6.45
NPAR(2)	IAMT4	4	4	8	New devellopment in v6.45
NPAR(3)	IQCD	3	3	3	
NPAR(4)	IMOMS	1	1	1	
NPAR(5)	IMASS	0	0	0	
NPAR(6)	ISCRE	0	0	0	
NPAR(7)	IALEM	3	3	3	
NPAR(8)	IMASK	0	0	0	Not used since v6.21
NPAR(9)	ISCAL	0	0	0	
NPAR(10)	IBARB	2	2	2	
NPAR(11)	IFTJR	1	1	1	
NPAR(12)	IFACR	0	0	0	
NPAR(13)	IFACT	0	0	0	
NPAR(14)	IHIGS	0	0	0	
NPAR(15)	IAMFT	1	3	3	
NPAR(16)	IEWLC	1	1	1	
NPAR(17)	ICZAK	1	1	1	
NPAR(18)	IHIG2	1	1	1	
NPAR(19)	IALE2	3	3	3	
NPAR(20)	IGREF	2	2	2	
NPAR(21)	IDDZZ	1	1	1	
NPAR(22)	IAMW2	0	0	0	
NPAR(23)	ISFSR	1	1	1	
NPAR(24)	IDMWW	0	0	0	
NPAR(25)	IDSWW	0	0	0	

Table 2: The `Dizet` v6.45 recalculated parameters: masses, couplings, etc.., with initialisation as in Tables 1, 7 and 8.

Parameter	Value	Description
$\alpha_{QED}(M_Z^2)$	0.0077549256	calculated using $\Delta\alpha_h^{(5)}(m_Z^2)$ from *Jegerlehner:2017zsb*
$1/\alpha_{QED}(M_Z^2)$	128.950302056	
M_W (GeV)	80.3589356	W mass
$ZPAR(1) = \delta r$	0.03640338	the loop corrections to G_μ
$ZPAR(2) = \delta r_{rem}$	0.01167960	the remainder contribution $O(\alpha)$
$ZPAR(3) = s_W^2$	0.22340108	weak mixing angle defined by weak masses
$ZPAR(4) = G_\mu$ (GeV^{-2})	$1.16614173 \cdot 10^{-5}$	G_μ with loop correct.
$ZPAR(6) = \sin^2\theta_{eff}^{\ell}(M_Z^2)$	0.231499	effective weak mixing angle
$ZPAR(9) = \sin^2\theta_{eff}^{up}(M_Z^2)$	0.231392	effective weak mixing angle
$ZPAR(10) = \sin^2\theta_{eff}^{down}(M_Z^2)$	0.231265	effective weak mixing angle
$ZPAR(14) = \sin^2\theta_{eff}^{bottom}(M_Z^2)$	0.232733	effective weak mixing angle

Extra slides on Dizet initialization and results.

Table 3: The `Dizet v6.45` predictions for two different parametrisations of $\Delta\alpha_h^{(5)}(M_Z^2)$. Other flags as in Table 1.

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2)$ = 0.0280398 (param. Jegerlehner 1995)	$\Delta\alpha_h^{(5)}(M_Z^2)$ = 0.0275762 (param. Jegerlehner 2017)	Δ
$\alpha(M_Z^2)$	0.0077587482	0.0077549256	
$1/\alpha(M_Z^2)$	128.8867699646	128.95030224	
s_W^2	0.22356339	0.22340108	- 0.00016
$sin^2\theta_{eff}(M_Z^2)$ (lepton)	0.23166087	0.23149900	- 0.00023
$sin^2\theta_{eff}(M_Z^2)$ (up-quark)	0.23155425	0.23139248	- 0.00016
$sin^2\theta_{eff}(M_Z^2)$ (down-quark)	0.23142705	0.23126543	- 0.00016
M_W (GeV)	80.3505378	80.358936	+8.4 MeV
Δr	0.03690873	0.03640338	
Δr_{rem}	0.01168001	0.01167960	

Table 4: The `Dizet v6.45` predictions: uncertainty from $\Delta\alpha_h^{(5)}(M_Z^2 = 0.0275762)$ (param. *Jegerlehner:2017zsb*), varied by $\pm$ 0.0001.

