What’s the matter with $V_{ud}$?

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New Physics on the Low-Energy Precision Frontier - CERN - Jan 20- Feb 7, 2020
1950’s: V - A Fermi theory:

\[
\mathcal{L} = -\frac{G\mu}{\sqrt{2}}[\bar{\psi}_\mu \gamma^\mu(1-\gamma_5)\psi_\mu][\bar{\psi}_e \gamma^\mu(1-\gamma_5)\psi_\nu] + \text{h.c.}
\]

Calculating radiative corrections to muon decay: important evidence for V-A theory

RC to muon decay - UV finite for V and A interactions but UV divergent for S, PS

\[
\frac{1}{\tau_\mu} = \frac{G^2 m_\mu^5}{192\pi^3} F(x)[1 + \delta_\mu]
\]

Tree-level phase space: \( F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4 \quad x = m_e^2/m_\mu^2 \)

RC (2-loop): \( \delta_\mu = \frac{\alpha}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \left[ 1 + \frac{2\alpha}{3\pi} \ln \left( \frac{m_\mu}{m_e} \right) \right] + 6.700 \left( \frac{\alpha}{\pi} \right)^2 + \cdots = -4.19818 \times 10^{-3} \)

Precise measurement of muon lifetime: \( \tau_\mu = 2196980.3(2.2)\text{ps} \)

Precise determination of Fermi constant: \( G_F = G_\mu = 1.1663788(7) \times 10^{-5}\text{GeV}^{-2} \)
History of Radiative Corrections to $\beta$ Decay

\[ \mathcal{L}_{\beta-\text{decay}} = -\frac{G_V}{\sqrt{2}} [\bar{\psi}_p \gamma^\mu (1 - \gamma_5) \psi_n] [\bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_e] + \text{h.c.} \]

However, RC to neutron decay - UV divergent even in V-A theory

Uncorrected spectrum for Fermi transition: \[ P^0 d^3p = \frac{8G_V^2}{(2\pi)^4} (E_m - E)^2 d^3p \]

RC to spectrum: \[ \Delta P d^3p = \frac{\alpha}{2\pi} P^0 d^3p \left\{ 6 \ln \left( \frac{\Lambda}{m_p} \right) + g(E, E_m) + \frac{9}{4} \right\} \]

Sirlin’s function:
(QED beyond Coulomb distortion)
\[ g(E, E_m) = 3 \ln \left( \frac{m_p}{m_e} \right) - \frac{3}{4} - \frac{4}{\beta} \text{Li}_2 \left( \frac{2\beta}{1 + \beta} \right) + 4 \left[ \frac{\tanh^{-1} \beta}{\beta} - 1 \right] \left[ \frac{(E_m - E)}{3E} - \frac{3}{2} + \ln \left( \frac{2(E_m - E)}{m_e} \right) \right] \]
\[ + \frac{\tanh^{-1} \beta}{\beta} \left[ 2(1 + \beta^2) + \frac{(E_m - E)^2}{6E^2} - 4\tanh^{-1} \beta \right], \]

Together with the finding that $G_V < G_\mu$ cast doubts of universality of weak interaction!

Strong interaction effects?

Current algebra: UV div. part \[ \frac{\alpha}{2\pi} P^0 d^3p \left[ 3[1 + 2\tilde{Q}] \ln(\Lambda/M) \right] \]

$\tilde{Q}$ : average charge of fields involved: $1 + 2\tilde{Q}_{\mu,\nu} = 0$ but $1 + 2\tilde{Q}_{n,p} = 2$

Cabibbo: the strength of weak interaction is distributed among $\Delta S=0,1$ transitions:

\[ G_V(\Delta S = 0) = G_\mu \cos \theta_c; \quad G_V(\Delta S = 1) = G_\mu \sin \theta_c \]
\[ G^2_\mu = G^2_V(\Delta S = 0) + G^2_V(\Delta S = 1) \]
Precision determination of $V_{ud}$

1960’s: electroweak theory - $SU(2)_L \times U(1)_Y$, massive $W$, $Z$ bosons, EW mixing, ...

Charged current interaction - $\beta$-decay ($\mu$, $\pi^\pm$, $n$)

$\mu^-$, $e^-$, $\nu_\mu$ → $\nu_e$

$\pi^\pm$ → $\mu^\pm$, $\nu_e$

$\nu$ (anti-$\nu$)

$W$ coupling to leptons and hadrons very close but not exactly the same:

quark mixing - Cabbibo-Kabayashi-Maskawa matrix

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
= V_{CKM}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
$$

CKM - Determines the relative strength of the weak CC interaction of quarks vs. that of leptons

CKM unitarity - measure of completeness of the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
Vud and CKM unitarity in early 2018

From neutron decay
\[ |V_{ud}|^2 = \frac{5099.34s}{\tau_n(1 + 3g_A^2)(1 + \Delta_R)} \]

From superallowed decays
\[ |V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1 + \Delta_R^V)} \]

CKM unitarity: \( V_{ud} \) the main contributor to the sum and to the uncertainty

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005 \]

0\(^+-\)0\(^+\) nuclear decays

KI3 and KI2 average

B decays
Why are superallowed decays special?

