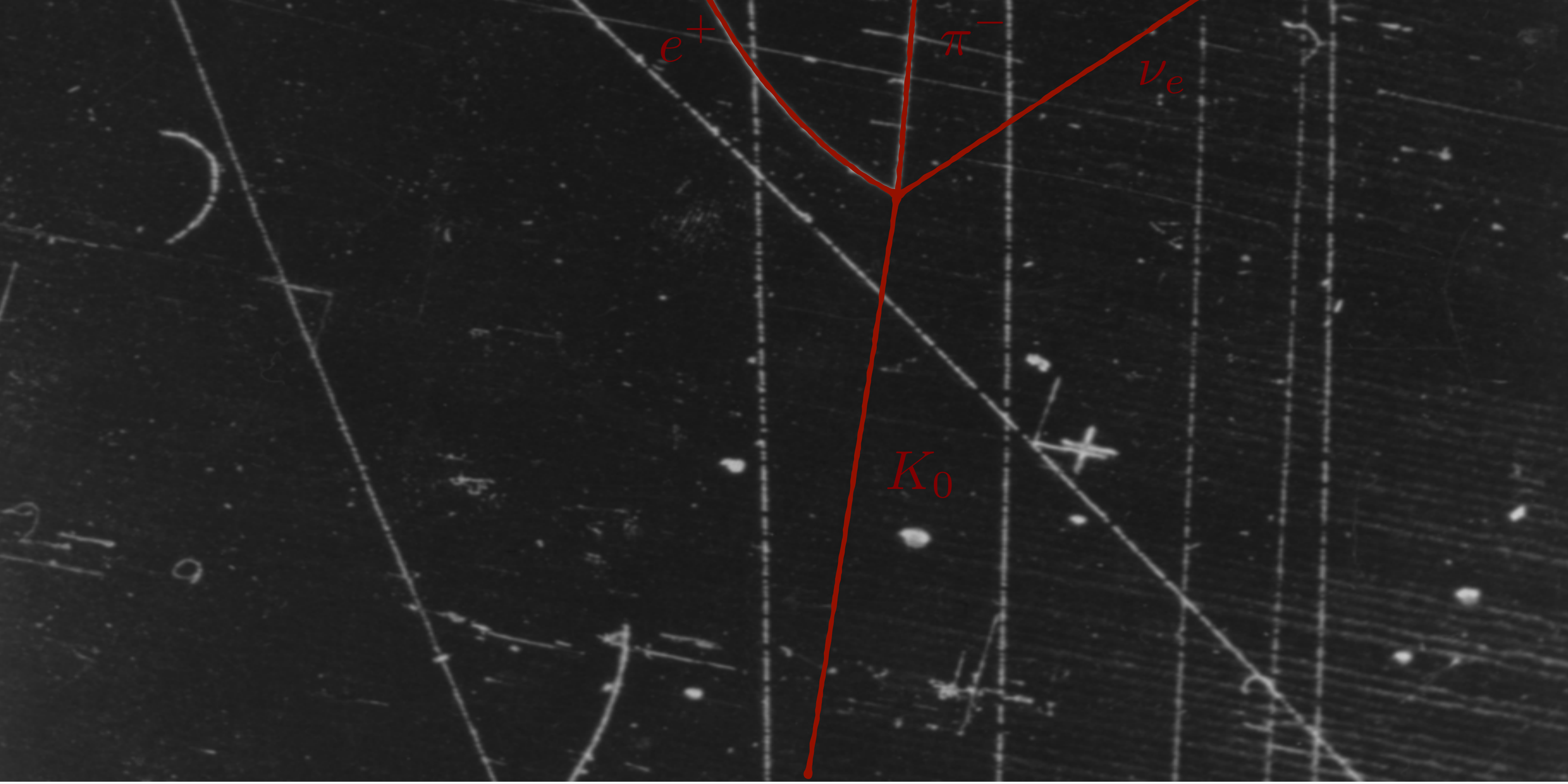

Recent developments in light flavour physics

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3rd of February 2020
CERN

THE UNIVERSITY of EDINBURGH

Picture: Berkley Lab 26/03/1959 - US National Archive

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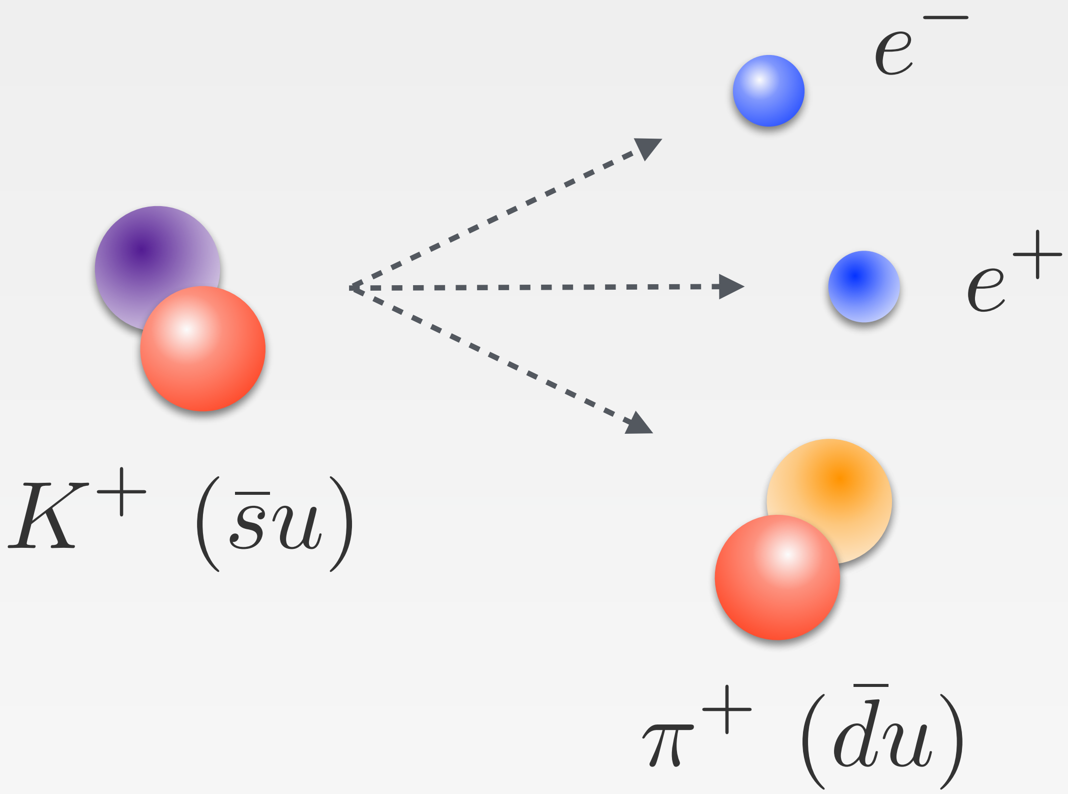
- Rare kaon decays at the physical point

- Isospin corrections to $K_{\ell 2}$ decays at the physical point

Rare kaon decays at the physical point

Rare kaon decays



Flavour Changing Neutral Current

Extremely rare in the SM

⇒ sensitive to new physics

Experimental measurement **in progress at NA62 (CERN)**.
Important results are expected in the next five years.

Improved theory predictions are needed.

Decay channels

- $K^+ \to \pi^+ \ell^+ \ell^-$
  Long-distance dominated, "easy" to see experimentally.
- $K^0_{L/S} \to \pi^0 \ell^+ \ell^-$
  Long-distance dominated, interesting CP violations.
- $K^+ \to \pi^+ \bar{\nu} \nu$
  Mainly short-distance (top loop), **NA62 Run 1**.
  Long-distance charm effects?
- $K^0_{L/S} \to \pi^0 \bar{\nu} \nu$
  Short-distance (top loop) dominated. KOTO experiment.

Lattice

Long-distance amplitude

$K^c \to \pi^c \gamma^*$

EM current

$$\mathscr{A}^c_\mu(q^2) = \int \mathrm{d}^4x \, \langle \pi^c(\mathbf{p}) | \, \mathrm{T}[J_\mu(0) H_W(x)] \, | K^c(\mathbf{k}) \rangle$$

$\Delta S = 1$ Effective weak Hamiltonian

$$\mathscr{A}^c_\mu(q^2) = -i \frac{G_F}{(4\pi)^2} [q^2(k+p)_\mu - (M_K^2 - M_\pi^2) q_\mu] V_c(z)$$

$$V_c(z) = a_c + b_c z + V_c^{\pi\pi}(z) \qquad z = q^2/M_K^2$$

SM prediction?

[D'Ambrosio et al., JHEP08 (1998) 004]

Phenomenological relevance

- LFUV can be probed through the difference $a_+^{\mu\mu} - a_+^{ee}$.
- Assuming MFV, related to B decays.

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2}\lambda_t}$$

[Crivellin et al., PRD 93(7) 074038, 2016]

- SM value important for disambiguation, experiment only allow access to $|a_+ + b_+ z|^2$.
(although combined e/μ analysis clearly favour - sign)

[D'Ambrosio et al., JHEP02 (2019) 49]

Minkowski spectral representation

$$\mathscr{A}_\mu^c(q^2) = i \int_0^{+\infty} \mathrm{d}E \, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})| \, J_\mu \, |E, \mathbf{k}\rangle \langle E, \mathbf{k}| \, H_W \, |K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E + i\varepsilon}$$

$$- \, i \int_0^{+\infty} \mathrm{d}E \, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})| \, H_W \, |E, \mathbf{p}\rangle \langle E, \mathbf{p}| \, J_\mu \, |K^c(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p}) + i\varepsilon}$$

[RBC-UKQCD, PRD 92(9), 094512, 2015]

Euclidean spectral representation

$$\mathscr{A}_\mu^c(q^2, T_a, T_b) = -\int_0^{+\infty} \mathrm{d}E \, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})| J_\mu |E, \mathbf{k}\rangle \langle E, \mathbf{k}| H_W |K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E} \times (1 - e^{[E_K(\mathbf{k}) - E]T_a}) + \int_0^{+\infty} \mathrm{d}E \, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})| H_W |E, \mathbf{p}\rangle \langle E, \mathbf{p}| J_\mu |K^c(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})} \times (1 - e^{-[E - E_\pi(\mathbf{p})]T_b})$$

Time integration range: $[-T_a, T_b]$.

