

# Recent Progress on Form Factor and Charm Loop Uncertainties

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08.06.2020

Danny van Dyk

Technische Universität München

based on

arXiv:1908.09398 with M. Bordone, M. Jung

arXiv:1912.09335 with M. Bordone, N. Gubernari, M. Jung

arXiv:20xy.abcde with N. Gubernari, J. Virto

funded by



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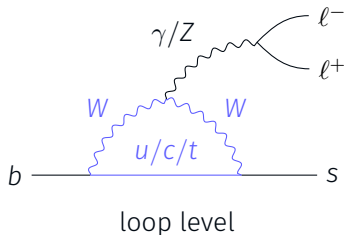
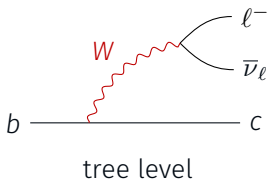
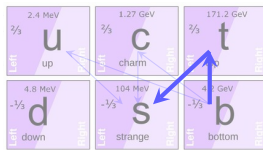
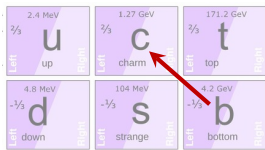
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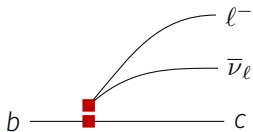
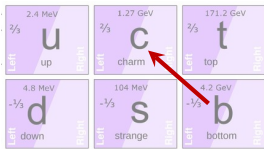
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# Introduction

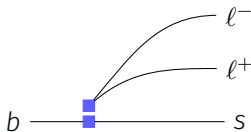
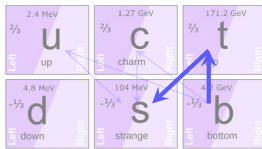
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$$\sum_i C_i \times [\bar{c} \Gamma_i b] \times \dots$$

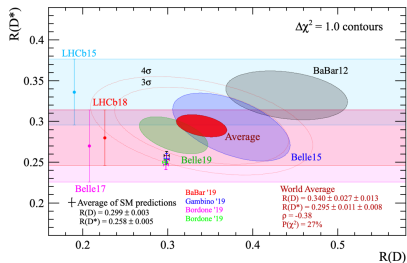
tree level



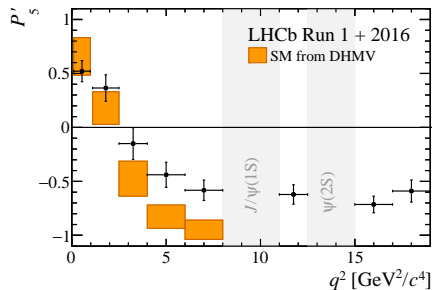
$$\sum_i C_i \times [\bar{s} \Gamma_i b] \times \dots$$

loop level

# The anomalies in plots



[M. Rotondo for upcoming HFLAV update]



[E.A. Smith CERN Seminar '20; LHCb 2003.04831]

$$\sum_i \mathcal{C}_i \times [\bar{c} \Gamma_i b] \times \dots$$

$$\sum_i \mathcal{C}_i \times [\bar{s} \Gamma_i b] \times \dots$$

Interpretation of the Wilson coefficients  $\mathcal{C}_i$  requires knowledge of the hadronic matrix elements  $\langle \dots | \bar{c} \Gamma_i b | \dots \rangle$  and  $\langle \dots | \bar{s} \Gamma_i b | \dots \rangle$ .

$\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}$  Form Factors

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# Definitions

- ▶ SM requires 2 + 4 form factors for description of  $D + D^*$  final states
  - ▶ form factors are functions dependent on  $q^2$ : the  $\ell^- \bar{\nu}$  mass squared

$$\langle D(k) | \bar{c} \gamma^\mu b | B(p) \rangle = \left[ (p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] f_+(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0(q^2)$$

$$\langle D(k) | \bar{c} \sigma_{\mu\nu} b | B(p) \rangle = \frac{2i}{M_B + M_D} (k^\mu p^\nu - p^\mu k^\nu) f_T(q^2)$$

$$\langle D^*(k, \eta) | \bar{c} \gamma^\mu b | B(p) \rangle = -\epsilon^{\mu\nu\rho\sigma} \eta_{*\nu}(k) p_\rho k_\sigma \frac{2V(q^2)}{M_B + M_{D^*}}$$

