SEMILEPTONIC B DECAYS
$|V_{xb}|$ and $R(D,D^*)$

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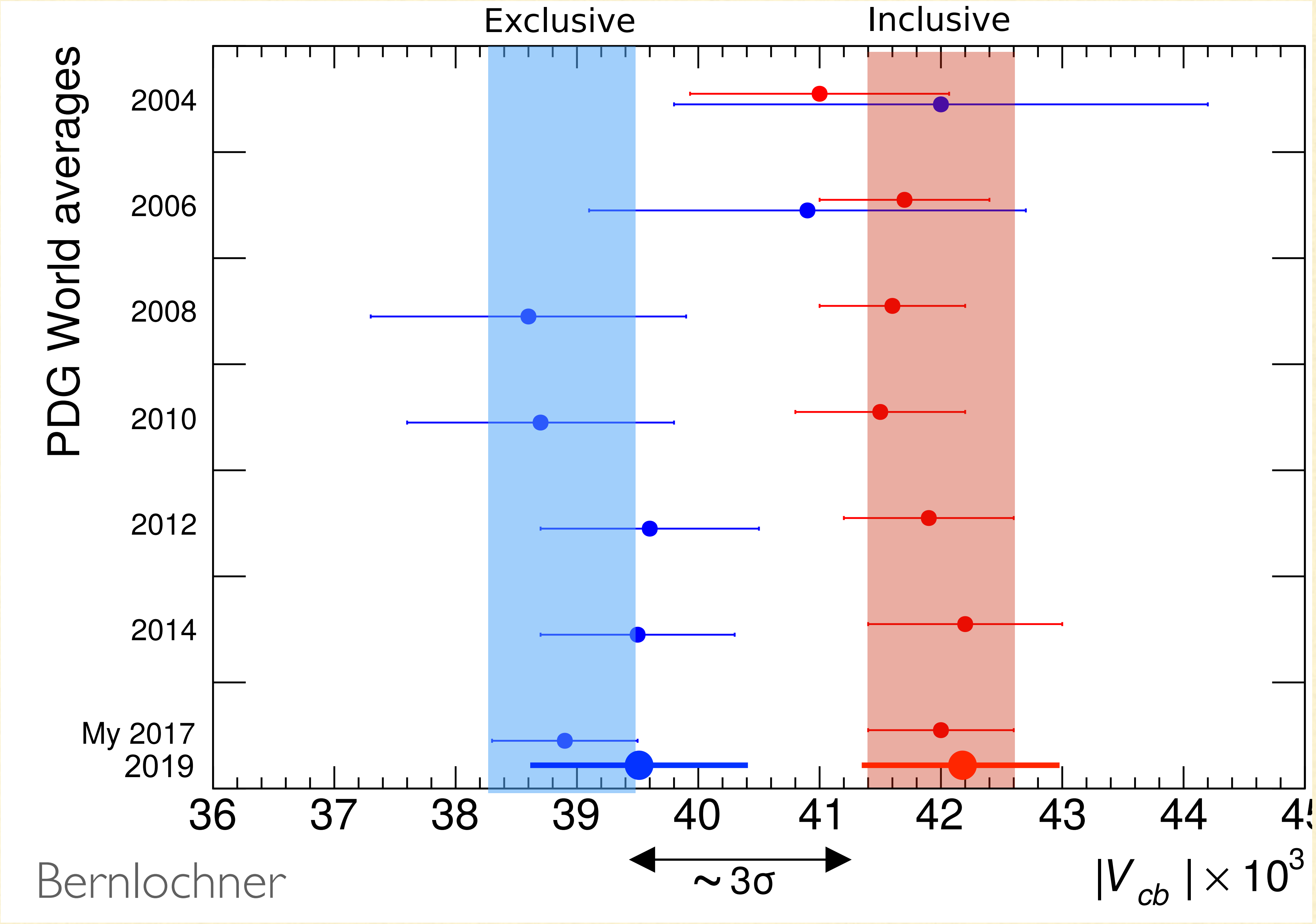
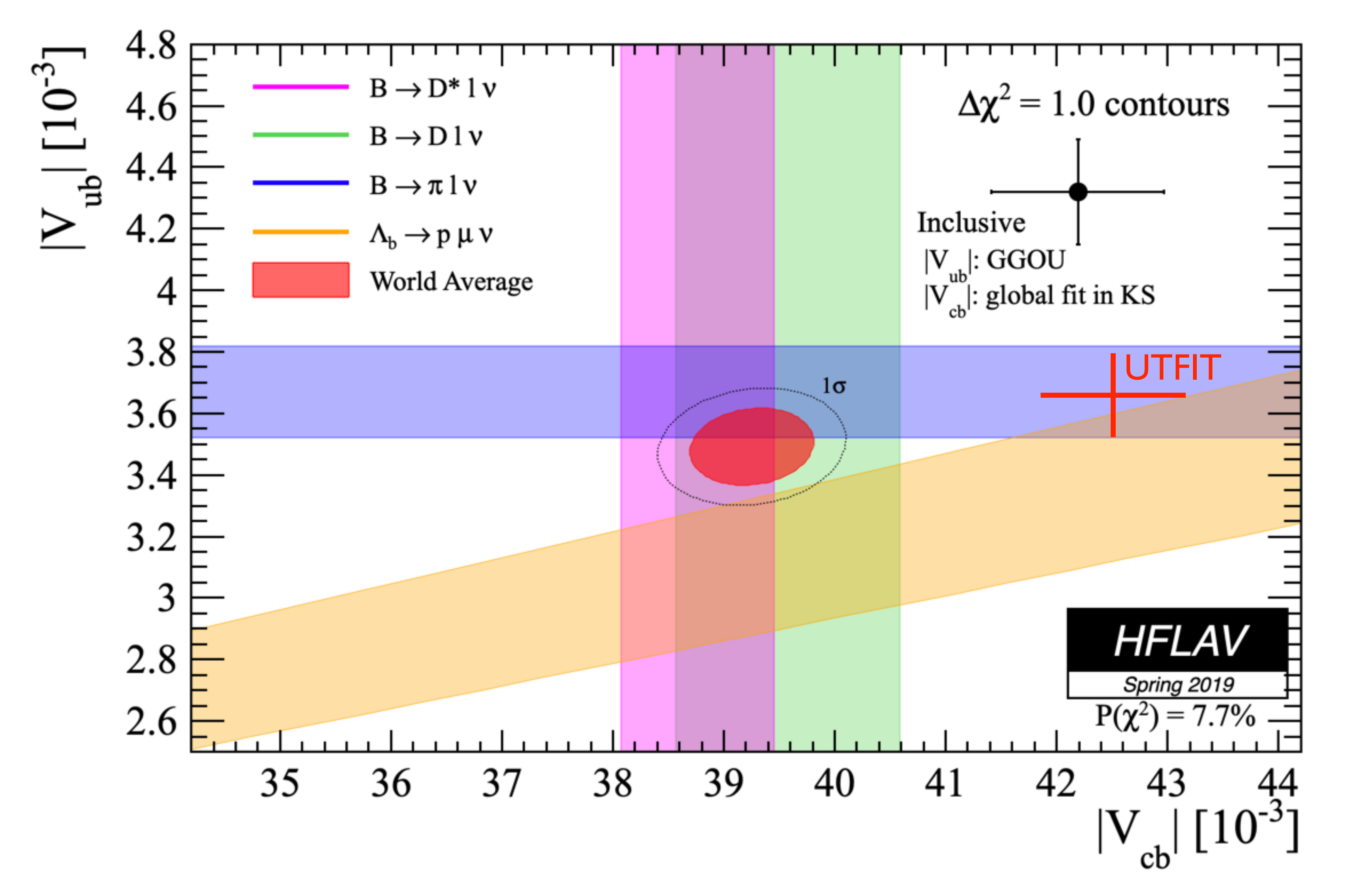
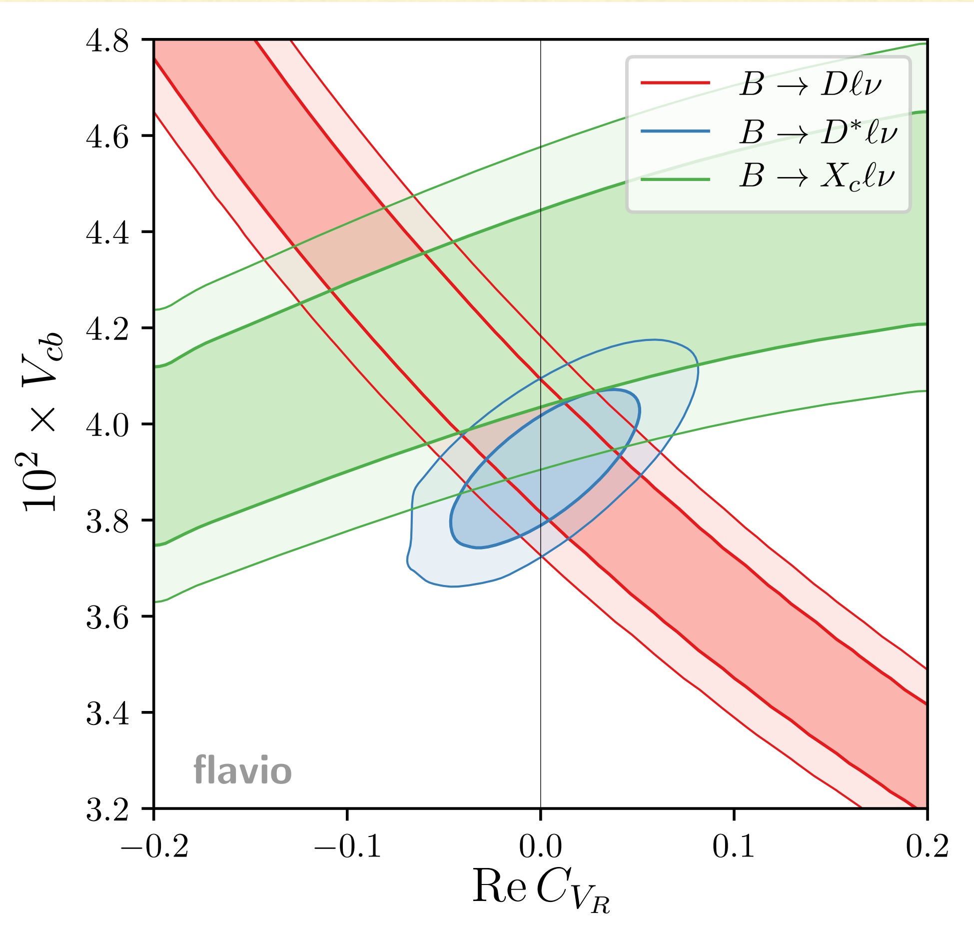
PDG AVERAGES

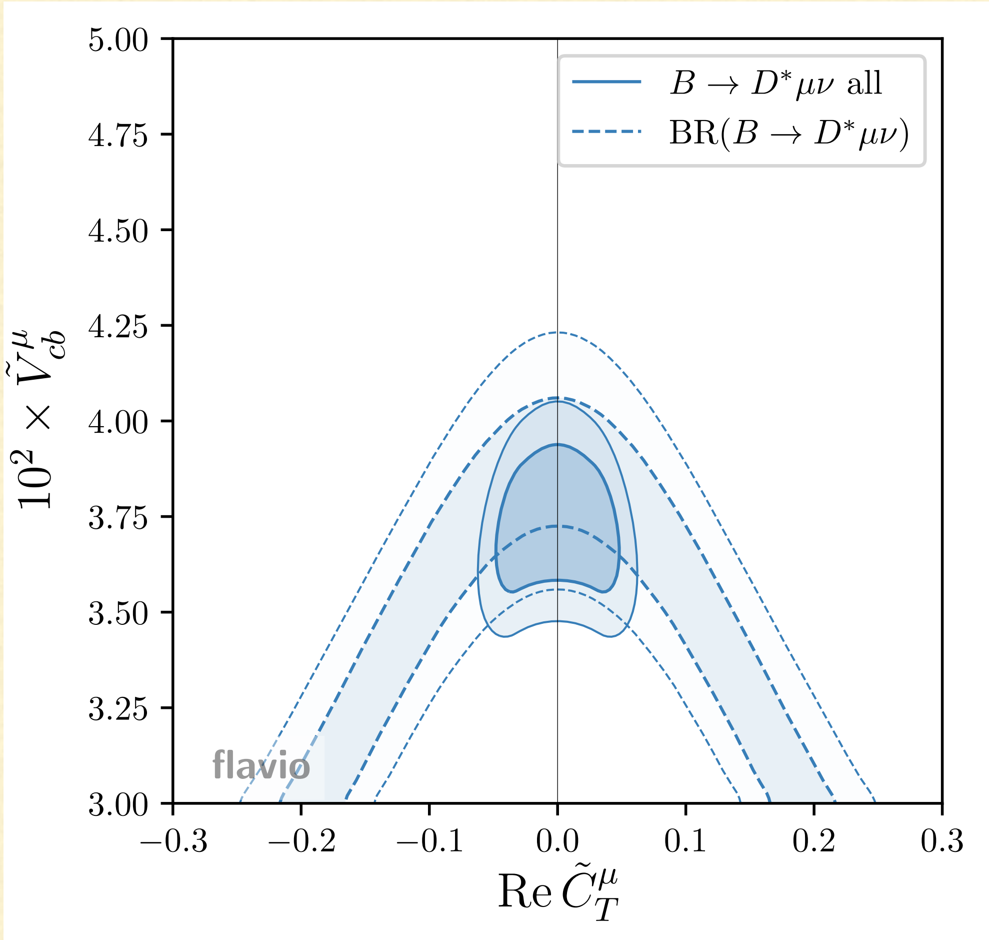




Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement

NEW PHYSICS?





Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data (Crivellin, Pokorski etc.)

For a recent analysis see Jung & Straub 1801.01112

The importance of $|V_{xb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

V_{cb} plays an important role in UT

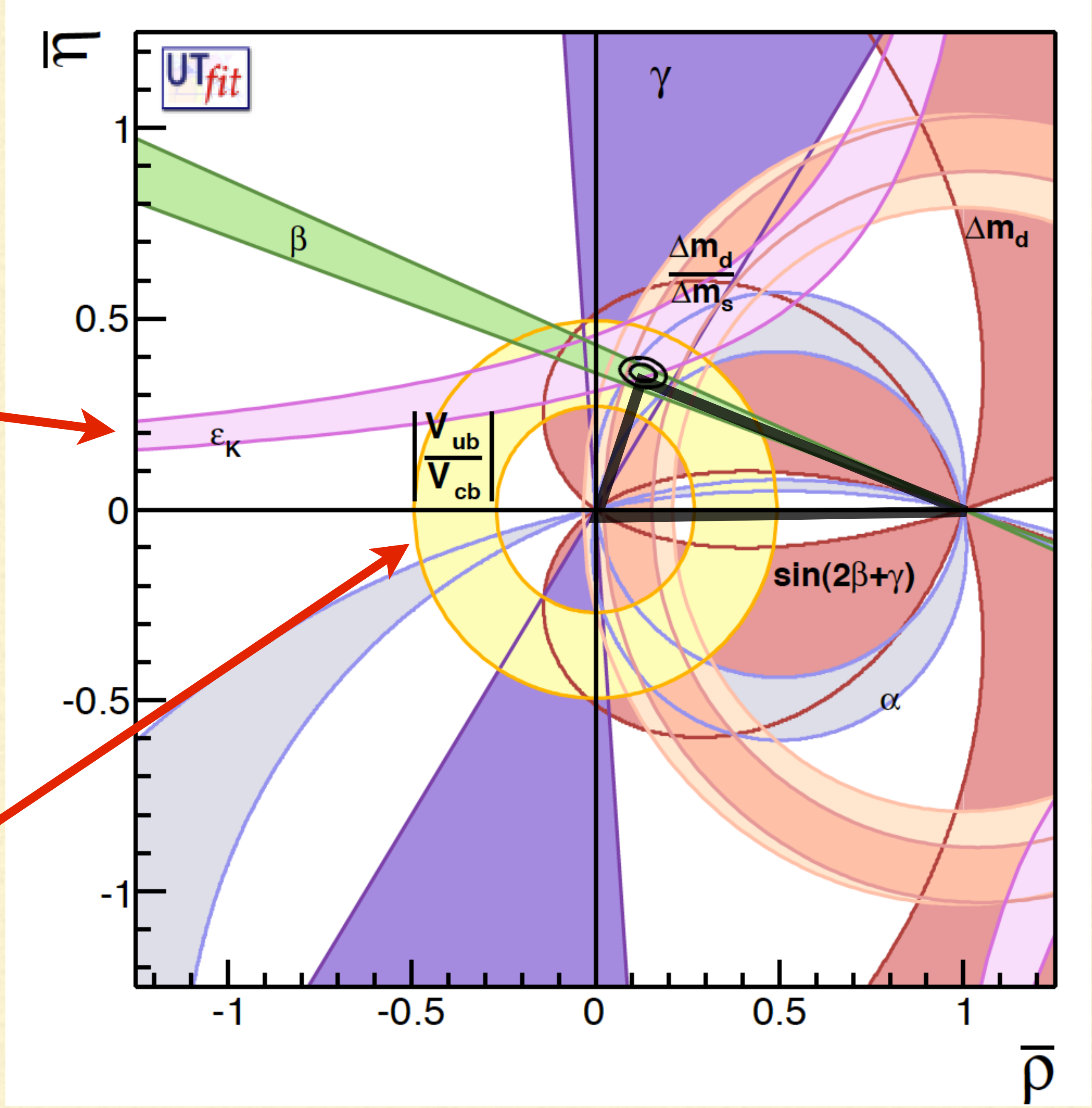
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty.