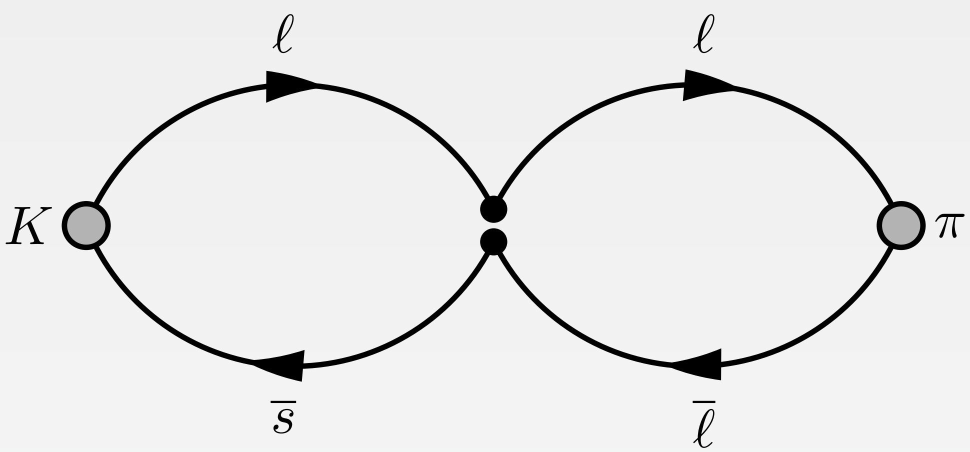
Diverges at infinite time for $E < E_K(\mathbf{k})$.

"Simple" here (only π, $\pi\pi\pi$).

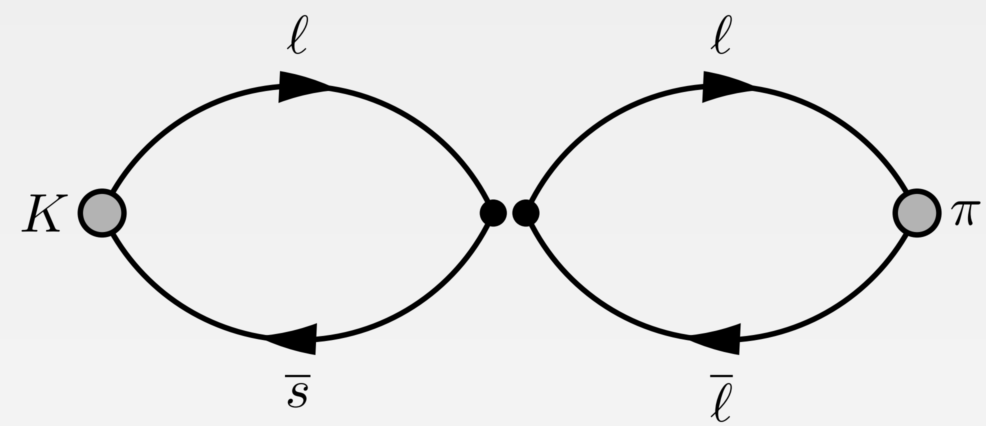
Try to think about rare B decays!

[RBC-UKQCD, PRD 92(9), 094512, 2015]

Lattice correlators

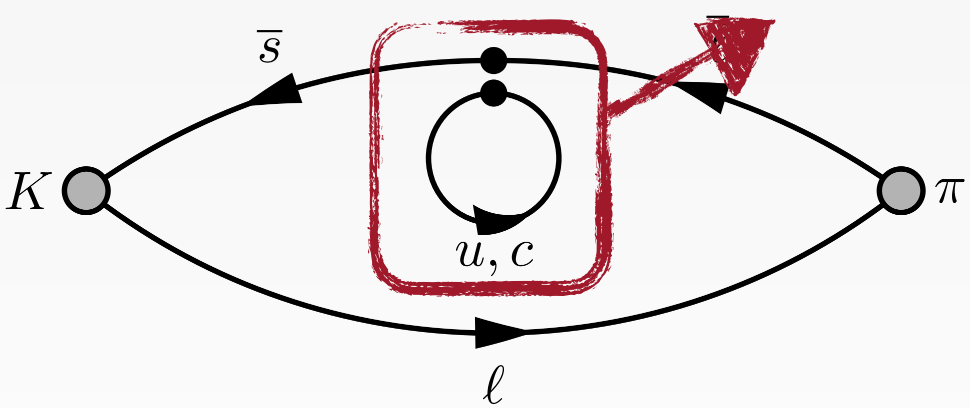


C: "Connected"

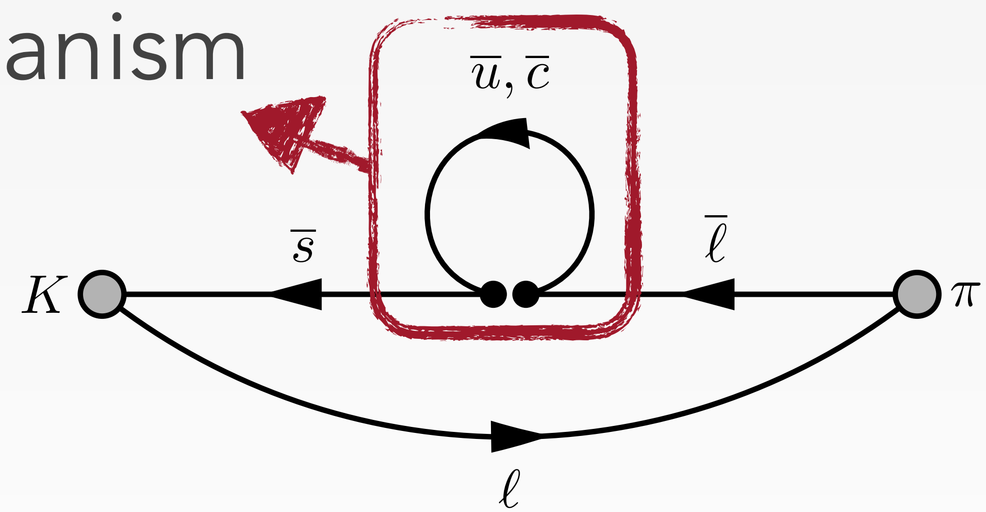


W: "Wing"

GIM mechanism



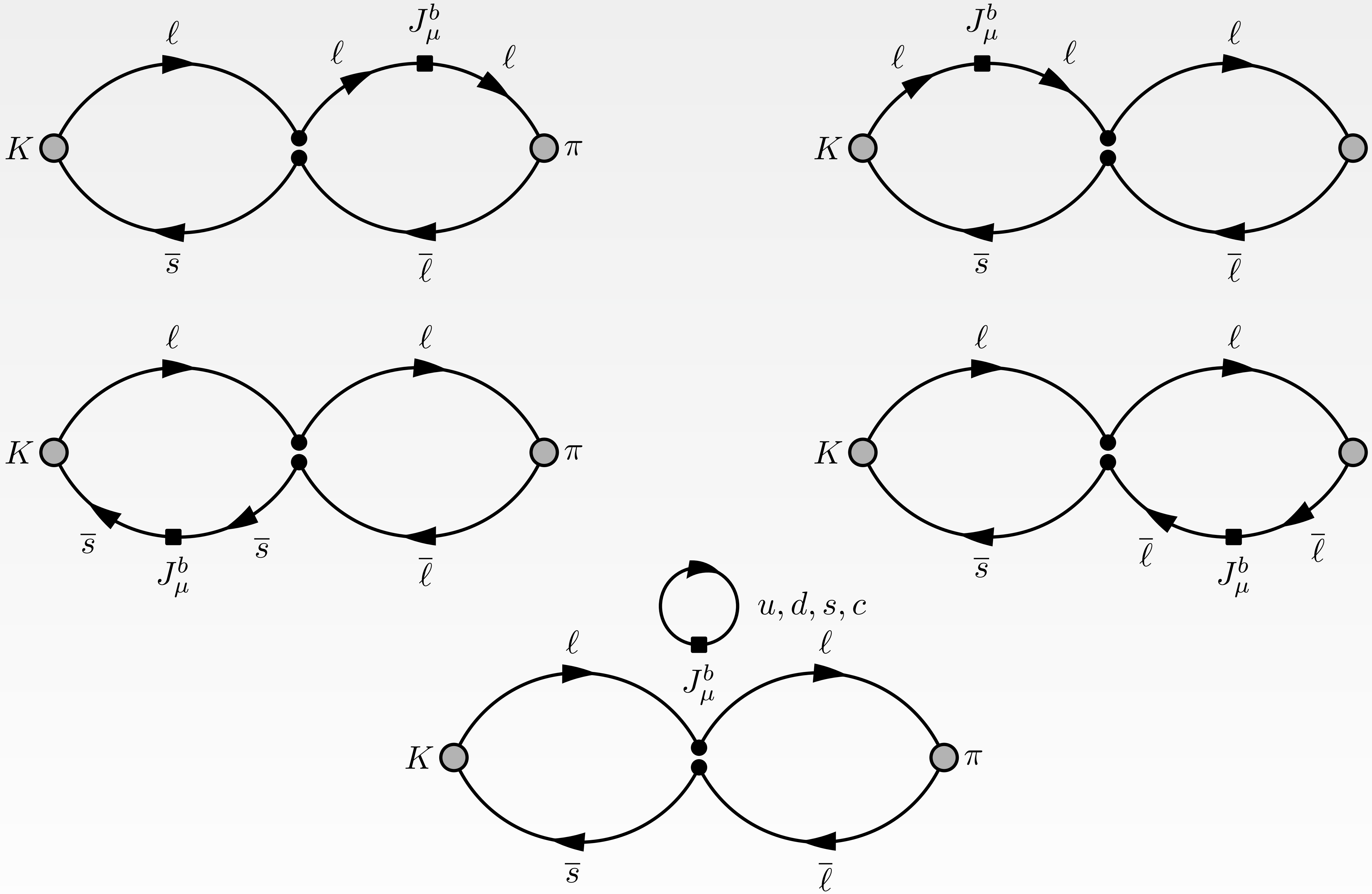
E: "Eye"



S: "Saucer"

[RBC-UKQCD, PRD 92(9), 094512, 2015]