Superallowed 0+-0+ nuclear decays:
- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment: \( f \) - phase space (Q value) and \( t \) - partial half-life (\( t_{1/2} \), branching ratio)

- 8 cases with \( ft \)-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.
- ~220 individual measurements with compatible precision

\[ ft \text{ values: same within } \sim 2\% \text{ but not exactly! Reason: SU}(2) \text{ slightly broken} \]
  a. RC (e.m. interaction does not conserve isospin)
  b. Nuclear WF are not SU(2) symmetric (proton and neutron distribution not the same)
Why are superallowed decays special?

$$|V_{ud}|^2 = \frac{2984.432(3)}{\cal{F}t(1 + \Delta_V^R)}$$

Modified ft-values to include these effects

$$\cal{F}t = ft(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]$$

$\delta'_R$ - “outer” correction (depends on e-energy) - QED

$\delta_C$ - SU(2) breaking in the nuclear matrix elements
- mismatch of radial WF in parent-daughter
- mixing of different isospin states

$\delta_{NS}$ - RC depending on the nuclear structure

$\delta_C, \delta_{NS}$ - energy independent

Average

$$\overline{\cal{F}t} = 3072.1 \pm 0.7$$
RC on the free neutron

Outer (depend on e-energy): retain only IR divergent pieces

Separation due to scale hierarchy: \( m_e = 0.511 \) MeV, \( Q = M_n - M_p = 1.3 \) MeV;
\( Q/m_e \) not small, need to account for exactly.
Coulomb distortion: resummation of \((Z\alpha)^n \rightarrow \) Dirac equation in the Coulomb field
IR finite piece: can set \( m_e = 0 \) \( \rightarrow \) if energy-dependent \( \sim (\alpha/2\pi) \times (E/\Lambda_{\text{had}}) \)
Hadronic structure: relevant scale \( \sim m_\pi = 140 \) MeV - on top of \( \alpha/2\pi \sim 10^{-3} \) \( \rightarrow \) 10^{-5} effect <<

Inner (energy-independent - take E=0)

\( W,Z \)-exchange:
UV-sensitive, pQCD;
model-independent

When \( \gamma \) involved:
sensitive to long range physics;
model-dependent!

\( V \times V \) correlator protected by CVC - no hadronic uncertainty
Axial vector not conserved \( \rightarrow \) A \( \times V \) correlator from \( \gamma W \) box sensitive to hadron structure


\[ \Delta^V_R = 0.02361(38) \]
\[ |V_{ud}|^2 = \frac{5099.34\tau}{(1 + 3g_A^2)(1 + \Delta_R)} \]

\[ |V_{ud}|^2 = \frac{2984.43s}{\mathcal{F} t (1 + \Delta_R^V)} \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005 \]

CKM unitarity: \( V_{ud} \) the main contributor to the sum and to the uncertainty

\[ |V_{ud}|^2 = 0.97420 \pm 0.00021 \]

\[ |V_{us}|^2 = 0.05031 \pm 0.00022 \]

\[ |V_{ub}|^2 = 0.00002 \]

0\(^+\)-0\(^+\) nuclear decays

KI3 and KI2 average

B decays
\( V_{ud} \) and CKM unitarity in Fall 2018

\[ V_{ud} = 0.97366 \pm 0.00015 \]

\[ V_{ud} = 0.97420 \pm 0.00021 \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9979 \pm 0.0004 \]

CKM unitarity: \( V_{ud} \) und \( V_{us} \) contribute equally to the uncertainty

\[ 0^{-+}0^{+} \text{ nuclear decays} \]
\[ |V_{ud}|^2 = 0.94801 \pm 0.00029 \]

\[ \text{K}3 \text{ decays} \]
\[ |V_{us}|^2 = 0.04987 \pm 0.00027 \]

\textit{Bazavov et al. (FNAL/MILC), 1809.02827}
**V_{ud} and CKM unitarity in December 2018**

**Major improvement in exp. determination of g_A**

\[
\text{PDG2018} \quad \quad \text{PERKEO-III} \quad \quad \text{Märkisch et al., 1812.04666}
\]

\[
g_A = -1.2723(23) \quad \quad \quad \quad g_A = -1.2764(6)
\]

\[
V_{ud} \text{ from free neutron decay}
\]

\[
V_{ud} = 0.9763(5)\tau_n(15)g_A(2)_{RC} \quad \quad \quad \quad V_{ud} = 0.9735(5)\tau_n(3)g_A(1)_{RC}
\]

**Revision of nuclear corrections to 0^+ - 0^+-beta decay**

\[
\text{Seng, MG, Ramsey-Musolf, 1812.03352; MG 1812.04229}
\]

\[
V_{ud} = 0.97366(10)Ft(10)_{RC} \quad \quad \quad \quad V_{ud} = 0.97366(32)Ft(10)_{RC}
\]

Free neutron decay becomes competitive;
Limitation: the lifetime (also, beam vs. bottle)

Scrutiny of nuclear corrections with new methods

Top-row unitarity: 2.5-3.5σ deficit

\[
\Delta_u = -(0.0016 - 0.0021) \pm 0.0006
\]

(ddepending on V_{us})
\textbf{γW-box from Dispersion Relations}

Box at zero momentum transfer\(^*\) (but with energy dependence)

\[
T_{γW} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \bar{u}_εγ^μ(k - q + m_e)γ^ν(1 - γ_5)u_ν \frac{M_W^2}{q^2 - M_W^2} T_{γW}^{µν}
\]

\(^*\text{Precision goal: } 10^{-4}; \text{ RC } \sim α/2π \sim 10^{-3}; \text{ recoil on top } - \text{ negligible}