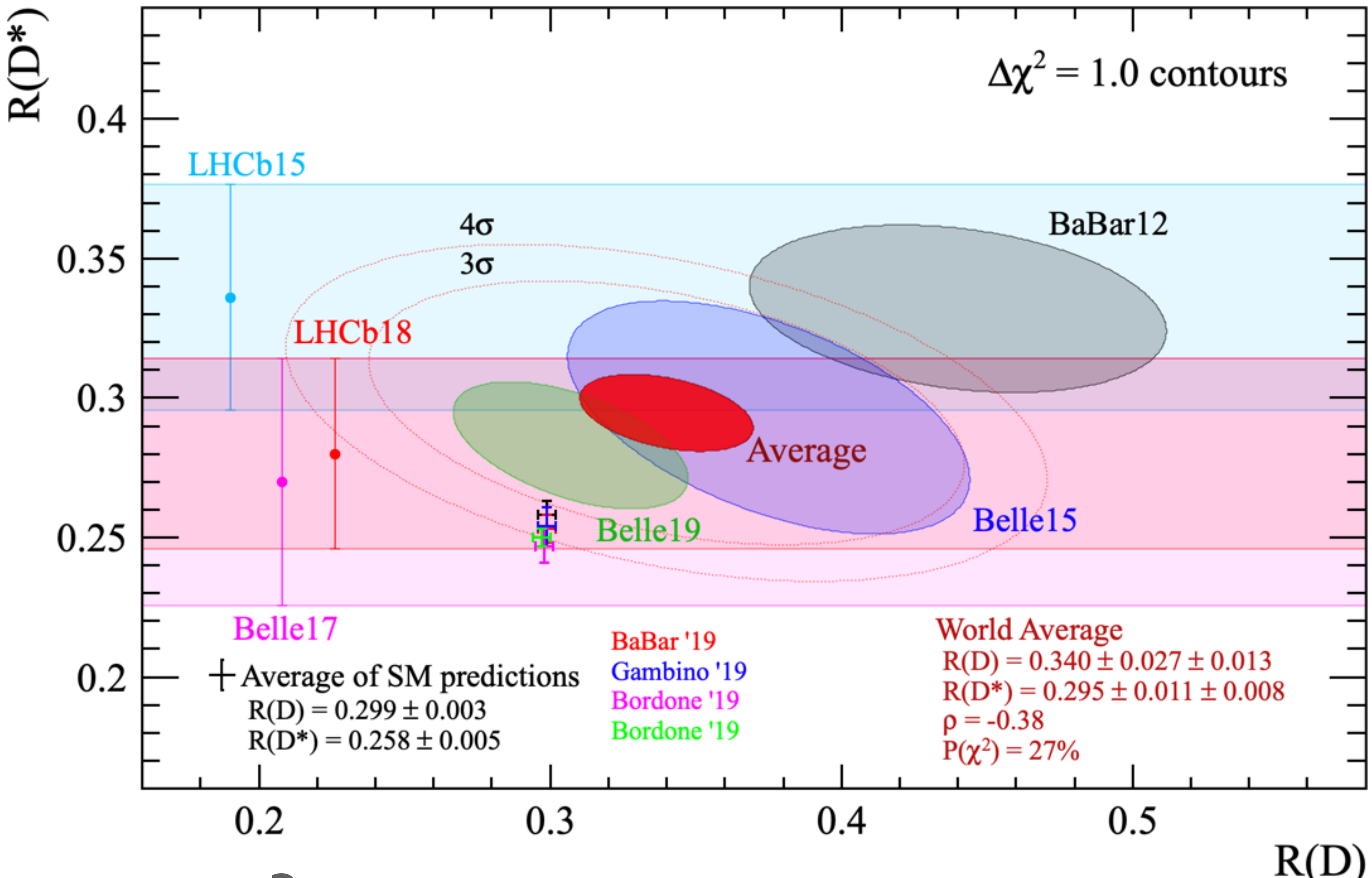
V_{ub}/V_{cb} constrains directly the UT



Our inability to determine precisely V_{xb} hampers significantly NP searches

VIOLATION OF LFU WITH TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



~3σ

M. Rotondo, Beauty 2019

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.

kinetic scheme fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice

FIT RESULTS

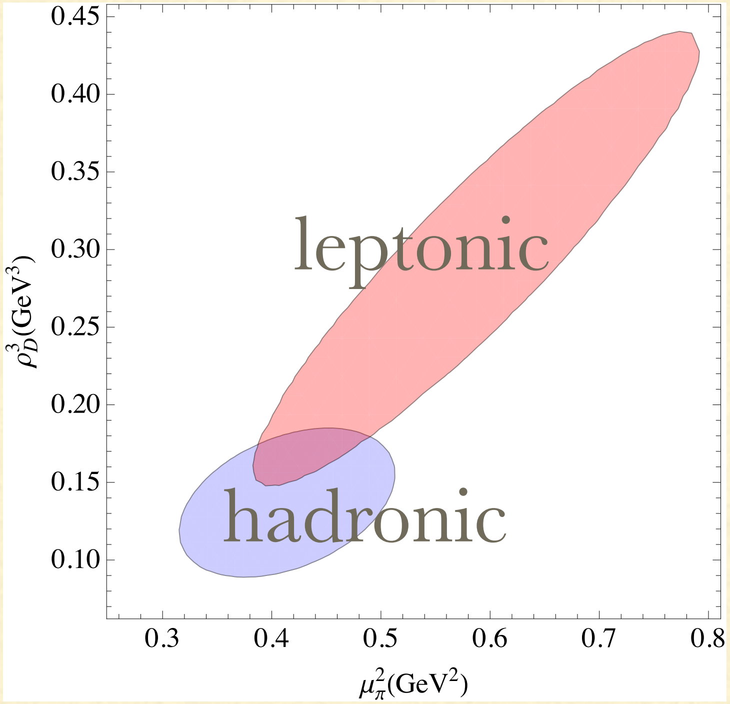
m_b^{kin}	$\overline{m}_c(3\,\text{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

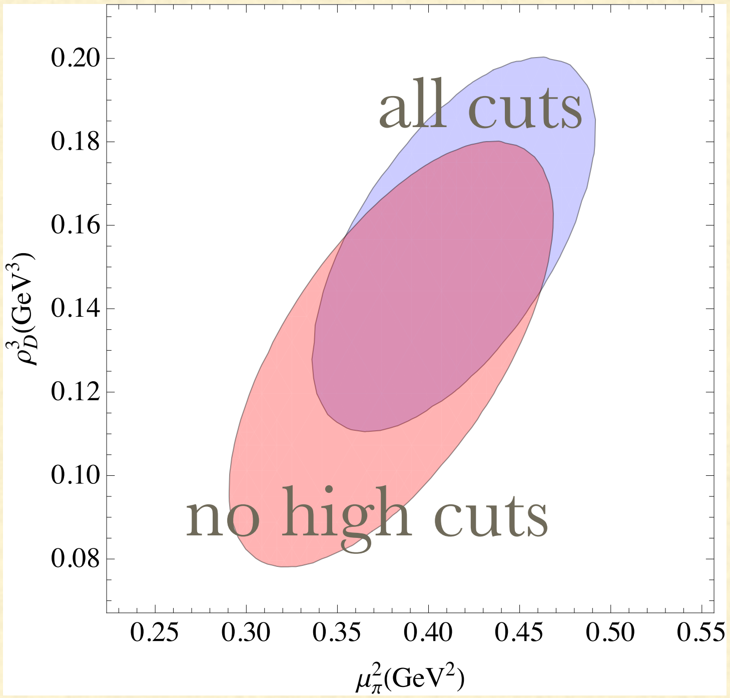
this is HFLAV fit

Alberti, Healey, Nandi, PG, 1411.6560

Without mass constraints $m_b^{kin}(1\text{GeV}) - 0.85\,\overline{m}_c(3\text{GeV}) = 3.714 \pm 0.018\,\text{GeV}$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results





HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Mannel,Turczyk,Uraltsev 1009.4622

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\,\mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers. The rest of the fit is unchanged, with slightly smaller theoretical errors

$$\boxed{|V_{cb}| = 42.00(64) \times 10^{-3}}$$

Healy,Turzcyk,PG 1606.06174

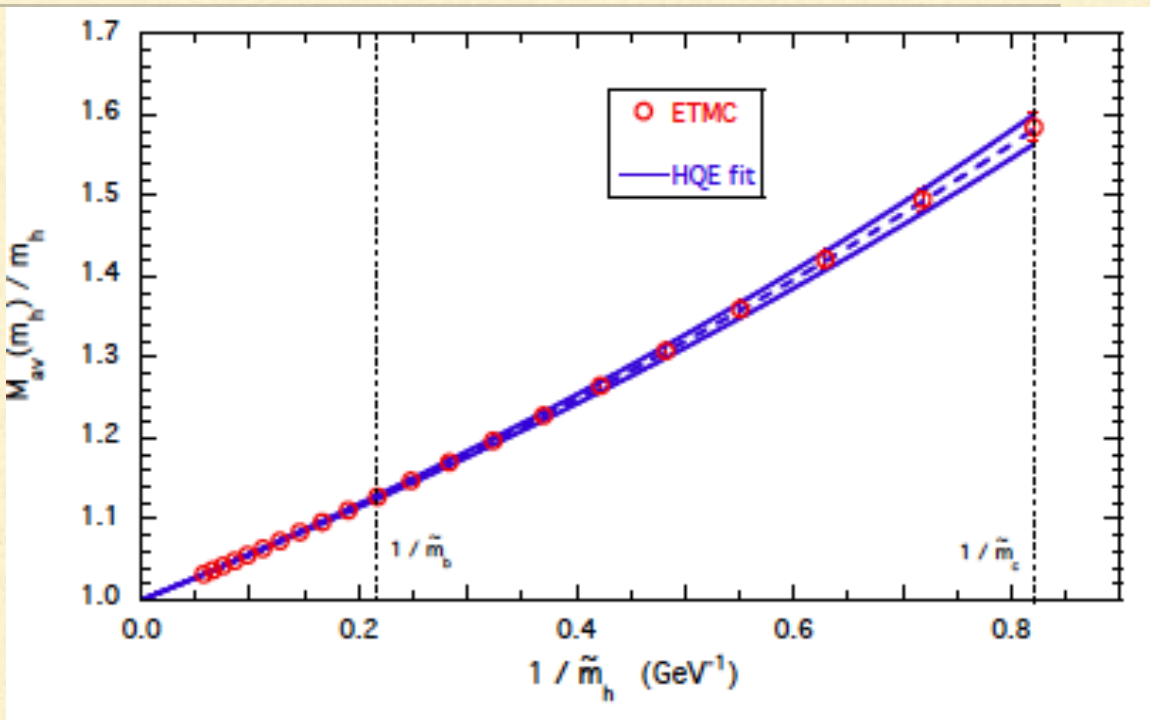
PROSPECTS for INCLUSIVE V_{cb}

- Theoretical uncertainties dominate
- $O(\alpha_s \, \rho_D^3/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- 3loop relation between MS and kin scheme just completed, 2005.06487
- $O(\alpha_s^3)$ corrections to total width feasible, needed for 1% uncertainty
- Electroweak (QED) corrections require attention
- New observables in view of Belle II: FB asymmetry (Turczyk) could be measured already by Babar and Belle now, q^2 moments (Fael, Mannel, Vos)…
- **Lattice QCD** is the next frontier

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \ldots$$



- on the lattice one can compute mesons for arbitrary quark masses
  see also Kronfeld & Simone hep-ph/0006345, Fermilab-MILC 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections
- Results consistent with s.l. fits, improvements under way, also following new 3loop calculation of pole-kinetic mass relation

INCLUSIVE SL DECAYS ON THE LATTICE

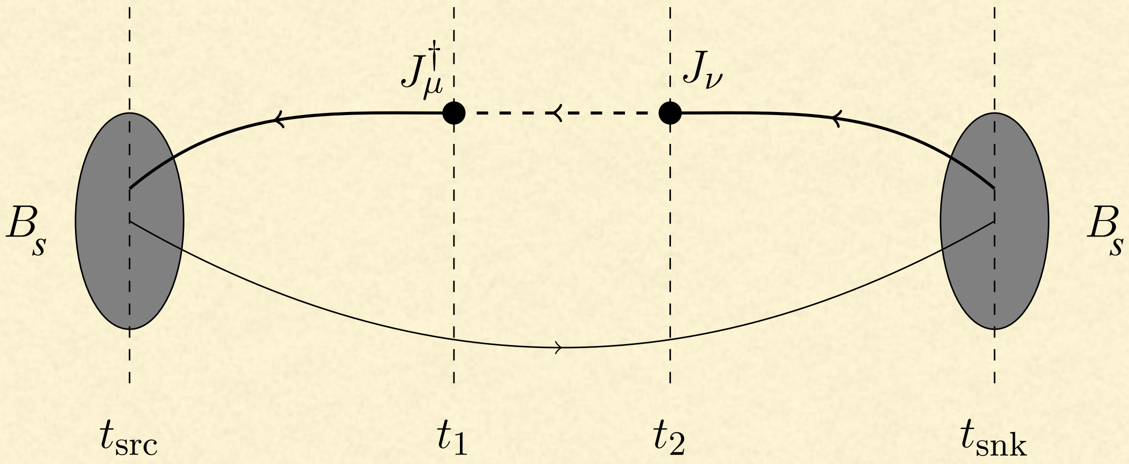
S. Hashimoto, PG 2005.13730

$$W^{\mu\nu} \sim \sum_{X_c} \delta^{(4)}(p-q-r)\langle B_s | J^{\mu\dagger} | X_c \rangle \langle X_c | J^{\nu} | B_s \rangle \sim \text{disc} \int d^4x\, e^{-iqx} \langle B_s | T J^{\mu\dagger}(x) J^{\nu}(0) | B_s \rangle$$

$$\frac{d\Gamma}{d\mathbf{q}^2 d\omega dE_\ell} = \frac{G_F^2 |V_{cb}^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}$$ (ω hadr. energy)

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q}^2 \sqrt{\boldsymbol{q}^2} \sum_{l=0}^{2} \bar{X}^{(l)} \qquad \bar{X}^{(l)} \equiv \int_{\sqrt{m_{D_s}^2+\boldsymbol{q}^2}}^{m_{B_s}-\sqrt{\boldsymbol{q}^2}} d\omega\, X^{(l)}$$

4point functions on the lattice are related to the hadronic tensor in unphysical region



$$\sum_{\boldsymbol{x}} e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \frac{1}{2m_{B_s}} \langle B_s(\mathbf{0}) | J_\mu^\dagger(\boldsymbol{x}, t) J_\nu(\mathbf{0}, 0) | B_s(\mathbf{0}) \rangle$$

$$\sim \langle B_s(\mathbf{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) e^{-\hat{H}t} \tilde{J}_\nu(\boldsymbol{q}) | B(\mathbf{0}) \rangle$$

$$\int_0^\infty d\omega\, K(\omega, \boldsymbol{q}) \langle B_s(\mathbf{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) \delta(\hat{H} - \omega) \tilde{J}_\nu(\boldsymbol{q}) | B_s(\mathbf{0}) \rangle$$

$$= \langle B_s(\mathbf{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) K(\hat{H}, \boldsymbol{q}) \tilde{J}_\nu(\boldsymbol{q}) | B_s(\mathbf{0}) \rangle$$