$$\begin{aligned} \langle D^*(k, \eta) | \bar{c} \gamma^\mu \gamma_5 b | B(p) \rangle = & i\eta_\nu^* \left\{ 2M_D \frac{A_0(q^2)}{q^2} q^\mu q^\nu + 16 \frac{M_B M_{D^*}^2}{\lambda} \frac{A_{12}(q^2)}{q^2} \left[ 2p^\mu q^\nu - \frac{M_B^2 - M_{D^*}^2 + q^2}{q^2} q^\mu q^\nu \right] \right. \\ & \left. + (M_B + M_{D^*}) \frac{A_1(q^2)}{q^2} \left[ g^{\mu\nu} + \frac{2(M_B^2 + M_{D^*}^2 - q^2)}{\lambda} q^\mu q^\nu - \frac{2(M_B^2 - M_{D^*}^2 - q^2)}{\lambda} p^\mu q^\nu \right] \right\} \end{aligned}$$

$$\begin{aligned} \langle D^*(k, \eta) | \bar{c} \sigma_{\mu\nu} b | B(p) \rangle = & i\eta_\alpha^* \epsilon_{\mu\nu\rho\sigma} \left\{ - \left[ \left( (p+k)^\rho - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\rho \right) g^{\alpha\sigma} + \frac{2}{q^2} p^\alpha p^\rho k^\sigma \right] T_1(q^2) \right. \\ & \left. \left( \frac{2}{q^2} p^\alpha p^\rho k^\sigma - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\rho g^{\alpha\sigma} \right) T_2(q^2) + \frac{2}{M_B^2 - M_{D^*}^2} p^\alpha p^\rho k^\sigma T_3(q^2) \right\}. \end{aligned}$$

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- ▶ **assuming** the SM, **1 + 3** of these form factors can be *partially* inferred from  $\bar{B} \rightarrow D^{(*)} \mu^- \bar{\nu}$  measurements

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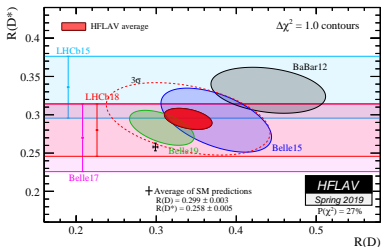
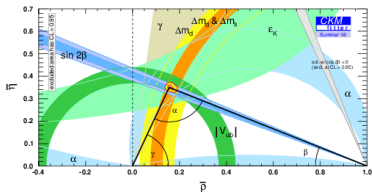
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- ▶ SM assumption affects extraction of  $|V_{cb}|$  and  $R(D^{(*)})$  predictions  
 aim: remove dependence on experimental data in SM predictions

ordered from most stringent to least stringent

- ▶ lattice QCD

[HPQCD, FNAL/MILC]


- ▶ QCD light-cone sum rules (LCSRs)

[Gubernari,Kokulu,DvD '18, Bordone et al. '19]

- ▶ QCD three-point sum rules

[Ligeti,Neubert,Nir '92+'92]

ordered from most stringent to least stringent

- ▶ lattice QCD [HPQCD, FNAL/MILC]
  - ▶ for **two out of three**  $\bar{B} \rightarrow D$  form factors for the **physical  $q^2$  range**
  - ▶ for **one out of seven**  $\bar{B} \rightarrow D^*$  form factor and only in **one  $q^2$  point**
- ▶ QCD light-cone sum rules (LCSRs) [Gubernari,Kokulu,DvD '18, Bordone et al. '19]
  -  for **all**  $\bar{B}_{(s)} \rightarrow D_{(s)}^{(*)}$  form factors, but only at  **$q^2 \leq 0$**
- ▶ QCD three-point sum rules [Ligeti,Neubert,Nir '92+'92]
  - ▶ for some matrix elements (to be explained!) emerging in **all**  $\bar{B}_{(s)} \rightarrow D_{(s)}^*$  form factors, but limited to **one  $q^2$  point**

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▶ QCD light-cone sum rules (LCSRs)

[Gubernari,Kokulu,DvD '18, Bordone et al. '19]