Hadronic tensor: two-current correlator

\[
T_{γW}^{µν} = \int dx e^{iqx} \langle f | T[J_{em}^µ(x)J_W^{ν,±}(0)] | i \rangle
\]

General gauge-invariant decomposition of a spin-independent tensor

\[
T_{γW}^{µν} = \left( -g^{µν} + \frac{q^{µ}q^{ν}}{q^2} \right) T_1 + \frac{1}{(p \cdot q)} \left( p - \left( \frac{p \cdot q}{q^2} \right) q \right)^µ \left( p - \left( \frac{p \cdot q}{q^2} \right) q \right)^ν T_2 + \frac{iε^{µναβ}p_αq_β}{2(p \cdot q)} T_3
\]

Loop integral with generally unknown forward amplitudes

\[
T_{γW} = -\frac{α}{2π} \sqrt{2} G_F V_{ud} \int \frac{d^4q M_W^2}{q^2(M_W^2 - q^2)} \bar{u}_εγ_ρ(1 - γ_5)u_ν \sum_i C_i^β(E, ν, q^2) T_i^{γW}(ν, q^2)
\]

Known algebraic functions of external energy \(E\) and loop variables \(ν, q^2\)

\[p^µ = (M, \vec{0}) \quad E = (pk)/M \quad ν = (pq)/M\]
Marciano & Sirlin's approach involves loop techniques to calculate the

\[ \Box_{\gamma W} = 4\pi \alpha \text{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2 + \nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu} \]

Marciano & Sirlin used loop techniques:

\[ \Box_{\gamma W} = \frac{\alpha}{8\pi} \int_0^\infty \frac{dQ^2M_W^2}{M_W^2 + Q^2} F(Q^2) \]

Short distance \( Q^2 \gg \)

\[ F^{\text{DIS}}(Q^2) = \frac{1}{Q^2} \]

\[ \Box_{\gamma W}^{\text{DIS}} = \frac{\alpha}{8\pi} \int_\Lambda^2 \frac{dQ^2M_W^2}{M_W^2 + Q^2} F^{\text{DIS}}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda} \]

Finite \( Q^2 \)-
pQCD corrections:

\[ F^{\text{DIS}} = \frac{1}{Q^2} \rightarrow \frac{1}{Q^2} \left[ 1 - \frac{\alpha_s^{\text{MS}}}{\pi} \right] \]

Long distance \( Q^2 \ll \) - elastic box

MS 1987: asymptotic + pQCD + Born

\[ \left[ \Delta_{R}^{\gamma W} \right] = \frac{\alpha}{2\pi} \left[ \ln \frac{M_W}{\Lambda} + A_g + 2C_B \right] \]

\(~4.1\quad -0.24\quad 1.85\)

Problem: connecting short and long distances
\[ F^{\text{DIS}} = \frac{1}{Q^2} \rightarrow \frac{1}{Q^2} \left[ 1 - \frac{\alpha_{s}^{\overline{\text{MS}}}}{\pi} - C_2 \left( \frac{\alpha_{s}^{\overline{\text{MS}}}}{\pi} \right)^2 - C_3 \left( \frac{\alpha_{s}^{\overline{\text{MS}}}}{\pi} \right)^3 \right] \]

GLS and Bjorken SR to N3LO

**MS 2006 update**

**Short distance:** DIS to N$^3$LO

\[ Q^2 > Q_2^2 \]

Interpolate between them

**Vector Dominance Model Ansatz**

\[ F^{\text{INT}}(Q^2) = -\frac{1.490}{Q^2 + m_\rho^2} + \frac{6.855}{Q^2 + m_A^2} - \frac{4.414}{Q^2 + m_{\rho'}^2} \]

**Long distance:** Born

\[ F(Q^2) = F^B(Q^2) \]

\[ \left[ \Delta_R^V \right]^{\gamma W} = \frac{\alpha}{2\pi} \left[ \ln \frac{M_W}{\Lambda} + A_g + 2C_B \right] \]

\[ \sim 3.86 \quad 1.78 \]

\[ \left[ \Delta_R^V \right]^{\gamma W} = \frac{\alpha}{2\pi} \left[ \ln \frac{M_W}{\Lambda} + A_g + C^{\text{Int}} + 2C_B \right] \]

\[ \sim 3.77 \quad 0.14(14) \quad 1.66 \]

Uncertainty reduced by a factor \( \sim 2 \)
\gamma W\text{-box from Dispersion Relations}

T_{1,2,3} - analytic functions inside the contour C in the complex \nu-plane determined by their singularities on the real axis - poles + cuts

\[
T_i^{\gamma W}(\nu, Q^2) = \frac{1}{2\pi i} \oint dz \frac{T_i^{\gamma W}(z, Q^2)}{z - \nu}, \quad \nu \in C
\]

Forward amplitudes T_i - unknown;
Their absorptive parts can be related to production of on-shell intermediate states
\[\rightarrow\] a \gamma W\text{-analog of structure functions } F_{1,2,3}

X = inclusive strongly-interacting on-shell physical states

\[\text{Structure functions } F_i^{\gamma W} \text{ are NOT data} \]
\[\text{But they can be related to data} \]

\[\text{Im } T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)\]
\( \gamma W \)-box from Dispersion Relations