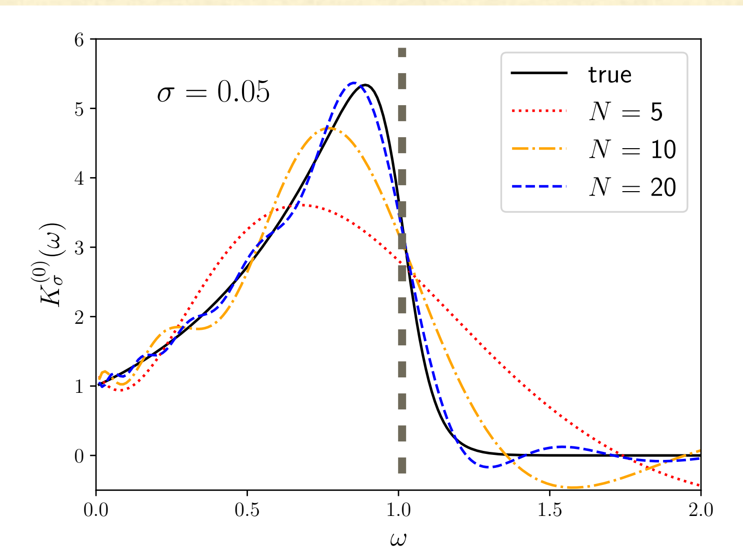
K approximated by polynomial $\quad K(\hat{H}, \boldsymbol{q}) = k_0(\boldsymbol{q}) + k_1(\boldsymbol{q}) e^{-\hat{H}} + \cdots + k_N(\boldsymbol{q}) e^{-N\hat{H}}$

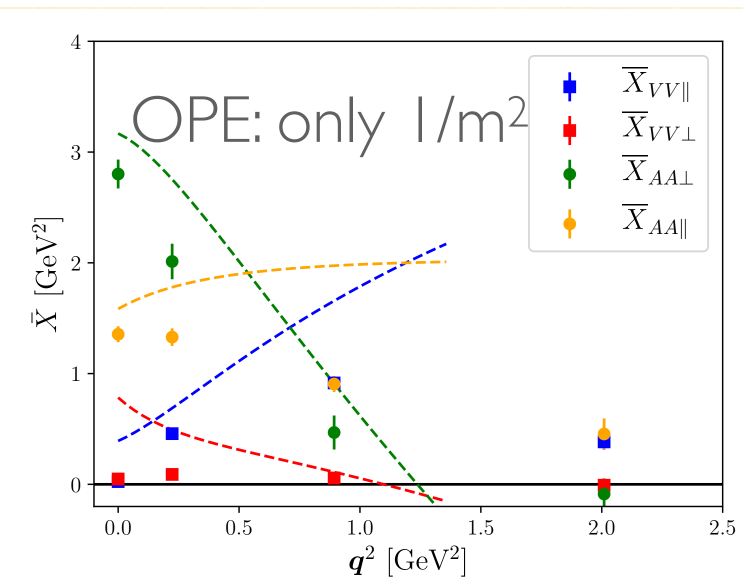
A PILOT NUMERICAL STUDY

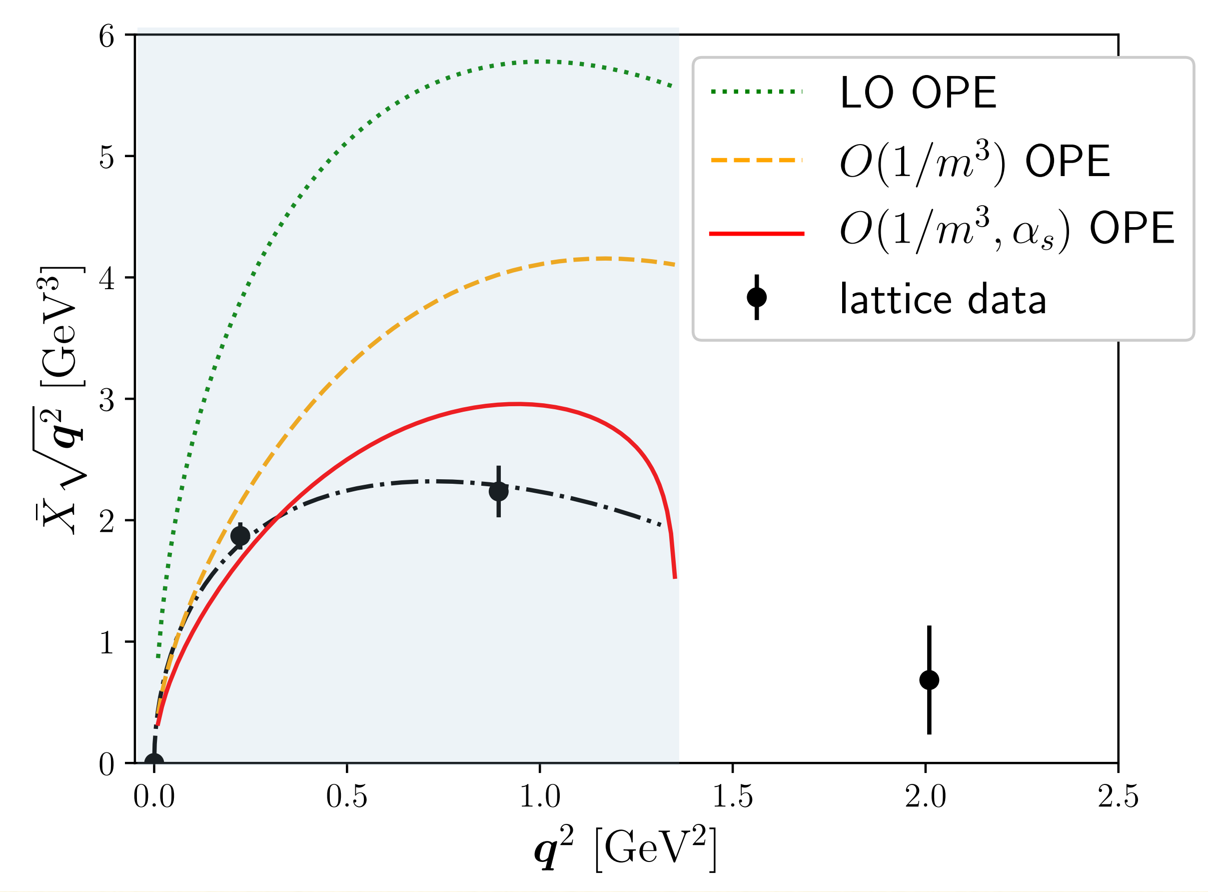
Smeared spectral functions can be computed on the lattice, see also 1704.08993, 1903.06476, 2001.11779

Using Chebychev polynomials to approx the kernel, 2+1 flavours of Moebius domain wall fermions with 1/a=3.6GeV, one gets

M_{B_s}=3.45 GeV, i.e. $m_b \approx$2.70GeV, $\omega_{part} = \sqrt{m_c^2 + \mathbf{q}^2} \sim$ 1.0-1.5 GeV







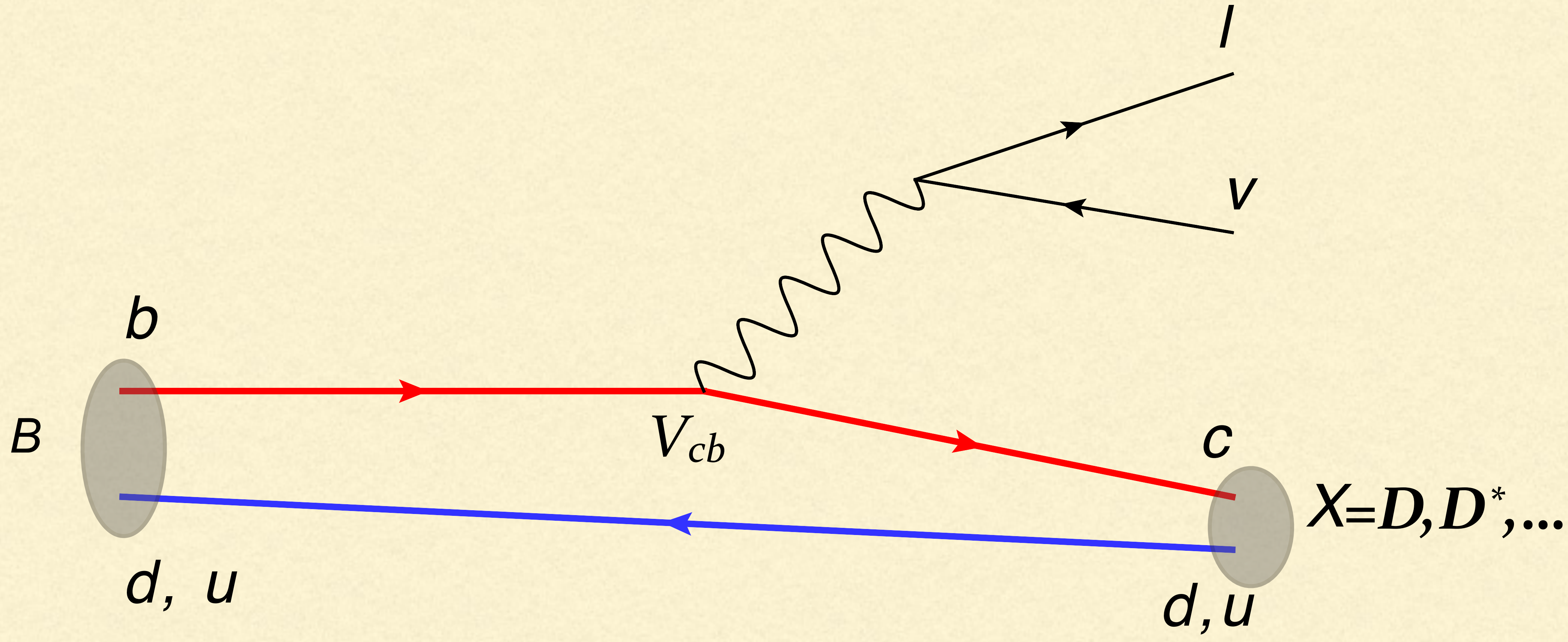
Promising! many applications to *B*,*D* incl. decays to $X_{u,c}$ *etc*

$\Gamma/|V_{cb}|^2 \; = \; 4.88(57) \times 10^{-13}$ GeV Lattice

$\Gamma/|V_{cb}|^2 = 5.41(82) \times 10^{-13}$ GeV OPE

including all known corrections

EXCLUSIVE DECAYS



There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

$$\langle D(k)|\,\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2}q^\mu\right] f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2}q^\mu f_0^{B\to D}(q^2)$$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp…

FORM FACTOR PARAMETRIZATION

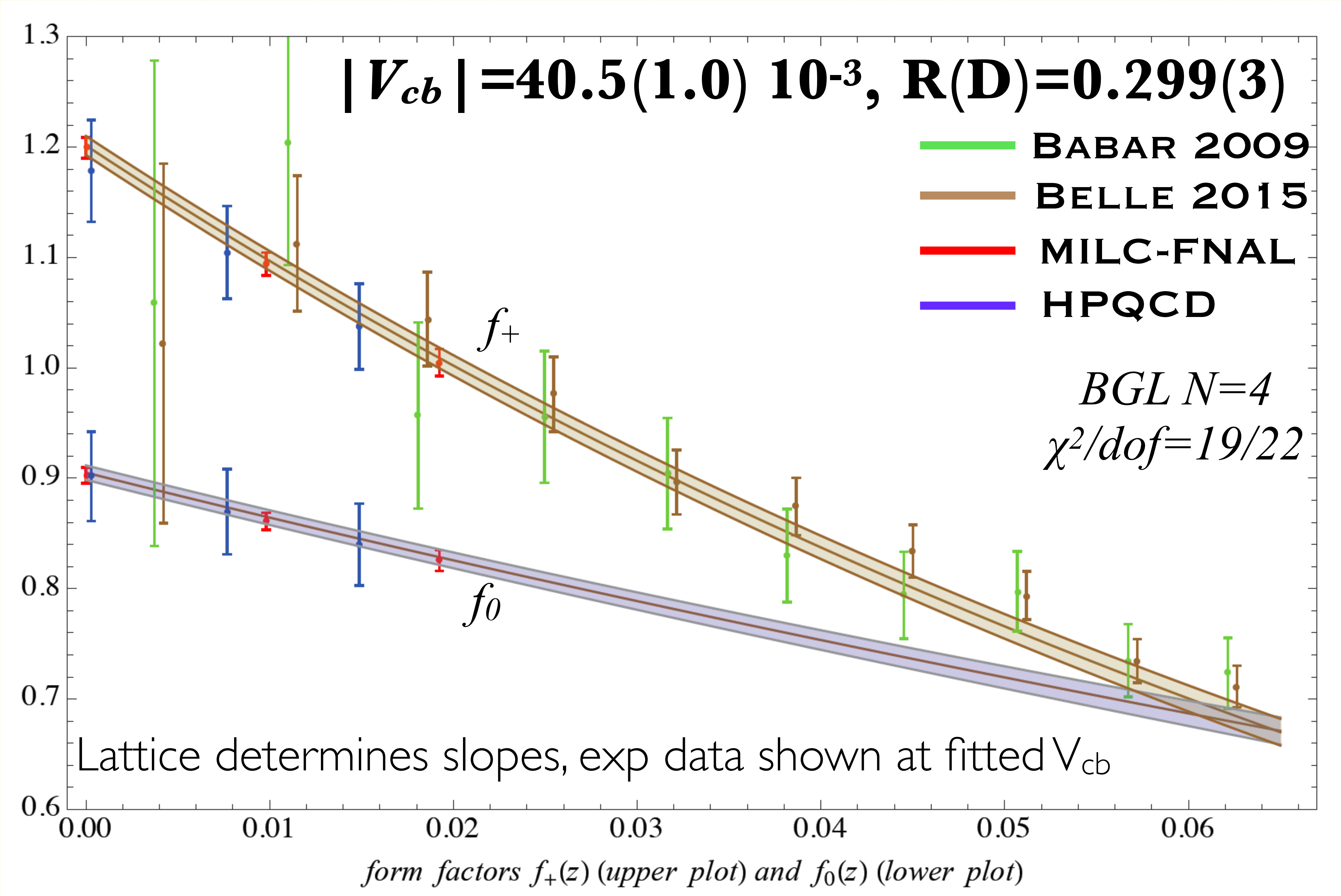
- A model independent parametrization is necessary
- Boyd-Grinstein-Lebed (**BGL** 1995-7) is based on crossing & analyticity, power series in $z(q^2)$ with coefficients a_i that satisfy unitarity bounds $\Sigma_i\, a_i^2 \leq 1$
- HQET for $B^{(*)} \to D^{(*)}$ form factors:

$$F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \ldots\right] \quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$$

- $c_{b,c}$ can be computed using subleading IW functions from QCD sumrules Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330
- Ratios free of Isgur-Wise function: can use to get ***strong unitarity bounds*** but $1/m_c^2$ corrections can be significant as shown by lattice calculations
- Caprini-Lellouch-Neubert (**CLN,** 1998) parametrization is simpler with fewer parameters, but relies on NLO HQET. All exp analyses before 2017 were based on CLN, did not include uncertainty.