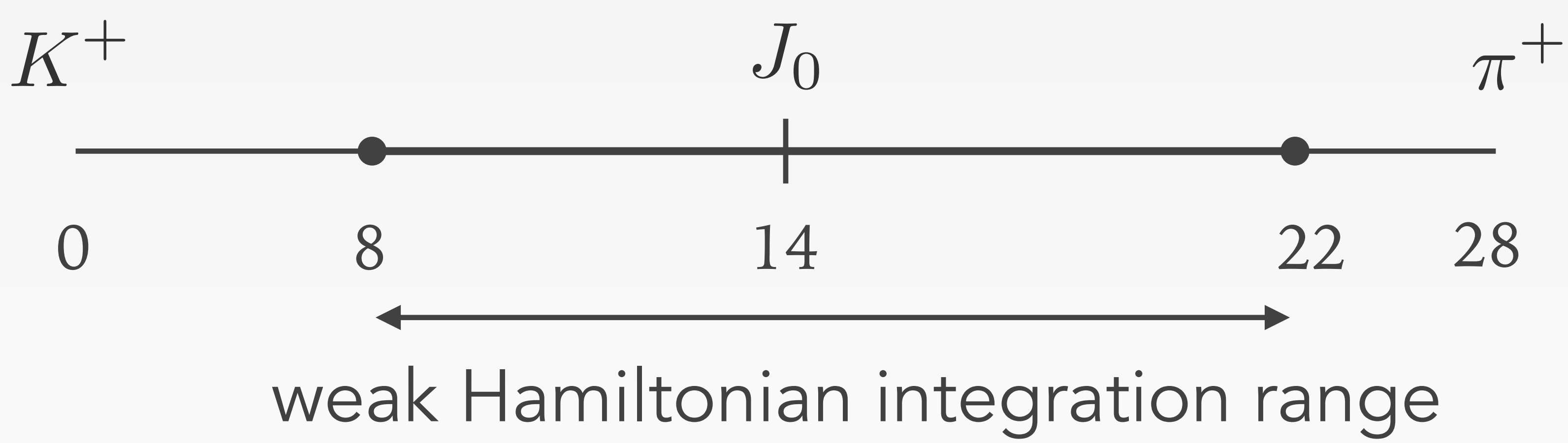
Lattice correlators



[RBC-UKQCD, PRD 92(9), 094512, 2015]

Unphysical lattice setup

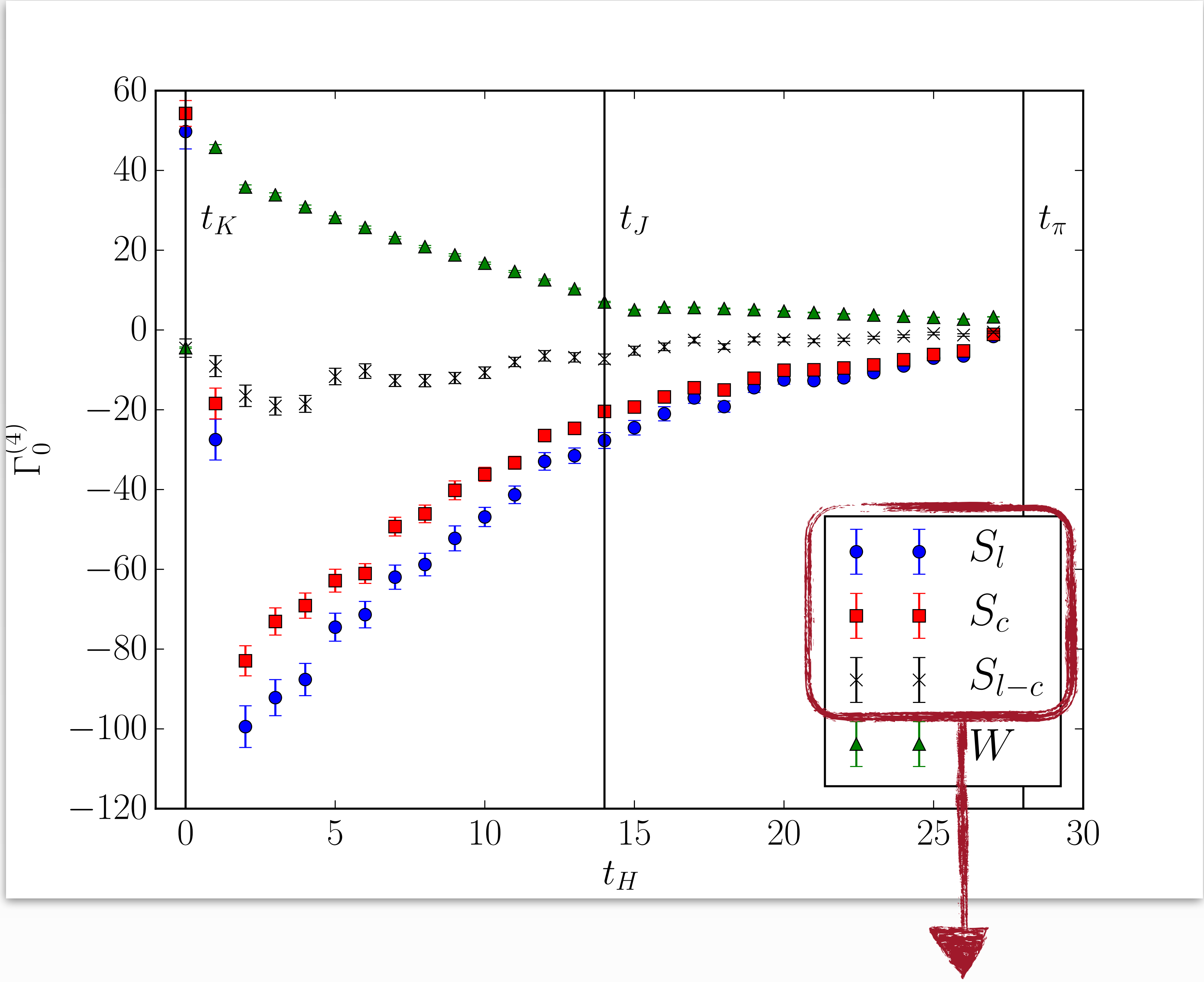
- DWF action, $24^3 \times 64$ lattice with spacing ~0.12 fm.
- $N_f = 2+1,\ M_\pi \simeq 420\ \mathrm{MeV}$ and $M_K \simeq 600\ \mathrm{MeV}$.



- For this kinematics only single π state is problematic.

[RBC-UKQCD, PRD 94(1), 114516, 2016]