Crossing behavior: relate the left and right hand cut
Mismatch between the initial and final states - asymmetric;
Symmetrize - \( \gamma \) is a mix of \( I=0 \) and \( I=1 \)

\[
T_{i}^{\gamma W,a} = T_{i}^{(0)}a + T_{i}^{(-)}\frac{1}{2}[\tau^3, \tau^a]
\]

\[
T_{i}^{(I)(-\nu, Q^2)} = \xi_{i}^{(I)}T_{i}^{(I)(\nu, Q^2)}
\]

\[\xi_{1}^{(0)} = +1, \quad \xi_{2,3}^{(0)} = -1; \quad \xi_{i}^{(-)} = -\xi_{i}^{(0)}\]

Two types of dispersion relations for scalar amplitudes

\[
T_{i}^{(I)(\nu, Q^2)} = 2 \int_{0}^{\infty} d\nu' \left[ \frac{1}{\nu' - \nu - i\epsilon} + \frac{\xi_{i}^{(I)}}{\nu' - \nu - i\epsilon} \right] \xi_{i}^{(I)} F_{i}^{(I)}(\nu', Q^2)
\]

Substitute into the loop and calculate leading energy dependence

\[
\text{Re} \square_{\gamma W}^{\text{even}} = \frac{\alpha}{\pi N} \int_{0}^{\infty} dQ^2 \int_{\nu_{\text{thr}}}^{\infty} d\nu \frac{F_{3}^{(0)}}{M \nu} \frac{\nu + 2q}{(\nu + q)^2} + O(E^2)
\]

\[
\text{Re} \square_{\gamma W}^{\text{odd}}(E) = \frac{8\alpha E}{3\pi NM} \int_{0}^{\infty} dQ^2 \int_{\nu_{\text{thr}}}^{\infty} d\nu \frac{F_{1}^{(0)}}{(\nu + q)^3} \left[ + F_{1}^{(0)} \pm \left( \frac{3\nu(\nu + q)}{2Q^2} + 1 \right) \frac{M}{\nu} F_{2}^{(0)} + \frac{\nu + 3q}{4\nu} F_{3}^{(-)} \right] + O(E^3)
\]
Input into dispersion integral

Dispersion in energy: \( W^2 = M^2 + 2M\nu - Q^2 \)
scanning hadronic intermediate states

Dispersion in \( Q^2 \):
scanning dominant physics pictures

Boundaries between regions - approximate
Input in DR related (directly or indirectly)
to experimentally accessible data
Input into dispersion integral

\[ F_3^{(0)} \propto \int dx e^{iqx} \langle p \mid [J_{em}^{\mu,(0)}(x), J_{W}^{\nu,+}(0)] \mid n \rangle \sim \int dx e^{iqx} \sum_X \langle p \mid J_{em}^{\mu,(0)}(x) \mid X \rangle \langle X \mid J_{W}^{\nu,+}(0) \mid n \rangle \]

Our parametrization of the needed SF follows from this diagram

\[ F_3^{(0)} = F_{\text{Born}} + \begin{cases} F_{\text{pQCD}}, & Q^2 \gtrsim 2 \text{ GeV}^2 \\ F_{\pi N} + F_{\text{res}} + F_{\text{R}}, & Q^2 \lesssim 2 \text{ GeV}^2 \end{cases} \]

Born: elastic FF from e\(^{-}\), ν scattering data

\[ \Box^{V_{A,\text{Born}}} = -\frac{\alpha}{\pi} \int_0^\infty dQ \frac{2\sqrt{4M^2 + Q^2 + Q}}{\left(\sqrt{4M^2 + Q^2 + Q}\right)^2} G_A(Q^2)G_M^S(Q^2) \]

πN:
relativistic ChPT calculation plus nucleon FF

Resonances:
axial excitation from PCAC (Lalakulich et al 2006) - neutrino scattering
isoscalar photo-excitation from MAID and PDG - electron and γ inelastic scattering

Above resonance region:
multiparticle continuum economically described by Regge exchanges
Input into dispersion integral

Unfortunately, no data can be obtained for $F_3\gamma W^{(0)}$

Data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{G_F^2 ME}{\pi} \left[ xy^2 F_1 + \left( 1 - y - \frac{Mxy}{2E} \right) F_2 \pm x \left( y - \frac{y^2}{2} \right) F_3 \right]$$

$$\sigma^{\nu p} - \sigma^{\bar{\nu} p} \sim F_3^{\nu p} + F_3^{\bar{\nu} p} = u^p_v(x) + d^p_v(x)$$

Gross-Llewellyn-Smith sum rule

$$\int_0^1 dx (u^p_v(x) + d^p_v(x)) = 3$$

Validate the model for CC process; apply an isospin rotation to obtain $\gamma W$

$$F^{\nu p+\bar{\nu} p}_{3, low-Q^2} = F^{\nu p+\bar{\nu} p}_{3, el.} + F^{\nu p+\bar{\nu} p}_{3, \pi N} + F^{\nu p+\bar{\nu} p}_{3, R} + F^{\nu p+\bar{\nu} p}_{3, Regge}$$

Low-W part of spectrum:
- neutrino data from MiniBooNE, Minerva, ...
- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^\nu \sim x^{-\alpha}$, $\alpha \sim 0.5-0.7$
Inelastic states - low $Q^2$, high $W$

Scattering at high energy can be very effectively described by Regge exchanges

$$F_{3}^{(0),\text{Regge}}(\nu, Q^2) = C_R(Q^2) \left( \frac{\nu}{\nu_0} \right)^{\alpha_\rho}$$