LATTICE + EXP BGL FIT for $B \to Dl\nu$

Bigi, PG 1606.08030



R(D) 1.3σ from exp

FLAG has very similar results

CLN cannot fit both ff

$|V_{cb}|$ from $B\to D^* l\nu$ new HFLAV (2019)

LQCD provides only light lepton FF at zero recoil, w=1, where rate vanishes.
Experimental results must therefore be **extrapolated to zero-recoil**

Exp error only ~1.1% $\mathcal{F}(1)\eta_{ew}|V_{cb}| = 35.27(38) \times 10^{-3}$

Beware: HFLAV extrapolate with CLN (w/o error)

Two unquenched lattice calculations

$\mathcal{F}(1) = 0.906(13)$
Bailey et al 1403.0635 (FNAL/MILC)

$\mathcal{F}(1) = 0.895(26)$
Harrison et al 1711.11013 (HPQCD)

Using their average 0.904(12): $\boxed{|V_{cb}| = 38.76(69)\ 10^{-3}}$

~ 3.4σ or ~ 8% from inclusive determination

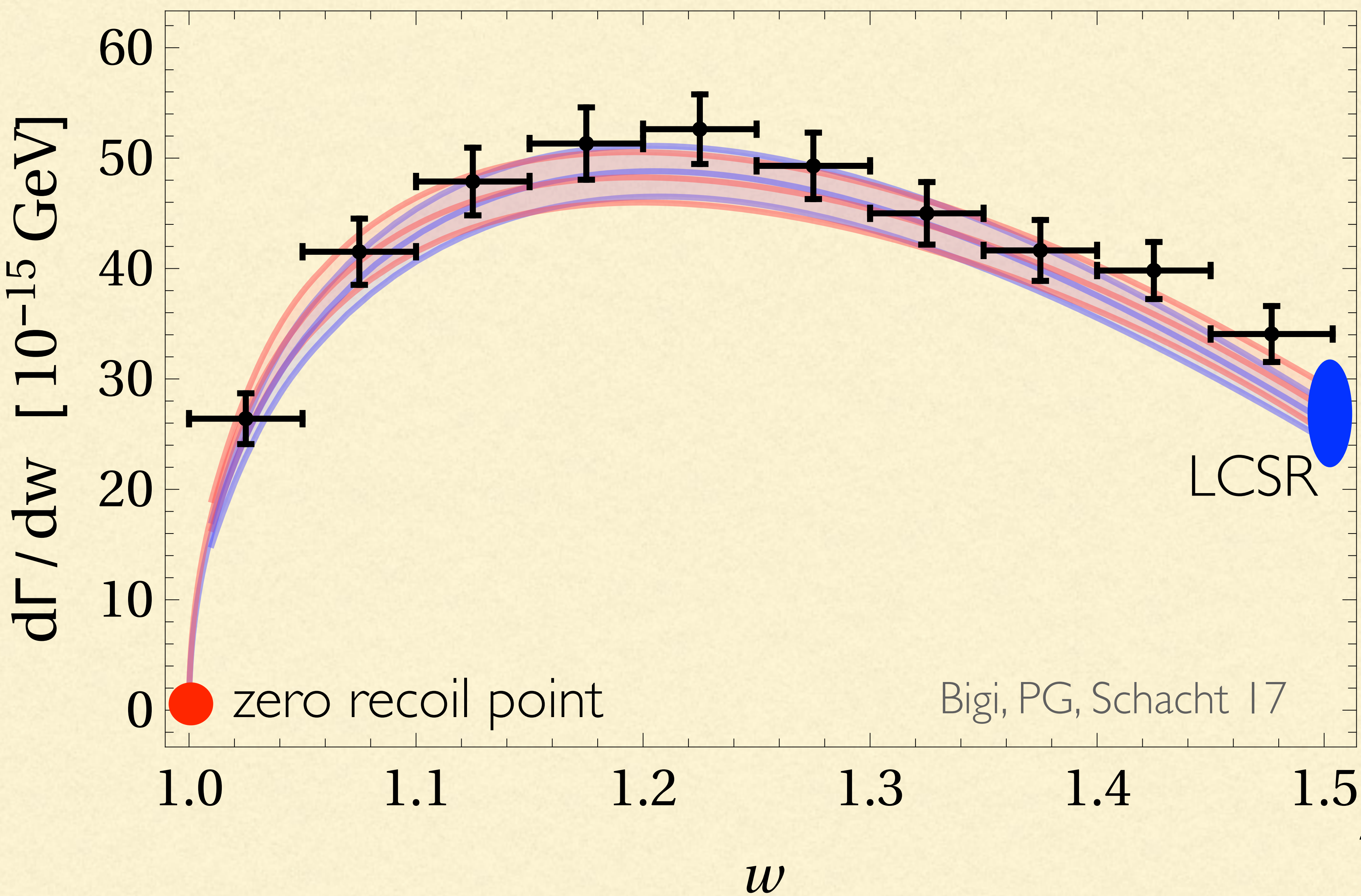
Heavy quark sum rules $F(1) < 0.925$ and estimate of inelastic contribution $F(1) \approx 0.86$

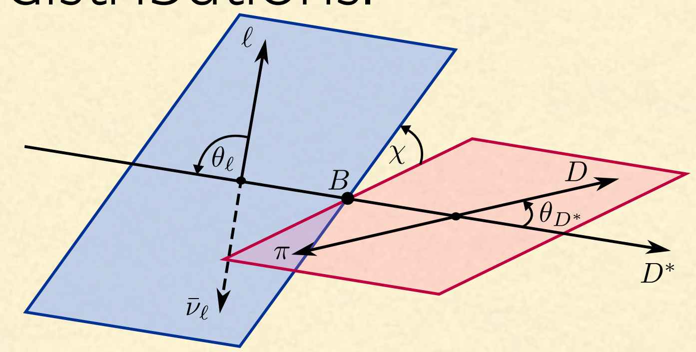
Mannel, Uraltsev, PG, 2012

2017 tagged Belle analysis (still preliminary)

1702.01521

w and angular deconvoluted distributions (independent of parameterization).
All previous analyses are CLN based and do NOT provide diff distributions.



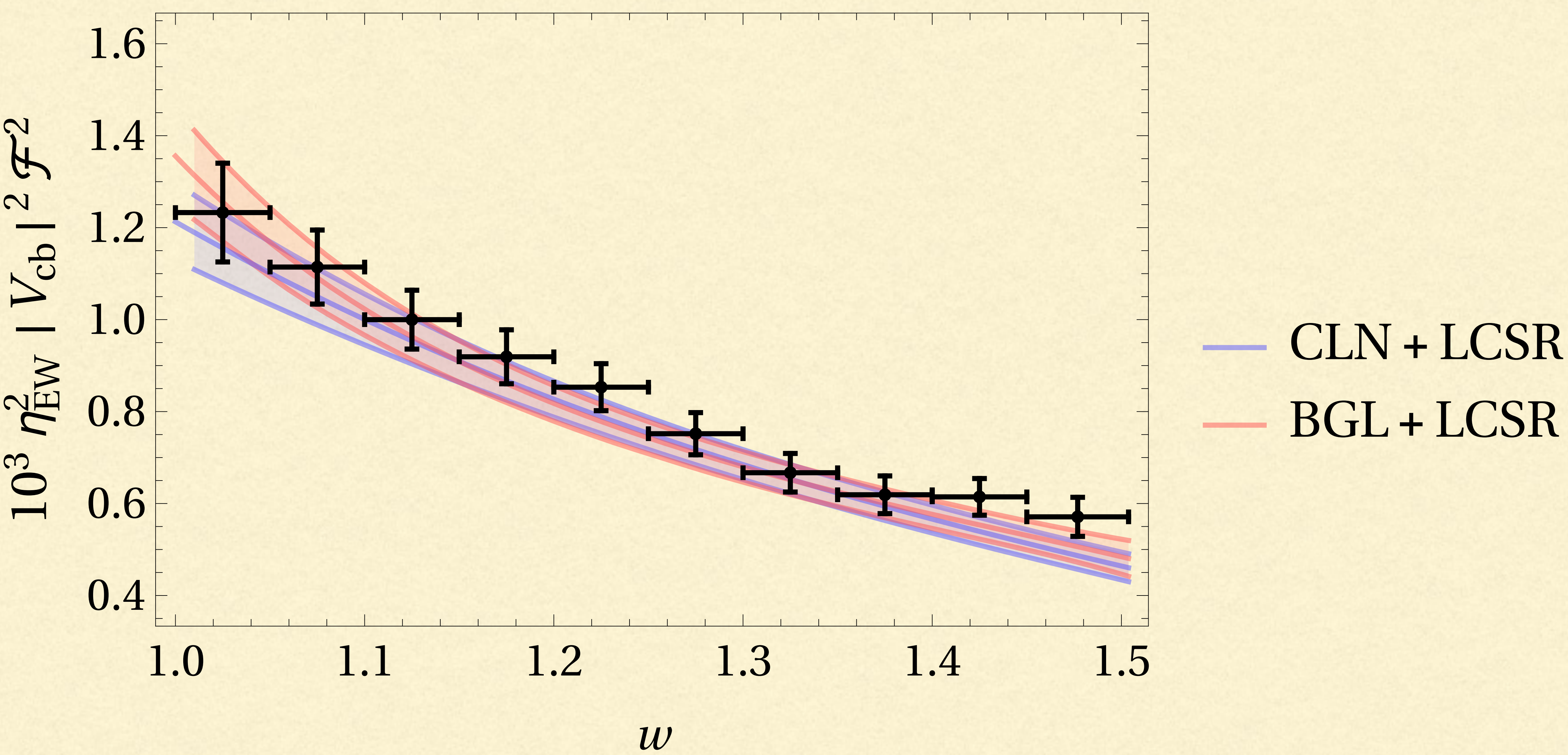


see also
Kobach & Grinstein

$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

Bands show two parametrizations both fitting data well, with 6% different V_{cb}

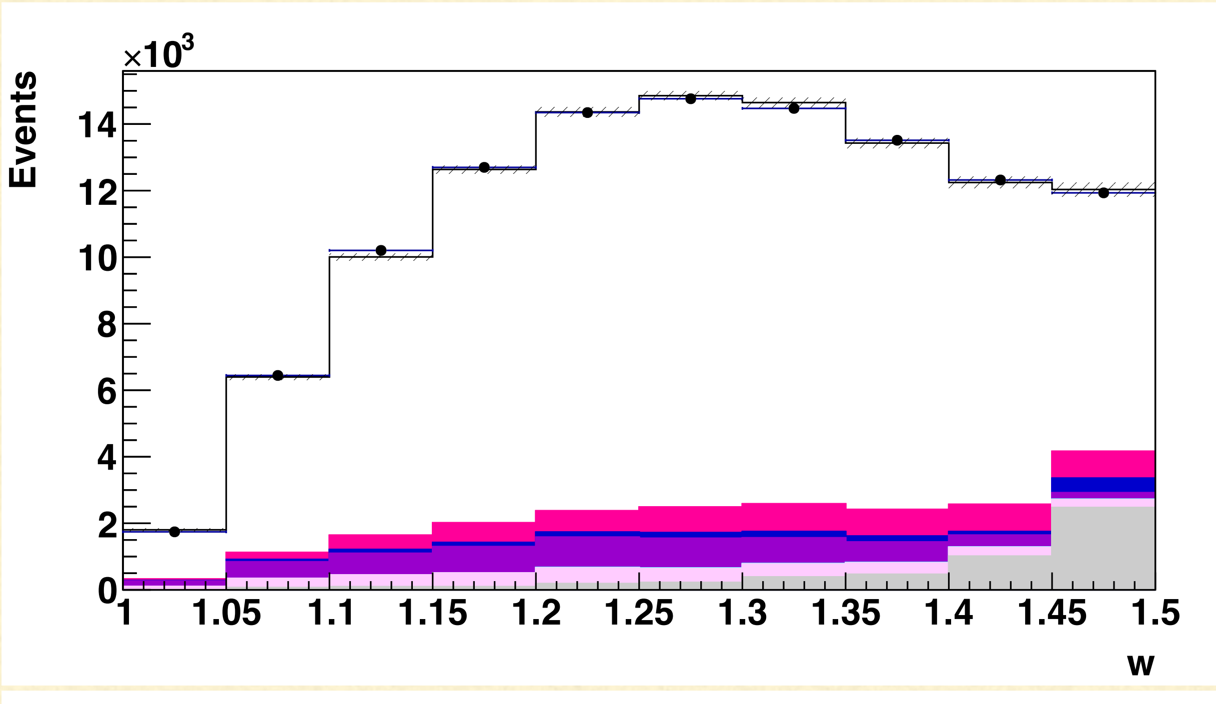
EXTRAPOLATING TO ZERO RECOIL

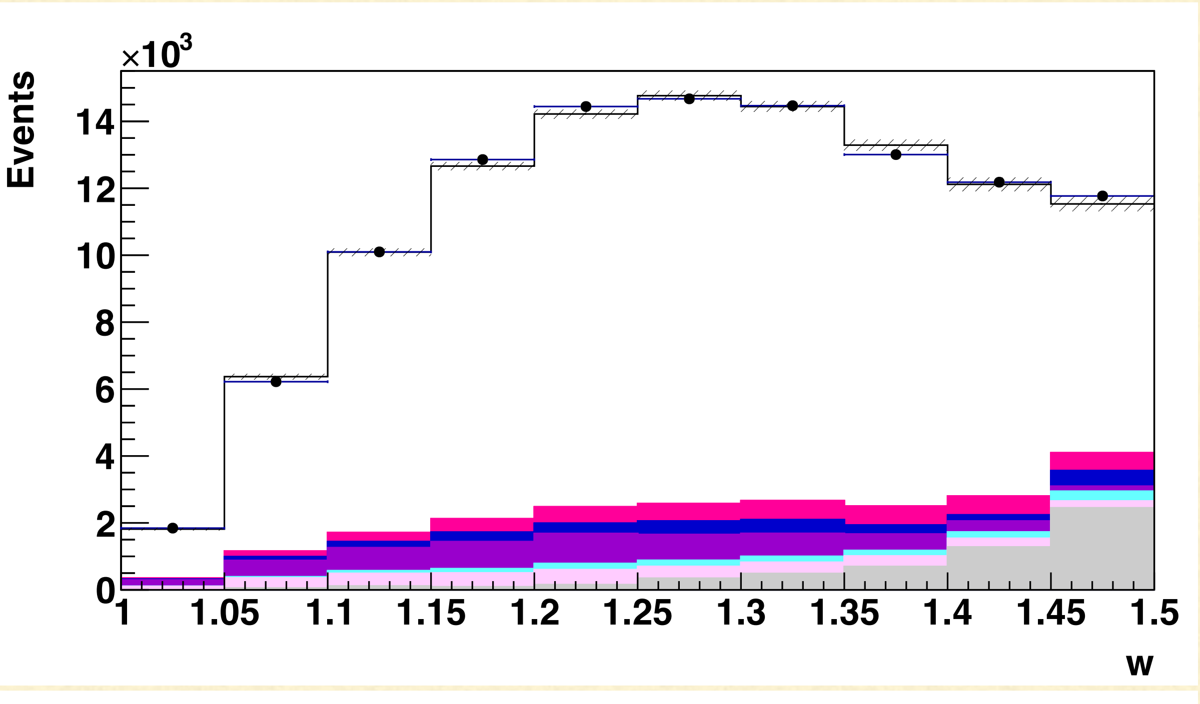


+30 additional angular bins

2018 UNTAGGED BELLE ANALYSIS

1809.03290v3

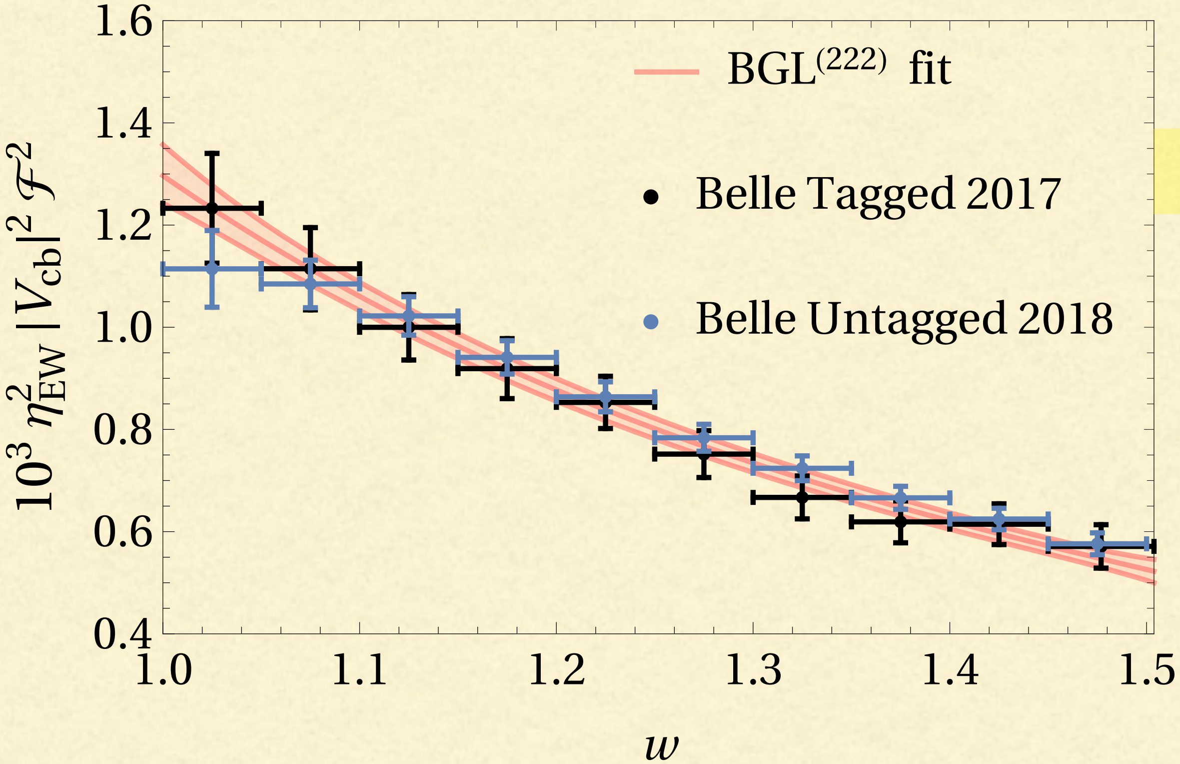




- Full Belle dataset, most precise study to date; provides data in a way that can be reanalysed with different assumptions. Central values obtained without systematics.
- CLN and BGL(102) analysis lead to very similar results, suggesting low **$|V_{cb}|$=38.4(0.9) 10^{-3}** .
- We used BGL(222) to fit the data, taking into account *D'Agostini effect* and got **$|V_{cb}|$=39.1($+1.5-1.3$) 10^{-3}** 1905.08209

A GLOBAL FIT TO 2017 & 2018 DATA

Jung, Schacht, PG 1905.08209



BGL$^{(222)}$	Data + lattice (weak)	Data + lattice (strong)
χ^2/dof	80.1/72	80.1/72
$\lvert V_{cb}\rvert 10^3$	$39.6\,\binom{+1.1}{-1.0}$	$39.6\,\binom{+1.1}{-1.0}$
a_0^f	0.01221(16)	0.01221(16)
a_1^f	$0.006\,\binom{+32}{-45}$	$0.006\,\binom{+20}{-32}$
a_2^f	$-0.2\,\binom{+12}{-8}$	$-0.2\,\binom{+7}{-3}$
$a_1^{\mathcal{F}_1}$	$0.0042\,\binom{+22}{-22}$	$0.0042\,\binom{+19}{-22}$
$a_2^{\mathcal{F}_1}$	$-0.069\,\binom{+41}{-37}$	$-0.068\,\binom{+41}{-30}$
a_0^g	$0.024\,\binom{+21}{-9}$	$0.024\,\binom{+12}{-4}$
a_1^g	$0.05\,\binom{+39}{-72}$	$0.05\,\binom{+21}{-41}$
a_2^g	$1.0\,\binom{+0}{-20}$	$0.9\,\binom{+0}{-18}$

- No parametrization dependence (CLN and BGL give ~same central value)
- About 1σ higher than HFLAV, larger uncertainty on firmer ground, p-value ~24% ***1.9σ from inclusive, consistent with B to Dlv***
- We truncate the BGL series when additional terms do not change the fit (no overfitting!). Fit stable. Strong constraints irrelevant.