**NEW!**

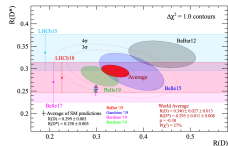
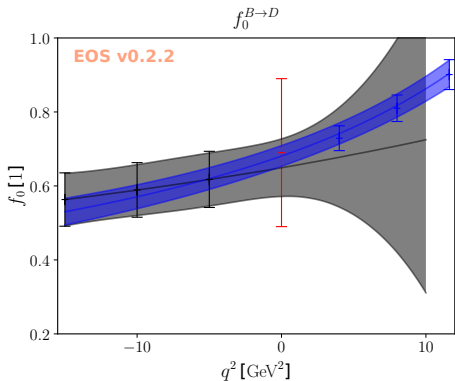
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- ▶ for some matrix elements (to be explained!) emerging in all  $\bar{B}_{(s)} \rightarrow D_{(s)}^{(*)}$  form factors, but limited to one  $q^2$  point

- ▶ eagerly awaiting results by FNAL/MILC on  $\bar{B} \rightarrow D^*$  for the full (or at least a large part of) the  $q^2$  range!

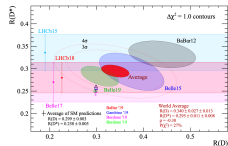
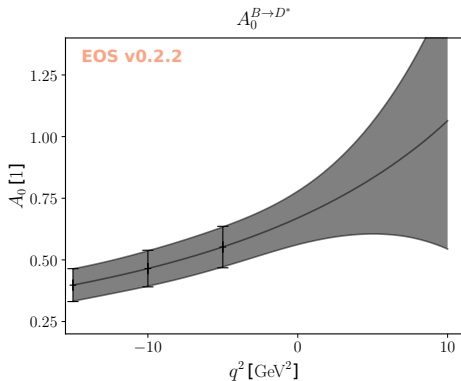


LCSR '18 only

LCSR '18 + Lattice

LCSR '08

- ▶ for  $f_+$  and  $f_0$ , good agreement between both approaches



LCSR '18 only

LCSR '18 + Lattice

LCSR '08

- ▶ for  $f_+$  and  $f_0$ , good agreement between both approaches
- ▶ for  $A_0$ , this is the first and only QCD-based determination at non-maximal  $q^2$
- ▶ **large** uncertainties in LCSRs, which can be reduced by using a parametrization that accounts for additional theory inputs: the heavy quark expansion

# Heavy-Quark Expansion to $1/m_c^2$

- ▶ any  $\bar{B}^{(*)} \rightarrow D^{(*)}$  form factor  $h$  can be expanded in  $\alpha_s$ ,  $\varepsilon_b \equiv \Lambda/m_b$ , and  $\varepsilon_c \equiv \Lambda/m_c$

$$h(w) = \xi(w)\hat{h}(w) = \xi(w) \left( a + \hat{\alpha}_s b + \varepsilon_b c_b^{(i)} [\hat{L}_i(w)] + \varepsilon_c c_c^{(i)} [\hat{L}_i(w)] + \varepsilon_c^2 d^{(i)} [\hat{\ell}_i(w)] \right)$$

$$\text{using } q^2 = M_B^2 + M_D^2 - 2M_B M_D w$$

- ▶ adopting the power-counting  $\varepsilon_b \sim \varepsilon_c^2 \sim \varepsilon^2$  on encounters
  - ▶ one function  $\xi$  at leading power in  $\varepsilon$  [Isgur,Wise '90]
  - ▶ three independent functions  $\vec{L} \sim \{\chi_2, \chi_3, \eta\}$  at  $\varepsilon^1$  [Falk,Neubert '92]
  - ▶ six independent functions  $\vec{\ell} = \{\ell_1, \dots, \ell_6\}$  at  $\varepsilon^2$  [Falk,Neubert '92]
- ▶ common to expand these function in a new variable  $z$  with  $|z| \ll 1$
- ▶ QCD three-point sum rules constrain  $\chi_2, \chi_3, \eta$  at  $w = w_{\min}$



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using  $q^2 = M_B^2 + M_D^2 - 2M_B M_D w$

NEW!

analysis incl. all  $\varepsilon_c^2$  terms with *a posteriori* comparisons of the fit models

[Bordone, Jung, DvD '19]

- ▶ model  $k/l/m$  expands  $\xi$  to  $k$ th order,  $\chi_{2,3}$  and  $\eta$  to  $l$ th order, and  $\ell_{1,\dots,6}$  to  $m$ th order in  $z(w)$
- ▶ minimal viable model is 2/1/0 #par = 13 + 13
- ▶ use 3/2/1 model to account for systematic uncertainties, increasing each order by one above the minimally viable model #par = 23 + 23

NEW!