Regge behavior in EW processes: hadron-like behavior of HE electroweak probes - Vector/Axial Vector Dominance is the proper language

γW-box: conversion of $W^\pm$ (charged, $I=1$, axial) to $\gamma$ (neutral, vector, $I=0$) requires charged vector exchange w. $I=1$ - $\rho^\pm$ effective $a_1 - \rho - \omega$ vertex

Inclusive $\nu$ scattering: conversion of $W^\pm$ (charged, $I=1$, axial) to $W^\pm$ (charged, $I=1$, axial) requires neutral vector exchange w. $I=0$ - $\omega$ effective $a_1 - \omega - \rho$ vertex

Minimal model for both reactions - check with data.
Parameters of the Regge model from neutrino scattering

Low $Q^2 < 0.1 \text{ GeV}^2$: Born + $\Delta(1232)$ dominate
Not fitted: modern data more precise but cover only limited energy range
Fit driven by 4 data points between 0.2 and 2 GeV$^2$

Model & Uncertainty fully specified
- compare M&S vs This work

\[ M_{3WW}^{(1,Q^2)} \]
\[ M_{3^W}^{WW} (1,Q^2) \]
Isospin symmetry

Log scale for x-axis: integral = surface under the curve

- MS Total: $\gamma_{W}^{(0)} = 0.00324 \pm 0.00018$
- New Total: $\gamma_{W}^{(0)} = 0.00379 \pm 0.00010$

Uncertainty reduced by almost factor 2;
~ 3-5 sigma shift from the old value
Δ^V_R and the V_{ud} extraction

Δ^V_R = 2\text{Re} \Box^{\text{even}}_{\gamma W} (E = 0) + \ldots

Marciano, Sirlin 2006: Δ^V_R = 0.02361(38) \rightarrow |V_{ud}| = 0.97420(10)_{F_1(18)}^{RC}

DR (Seng et al. 2018): Δ^V_R = 0.02467(22) \rightarrow |V_{ud}| = 0.97370(10)_{F_1(10)}^{RC}

In July 2019 Czarnecki, Marciano and Sirlin published an update in which they largely agreed that the RC was underestimated:

*Czarnecki, Marciano, Sirlin, 1907.06737*

What was done exactly:
Interpolator was continued down to Q^2=0;
4-loop pQCD running included;
Matching to pQCD was done already at Q^2=1 \text{ GeV}^2
Holographic QCD model to continue to Q^2=0 used

Note that the 4-loop pQCD curve undershoots data

C-M-S 2019: Δ^V_R = 0.02421(32) \rightarrow |V_{ud}| = 0.97391(10)_{F_1(15)}^{RC}
What’s the matter with $\delta_{\text{NS}}$?
Splitting the $\gamma W$-box into Universal and Nuclear Parts

General structure of RC for nuclear decay

$$ft(1 + RC') = Ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta^V_R)$$

RC on a free neutron

$$\Delta^V_R \propto F^\text{free}_3 \propto \int dx e^{iqx} \sum_X \langle p | J^\mu_\text{em}(0)(x) | X \rangle \langle X | J^{\nu, +}(0) | n \rangle$$

RC on a nucleus

$$\Delta^V_R + \delta_{NS} \propto F^\text{Nucl}_3 \propto \int dx e^{iqx} \sum_{X'} \langle A' | J^\mu_\text{em}(0)(x) | X' \rangle \langle X' | J^{\nu, +}(0) | A \rangle$$

NS correction reflects this extraction of the free box

$$\Box_{\gamma W}^{\text{VA, Nucl.}} = \Box_{\gamma W}^{\text{VA, free n}} + \left[ \Box_{\gamma W}^{\text{VA, Nucl.}} - \Box_{\gamma W}^{\text{VA, free n}} \right]$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus
Splitting the $\gamma W$-box into Universal and Nuclear Parts

Need to know the full nuclear Green’s function

$$T_{\mu\nu}^{\gamma W_{\text{nuc}}} \sim \sum_{k,\ell} \langle f | J^W_\mu (k) G_{\text{nuc}} J^{\text{EM}}_{\nu} (\ell) | i \rangle$$

(A) same active nucleon

(B) two nucleons correlated by $G$

$$\delta_{NS} =$$

Case (A): non-interacting (=on-shell) neutron propagating between interaction vertices
Case (B): all two-nucleon contributions (QE 2p2h and nuclear excitations)

Insert on-shell intermediate states:

$$T_{\mu\nu}^A \rightarrow \sum_k \langle f | J^W_\mu (k) [S_F^N \otimes G_{\text{nuc}}^A] J^{\text{EM}}_{\nu} (k) | i \rangle$$

The elastic nucleon box is replaced by a single N QE knockout
Universal vs. Nuclear Corrections

Towner 1994 and ever since:

\[ \delta_{\text{quenched Born}} = [q_S^{(0)}q_A - 1] \delta_{\text{Born}} \]

Idea: calculate Gamow-Teller and magnetic nuclear transitions in the shell model; Insert the single nucleon spin operators —> predict the strength of nuclear transitions “Quenching of spin operators in nuclei”: shell model overestimates those strengths! Each vertex is suppressed by 10-15%

Numerically: on average between the 14 superallowed decays

\[ \delta_{\text{quenched Born}} = [q_S^{(0)}q_A - 1]2 \delta_{\text{free n, Born}} \approx -0.058(14) \%

