$K \to \pi\pi$ decays from lattice QCD

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1. Directly computing $K \to \pi\pi$ decay amplitudes
2. Evaluation of A_2
3. Evaluation of A_0
4. Conclusions

- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.

$$_{I=2}\langle \pi\pi | H_W | K^0 \rangle = A_2\, e^{i\delta_2}\,, \qquad _{I=0}\langle \pi\pi | H_W | K^0 \rangle = A_0\, e^{i\delta_0}\,.$$

- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule (Re A_0/Re $A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.

- CP-violating experimental amplitudes:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | H_W | K_L \rangle}{\langle \pi^+\pi^- | H_W | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H_W | K_L \rangle}{\langle \pi^0\pi^0 | H_W | K_S \rangle} = \epsilon - 2\epsilon'$$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1}{6}\left(1 - \frac{|\eta_{00}|^2}{|\eta_{+-}|^2}\right)$$

- Theoretically (without isospin breaking corrections),

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right)$$

where $\omega = \mathrm{Re}\,A_2/\mathrm{Re}A_0 \simeq 1/22$.

- Indirect CP-violation: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$
- Direct CP-violation: Re $(\epsilon'/\epsilon) = (16.6 \pm 2.3) \times 10^{-4}$

- The effective $\Delta S = 1$ Hamiltonian can be written in the standard form:

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left\{ z_i(\mu) + \tau\, y_i(\mu) \right\} Q_i(\mu) \,,$$

  where

- G_F and V_{ij} are the Fermi Constant and CKM matrix elements respectively;
- τ is the ratio of CKM matrix elements

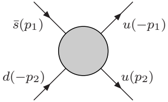
$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \simeq \left(1.543 + 0.635i\right) \times 10^{-3};$$

  (to be refined)

- $Q_i(\mu)$ are four-quark operators defined at the renormalisation scale μ with Wilson Coefficients $z_i(\mu)$ and $y_i(\mu)$.

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \{ z_i(\mu) + \tau\, y_i(\mu) \} \, Q_i(\mu)$$

- Role of lattice computations is to evaluate the hadronic matrix elements $\langle \pi\pi | Q_i(\mu) | K \rangle$.
- We can evaluate these matrix elements entirely non-perturbatively, but only in renormalisation schemes which can be defined beyond perturbation theory e.g. RI-(S)MoM.



  - $p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$.
  - Renormalisation condition chosen such that a suitable trace of this vertex = tree-level value.

  - This does not include schemes based on dimensional regularisation, such as $\overline{\text{MS}}$.
  - Since the Wilson coefficients were calculated using the $\overline{\text{MS}}$ scheme, there is necessarily an additional perturbative matching calculation from e.g. RI-(S)MoM$\to \overline{\text{MS}}$.
- The uncertainties on the Wilson coefficients and also on the SM parameters are significant in the determination of ϵ'/ϵ.

1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

   "K to $\pi\pi$ decay amplitudes from lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

   "Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"

   Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

   "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm

   Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

   "Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2"

   Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

   "Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"

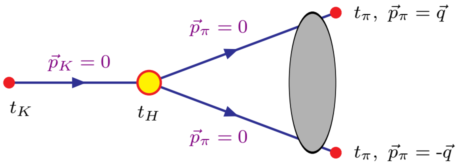
   P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D.Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3 A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
"$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit,"
T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].

4 A_0 at physical kinematics and a single coarse lattice spacing.
"Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

See also: "Calculation of ϵ'/ϵ on the lattice" C.Kelly et al. PoS FPCP2016 (2017) 017

5 "An Improved standard model calculation of direct CP-violation on the lattice,"
RBC-UKQCD Collaborations, arXiv:20??.?????

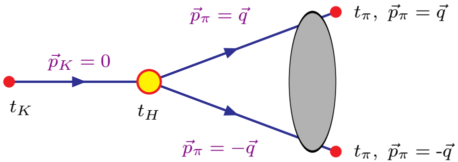


- $K \to \pi\pi$ correlation function is dominated by the lightest state, i.e. the state with two-pions at rest (or the vacuum for $I = 0$). Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003
    N.Christ, C.Kelly, D.Zhang, arXiv:1908.08640

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.



- Imagine now that we chosen the boundary conditions so that the ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$.
  - In a finite volume each component of $\vec{q}$ is quantised, with allowed values separated by $2\pi/L$.
  - Thus in order to obtain the physical value of $|\vec{q}|$ the volume must be chosen appropriately.
  - Moreover, the $J=0$, $I=0$ and $I=2$ channels are attractive and repulsive respectively and so the two cases must be studied on lattices of different volumes.