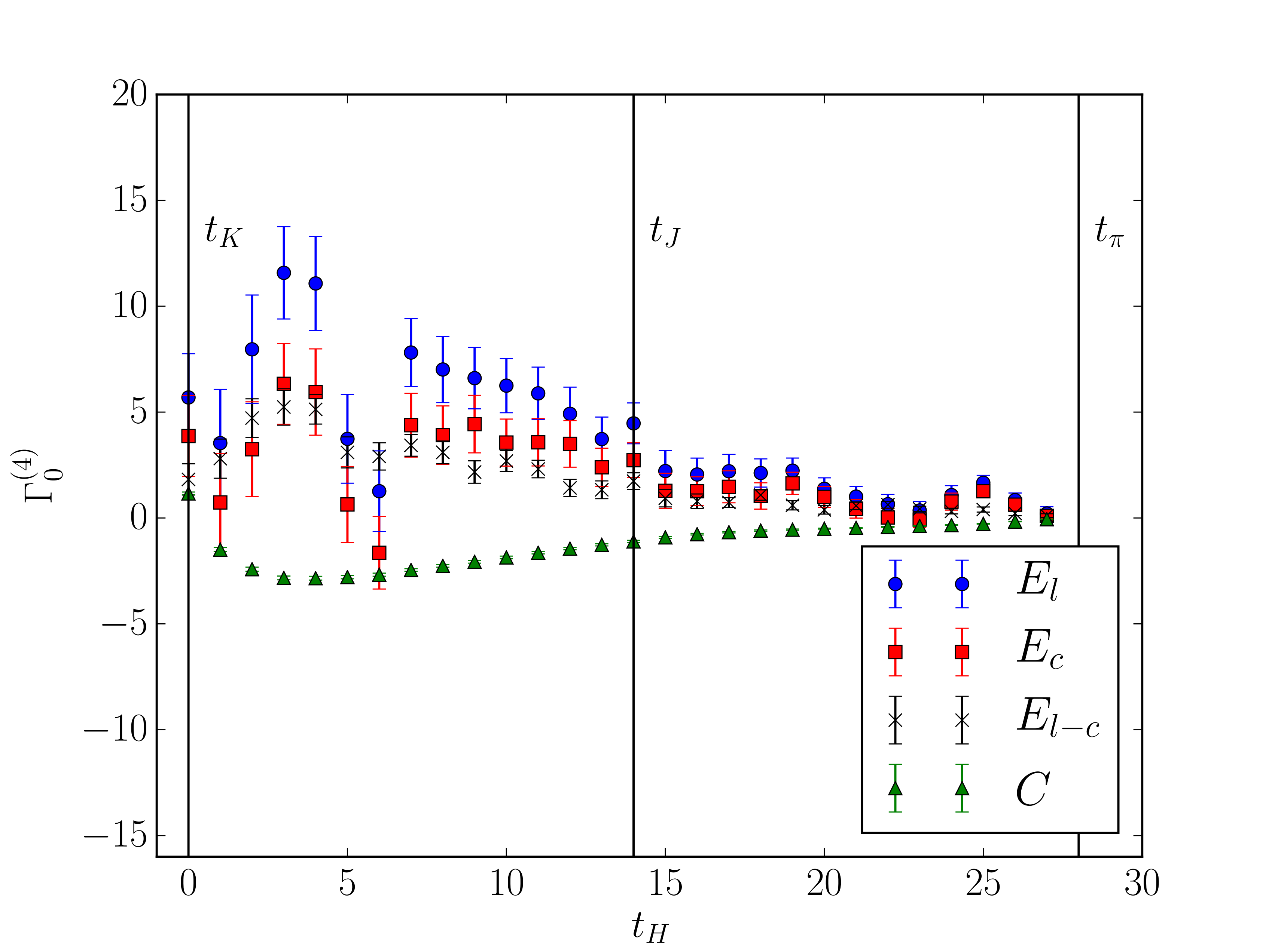
Results: correlators



GIM mechanism

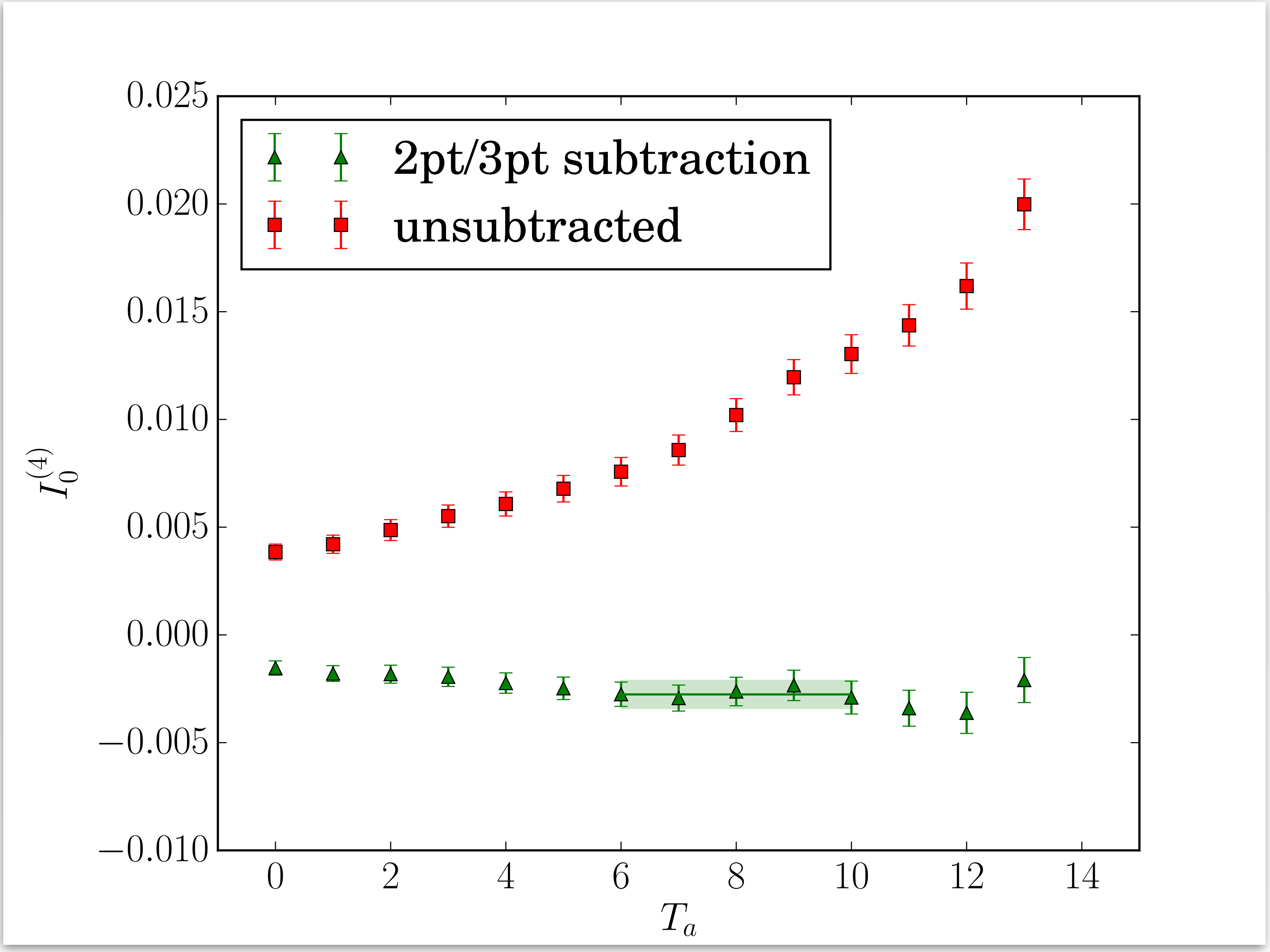
[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: correlators



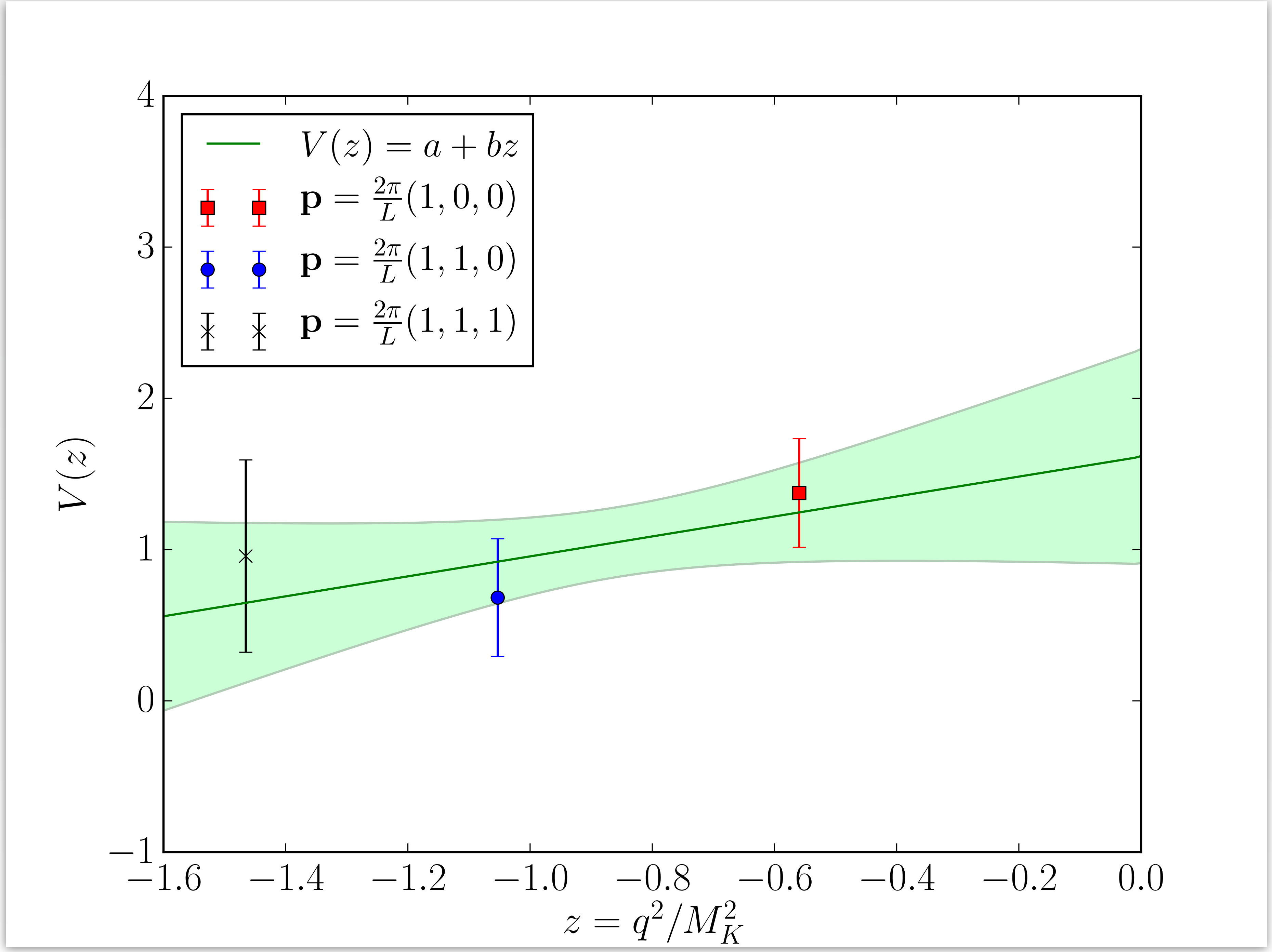
[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: exponential subtraction



[RBC-UKQCD, PRD 94(1), 114516, 2016]

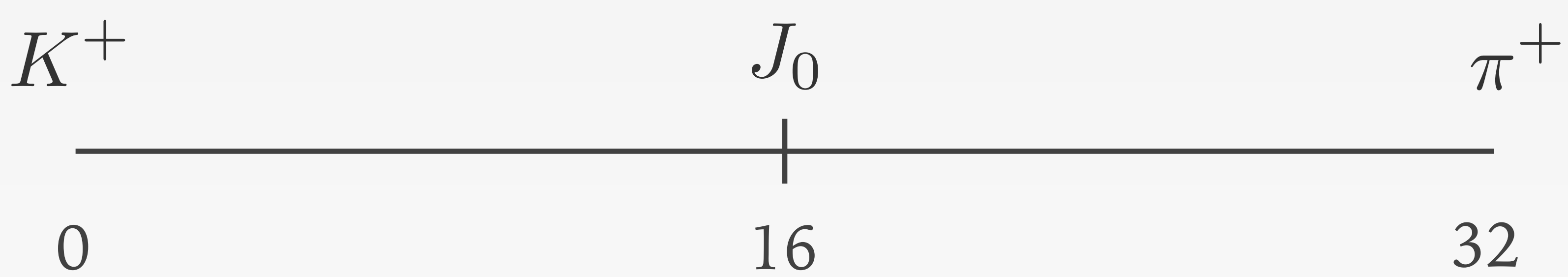
Results: form factor



[RBC-UKQCD, PRD 94(1), 114516, 2016]

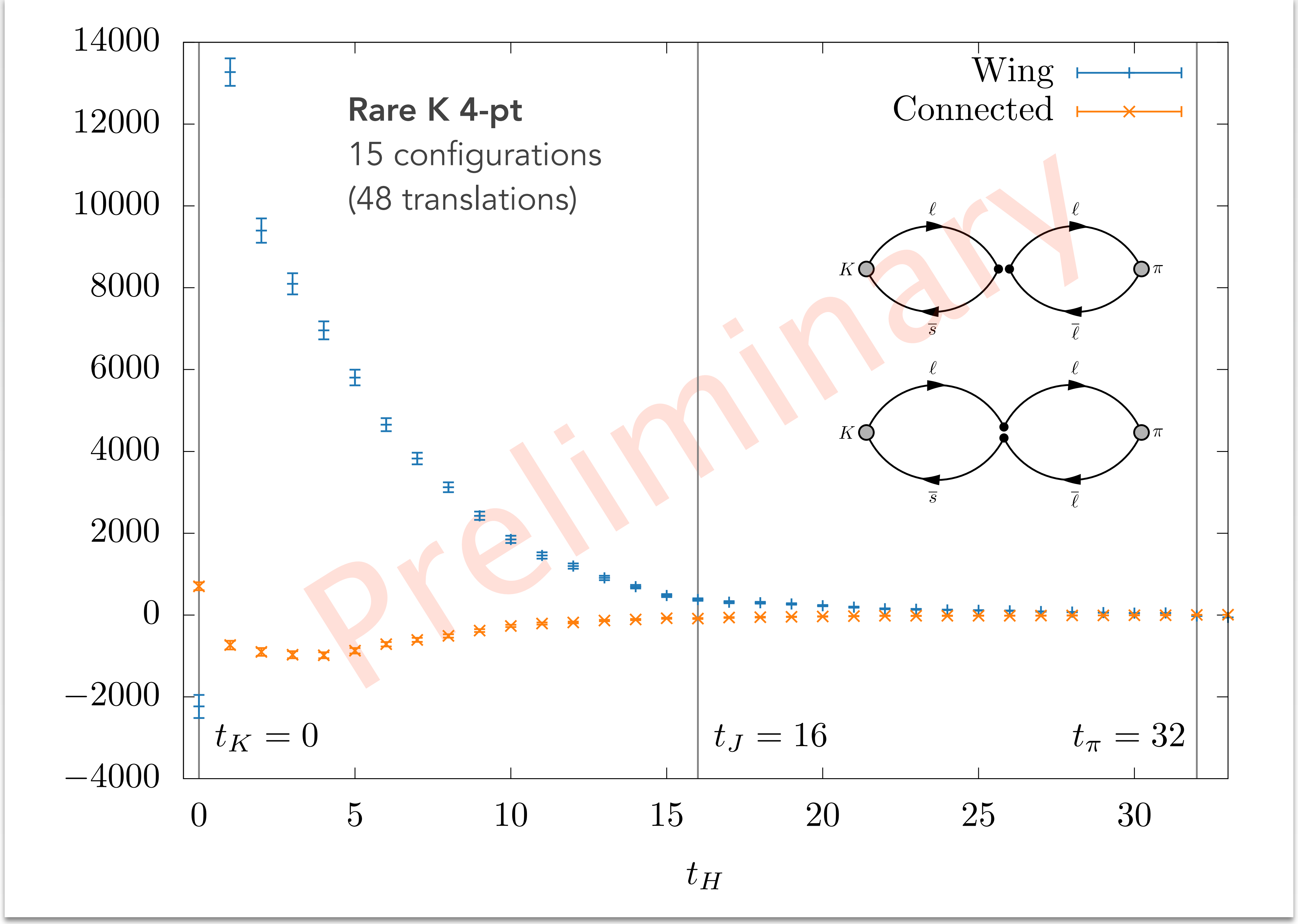
Physical lattice setup

- DWF action, $48^3 \times 96$ lattice with spacing ~0.12 fm.
- $N_f = 2 + 1, \; M_\pi \simeq 140 \text{ MeV}$ and $M_K \simeq 495 \text{ MeV}$.