But from dispersion relation perspective it corresponds to a contribution of an excited nuclear state, not to the modified box on a free nucleon! The correct estimate should base on quasielastic knockout with an on-shell N + spectator in the intermediate state

Note that continuum is outside shell model Hilbert space!
Splitting the γW-box into Universal and Nuclear Parts

$$\delta_{NS} = \frac{\alpha}{\pi} \left[ C_{NS} + C_B^{\text{quenched}} \right] \approx 0.22\% \left[ C_{NS} + C_B^{\text{quenched}} \right]$$

Hardy, Towner 2002 review

<table>
<thead>
<tr>
<th>Parent nucleus</th>
<th>Unquenched $C_{NS}$</th>
<th>Quenched $C_{NS}$</th>
<th>$(q-1) \times C_{\text{Born free}}$</th>
<th>$\delta_{NS}(%)$</th>
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<tbody>
<tr>
<td>$T_z = -1$:</td>
<td></td>
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<tr>
<td>$^{10}$C</td>
<td>-1.669</td>
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<td>$^{14}$O</td>
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<td>$^{18}$Ne</td>
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<tr>
<td>$^{22}$Mg</td>
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<td>-0.222</td>
<td>-0.067</td>
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<tr>
<td>$^{26}$Si</td>
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<td>-0.007</td>
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<td>-0.086</td>
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<tr>
<td>$^{30}$S</td>
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<td>0.002</td>
<td>-0.287</td>
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<td>$^{34}$Ar</td>
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<td>0.014</td>
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<tr>
<td>$^{38}$Ca</td>
<td>-0.693</td>
<td>0.041</td>
<td>-0.358</td>
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<tr>
<td>$^{42}$Ti</td>
<td>-1.011</td>
<td>-0.016</td>
<td>-0.181</td>
<td>-0.225</td>
</tr>
<tr>
<td>$T_z = 0$:</td>
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<tr>
<td>$^{26m}$Al</td>
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<td>-0.224</td>
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<td>$^{34}$Cl</td>
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<td>$^{38m}$K</td>
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<td>$^{42}$Sc</td>
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<td>$^{46}$V</td>
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<td>$^{50}$Mn</td>
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<td>$^{66}$As</td>
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<td>$^{70}$Br</td>
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<tr>
<td>$^{74}$Rb</td>
<td>0.155</td>
<td>0.009</td>
<td>-0.261</td>
<td>0.006</td>
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</table>
Modification of $C_B$ in a nucleus - QE

Integral is peaked at low $\nu$, $Q^2$

Born on free n:

$$F_{3}^{(0), B} = -\frac{Q^2}{4} G_A G_M^S \delta(2M\nu - Q^2)$$

Reduction for QE: finite threshold $\epsilon$ (binding energy) + Fermi momentum $k_F$

QE calculation in free Fermi gas model with Pauli blocking assign a generous 30% model uncertainty


New $\delta_{NS}^{QE} = -0.11(3)\%$ instead of the previous estimate $\delta_{NS}^{\text{quenched B}} = -0.058(14)\%$
Splitting the RC into “inner” and “outer”

Radiative corrections $\sim \frac{\alpha}{2\pi} \sim 10^{-3}$

Precision goal: $\sim 10^{-4}$

When does energy dependence matter?
Correction $\sim E_e/\Lambda$, with $\Lambda \sim$ relevant mass ($m_e; M_p; M_A$)
Maximal $E_e$ ranges from 1 MeV to 10.5 MeV

Electron mass regularizes the IR divergent parts - ($E_e/m_e$ important) - “outer” correction

If $\Lambda$ of hadronic origin (at least $m_\pi$) $\rightarrow$ $E_e/\Lambda$ small, correction $\sim 10^{-5} \rightarrow$ negligible
- certainly true for the neutron decay
- hadronic contributions do not distort the spectrum, may only shift it as a whole

However, in nuclei binding energies $\sim$ few MeV --- similar to Q-values

A scenario is possible when RC $\sim (\frac{\alpha}{2\pi}) \times (E_e/\Lambda^{\text{Nucl}}) \sim 10^{-3}$

Nuclear structure may distort the electron spectrum

With dispersion relations can be checked straightforwardly!
\[ \delta_{NS} = \frac{2\alpha}{\pi NM} \int_{0}^{1 \text{GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_{\pi}} d\nu \left[ \frac{\nu + 2q}{(\nu + q)^2} \left( F_{3}^{(0)\text{Nucl.}} - F_{3}^{(0),B} \right) + \frac{2\langle E \rangle}{3} \frac{\nu + 3q}{(\nu + q)^3} F_{3}^{(-)(\text{Nucl.})} \right] \]

\[ \langle E \rangle = \frac{\int_{m}^{Q} dEE_p (Q - E)^2 E}{\int_{m}^{Q} dEE_p (Q - E)^2} \]

Compare the effect on the average \( F_t \) value:

**HT value 2018:**
\[ F_t = 3072.1(7)s \]

**Old estimate:**
\[ \delta F_t = -(1.8 \pm 0.4)s + (0 \pm 0)s \]

**New estimate:**
\[ \delta F_t = -(3.5 \pm 1.0)s + (1.6 \pm 0.5)s \]

Two 2\( \sigma \) corrections that cancel each other;
The cancellation is delicate: the two terms are highly correlated
   - Larger E-dep. term will correspond to a smaller negative E-indep. term and vv.