COMPARISON WITH HQET+QCDSR

1905.08209

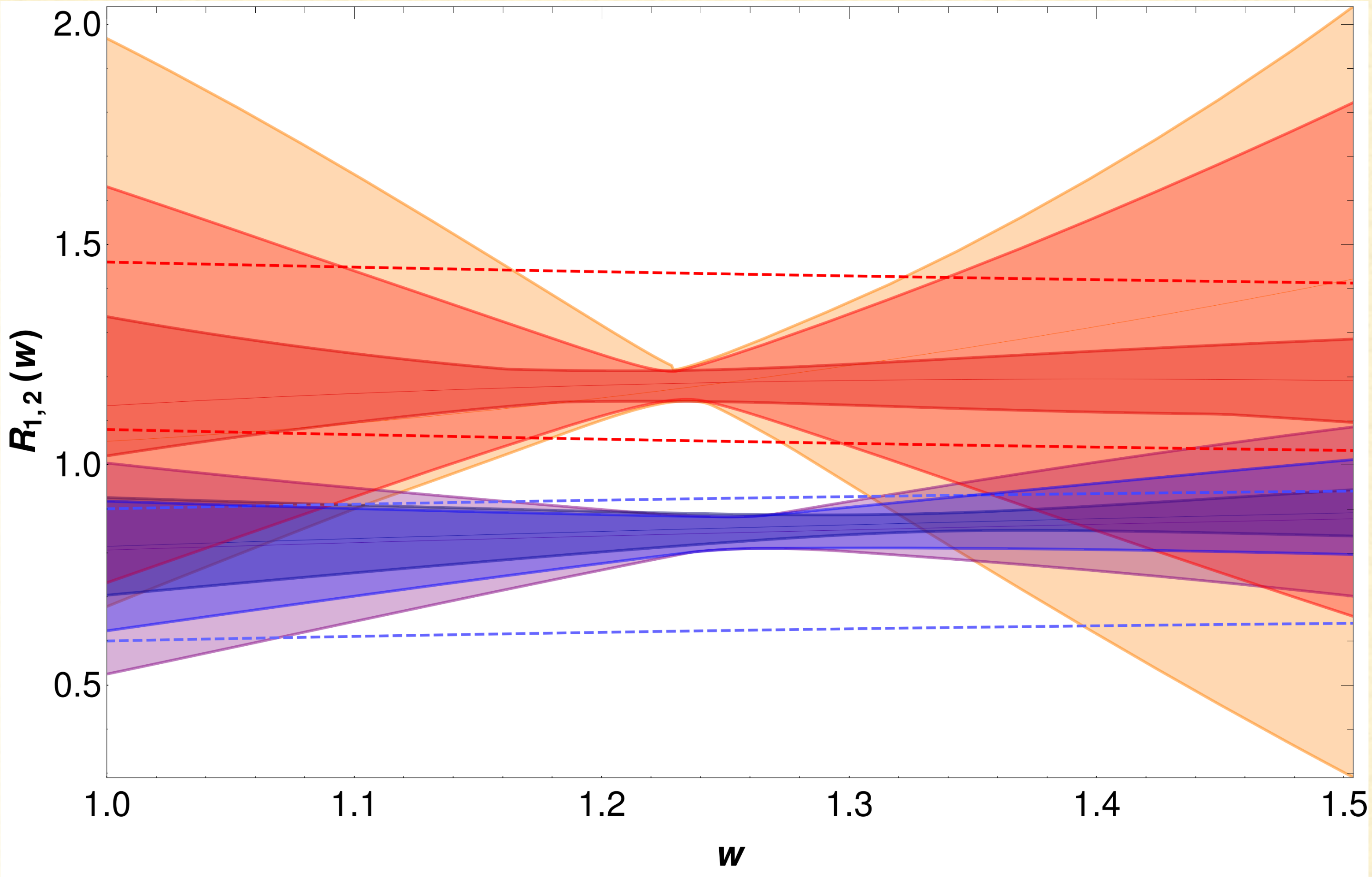
$$R_1(w) = \frac{V_4(w)}{A_1(w)} = 1 + O(\alpha_s, 1/m)$$

$$R_2(w) = \frac{w-r}{w-1}\left(1 - \frac{1-r}{w-r}\frac{A_5(w)}{A_1(w)}\right)$$

Comparison of $R_{1,2}$ from BGL fit to 2017+2018 data vs NLO HQET+QCDSR predictions (with parametric + 15% th uncertainty)

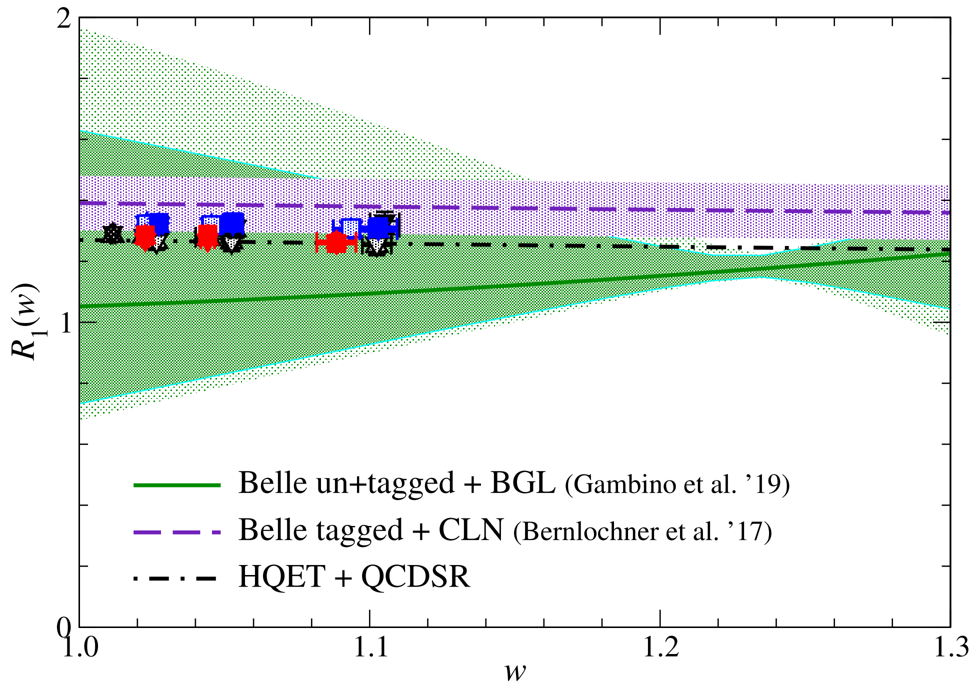
Very good agreement in both cases.

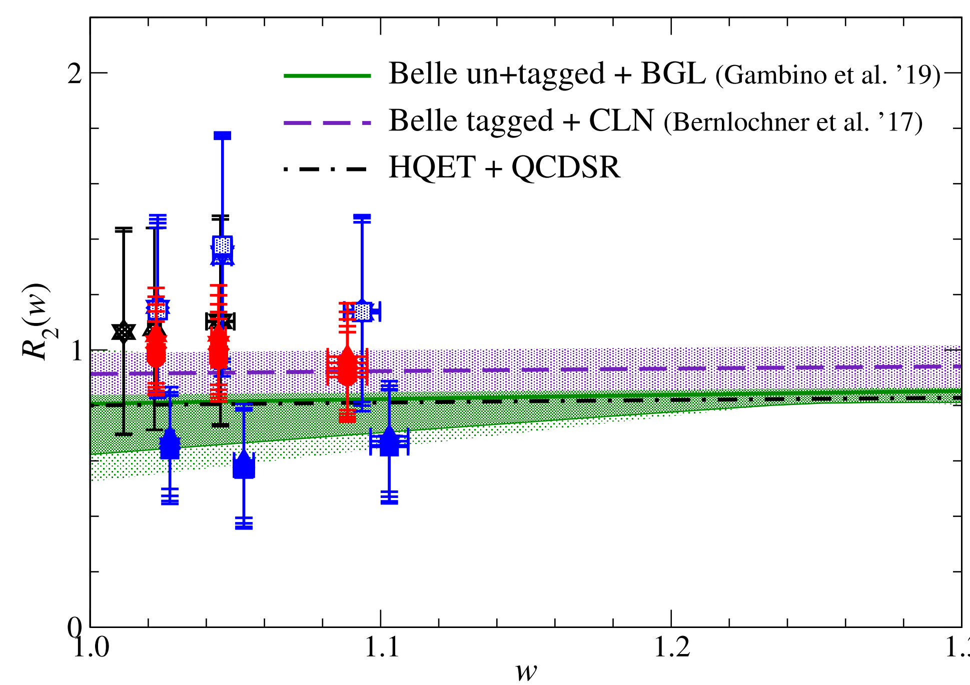
Narrower bands correspond to adding strong unitarity and LCSRs



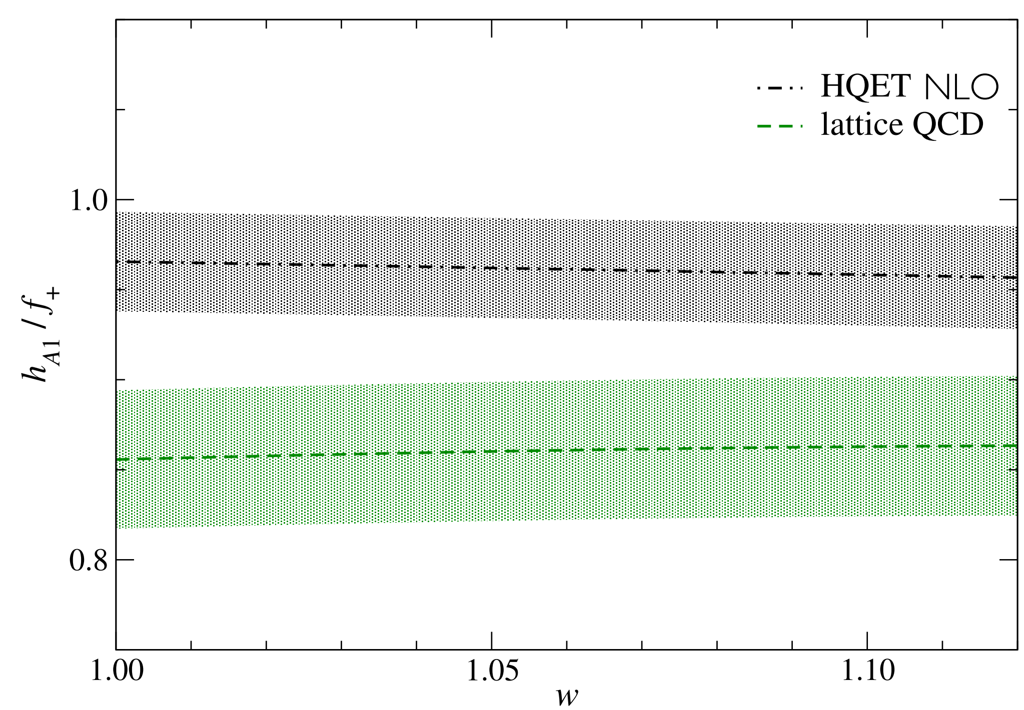
PRELIMINARY JLQCD RESULTS

1912.11770

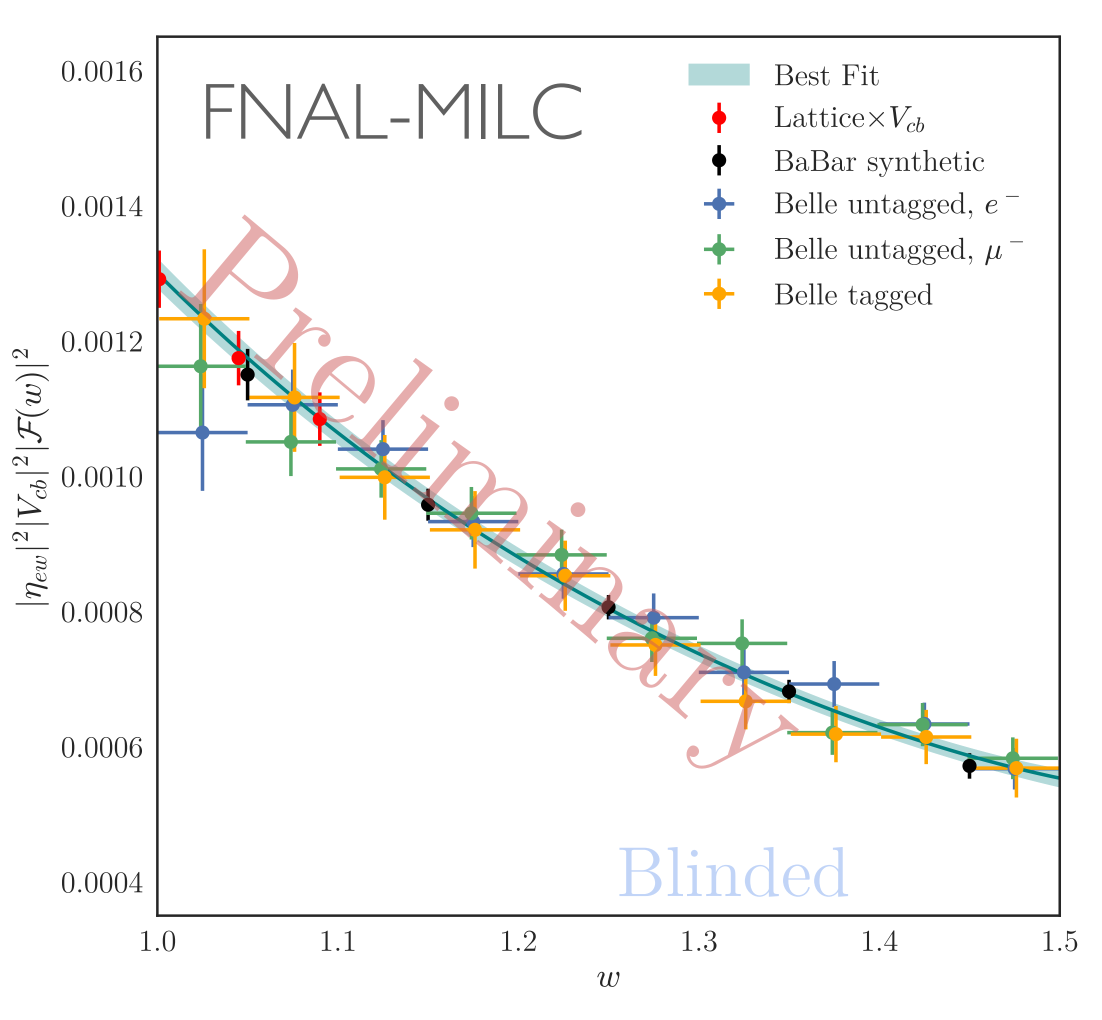




JLQCD results for $R_{1,2}$ are consistent with our latest fit and with NLO HQET+QCDSR, and confirm large power corrections in h_{A1}/f_+



THE IMPORTANCE OF THE SLOPE



A.Vaquero, BNL 9/2019

Here we use **new improved LCSR** results by Gubernari, Kokulu, van Dyk, 1811.00983 that improve upon 0809.0222

Blinding affects only marginally the slope of the ff F, which is key to V_{cb}.