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2)$ - 0.0001	$\Delta\alpha_h^{(5)}(M_Z^2)$ = 0.0275762	$\Delta\alpha_h^{(5)}(M_Z^2)$ + 0.0001	$\Delta/2$
$\alpha(M_Z^2)$	0.0077541016	0.0077549256	0.0077557498	
$1/\alpha(M_Z^2)$	128.9640056546	128.95030224	128.9365984574	
s_W^2	0.22336607	0.22340108	0.22343610	0.000035
$sin^2\theta_{eff}(M_Z^2)$ (lepton)	0.23146409	0.23149900	0.23153392	0.000035
$sin^2\theta_{eff}(M_Z^2)$ (up-quark)	0.23135758	0.23139248	0.23142737	0.000035
$sin^2\theta_{eff}(M_Z^2)$ (down-quark)	0.23123057	0.23126543	0.23130029	0.000035
M_W (GeV)	80.3607471	80.358936	80.357124	1.8 MeV
Δr	0.03629414	0.03640338	0.03651261	
Δr_{rem}	0.01167983	0.01167960	0.01167938	

Table 5: The `Dizet v6.45` predictions with improved treatment of two-loop corrections. Other flags as in Table 1.

Parameter	AMT4= 4	AMT4 = 8	$\triangle$
$\alpha(M_Z^2)$	0.0077549256	0.0077549256	
$1/\alpha(M_Z^2)$	128.9503020560	128.95030224	
s_W^2	0.22333971	0.22340108	+ 0.00006
$sin^2\theta_{eff}(M_Z^2)$ (lepton)	0.23157938	0.23149900	-0.00008
$sin^2\theta_{eff}(M_Z^2)$ (up-quark)	0.23147290	0.23139248	-0.00008
$sin^2\theta_{eff}(M_Z^2)$ (down-quark)	0.23134590	0.23126543	-0.00008
M_W (GeV)	80.361846	80.358936	- 2.9 MeV
Δr	0.03640338	0.03640338	
Δr_{rem}	0.01167960	0.01167960	

Table 6: The `Dizet v6.45` predictions: uncertainty from changing top-quark mass by ± 0.5 GeV. Other flags as in Table 1.

Parameter	m_t - 0.5 GeV	m_t = 173.0 GeV	m_t + 0.5 GeV	$\Delta/2$
$\alpha(M_Z^2)$	0.0077549221	0.0077549256	0.0077549291	
$1/\alpha(M_Z^2)$	128.9503600286	128.95030224	128.9502446106	
s_W^2	0.22345908	0.22340108	0.22334300	0.000058
$sin^2\theta_{eff}(M_Z^2)$ (lepton)	0.23151389	0.23149900	0.23148410	0.000016
$sin^2\theta_{eff}(M_Z^2)$ (up-quark)	0.23140736	0.23139248	0.23137758	0.000016
$sin^2\theta_{eff}(M_Z^2)$ (down-quark)	0.23128031	0.23126543	0.23125053	0.000016
M_W (GeV)	80.355935	80.358936	80.361941	3 MeV
Δr	0.03658500	0.03640338	0.03622132	
Δr_{rem}	0.01167011	0.01167960	0.01168907	

Table 7: Values of fermions and Higgs boson masses used for calculating EW corrections.

Parameter	Mass (GeV)	Description
m_e	5.1099907e-4	mass of electron
m_μ	0.1056583	mass of muon
m_τ	1.7770500	mass of tau
m_u	0.0620000	mass of up-quark
m_d	0.0830000	mass of down-quark
m_c	1.5000000	mass of charm-quark
m_s	0.2150000	mass of strange-quark
m_b	4.7000000	mass of bottom-quark
m_t	173.0	mass of top quark
m_H	125.0	mass of Higgs boson

Table 8: The EW parameters used at tree-level EW, with on-mass-shell definition (LEP convention).

Parameter	$(\alpha(0), G_\mu, M_Z)$
M_Z (GeV)	91.1876
Γ_Z (GeV)	2.4952
Γ_W (GeV)	2.085
$1/\alpha$	137.035999139
α	0.007297353
G_μ (GeV^{-2})	$1.1663787 \cdot 10^{-5}$
M_W (GeV)	80.93886
s_W^2	0.2121517
$\alpha_s(M_Z)$	0.120178900000