Neutrino counting measured at LEP with/without radiative $\gamma$:

$$N_\nu = 2.984 \pm 0.008$$

**Beam-beam effect correction**

G. Voutsinas et al., arXiv:1908.01704

$$N_\nu = 2.9975 \pm 0.0074$$


No distinction between neutrino flavor
Motivation: Complementing tests of lepton universality

$$\Delta_{W}^{\tau/\ell} = BR(W \to \tau\nu) - BR(W \to \ell\nu) = 0.00711 \pm 0.00237 \quad (PDG: \approx 3\sigma) \quad (\ell = e, \mu)$$

$$R_{D*}^{\tau/\ell} = \frac{BR(B \to D^{*}\tau\nu)_{exp}}{BR(B \to D^{*}\ell\nu)_{exp}} \div \frac{BR(B \to D^{*}\tau\nu)_{SM}}{BR(B \to D^{*}\ell\nu)_{SM}} = 1.28 \pm 0.08 \quad (3.8 \sigma)$$

$$R_{D}^{\tau/\ell} = \frac{BR(B \to D\tau\nu)_{exp}}{BR(B \to D\ell\nu)_{exp}} \div \frac{BR(B \to D\tau\nu)_{SM}}{BR(B \to D\ell\nu)_{SM}} = 1.37 \pm 0.18 \quad (2.0 \sigma)$$

$$R_{K}^{\mu/e} = \frac{BR(B \to K\mu^{+}\mu^{-})_{exp}}{BR(B \to K\ell^{+}\ell^{-})_{exp}} = 0.745 \pm 0.080 \pm 0.036 \quad (2.6 \sigma)$$

$$= 0.846^{+0.060}_{-0.054}^{+0.016}_{-0.014}(syst) \quad (2.5\sigma, LHCb)$$
How to do better at FCC-ee?

In the following we assume $N_{inv} \equiv 3 \nu$ since it will be measured at FCC with negligible error.
Can one measure Neutrino flavor directly?

With 150 ab$^{-1}$ at Z-pole, $2.4 \times 10^{12}$ neutrinos are produced.

Unfortunately, the cross section for $E_{\nu} = 45 \text{ GeV}$ is low, $\sim 0.3 \text{ pb}$

With 1 $X_0$ in the tracking area (much more than any reasonable tracker), only $\sim 3$ interactions expected!

$\Rightarrow$ A dedicated detector with some 100 $X_0$ would be needed
Search for interference with diagrams with well known couplings

Only $\nu_e$ interfere $\Rightarrow$ interference effect measures $g_{Z\nu_e}^\nu_e$
We concentrate on $\sqrt{S} = 161 \text{ GeV}$ with $L=10 \text{ab}^{-1}$ (i.e. with 2 detectors) MC used KKMC (see Staszek Jadach et al.)

$\frac{d\sigma}{dv_{\nu}}$; KKMC $e^+e^- \rightarrow \nu\bar{\nu}+n\gamma$, $\nu=\nu_e+\nu_\mu+\nu_\tau$ 

$\sqrt{S} = 161.00 \text{GeV}$ 

(a) $\gamma$'s untagged, $v=1-M_{\nu\nu}/s$  

(b) $\gamma$'s tagged, $v=E_\gamma/E_{\text{beam}}$

Cuts for (b) curve

$$\sum E_\gamma > 0.1E_{\text{beam}}$$  

$$\theta_\gamma > 15^\circ$$  

$$E_{T\gamma} > 0.02 E_{\text{beam}}$$

Essentially 1 $\gamma$ after cuts
Zoom on Z Radiative Return (ZRR)

Difference between $\nu_{\mu(\tau)}$ and $\nu_e$

$\frac{d\sigma}{dv}$ [nb], $e^+e^- \rightarrow W^+W^-$, $\gamma$'s tagged

$t$-channel $W$ contrib. $R_t(v) = (\nu_{\mu} - \nu_{\tau}) / (3 \times \nu_e)$

Integrated $R_t = -0.0010$

$V_Z = 1 - \frac{M_Z^2}{S}$
Interference effects may look small but huge statistics is available ~\(25 \times 10^6\) events.

For simplicity let’s define the Asymmetry \(S = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}\) with \(\sigma_+ = \sigma(v>v_z)\), \(\sigma_- = \sigma(v<v_z)\)

\[
\Delta S = S(\eta) - S(0)
\]

\[
S = (\sigma_+ - \sigma_0) / (\sigma_+ + \sigma_0)
\]

\[
\sigma_+ = \sigma(v>v_z), \quad \sigma_- = \sigma(v<v_z)
\]

\[
v_z = 1 - M_Z^2/s = 0.67921
\]

\[
S(0) = -0.13457 \pm 0.00020
\]

Parametrization assuming \(N_{inv} \equiv 3\nu\) \(\Rightarrow \quad g_Z^{\nu_e} = \sqrt{1 + \eta}, \quad g_Z^{\nu_\mu} = 1, \quad g_Z^{\nu_\tau} = \sqrt{1 - \eta}\)
Without detector resolution dilution effects

\[ \delta(g_Z^{\nu e}) = \pm 1.0\% \]

With detector resolution dilution effects

\[ \frac{\delta E_{\gamma}}{E_{\gamma}} = \frac{0.05}{\sqrt{E_{\gamma}}} \oplus 0.002 \]

Can be calibrated with \( \mu \mu \gamma \) events

\[ \delta(g_Z^{\nu e}) = \pm 1.4\% \]

If stochastic term \( x2 \) (sampling detector) \( \Rightarrow \)

\[ \delta(g_Z^{\nu e}) = \pm 2.4\% \]
Summary

- The method proposed would lead to a considerable improvement on the precision on $g_Z^{\nu e}$
  \[ \delta(g_Z^{\nu e}) = \pm 1.4\% \]
- Assuming 3 $\nu$ and no new physics coupled to Z, one would derive
  \[ \delta(g_Z^{\nu\tau}) = \pm 4.8\% \]
- $\sqrt{S} = 161$ GeV not optimal (but we will run there anyway), e.g. 6 months at $\sqrt{S} = 105$ GeV would allow for twice smaller errors
Final remarks:
This is a preliminary study and several complementary studies needed

• virtual corrections for W contribution in KKMC matrix element has to be checked
• the size and shape of the QED deformation of the Z peak in ZRR obtained from KKMC should be cross-checked using independent calculation
• EW corrections were included in the presented KKMC calculation - their size and role should be examined quantitatively
• dominant $O(\alpha^3)$ QED non-soft corrections (in our convention) should be estimated/calculated.

There are also several other improvements in the analysis front, which needs to be studied:
• carrying a full fit of the $v$ spectrum instead of measuring its asymmetry
• optimizing the $v$ range.
• study of the interference effect at low and high $v$ range might be useful to improve the sensitivity on $g_Z^{\nu_e}$
• Carrying an analysis with full detector simulation will be ultimately needed