Plot suggests large and well determined slope, $dF/dw|_{w=1}$.

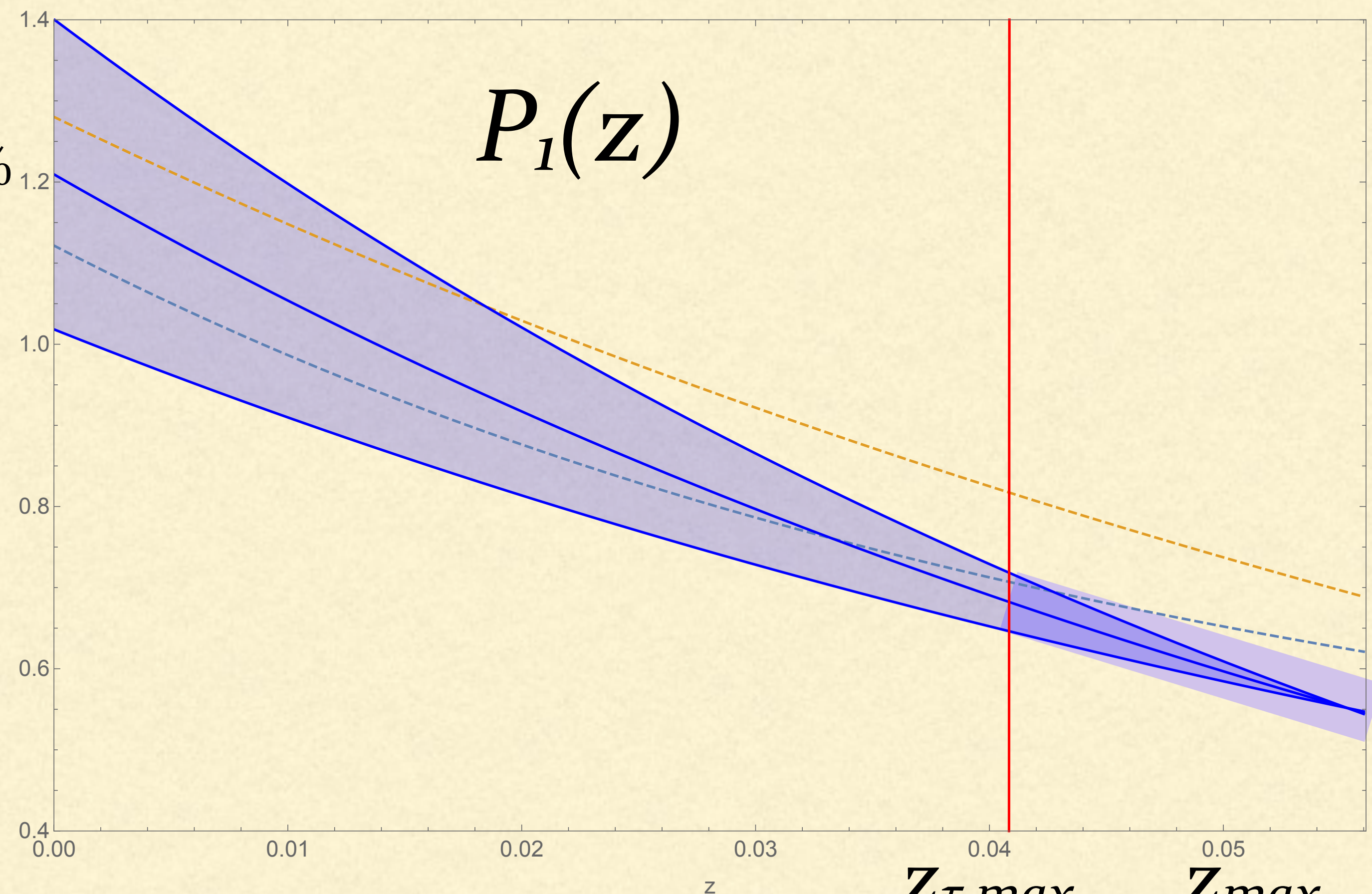
If it were -1.40(7) the fit could still accomodate a high V_{cb}

Constraints	$10^3\ V_{cb}$	χ^2
slope	40.8(0.8)	84.5/73
slope+LCSR	40.8(0.8)	88.0/76

SEMITAUONIC DECAYS

1905.08209

Decays with tau require pseudoscalar FF unconstrained from fit, no lattice calculation yet. We use kinematic constraint at q^2=0 and HQET with conservative uncertainty.



Endpoint constraint

$$P_1(w_{max}) = A_5(w_{max})$$

$$R(D^*) = 0.254^{+0.007}_{-0.006}\,,$$ 2.8σ from exp

$$P_\tau = -0.476^{+0.037}_{-0.034}\,,$$ 1.4σ from exp

$$F_L^{D^*} = 0.476^{+0.015}_{-0.014}\,,$$

Ref.	$R(D^*)$	Exp. deviation
[32]	$0.257(3)$	3.3σ
[13, 36]	$0.254\,\binom{7}{6}$	3.2σ
[34]	$0.257(5)$	3.1σ
[37]	$0.250(3)$	3.7σ

EXPLOITING HQET & LCSRS

1908.09398

Can HQET relations be further exploited, for instance including dominant NNLO terms $O(1/m_c^2)$?

$$F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}(w)\frac{\alpha_s}{\pi} + c^i_b(w)\epsilon_b + c^i_c(w)\epsilon_c + d^i\epsilon_c^2\right] \qquad \epsilon_{b,c} = \bar{\Lambda}/2m_{b,c}, \qquad \alpha_s \sim \epsilon_b \sim \epsilon_c^2$$

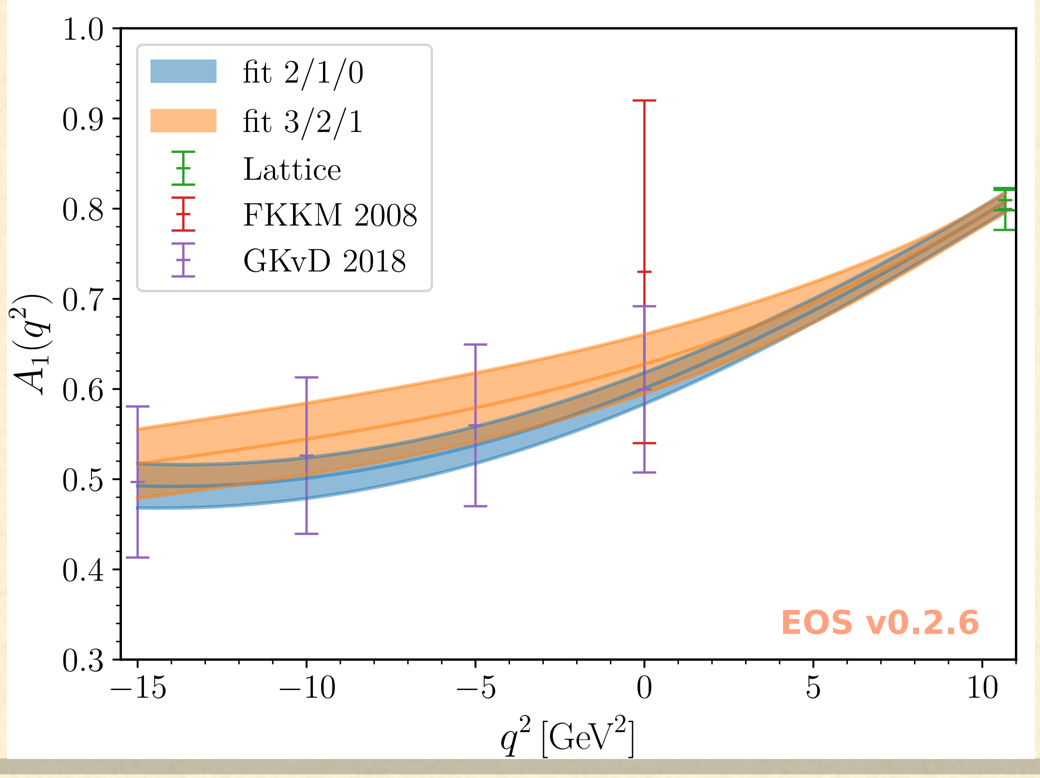
6 subsubleading IW functions, expanded around w=1. To fix them and for NP analyses we need additional info from LQCD or LCSR.

Bordone, Jung, Van Dyk, building on 1703.05330 (Bernlochner et al) and 1707.09977 (Jaiswal et al) use available LQCD and LCSR calculations and Belle B->D$^{(*)}$lv data using BGL with *strong unitarity bounds*

V_{cb}=40.3(0.8)10^{-3}

R(D)=0.297(3)

R(D*)=0.250(3)

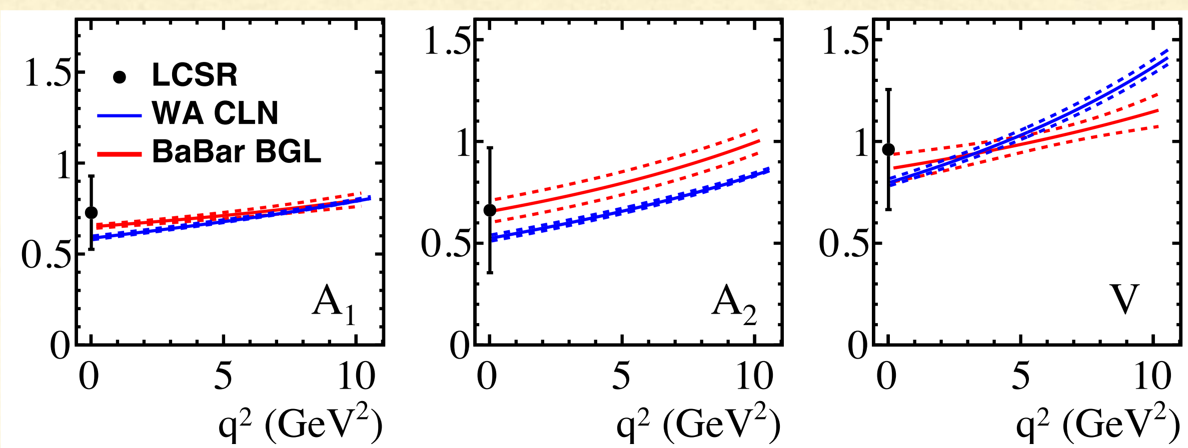


Same including B_s beyond SU(3), 1912.09335

NEW RESULTS BY BABAR AND LHCb

1903.10002, 2001.03225

Reanalysis of tagged B^0 and B^+ data, unbinned 4 dimensional fit with simplified BGL and CLN
About 6000 events
No data provided yet



No clear BGL$^{(111)}$/CLN difference

V_{cb}=0.0384(9)

BGL form factors compared with CLN HFLAV

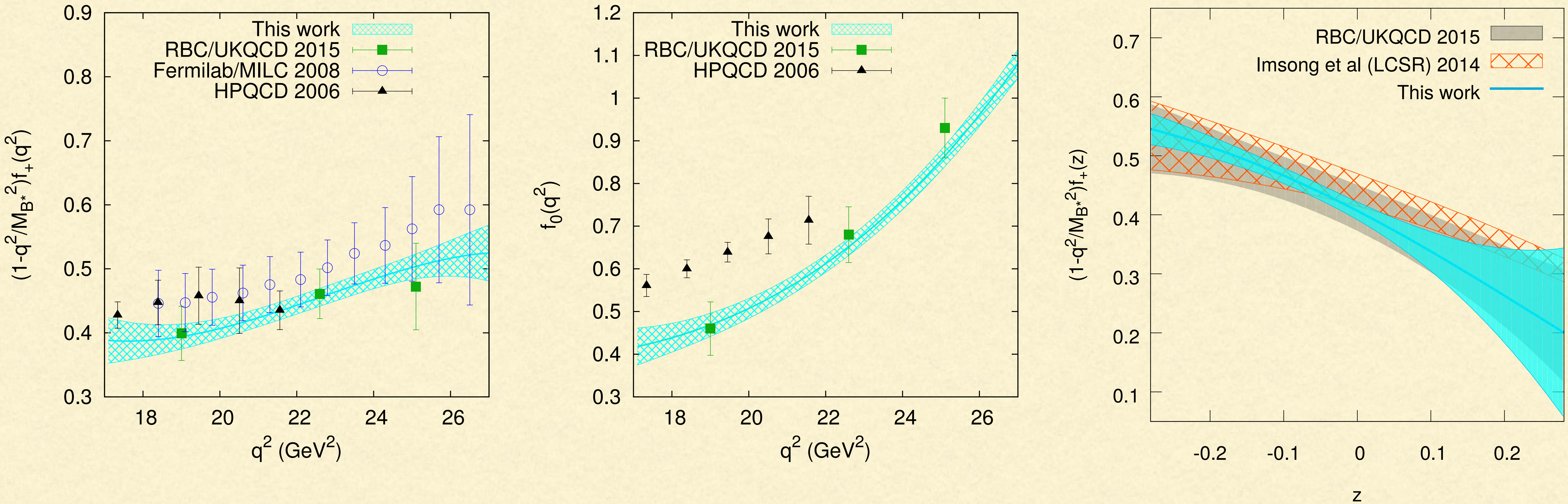
Measurement of $|V_{cb}|$ with $B^0_s \to D^{(*)-}_s \mu^+ \nu_\mu$ decays

$$\mathcal{R} \equiv \frac{\mathcal{B}(B^0_s \to D^-_s \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \to D^- \mu^+ \nu_\mu)},$$
$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B^0_s \to D^{*-}_s \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \to D^{*-} \mu^+ \nu_\mu)}$$