- For A_2, there is no vacuum subtraction and we can use the Wigner-Eckart theorem to write

$$\underbrace{\langle(\pi\pi)^{I=2}_{I_3=1}\,|}_{\frac{1}{\sqrt{2}}(\langle\pi^+\pi^0|+\langle\pi^0\pi^+|)}\quad Q^{\Delta I=3/2}_{\Delta I_3=1/2,i}\,|\,K^+\rangle = \frac{3}{2}\,\underbrace{\langle(\pi\pi)^{I=2}_{I_3=2}\,|}_{\langle\pi^+\pi^+|}\,Q^{\Delta I=3/2}_{\Delta I_3=3/2,i}\,|\,K^+\rangle\,,$$

  and impose anti-periodic conditions on the d-quark in one or more directions.
- If we impose the anti-periodic boundary conditions in all 3 directions then the ground state is

$$\left|\pi\left(\frac{\pi}{L},\frac{\pi}{L},\frac{\pi}{L}\right)\,\pi\left(\frac{-\pi}{L},\frac{-\pi}{L},\frac{-\pi}{L}\right)\right\rangle.$$

- With an appropriate choice of L and the number of directions, we can arrange that $E_{\pi\pi}=m_K$.
- Isospin breaking by the boundary conditions is harmless here.

CTS & G.Villadoro, hep-lat/0411033

- These are obtained using the Poisson summation formula:

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty}\frac{dp}{2\pi} f(p^2) + \sum_{n\neq 0}\int_{-\infty}^{\infty}\frac{dp}{2\pi} f(p^2)e^{inpL},$$

- For single-hadron states the finite-volume corrections decrease exponentially with the volume $\propto e^{-m_\pi L}$. For multi-hadron states, the finite-volume corrections generally fall as powers of the volume.
- For two-hadron states, there is a huge literature following the seminal work by Lüscher and the effects are generally understood.
  - The spectrum of two-pion states in a finite volume is given by the scattering phase-shifts. M. Luscher, Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B354 (1991) 531.
  - The $K \to \pi\pi$ amplitudes are obtained from finite-volume matrix elements by the Lellouch-Lüscher factor which contains the derivative of the phase-shift. L.Lellouch & M.Lüscher, hep-lat/0003023; C.J.D.Lin, G.Martinelli, CTS & M.Testa, hep-lat/0104006; C.h.Kim, CTS & S.R.Sharpe, hep-lat/0507006 $\cdots$
  - More recently we have also determined the finite-volume corrections for $\Delta m_K = m_{K_L} - m_{K_S}$. N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170
- For three-hadron states, there has been a major pioneering effort by Hansen and Sharpe leading to much theoretical clarification.

  see e.g. M.Hansen & S.Sharpe, arXiv:1901.00483

RBC-UKQCD, T.Blum et al., arXiv:1502:00263

Source	ReA_2	ImA_2
NPR (nonperturbative)	0.1%	0.1%
NPR (perturbative)	2.9%	7.0%
Finite volume corrections	2.4%	2.6%
Unphysical kinematics	4.5%	1.1%
Wilson coefficients	6.8%	10%
Derivative of the phase shift	1.1%	1.1%
Total	9%	12%

- *Wilson Coefficients* and *NPR(perturbative*) errors are not from our lattice calculation.
- Step-scaling can be used to increase the scale at which the matching is performed.

- The amplitude A_2 is considerably simpler to evaluate that A_0.
- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. arXiv:1111.1699, arXiv:1206.5142
- Our latest results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned} \mathrm{Re}(A_2) &= 1.50(4)_{\rm stat}(14)_{\rm syst} \times 10^{-8}\ \mathrm{GeV}. \\ \mathrm{Im}(A_2) &= -6.99(20)_{\rm stat}(84)_{\rm syst} \times 10^{-13}\ \mathrm{GeV}\,. \end{aligned}$$

arXiv:1502.00263

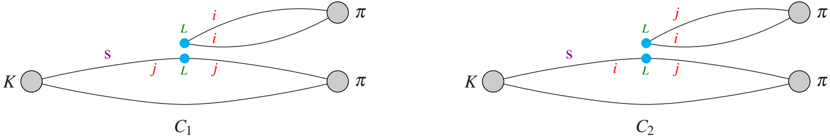
- The experimentally measured value is $\mathrm{Re}(A_2) = 1.479(4) \times 10^{-8}$ GeV.
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.
- We are not currently working towards improving this result.

RBC-UKQCD Collaboration, arXiv:1212.1474

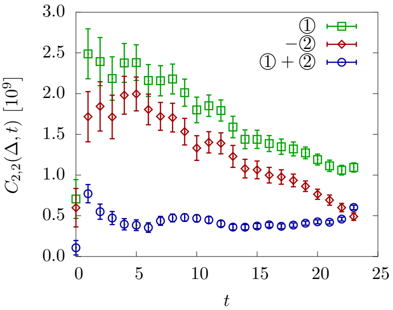
- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\right\} + (\bar{s}^i u^i)_L\,(\bar{u}^j d^j)_L$$

and two diagrams:



- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- The strong suppression of Re A_2 is a major factor in the $\Delta I = 1/2$ rule.