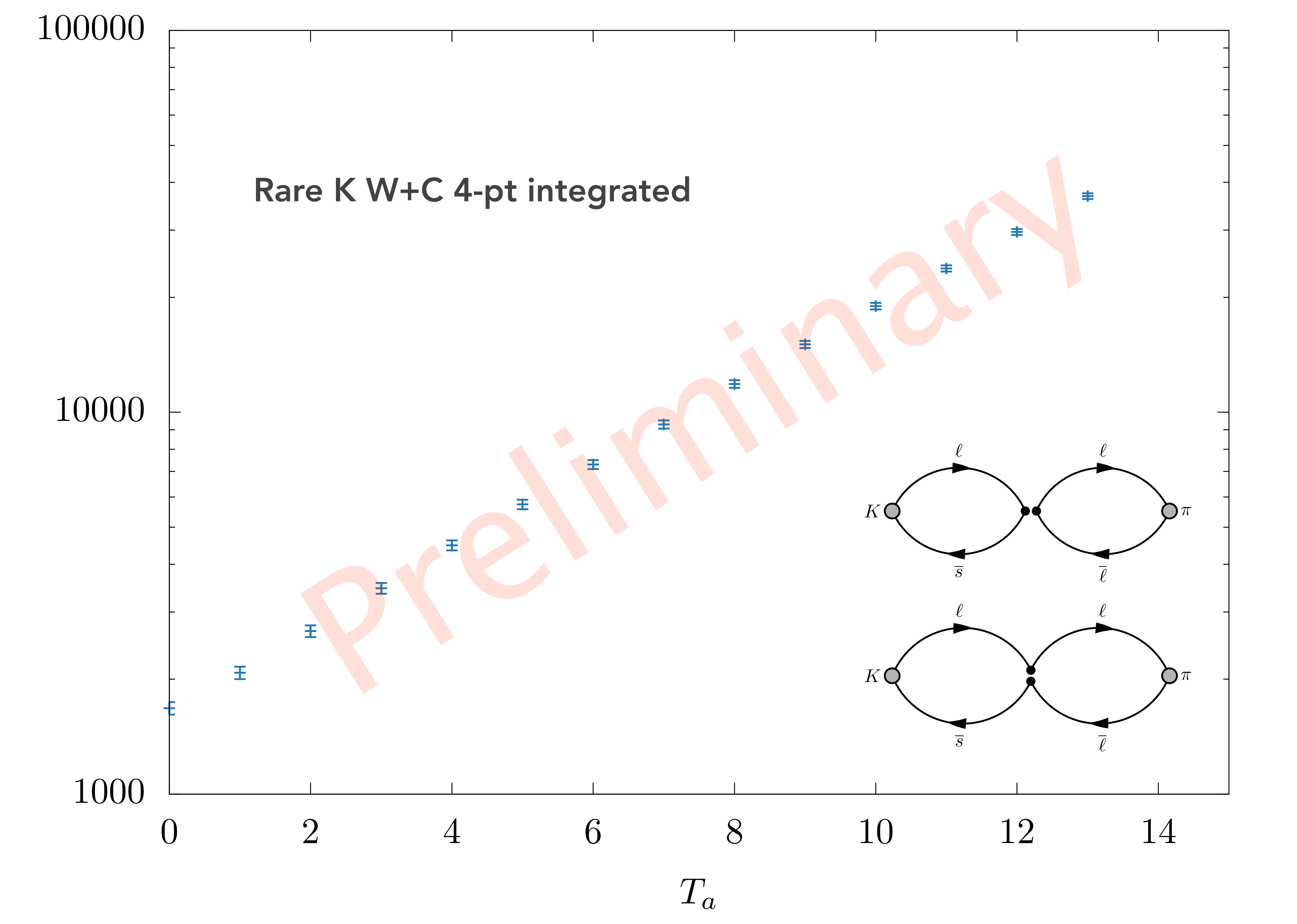


- Kaon at rest, pion momentum $|p_\pi| = \dfrac{2\pi}{L} \simeq 226 \text{ MeV}.$
- $z \simeq 0.01$, excellent to determine a_+ directly.