assuming linear  $SU(3)_F$  breaking  $\ell_i^{(s)} = \ell_i^{(u,d)} + \varepsilon_F \delta_i$

- ▶  $\varepsilon_F \sim \varepsilon_c \sim \varepsilon$  leaves breaking terms beyond accuracy of the ansatz
- ▶ reduces number of free parameter #par  $\rightarrow$  34

# Dispersive bounds

- ▶ causality and unitarity of the  $S$  matrix implies form factors are complex-valued functions on entire complex  $q^2$  plane
- ▶ dyn. singularities in the  $q^2$  plane are assoc. with phys. features:
  - ▶ poles are associated with  $\bar{c}b$  bound states
  - ▶ branch cuts are associated with production 2-body (e.g.  $BD$ ) and many-body production thresholds
- ▶ dispersive bounds arise from a carefully constructed two-point function with computable normalization [Boyd, Grinstein, Lebed '95]
- ▶ the bounds can be expressed through the parameters of the heavy-quark expansion [Caprini, Lellouch, Neubert '97]
- NEW! updated bounds to include parameters up to  $\epsilon^2$  [Bordone, Jung, DvD '19]
- NEW! expanded analysis to include both  $\bar{B} \rightarrow D^{(*)}$  and  $\bar{B}_s \rightarrow D_s^{(*)}$  form factors **simultaneously** and **without assumption of  $SU(3)_F$  symmetry** [Bordone, Gubernari, Jung, DvD '19]

- ▶ update SM Predictions for  $\bar{B}_q \rightarrow D_q^{(*)} \tau^- \bar{\nu}$ 
  - ▶ using **only theory** inputs

[Bordone,Gubernari,Jung,DvD '19]

$$\begin{aligned} R_D &= 0.2989 \pm 0.0032 & R_{D_s} &= 0.2970 \pm 0.0034 \\ R_{D^*} &= 0.2472 \pm 0.0050 & R_{D_s^*} &= 0.2450 \pm 0.0082 \end{aligned}$$

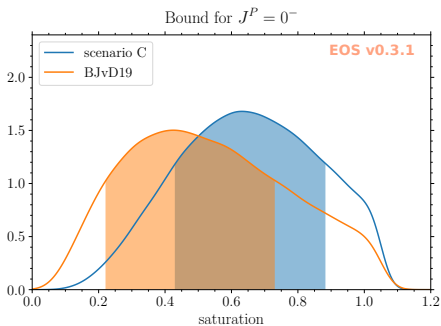
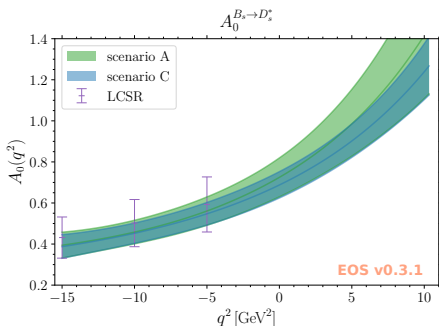
- ▶ using theory + **Belle measurements of  $e^-$ ,  $\mu^-$  final states**

[Bordone,Gubernari,Jung,DvD '19]

$$\begin{aligned} R_D &= 0.2981 \pm 0.0029 & R_{D_s} &= 0.2971 \pm 0.0034 \\ R_{D^*} &= 0.2504 \pm 0.0026 & R_{D_s^*} &= 0.2472 \pm 0.0077 \end{aligned}$$

- ▶ combined significance of  $R_D$  and  $R_{D^*}$  measurement now above  **$4\sigma$**

- ▶ gaussian constraint on the BGL form factor coefficients available as machine-readable files



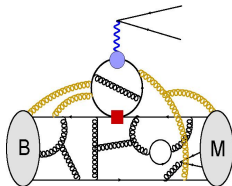
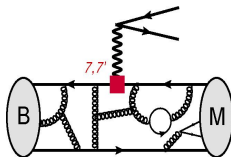
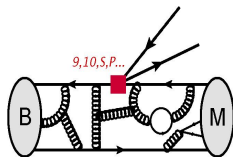
- ▶ using information on  $\bar{B}_s \rightarrow D_s^{(*)}$  form factors in the heavy-quark expansion and the dispersive bounds has become essential for accurate determination of  $\bar{B} \rightarrow D^{(*)}$  form factors