Conservative uncertainty estimate: 100%
\[ F_t = (3072 \pm 2)s \]
V_{ud} and CKM unitarity in December 2018

Major improvement in exp. determination of g_A

PDG2018

PERKEO-III

Märkisch et al., 1812.04666

\[ g_A = -1.2723(23) \rightarrow g_A = -1.2764(6) \]

V_{ud} from free neutron decay

\[ V_{ud} = 0.9763(5) \tau_n(15) g_A(2)_{RC} \rightarrow V_{ud} = 0.9735(5) \tau_n(3) g_A(1)_{RC} \]

Revision of nuclear corrections to 0^+ - 0^+ -beta decay

Seng et al., 1812.03352; MG 1812.04229

\[ V_{ud} = 0.97366(10) F_t(10)_{RC} \rightarrow V_{ud} = 0.97366(32) F_t(10)_{RC} \]

Free neutron decay becomes competitive;
Limitation: the lifetime (also, beam vs. bottle)

Scrutiny of nuclear corrections with new methods

Top-row unitarity: 2.5-3.5\sigma deficit

\[ \Delta_u = -(0.0016 - 0.0021) \pm 0.0006 \]

(depending on V_{us})
Summary on $\gamma W$-box and $\delta_{NS}$

- The $\gamma W$-box in the forward dispersion relation framework
- Hadronic and nuclear corrections in a unified framework
- Nuclear structure leaks in the outer correction, distorts the beta decay spectrum
- Independent test of the shell model $\delta_{NS}$: substantial changes observed; for now only partial evaluation done (QE contribution)
- Nuclear uncertainties shift the emphasis on free neutron decay
- Tensions with CKM unitarity: $\sum_{i=d,s,b} |V_{ui}|^2 - 1 = -0.0016(6)$
Beam - Bottle discrepancy

Instead of $V_{ud}$ look for self-consistency:

$$\tau_n (1 + 3g_A^2) = 5168.98(1.8) \text{s}$$

Assuming HT analysis of $Ft$ from superalloweds

Including PERKEO-III: $g_A = 1.2762(5)$

Preferred value: $\tau_n = 878$ s - very close to bottle

Exp. plans and sensitivities:
Outlook: continued

Improved precision in $\Delta V^\nu_R$:

Better data on $F_3^{WW}$: might be possible within the DUNE and T2HK programs

A direct calculation on the lattice possible with Feynman-Hellmann theorem

\[
H_\lambda = H_0 + 2\lambda_1 \int d^3 x \cos(q \cdot x) J^2_{em}(x) - 2\lambda_2 \int d^3 x \sin(q \cdot x) J_A^3(x)
\]

\[
\left( \frac{\partial^2 E_{N,\lambda}(p)}{\partial \lambda_1 \partial \lambda_2} \right)_{\lambda=0} = \frac{i q_x}{Q^2 \omega} T_N^3(\omega, Q^2)
\]

Via DR

\[
\left( \frac{\partial^2 E_{N,\lambda}(p)}{\partial \lambda_1 \partial \lambda_2} \right)_{\lambda=0} = \frac{4 q_x}{Q^2} \int_0^1 dx \frac{F_N^3(x, Q^2)}{1 - \omega^2 x^2}
\]

Seng, Meissner, PRL122 (2019)

Work in progress: $\gamma W$-box RC to $\pi \ell 3$ decay (Xu Feng, MG, C-Y Seng)

RC: short-range part the same as for neutron (quark counting + pQCD)

Long-range part somewhat different but closely related

“Blind theory analysis” - calculate independently with DR and on the lattice
Unbiased comparison - do the best job you can on central value and uncertainty
Compare after that: gives an almost clean systematic uncertainty of the theory
Outlook: continued

Direct and complete calculation of $\delta_{NS}$ in nuclear models viable!

Isospin symmetry: input to DR related to PV electron scattering data

\[ 4F_{3\gamma W}^{(0)n\rightarrow p} = F_{3\gamma Z}^{p} - F_{3\gamma Z}^{n} \]

Nucleon level:

\[ 4F_{3\gamma W}^{(Z,N)\rightarrow (Z-1,N+1)} = F_{3\gamma Z}^{(Z,N)} - F_{3\gamma Z}^{(Z-1,N+1)} \]

Nuclear level:

Advantage: direct calculation without singling out “quenched Born”, two nucleon contr. etc. Isospin symmetry good (isospin breaking on top of $\alpha/\pi \sim 10^{-5}$ - negligible)

Direct from PVES data - tough: F3 contribution in NC suppressed by $g_V^e = -1 + 4 \sin^2 \theta_W \approx 0.05$

Direct and complete calculation of $\delta_{NS}$ in nuclear models viable!
Isospin breaking in $\beta$-decay vs. neutron skins

However... the largest correction to $F_t$ is ISB $\delta_c$
Isospin breaking in $\beta$-decay vs. neutron skins

ISB in Fermi matrix el.

$$\tau^+$$

$$|i(Z, Z)\rangle$$

$$R_p^i(r)$$

$$|f(Z - 1, Z + 1)\rangle$$

$$R_n^f(r)$$

Radial functions:

$$\int d^3\vec{r} |R_p|^2 = \int d^3\vec{r} |R_n|^2 = 1$$

Fermi matrix element:

$$\int d^3\vec{r} R_p^i R_n^f = 1 - \frac{\delta C}{2}$$

ISB NC matrix element

$$\tau^3$$

$$|i(Z, Z)\rangle$$

$$|f(Z - 1, Z + 1)\rangle$$

$$R_p^i(r), R_n^i(r), R_n^f(r), R_p^f(r)$$

$$\tau^3 = \frac{1}{2} \sum_{\alpha} (b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha)$$

$$\int d^3\vec{r} (|R_p|^2 - |R_n|^2) = 0$$

Neutron skin is nonzero!