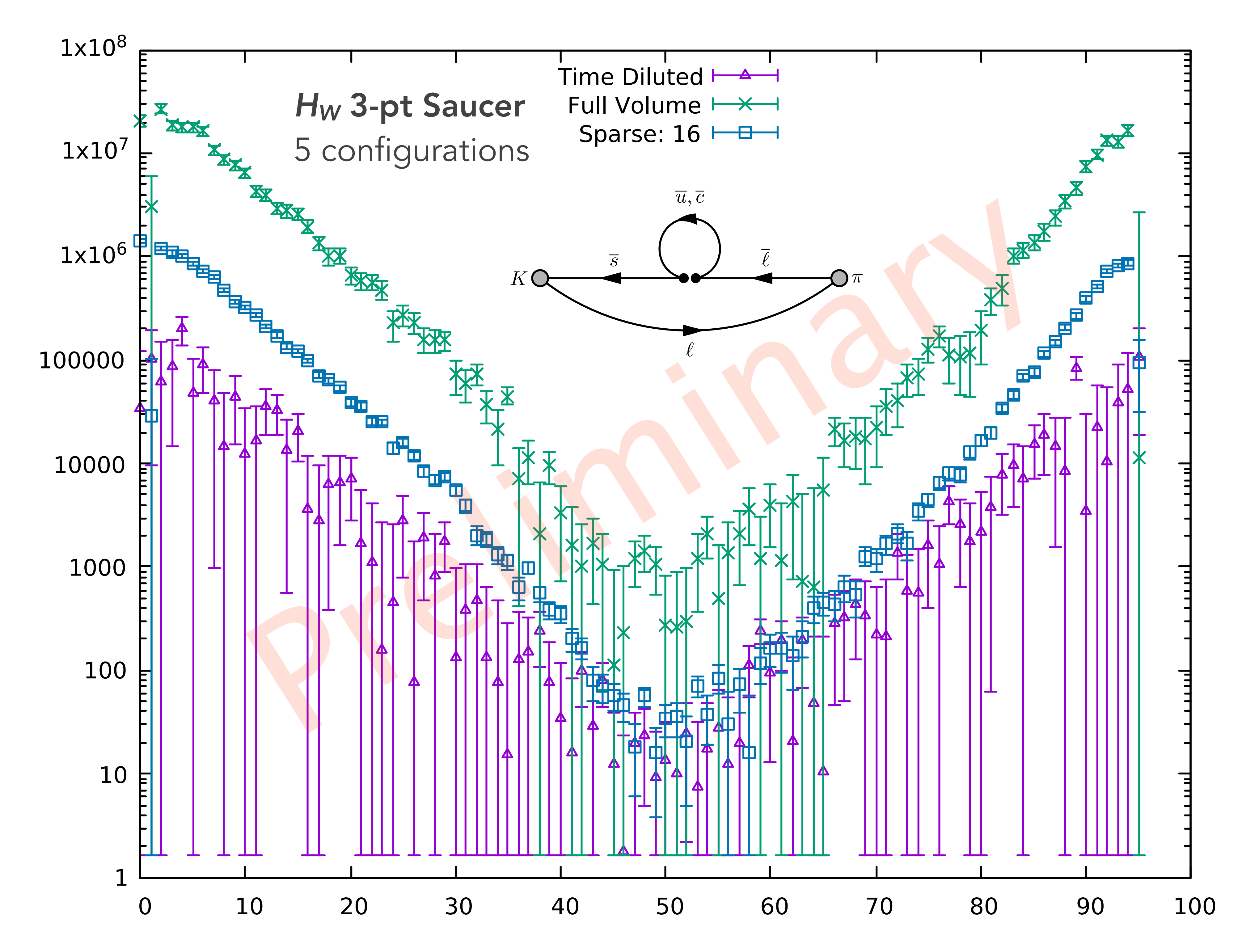
Preliminary results



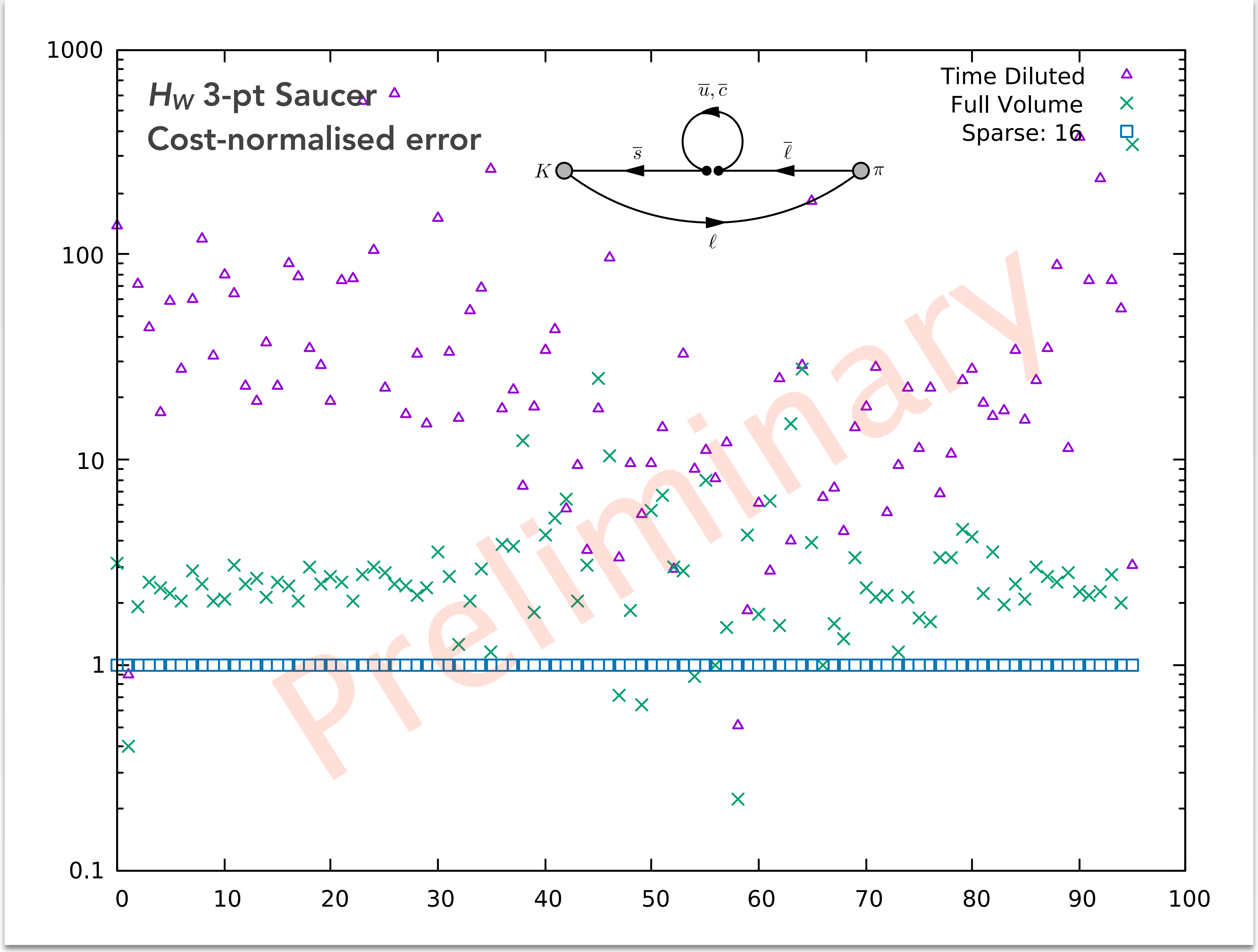
Preliminary results



Preliminary results



Preliminary results



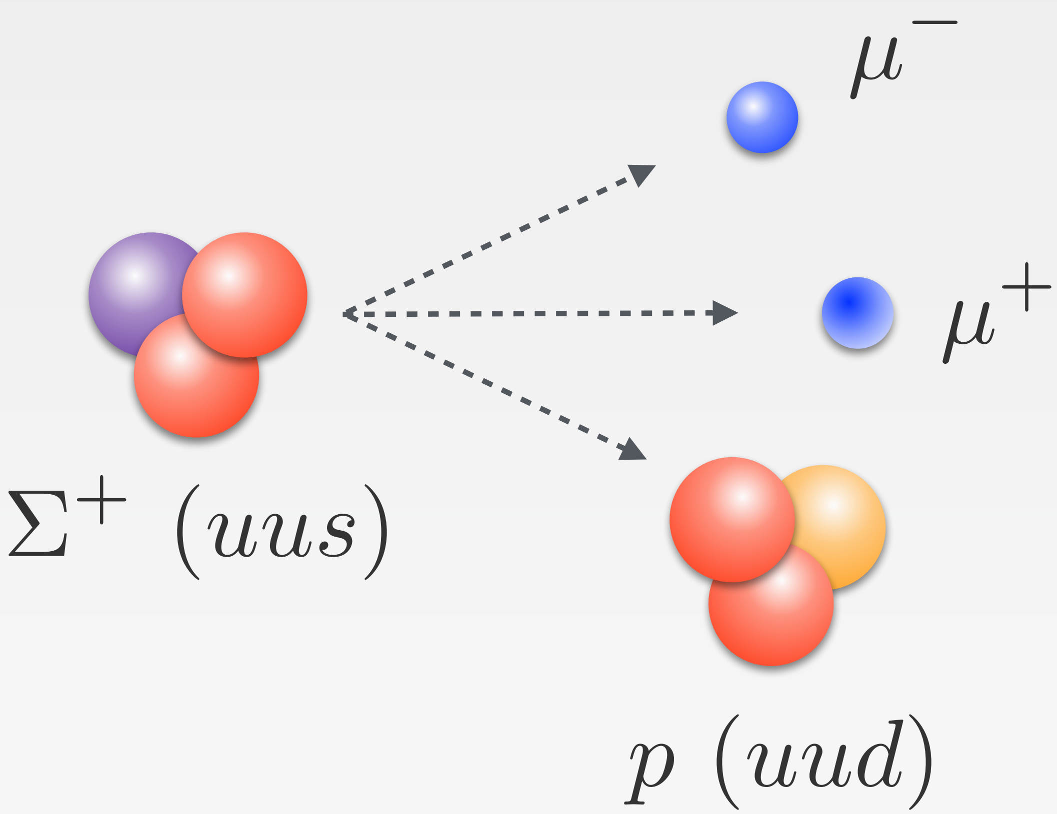
Conclusion

- Lattice framework for rare K decays achieved.
- Proof-of-concept calculations successful.
- Results comparison with phenomenology/experiment difficult because of unphysical parameters.

Perspectives

- Physical quark calculation: now!
  Direct determination of a_+ first, b_+ later.
- $\pi\pi$ & $\pi\pi\pi$ contamination problematic?
- We are excited with the NA62 $K^+ \to \pi^+ \bar{\nu}\nu$ and $K^+ \to \pi^+\mu^+\mu^-$ results.
- $K^+ \to \pi^+ e^+ e^-$ in future runs?

Rare hyperon decays



Still FCNC

"Rare kaons with a spectator"

- Observed at HyperCP in 2005 and LHCb in 2018.
- SM value poorly known.
- Lattice project in progress, very much the beginning!

Current status

- LHCb 2018 measurement

$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) = (2.2^{+1.8}_{-1.3}) \cdot 10^{-8}$$

[LHCb, PRL 120(2), 2018]

- Theory state-of the art

$$1.6 \cdot 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 9.0 \cdot 10^{-8}$$

[He, Tandean and Valencia, PRD 72(7), 2005]

Mix of baryon ChiPT, dispersive analysis, VMD, …
Essentially 100% long-distance.
Most poorly described part: real part.

On the menu…

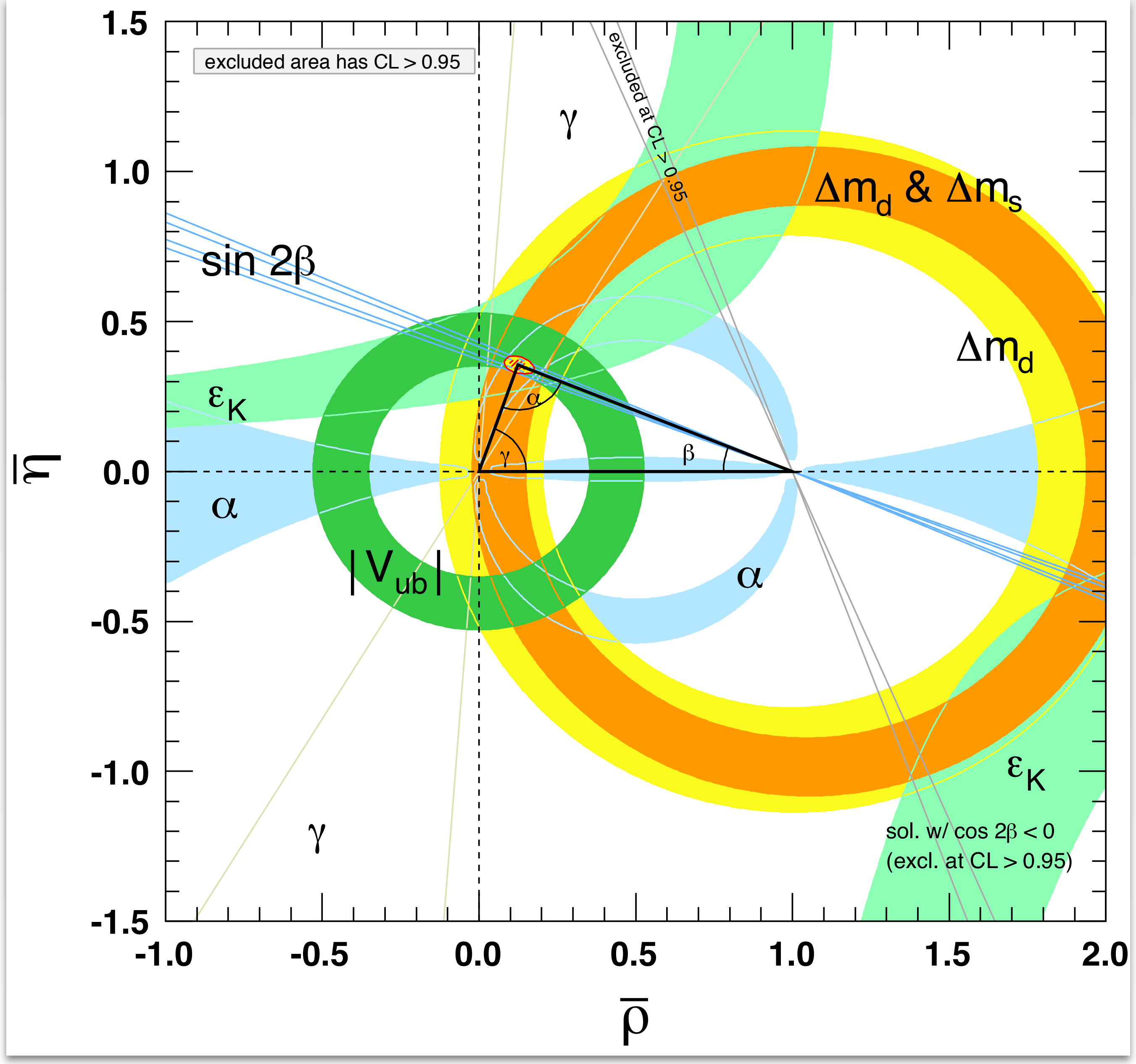
- 4 form factors to determine

$$\mathcal{A}_\mu = i\sigma_{\mu\nu}q_\nu[a(q^2) + \gamma_5 b(q^2)] + (q^2\gamma_\mu - q_\mu \not{q})[c(q^2) + \gamma_5 d(q^2)]$$