Physical Kinematics

- Notation ⓘ $\equiv C_i,\; i = 1, 2$.

3. Evaluation of A_0 and ϵ'/ϵ

- In 2015 RBC-UKQCD published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large relative errors:

  Z.Bai et al. (RBC-UKQCD), arXiv:1505.07863

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

  to be compared with

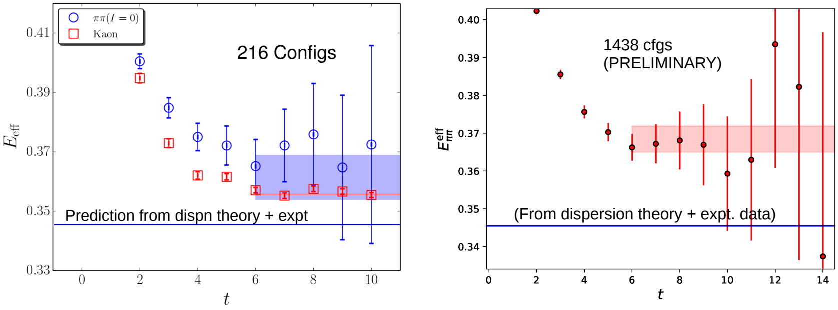
$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

  - Is this 2.1σ deviation real? $\Rightarrow$ must reduce the uncertainties.
  - The matrix elements themselves are calculated with a smaller relative error.
- This is by far the most complicated project that I have ever been involved with.
- Puzzle: For the $I=0$ s-wave $\pi\pi$ phase shift we obtained $\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ$, to be compared with the dispersive results of 34°. G.Colangelo et al.

Overview of 2015 calculation

- $32^3 \times 64$ ensemble (Möbius DWF and Iwasaki + DSDR gauge action)
- $a^{-1} = 1.3784(68)$ GeV, $L = 4.53$ fm.
- G-parity boundary conditions in 3-directions
- 216 configurations
- Almost physical kinematics:
  - $m_\pi = 143.1(2.0)$ MeV
  - $m_K = 490.6(2.2)$ MeV
  - $E_{\pi\pi} = 498(11)$ MeV

- Increase the statistics: 216 $\rightarrow$ 1438 configurations.
    - Reduce the statistical error;
    - Improved statistics allows for an in-depth study of the systematics.
- Use an expanded set of operators to create the $\pi\pi$ state.
- Improve the non-perturbative renormalisation, including step-scaling to match at a higher energy.
- Significantly improve the analysis techniques. C.Kelly and T.Wang, arXiv:1911.04582
- In addition there are improvements in non-lattice elements of the determination of ϵ'/ϵ.



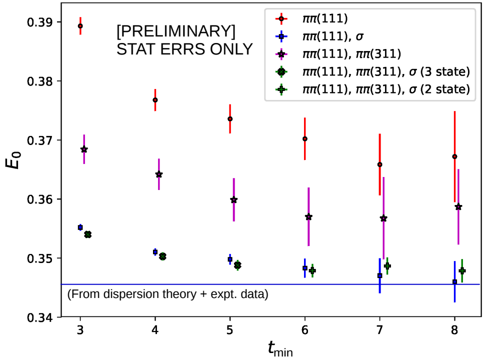
- Increasing the statistics from 216 to 1438 configurations, the $\pi\pi$ correlation function is still well described by a single $\pi\pi$ state.
  - It does not solve the δ_0 puzzle however:

$$\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ \quad \to \quad \delta_0 = (19.1 \pm 2.5 \pm 1.2)^\circ \qquad (\chi^2/\mathrm{dof} = 1.6)$$

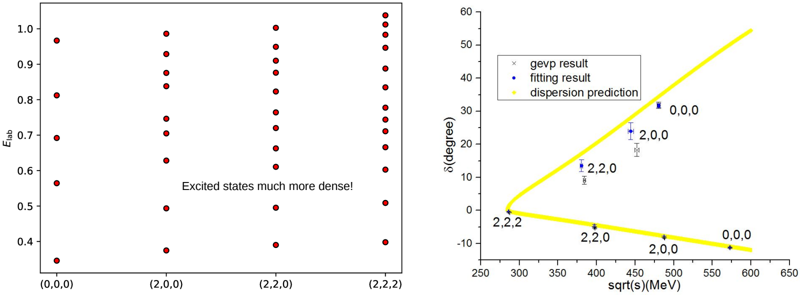
- The δ_0-puzzle has been resolved by adding more interpolating operators for the $\pi\pi$ states.
- In particular the inclusion of a σ-like two-quark operator ($\bar{u}u + \bar{d}d$) has exposed a second state, e.g. for $t_f - t_i = 5$