Non-local matrix elements in

$$\bar{B} \rightarrow \bar{K}^* l^+ l^-$$

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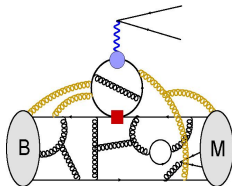
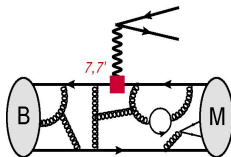
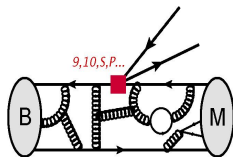
# Amplitudes for $\bar{B} \rightarrow \bar{K}^{(*)} \ell^+ \ell^-$



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- ▶ non-local :  $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T \{ \mathcal{J}_{em}^\mu(x), C_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$ 
  - ▶ major source of systematic uncertainty
  - ▶ several approaches dependent on the phase space region (i.e.:  $q^2$ )
  - ▶ here: only discuss impact of four-quark operators  $\mathcal{O}_{1c}$  and  $\mathcal{O}_{2c}$

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► non-local:  $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T \{ \mathcal{J}_{em}^\mu(x), C_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

below the  $J/\psi$ : Light-Cone OPE

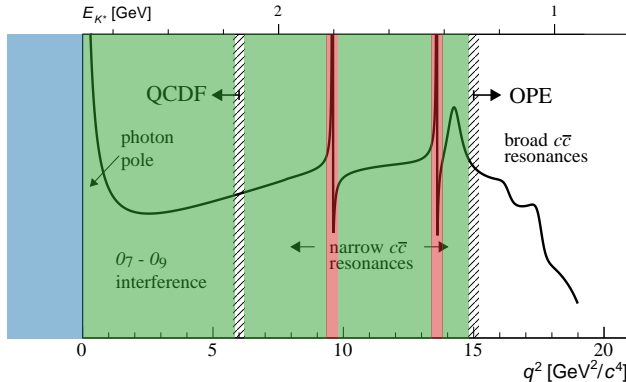
- leading (local) terms. in QCD Factorization
- subleading terms from LCSRs

[Khodjamirian, Mannel, Pivovarov, Wang '10]

[Beneke, Feldmann, Seidel '01&'04]

[Khodjamirian, Mannel, Pivovarov, Wang '10]

# Regions of phase space



[sketch from Blake, Gershon, Hiller 1501.03309]

- ▶ accessible in light-cone OPE
- ▶ needed here for phenomenology
- ▶ (partially) constrained from data on  $\bar{B} \rightarrow \bar{K}^{(*)} \psi_n$

Ansatz: parametrize, expand in small variable  $z$  as for local form factors, fit to theory and experimenta, and interpolate in central region 11/14



# Light-Cone OPE, matching, and matrix elements

$$4m_c^2 - q^2 \gg \Lambda_{\text{had}}^2$$

- expansion in operators at light-like

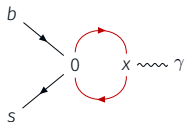
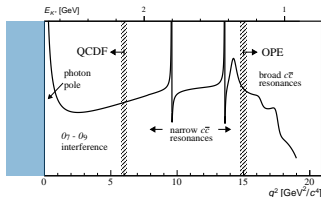
distances  $x^2 \simeq 0$

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

- employing light-cone expansion of charm

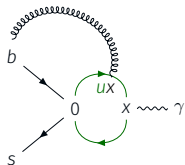
propagator

[Balitsky, Braun 1989]



$$\xrightarrow{4m_c^2 - q^2 \gg \Lambda_{\text{had}}^2} \underbrace{\left( \frac{C_1}{3} + C_2 \right) C_{LP}(q^2) [\bar{s} \Gamma^\mu b]}_{\text{coeff \#1}} + \dots$$

$$+ \underbrace{C_1 C_{NLP}^{\mu\rho\alpha\beta}(q^2; \omega_2)}_{\text{coeff \#2}} \times [\bar{s}_L \gamma_\rho \delta(\omega_2) \tilde{G}_{\alpha\beta} b_L]$$



w.i.p.

independently confirm result for  $C_{NLP}$

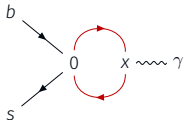
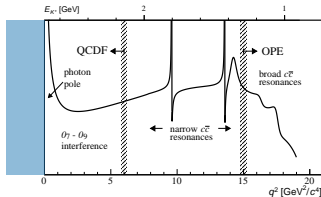
[Gubernari, DvD, Virto w.i.p.]