- Painful contractions.
- N and $N\pi$ intermediate state growing exponentials.
- Possibly a big signal-to-noise ratio problem.
- Now working on a $M_\pi \sim 350\ \mathrm{MeV}$ test run (no $N\pi$ contaminant here)

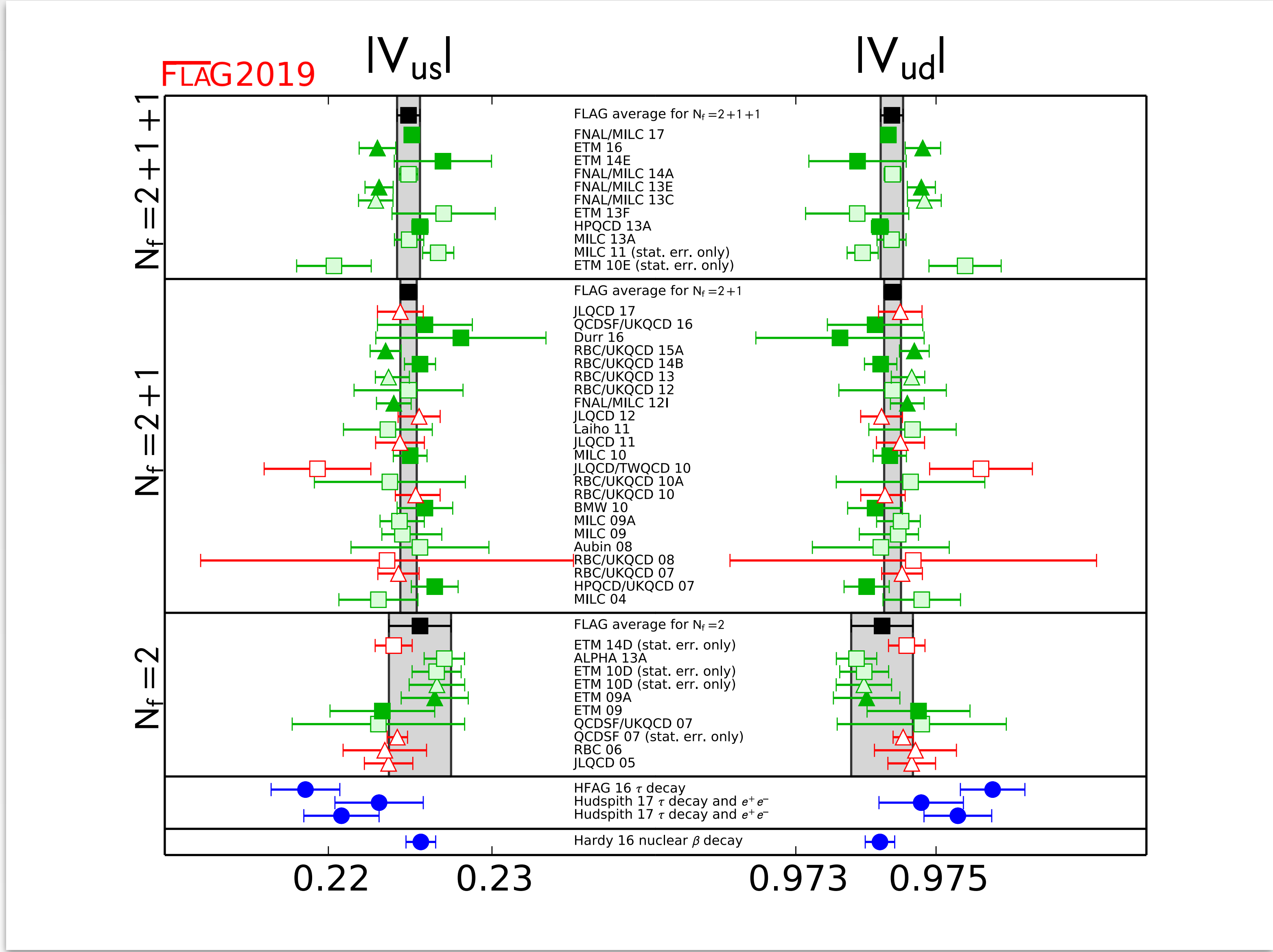
Isospin corrections to $K_{\ell 2}$ decays at the physical point

Motivations



- Search new physics through the SM flavour structure

Motivations



- Light CKM coefficients: percent level reached.
- Inclusion of isospin breaking effects required to progress.

Current lattice status

- Nice theory & lattice work by RM123

  [RM123, PRD 100(3), 034514, 2019]
  [RM123, PRL 120(7), 072001, 2018]
  [RM123, PRD 95(3), 034504, 2017]
  [RM123, PRD 91(7), 074506, 2015]

- New challenge here: **physical point calculation**.

Full width IB corrections

- Full width (rest frame)

$$\Gamma(P^+ \to \ell^+ \nu_\ell) = \Gamma = K \sum_{r,s} |\mathcal{M}^{rs}|^2$$

$$\mathcal{M}^{rs} = \langle \ell^+, r; \nu_\ell, s | \, H_W \, | P^+ \rangle = \overline{u}^s_\nu \, \widetilde{\mathcal{M}} \, v^r_\ell$$

- Tree-level matrix element (factorisable final state)

$$\mathcal{M}_0^{rs} = f_P M_P (\overline{u}^s_\nu \gamma_0^L v^r_\ell)$$

- Width first-order isospin breaking (IB) corrections

$$\delta\Gamma = \delta K \sum_{r,s} |\mathcal{M}_0^{rs}|^2 + 2K_0 \underbrace{\sum_{r,s} \Re(\mathcal{M}_0^{rs} \delta\mathcal{M}^{rs\,*})}_{f_P M_P \, \mathrm{tr}[\not{p}_\nu \delta\widetilde{\mathcal{M}} (-\not{p}_\ell + i m_\ell)\gamma_0^L]}$$

IB corrections classification

- $\mathcal{O}(m_u - m_d)$: strong IB corrections
- $\mathcal{O}(e_q^2)$: quark QED corrections
- ~~$\mathcal{O}(e_\ell^2)$: lepton QED corrections~~

  Absorbed in renormalisation

Factorisable

- $\mathcal{O}(e_\ell e_q)$: quark-lepton QED corrections

Non-factorisable

Factorisable corrections

$$\delta_{\text{fact.}}\mathcal{M}^{rs} = (\overline{u}^s_\nu \gamma^L_0 v^r_\ell)\delta\mathcal{M}_A$$

$$\delta\mathcal{M}_A = \delta(\langle 0| A_0 |P^+\rangle)$$

- "Standard" axial-pseudoscalar 2-point analysis

$$\delta C_{AP}(t) = C_{AP,0}(t)\left[\frac{\delta\mathcal{M}_P}{\mathcal{M}_{P,0}} + \frac{\delta\mathcal{M}_A}{\mathcal{M}_{A,0}} - \frac{\delta M_{P+}}{M_P} - t\delta M_{P+}\right]$$

$$\delta C_{PP}(t) = C_{PP,0}(t)\left[2\frac{\delta\mathcal{M}_P}{\mathcal{M}_{P,0}} - \frac{\delta M_{P+}}{M_P} - t\delta M_{P+}\right]$$

$$(t \gg 0)$$

Non-factorisable correction

- Amputated operator and matrix element

$$\overline{H}_W^{\alpha} = (\gamma_\mu^L \ell)^\alpha (\overline{q}_1 \gamma_\mu^L q_2)$$

$$\overline{\mathcal{M}}^{r\alpha} = \langle \ell^+, r | \overline{H}_W^\alpha | P^+ \rangle = (\widetilde{\mathcal{M}} v_\ell^r)^\alpha$$

- Euclidean 3-point function

$$C_3^{\alpha\beta}(t_\ell, t_H, t_P) = \langle \overline{\ell}^\alpha \overline{H}_W^\beta \phi_P^\dagger \rangle$$

- Asymptotic behaviour ($t_P \ll t_H$ and $t_\ell \gg t_H$)

$$\delta_{\ell q} C_3^{\alpha\beta} = \frac{\mathcal{M}_{P,0}[\delta_{\ell q}\widetilde{\mathcal{M}}(-\not{p}_\ell + i m_\ell)]^{\alpha\beta}}{4\omega_\ell M_P} e^{-(t_H - t_P)M_P} e^{-(t_\ell - t_H)\omega_\ell}$$

bit of the width correction

Infrared divergences

[RM123, PRD 95(3), 034504, 2017]

- Finite-volume (FV): IR regulator.
- FV effects $\sim \log(\Lambda_{\mathrm{IR}} L)$, infinite-volume regulator Λ_{IR} .
- FV subtraction: swap L and M_P .
- Add $\Gamma(P^+ \to \ell^+ \nu_\ell \gamma)$: swap Λ_{IR} and experimental cut.
- Performed in the point-like approximation.
  (we are working on the structure-dependent part)

Gauge ensemble

[RBC-UKQCD, PRD 93(7), 074505, 2016]

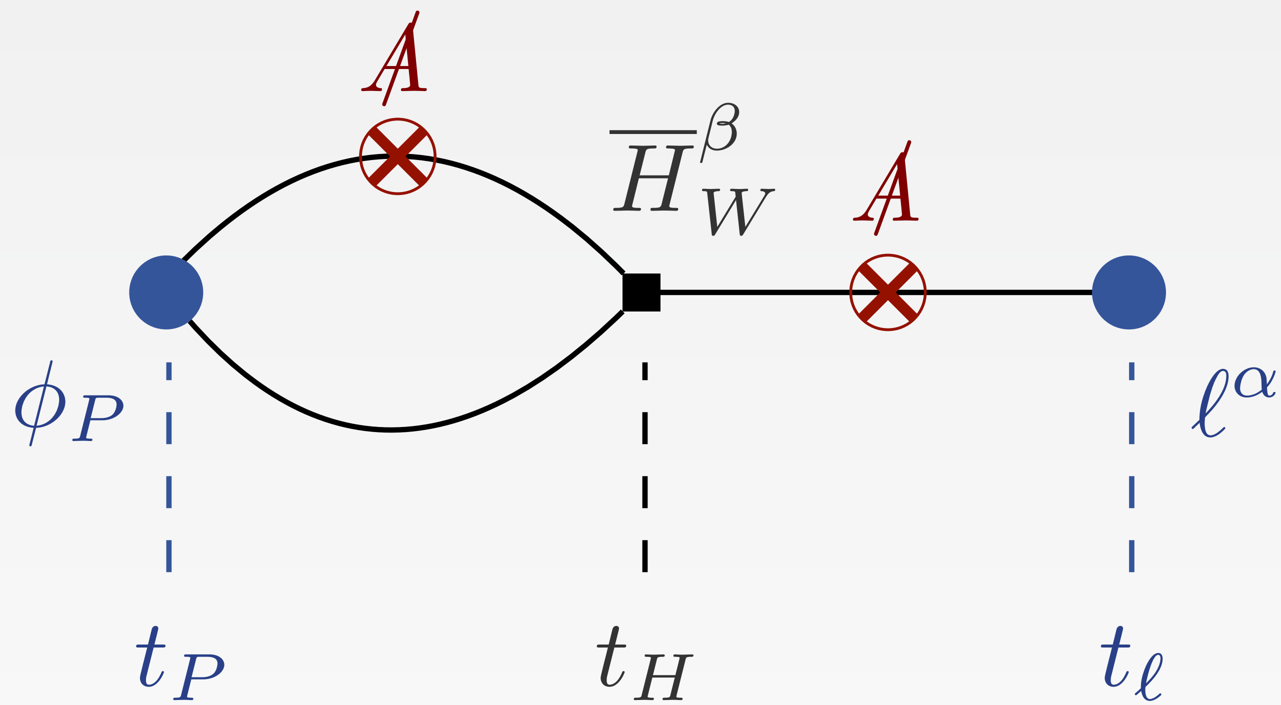
- Physical point Möbius domain-wall fermions.
- $a \simeq 0.12\ \mathrm{fm}\,,\ N_f = 2+1,\ \ 96\times 48^3,\ \ L_s = 24.$
- Valence light: physical mass z-Möbius $L_s = 10.$
- 98 configurations (20 trajectories spacing).
- 2000 eigenvectors/conf for the light red-black preconditioned matrix.