V_{cb}=0.0414(16) CLN
V_{cb}=0.0423(17) BGL$^{(222)}$

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL$^{(222)}$

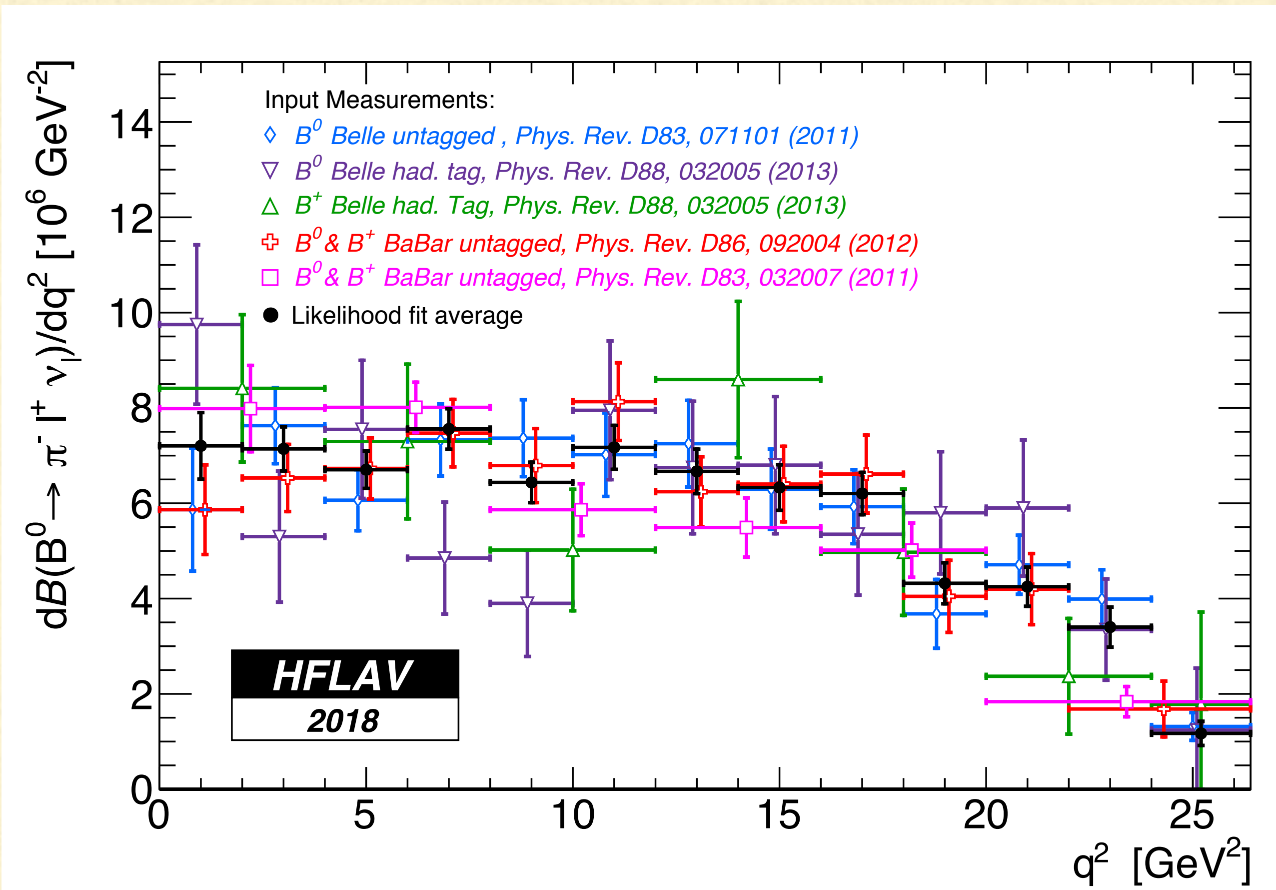
EXCLUSIVE V_{ub} $B \to \pi \ell \nu$

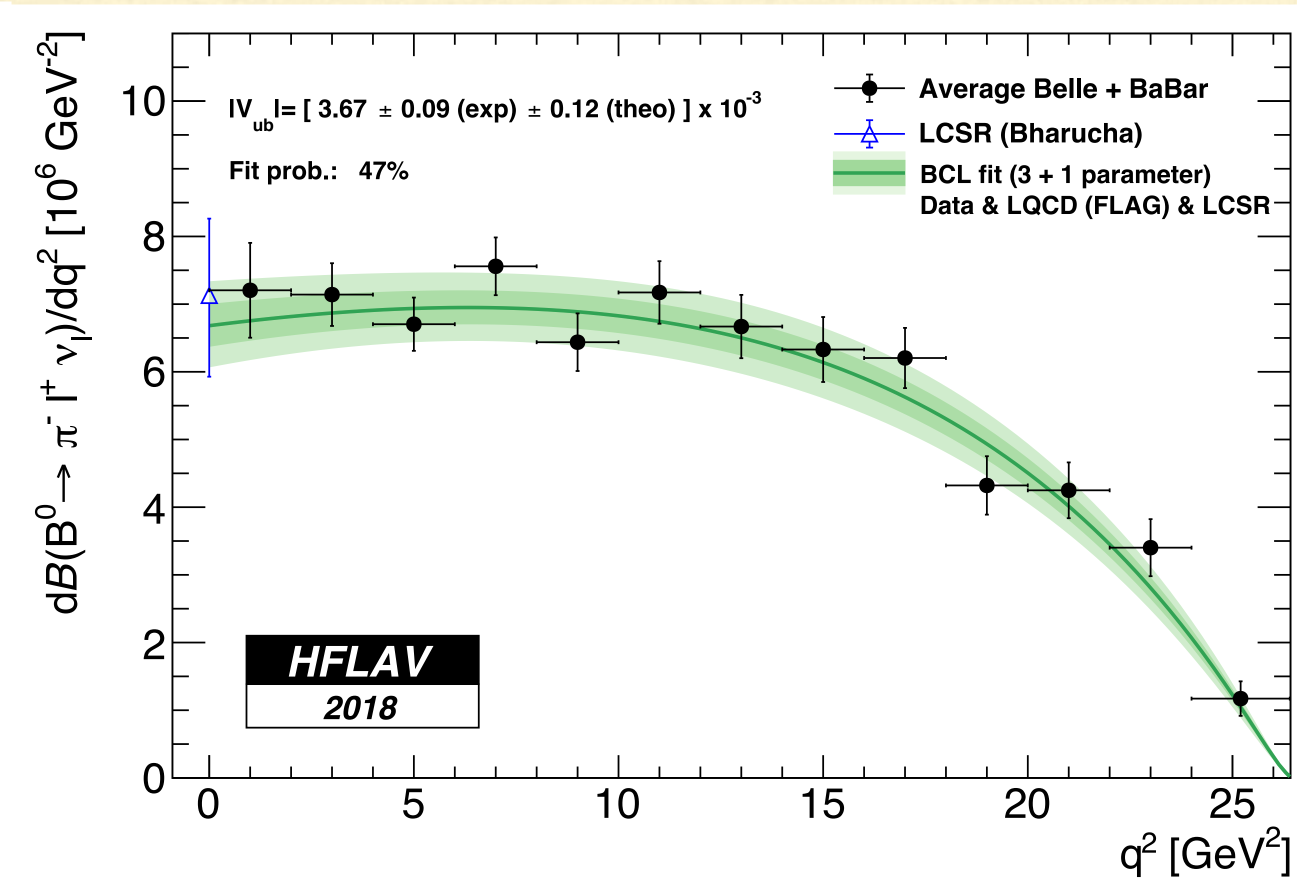


FNAL-MILC 1503.07839

- Theory looks fine: two LQCD and LCSR agree well

EXCLUSIVE V_{ub} $B \to \pi \ell \nu$





$$|V_{ub}| = (3.70 \pm 0.10\,(\mathrm{exp}) \pm 0.12\,(\mathrm{theo})) \times 10^{-3} \ \ (\mathrm{data} + \mathrm{LQCD}),$$
$$|V_{ub}| = (3.67 \pm 0.09\,(\mathrm{exp}) \pm 0.12\,(\mathrm{theo})) \times 10^{-3} \ \ (\mathrm{data} + \mathrm{LQCD} + \mathrm{LCSR}),$$

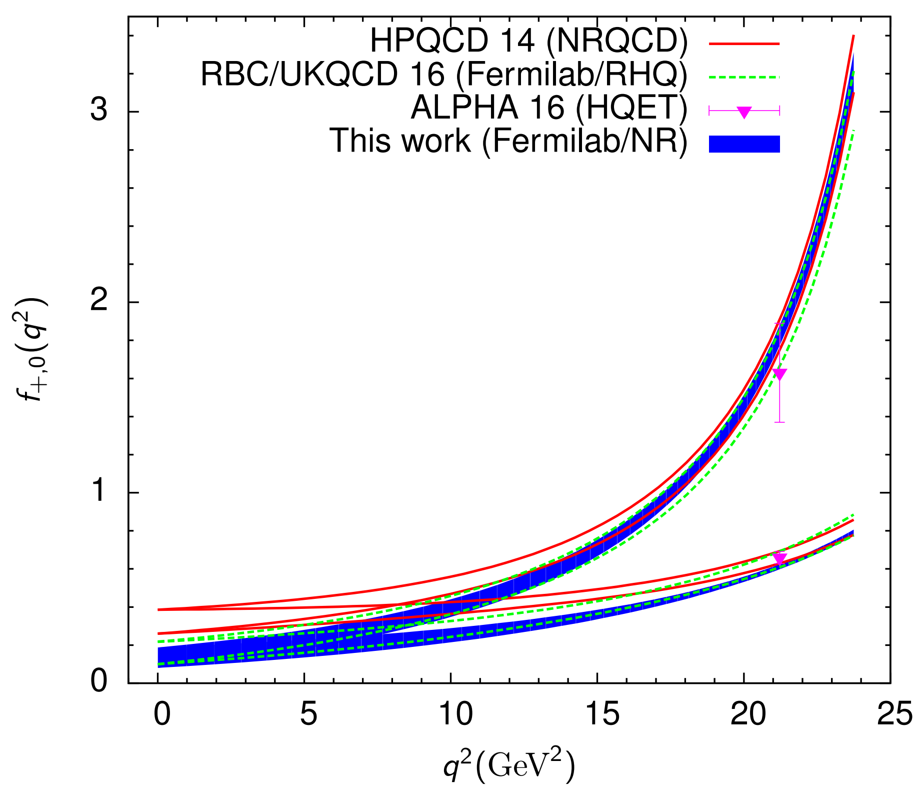
- bad chi2/dof, situation would improve (and V_{ub} would increase) by considering discrepant results with care

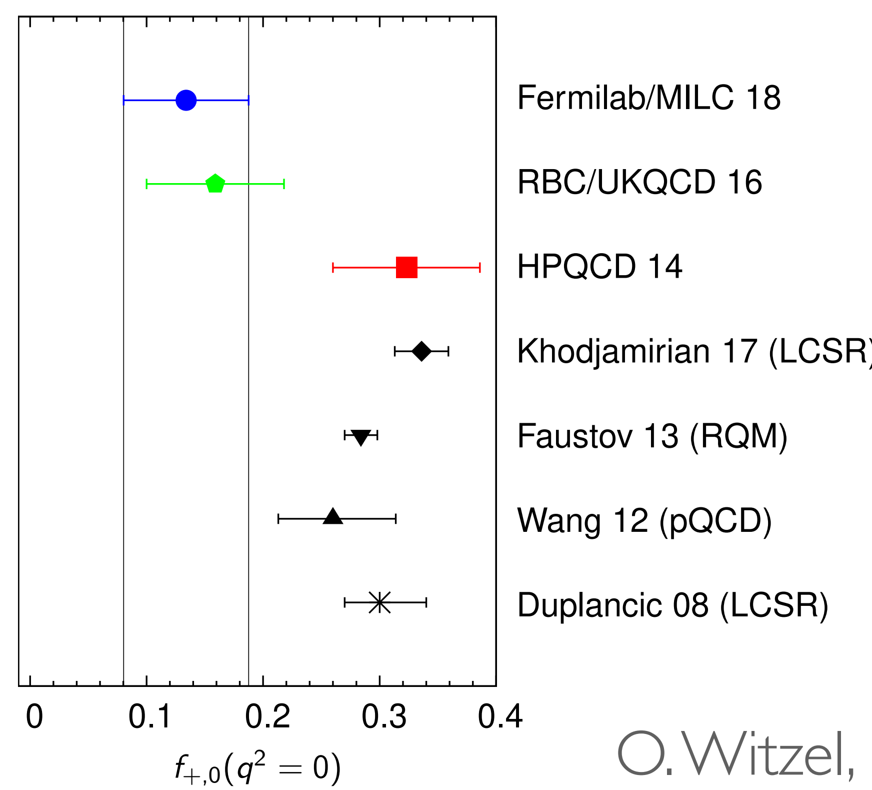
OTHER DECAY MODES

LHCb can access $|V_{ub}/V_{cb}|$ from Λ_b and B_s decays. Also $B \rightarrow \rho$ etc

$B_s \to K\ell\nu$

- HPQCD, RBC-UKQCD, ALPHA
  [Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473]
- New 2019: Fermilab/MILC [Bazavov et al. PRD100(2019)034501]





O. Witzel, Beauty 2019

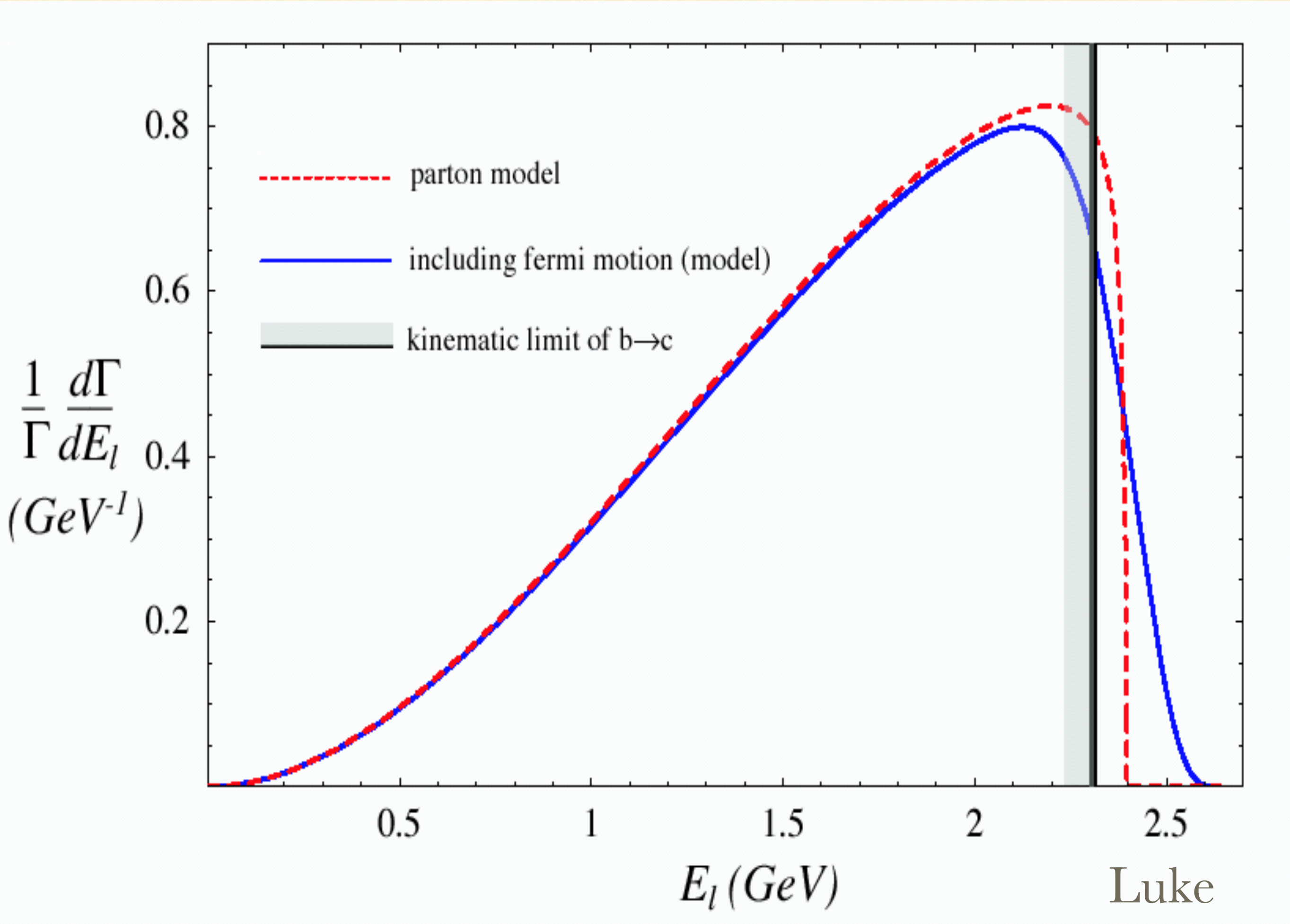
V_{ub} from inclusive decays: cuts

Experiments often use kinematic cuts to avoid the b→clν background:

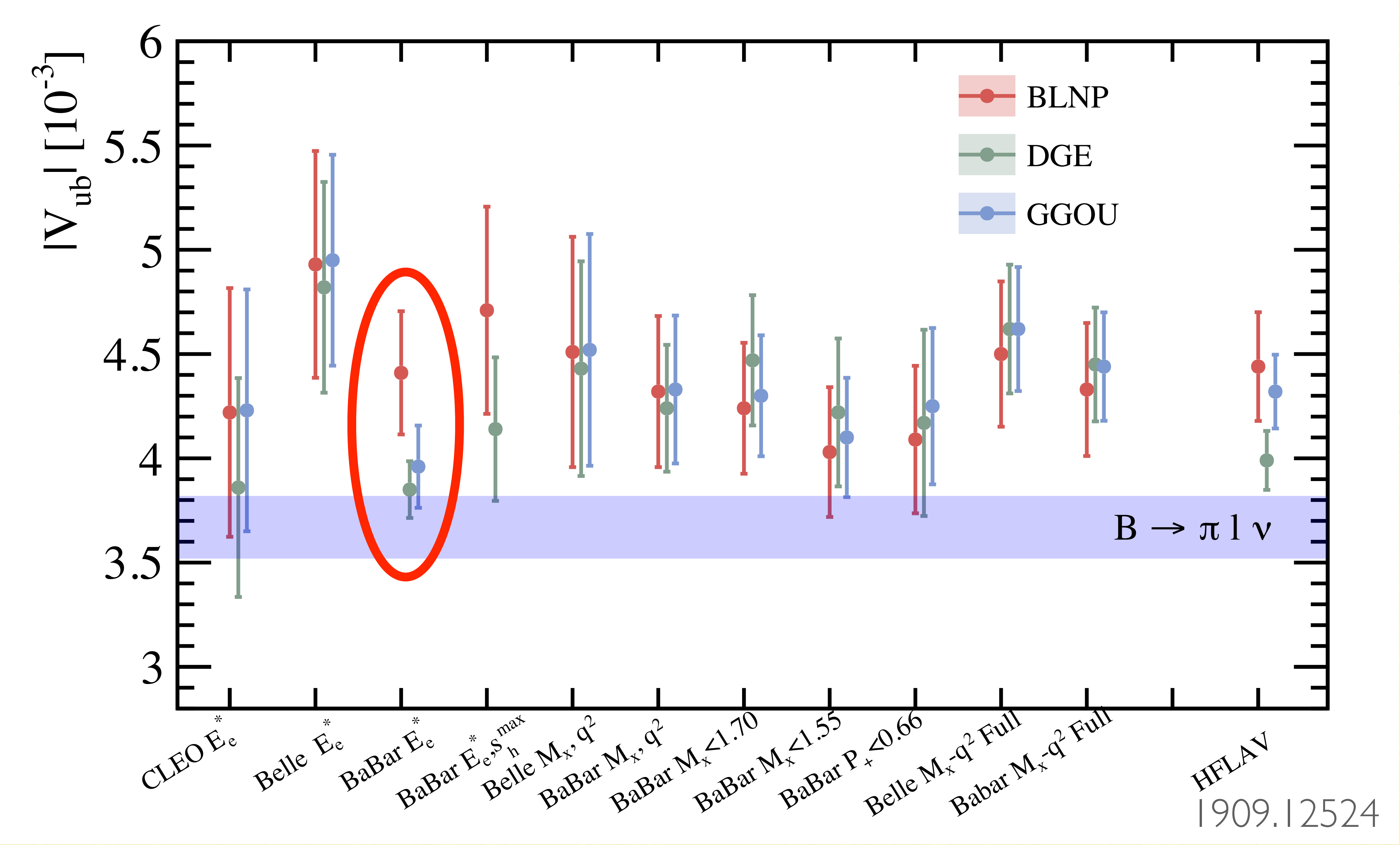
$$m_X < M_D \qquad E_\ell > (M_B^2 - M_D^2)/2M_B \qquad q^2 > (M_B - M_D)^2 \ldots$$

The cuts destroy convergence of the OPE that works so well in b→c.
OPE expected to work only
away from pert singularities

Rate becomes sensitive to
b-quark wave function properties
like Fermi motion. Dominant non-pert contributions can be resummed
into a ***SHAPE FUNCTION*** $f(k_+)$.
Equivalently the SF is seen to emerge
from soft gluon resummation



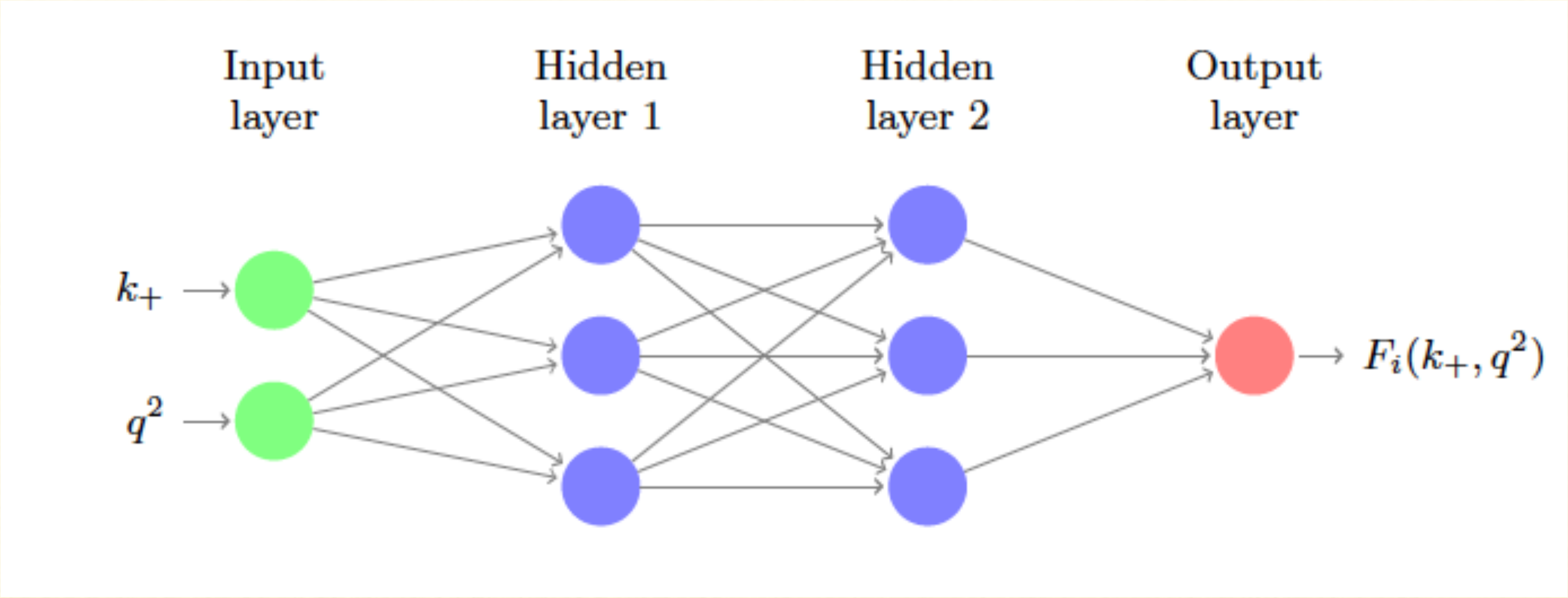
INCLUSIVE V_{ub}



Most precise measurement by Babar 2017.
Importance of the model used to simulate the signal

THE NNVUB PROJECT

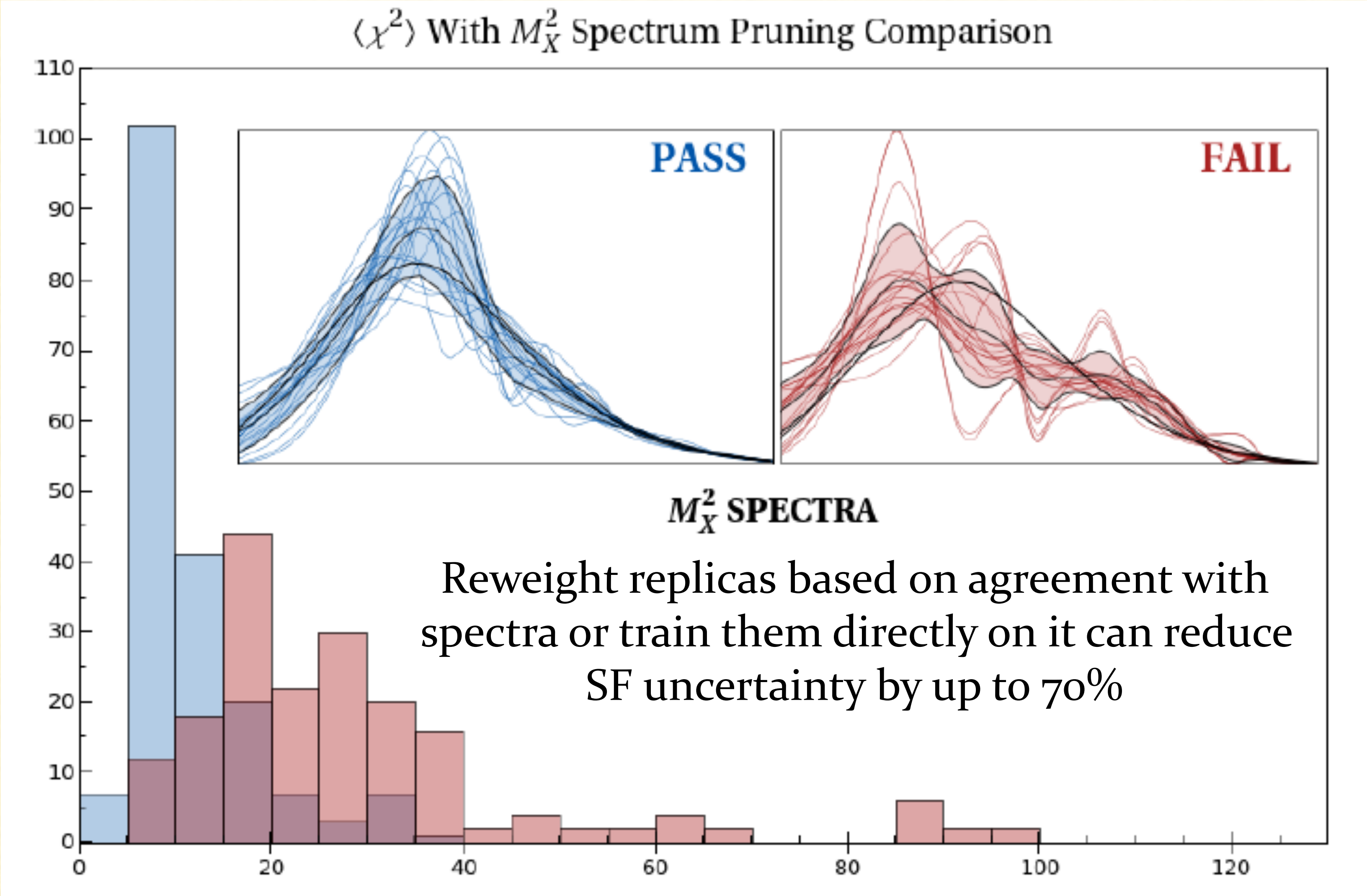
K.Healey, C. Mondino, PG, 1604.07598



- Use Artificial Neural Networks to parameterize shape functions without bias and extract V_{ub} from theoretical constraints and data, together with HQE parameters in a model independent way (without assumptions on functional form). Similar to NNPDF. Applies to $b \to u l \nu$, $b \to s \gamma$, $b \to s l^+ l^-$
- Belle II will be able to measure some kinematic distributions, thus constraining directly the shape functions. NNVub will provide a flexible tool to analyse data.

PROSPECTS @ BELLE-II

- Learning @ Belle-II from kinematic distributions, e.g. M_X spectrum
- OPE parameters checked/ improved in $b \to u l \nu$ (moments): global NN+OPE fit
- alternative approach SIMBA
  Bernlochner, Tackmann, Ligeti, Stewart
- include all relevant information with correlations
- check signal dependence at endpoint
- full phase space implementation of α_s^2 and α_s/m_b^2 corrections
- model/exclude high q^2 tail



At Belle-II we can hope to bring inclusive V_{ub} at almost the same level as V_{cb}

CONCLUSIONS

- **Inclusive/Exclusive tensions remain, but weaker, still puzzling.**
- Inclusive decays: ongoing effort to improve theory, promising progress on the lattice.
- Exclusive b→c decays have been revisited: uncertainties were underestimated. Several lattice groups are computing all necessary FFs at non-zero recoil.
- **SM predictions for R(D), R(D*) are robust, anomaly persists.**
- We expect improved measurements of both inclusive and exclusive decays at Belle II, with interesting new possibilities for V_{ub}
- Experiments should provide deconvoluted spectra or alternative but equivalent information. Theoretical prejudice is transient by definition and should never be hardwired into precision measurements.

UNITARITY CONSTRAINTS

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \qquad 0 < z < 0.056$$

$$f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n$$

BGL Boyd Grinstein Lebed 1997

BLASCHKE FACTORS REMOVE POLES BELOW THRESHOLD

PHASE SPACE FACTORS

FAST CONVERGING EXPANSION

TRUNCATED AT ORDER N

$$\sum_{n=0}^{N} (a_n^i)^2 < 1$$

WEAK UNITARITY CONSTRAINTS

assuming saturation by single hadron channel

STRONG UNITARITY CONSTRAINTS

Information on other channels with same quantum numbers makes the bounds tighter. HQS implies that all $B^{(*)}\to D^{(*)}$ ff either vanish or are prop to the Isgur-Wise function: any ff F_j can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\mathrm{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the a_i space for S, P, V, A currents

Caprini Lellouch Neubert (CLN, 1998) exploit NLO HQET relations between form factors + QCD sum rules to **reduce parameters** for ffs "up to 2% uncertainty", ***never included in exp analysis***. The *practical version of CLN* is

$$h_{A1}(z) = h_{A1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2,$$

only 2+2 parameters! but uncertainty? bias?

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \ldots\right] \quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$c_{b,c}$ can be computed using subleading IW functions from QCD sumrules
Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

RATIOS $\frac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \ldots\right] \quad w_1 = w - 1$

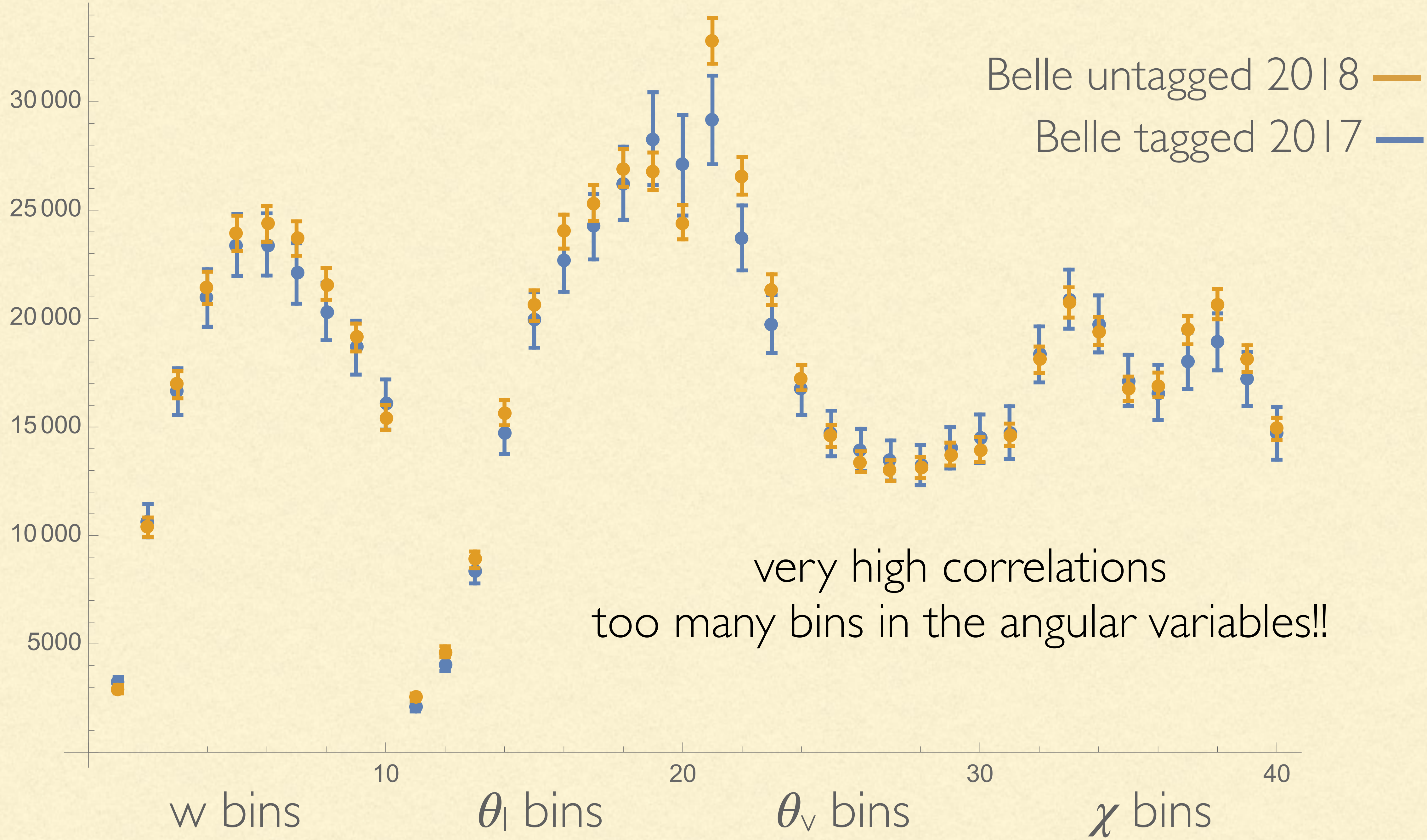
Roughly $\epsilon_c \sim 0.25, \quad \epsilon_c^2 \sim 0.06$ but coefficients??

In a few cases we can compare these ratios with recent lattice results: there are 5-13% differences, always > NLO correction. For ex.:

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET@NLO} = 0.966(28)$$

Looking at NLO HQET corrections, NNLO can be sizeable, naturally O(10-20)%

CONSISTENCY OF DATASETS (poor theorist approach)



very high correlations
too many bins in the angular variables!!