$$0 \leq u \leq 1$$

# Light-Cone OPE, matching, and matrix elements

$$4m_c^2 - q^2 \gg \Lambda_{\text{had}}^2.$$

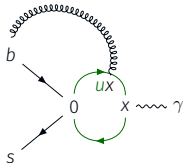
- ▶ expansion in operators at light-like distances  $x^2 \simeq 0$  [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- ▶ employing light-cone expansion of charm propagator [Balitsky, Braun 1989]



$$\mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}} + \text{coeff \#2} \otimes \mathcal{F}_\lambda^{\text{soft}}$$

- ▶ **leading** part identical to QCD Fact. results

[Beneke, Feldmann, Seidel '01&'04]



- ▶ **subleading** matrix element  $\mathcal{F}_\lambda^{\text{soft}}$  can be estimated using B-LCSRs, starting with three-particle B distribution amplitudes (DA)

[Khodjamirian, Mannel, Pivovarov, Wang '10]

- ▶ mistake in definition of DAs used for this purpose

[e.g. Braun et al. '17]

$$0 \leq u \leq 1$$

# Light-Cone OPE, matching, and matrix elements

$$4m_c^2 - q^2 \gg \Lambda_{\text{had}}^2.$$

- expansion in operators at light-like

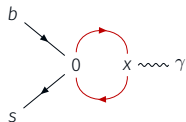
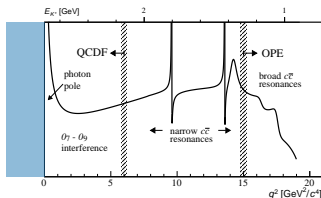
distances  $x^2 \simeq 0$

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

- employing light-cone expansion of charm

propagator

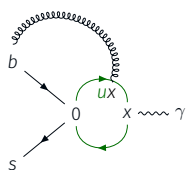
[Balitsky, Braun 1989]



w.i.p.

updating  $B$ -LC SR calculation of the matrix element of  $\mathcal{F}_\lambda^{\text{soft}}$

[Gubernari, DvD, Virto w.i.p.]



w.i.p.

**PRELIMINARY** at  $q^2 = 1 \text{ GeV}^2$  we find suppression by two orders of magnitude

- one from updated numerical inputs
- one from cancellations between “new” and “old” LCDAs

$$0 \leq u \leq 1$$

# Dispersive bounds of the $z$ parametrization

$$\mathcal{H} \propto \text{poles} \times \text{phase space} \times \left[ \sum_k \alpha_k z^k \right]$$

$$\sum_k |\alpha_k|^2 \leq 1$$

- ▶ BGL-like  $z$  parametrization useful due to **dispersive bound** on coefficients  $\alpha_k$  for *local form factors*
- ▶ for *non-local matrix elements* formulating the dispersive bound is more complicated than for local form factors

w.i.p.

**PRELIMINARY** able to formulate bound

[Gubernari,DvD,Virto w.i.p.]

- ▶ non-diagonal in coefficients, similar to situation for  $B \rightarrow \pi$  form factor
- ▶ analytic results not cross checked yet in time for this conference

[Bourelly,Caprini,Lellouch '08]

w.i.p.





existence of the bound implies homogeneous convergence of the parametrization

- ▶ systematic uncertainties due to cut-off of series expansion are bounded!

# Summary

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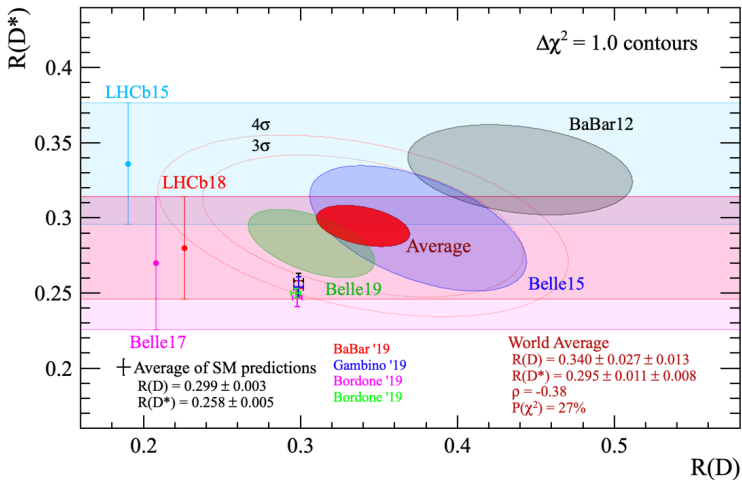
# Summary