QED implementation

- $\mathrm{QED_L}$ theory.
- Interaction vertices $\not{A}$ inserted on quark propagators using sequential solves.
- EM field Gaussian distributed from $\mathrm{QED_L}$ action.
- Local current vs. conserved: no new divergences for this process.

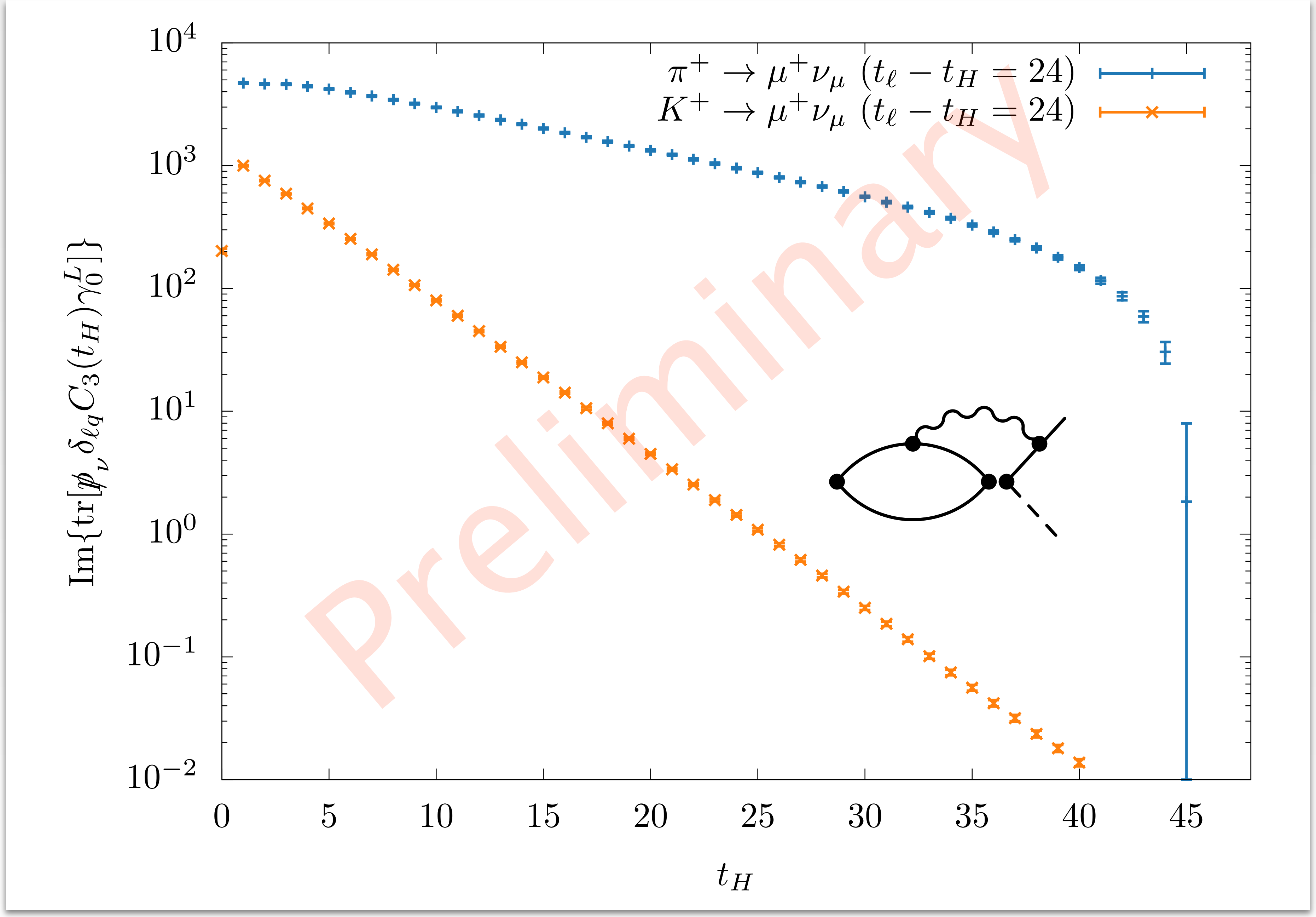
Correlation functions

- 2-point function: all-to-all with LMA.
- $C_3^{\alpha\beta}(t_H)$



- t_P, $t_\ell - t_H$ fixed, 96 translations. $\not{A}$ integrated.
- DWF lepton with twisted BC for energy conservation.

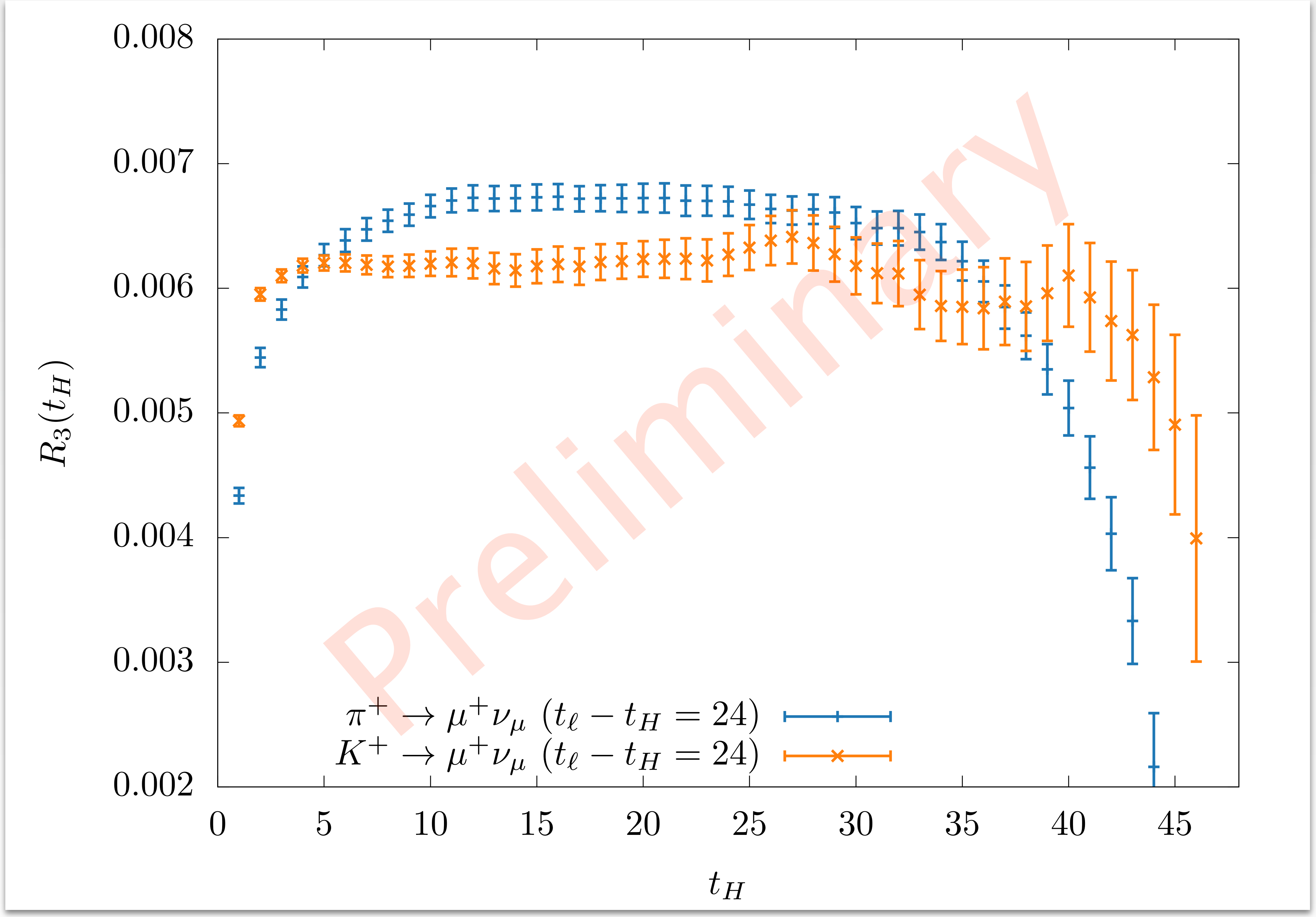
Non-factorisable corrections



22 configurations, 96 translations/conf.

24 consecutive translations binned

Non-factorisable corrections



$$R_3(t_H) = \frac{\mathrm{tr}[\not{p}_\nu \delta_{\ell q} C_3(t_H)\gamma_0^L]}{\mathrm{tr}[\not{p}_\nu C_{3,0}(t_H)\gamma_0^L]} \simeq \frac{\delta_{\ell q}|\mathcal{M}|^2}{|\mathcal{M}_0|^2}$$

Outlook

- Isospin breaking corrections to weak decays are necessary for **sub-percent CKM coefficients**.
- Working setup for **pion and kaon leptonic decays** directly at **physical quark masses**.
- Low-statistics results encouraging.

Perspectives

- **Soon: higher statistics and full analysis.**
- **Renormalisation** of the weak Hamiltonian.
- **Semi-leptonic** decays.

Thank you!