$$\int d^3\vec{r} r^2 (|R_p|^2 - |R_n|^2) = \langle r_n^2 \rangle - \langle r_p^2 \rangle$$
PVES on nuclei: weak charges and radii

For a spin-0 nucleus
\[
A^{PV} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{Q_W F_W(Q^2)}{Z F_C(Q^2)}
\]

Photons almost exclusively see protons \( Q(Z, N) = Z \)
Z’s almost exclusively see neutrons \( Q_W(Z, N) = Z(1 - 4\sin^2\theta_W) - N \approx -N \)

At low \( Q^2 \)
\[
\frac{F_W(Q^2)}{F_C(Q^2)} = 1 - \frac{R_W^2 - R_{Ch}^2}{6} Q^2 + \ldots
\]

Sensitive to the neutron skin \( R_W^2 - R_{Ch}^2 \approx R_n^2 - R_p^2 \)

PREX:
Weak radius to 3%
\[
A^{PV}(Pb - 208) = 0.656(60)(14) \text{ ppm}
\]
\[
R_n - R_p = 0.33^{+0.16}_{-0.18} \text{ fm} \quad (R_{Ch} = 5.5 \text{ fm})
\]

\[
R_{n,p}^2 = \int d^3r r^2 \rho_{n,p}(r) \quad R_{Ch}^2 = \int d^3r r^2 \rho_{Ch}(r) \quad R_W^2 - R_{Ch}^2 = \frac{R_n^2 - R_p^2}{1 - (1 - 4\sin^2\theta_W)Z/N}
\]

Usually small asymmetries \( \rightarrow \) can only measure one point in \( Q^2 \)
1. get rid of the FF to extract the weak charge (Q-Weak; P2@MESA) - measure \( \sin^2\theta_W \)
2. assume that \( Q_W \) is known - measure the FF (PREXI,II; CREX; MREX) - neutron skin

Plans at MESA: C-12 experiment may for the first time extract both with <1% precision!
P2 experiment @ MESA

Main goal: proton’s weak charge $\sim 0.07 \rightarrow$ test SM
200 days of data - 150 $\mu$A beam - 85% polarization

Additionally: $A_{PV}^\mu$ measurement on C-12
Weak charge is 15 times larger than $p$;
Cross sections 36 times larger than $p$;
2500h data - 0.3% on $\sin^2\theta_W$ possible!
PVES on C12: weak charge and radius

To extract weak charge and radius to sub-% precision - need to control RC and nuclear structure

Dedicated study of Coulomb distortion + ISB nuclear structure beyond neutron skin

MG et al, to be submitted!

\[ A^{PV} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{Q_W F_W(Q^2)}{Z F_C(Q^2)} \]

\[ F_{Ch}(Q^2) = 1 - (R_{Ch}^2 / 6) Q^2 + (\bar{R}_{Ch}^4 / 120) Q^4 - \ldots \]

\[ F_W(Q^2) = 1 - (R_{W}^2 / 6) Q^2 + (\bar{R}_{W}^4 / 120) Q^4 - \ldots \]

Idea: forward angle to measure the weak charge, backward to constrain skin

0.3% forward asymmetry —> requirement on the weak skin —> constrain it w. backward measurement

\[ \Delta R_W/R_{ch} \leq - (1.1 \pm 0.6) \% \]

\[ \delta A^{PV}(\theta = 145^\circ) \leq 7.5 \% \]
Proposal: neutron skins of stable 0+ daughters

Neutron skin C-12: per se not so interesting
Symmetric nucleus - cannot study symmetry energy, no connection to EOS of neutron-rich matter…
But in absence of large effects can study small effects - like isospin symmetry breaking

Fit fit dominated by 5 medium-Z decays: $^{26m}\text{Al} - ^{26}\text{Mg}$, $^{34}\text{Cl} - ^{34}\text{S}$, $^{38m}\text{K} - ^{38}\text{Ar}$, $^{42}\text{Sc} - ^{42}\text{Ca}$, $^{46}\text{V} - ^{46}\text{Ti}$
All 5 daughters are stable (but rare - $$$) - possible targets in PVES exp.

If works with C-12 at the planned precision - can measure $^{26}\text{Mg}$, $^{34}\text{S}$, $^{38}\text{Ar}$, $^{42}\text{Ca}$, $^{46}\text{Ti}$

Central questions:

what precision is interesting/necessary?
Is the connection skin - $\delta_C$ model-independent?
If not - a weaker statement: each model used to calculate $\delta_C$ should predict the skin - check with exp.
What about unstable parent nuclei? Scattering off heavy nuclei at FRIB? GSI?

Beyond this program: $\delta_C$ can be calculated in nuclear models (other than shell model)
Longstanding discussion (shell model vs. DFT vs. Hartree-Fock)
Novel methods - ab initio, QMC, … - discussed in Trento - what’s the status?

Fresh wind in nuclear corrections to beta decays!
New tools and richer context! (DR + PVES exp. + neutrino data) $\leftarrow$ Nuclear theory