- ▶ tensions between theory predictions and experimental results demand heightened scrutiny of all results
  - ▶ here: revisiting hadronic matrix elements needed for the theory predictions; Needs to understand old physics first!
- ▶ Form factors in  $\bar{B}_q \rightarrow D_q^{(*)} \ell^- \bar{\nu}$ 
  -  simultaneous analysis of theory and exp. constraints within heavy-quark expansion to  $1/m_c^2$ 
    - ▶  $b \rightarrow c \tau \bar{\nu}$  LFU SM predictions slightly lowered when incl.  $1/m_c^2$  effects, increasing tension w.r.t. measurements to  $\geq 4\sigma$
    - ▶ tension in  $V_{cb}$  puzzle reduced, but not entirely removed
- ▶ Non-local matrix elements in  $\bar{B} \rightarrow \bar{K}^* \ell^+ \ell^-$ 
  -  independent check of next-to-leading power coefficient ✓
  -  update of matrix elements at next-to-leading power, reduction in size by at least one order of magnitude
  -  formulating dispersive bound, thereby providing handle on systematic uncertainties arising from cut-off of the series expansion

## Backup Slides

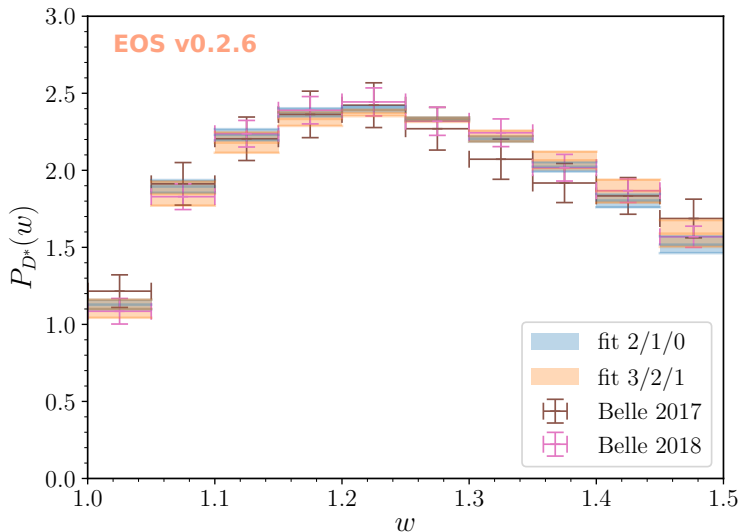
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# $R_D$ and $R_{D^*}$





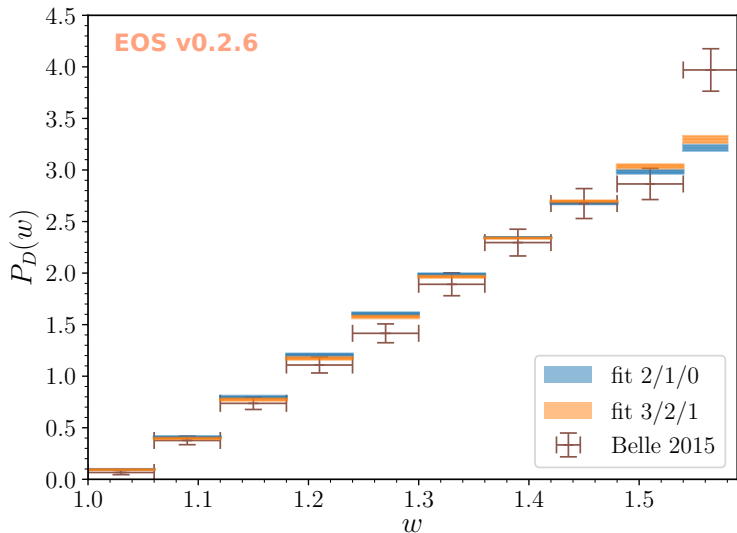
# Challenging data of $\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}$ decay



$$P \equiv \frac{1}{\Gamma} \frac{d\Gamma}{dw}$$

$$q^2 = M_B^2 + M_{D^{(*)}}^2 - 2M_B M_{D^{(*)}} w$$

# Challenging data of $\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}$ decay



$$P \equiv \frac{1}{\Gamma} \frac{d\Gamma}{dw}$$

$$q^2 = M_B^2 + M_{D^{(*)}}^2 - 2M_B M_{D^{(*)}} w$$

- ▶ LCSRs can be expressed as integrals over OPE result for the two-point function
- ▶ schematically

$$F(q^2; M^2) \sim \int_0^{\sigma_0} ds \exp[-(s - M_D^2)/M^2] \rho(s)$$

$M^2$ : Borel-conjugate to the final state mass square  $k^2$

- ▶ commonly, the threshold  $\sigma_0$  is determined from further sum rules

[Khodjamirian et al. '05, '06, '08]

- ▶ here, we determine the threshold by imposing that the dependence on  $M^2$  reproduces the final state mass (i.e.  $M_{D(*)}$ ):

$$M_D^2 = \frac{1}{F(q^2; M^2)} \frac{\partial F(q^2; M^2)}{\partial(-1/M^2)}$$

# Goodness of Fit

likelihood	#obs	#par	2/1/0	3/2/1	3/2/1	3/2/1	3/2/1
			13	23	23	23	23
lattice( $D$ )	12		11.15	7.06	7.29	7.29	7.36
lattice( $D^*$ )	1		0.00	0.01	0.00	0.01	0.00
QCDSR	5		4.58	0.04	0.04	0.01	0.02
LCSR	33		7.14	2.79	3.23	3.14	2.98
$\bar{B} \rightarrow D\{e^-, \mu^-\}\bar{\nu}$	(9)		—	—	—	—	6.75
$\bar{B} \rightarrow D^*\{e^-, \mu^-\}\bar{\nu}$ 2017	(9)		—	—	6.95	—	8.04
$\bar{B} \rightarrow D^*\{e^-, \mu^-\}\bar{\nu}$ 2018	(9)		—	—	—	4.42	4.84
	51		22.87	9.91	—	—	—
total	(60)		—	—	17.51	14.88	—
	(78)		—	—	—	—	30.00

# Determination of $|V_{cb}|$ from Exclusive $B$ Decays

- ▶ express all exclusive branching ratios  $\mathcal{B}^{(*)}$  as  $\bar{B}^0 \rightarrow D^{(*),+} \ell^- \bar{\nu}$
- ▶ predict correlated  $\mathcal{B}/|V_{cb}|^2$  and  $\mathcal{B}^*/|V_{cb}|^2$ , and extract  $V_{cb}$  from the world averages

[ALEPH, BaBar, Belle, CLEO2, Delphi, OPAL]

model	scenarios				
	2/1/0	3/2/1	3/2/1	3/2/1	3/2/1
exp. likelihood	—	—	2017	2018	all exp.
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \{e^-, \mu^-\} \bar{\nu})/ V_{cb} ^2$	$12.99 \pm 0.35$	$13.48 \pm 0.37$	—	—	$13.56 \pm 0.35$
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \{e^-, \mu^-\} \bar{\nu})/ V_{cb} ^2$	$32.33 \pm 1.28$	$33.16 \pm 2.15$	$31.74 \pm 1.46$	$32.19 \pm 1.03$	$32.00 \pm 1.03$
correlation	0.34	0.14	—	—	0.10
$ V_{cb}  \times 10^3$ from $\bar{B} \rightarrow D \{e^-, \mu^-\} \bar{\nu}$	$41.5 \pm 1.2$	$40.7 \pm 1.2$	—	—	$40.6 \pm 1.1$
$ V_{cb}  \times 10^3$ from $\bar{B} \rightarrow D^* \{e^-, \mu^-\} \bar{\nu}$	$39.8 \pm 1.2$	$39.3 \pm 1.7$	$40.1 \pm 1.3$	$39.8 \pm 1.0$	$40.0 \pm 1.1$
	—	—	( $39.5 \pm 1.9$ )	( $39.0 \pm 1.3$ )	—
$ V_{cb}  \times 10^3$ combined incl. corr.	$40.7 \pm 1.0$	<b><math>40.2 \pm 1.0</math></b>	—	—	<b><math>40.3 \pm 0.8</math></b>

form factor inputs: [theory only](#), [theory + Belle spectra](#)

- ▶ compatible with *inclusive* det.  $|V_{cb}| = (42.00 \pm 0.64) \times 10^{-3}$  at  $1.2\sigma$
- ▶ average of exclusive and inclusive det.  $|V_{cb}| = (41.3 \pm 0.5) \times 10^{-3}$