$$\det \begin{pmatrix} \langle \pi\pi(t_f)\pi\pi(t_i)\rangle & \langle \pi\pi(t_f)\sigma(t_i)\rangle \\ \langle \sigma(t_f)\pi\pi(t_i)\rangle & \langle \sigma(t_f)\sigma(t_i)\rangle \end{pmatrix} = 0.439(50) \neq 0$$

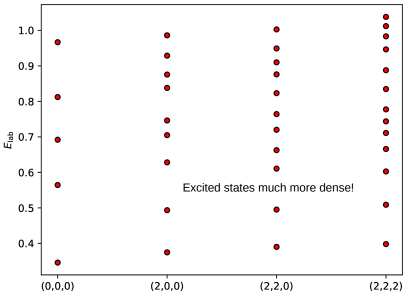
- We have also included a third operator giving each pion a larger momentum $\pm(3,1,1)\pi/L$.
- At present we have only analysed 741 configurations with the additional operators. Remainder will be done in the future.

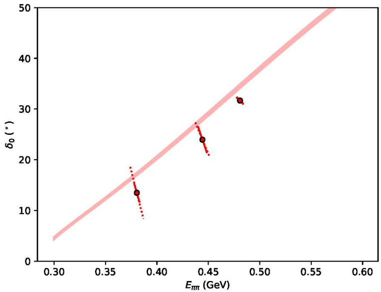


- $\delta_0 = (31.7 \pm 0.6)^\circ$ from a fit in the range $t = 5$ - 15 (statistical error only).
    - Recall that the fit from dispersion theory is about 34°.
- The $\pi\pi(3,1,1)$ operator turns out not to be very important.



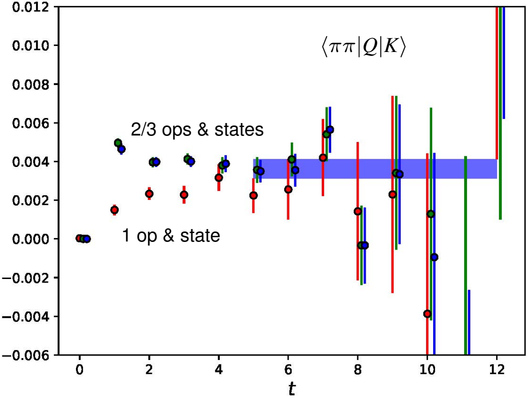
- We also evaluate the phase-shifts from the correlation functions at non-zero CoM momenta. (Components of momenta given in units of π/L.)
- The increasing density of excited states makes it difficult to separate the states $\Rightarrow$ poor plateaus.
- In the right-hand plot, only statistical errors are included and the curve comes from G.Colangelo, J.Gasser and H.Leutwyler, Nucl. Phys. B **603** (2001) 125





- We also evaluate the phase-shifts from the correlation functions at non-zero CoM momenta. (Components of momenta given in units of π/L.)
- The increasing density of excited states makes it difficult to separate the states $\Rightarrow$ poor plateaus.
- In the right-hand plot, the three points from right-to-left correspond to (0,0,0), (2,0,0) and (2,2,0) and the curve comes from

G.Colangelo, J.Gasser and H.Leutwyler, Nucl. Phys. B **603** (2001) 125



- We are currently completing the analysis of the $\langle\pi\pi|Q_i|K\rangle$ matrix elements, the amplitude A_0 and ϵ'/ϵ.
- The above is a sample plot for the matrix element of an unspecified (here) operator Q. $(t = t_{\pi\pi} - t_{\rm op})$

arXiv:1505.07863

Description	Error
Operator normalisation	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch - Lüscher factor	11%
Residual FV corrections	7%
Parametric errors	5%
Excited state contamination	5%
Unphysical kinematics	3%
Total	**27%**

- Representative fractional systematic errors for contributions to Re A_0 and Im A_0

Estimates of Systematic Errors in 2015 Calculation

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arXiv:1505.07863

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- Systematic error associated with the one-loop truncation in the SMoM $\to \overline{\text{MS}}$ matching at $\mu = 1.53$ GeV is the largest contribution.
- We now use step-scaling to perform this matching at $\mu = 4.0$ GeV.

Estimates of Systematic Errors in 2015 Calculation

arXiv:1505.07863

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- The uncertainty on the Wilson coefficients is estimated by taking the difference between the NLO and LO contributions.
- Partial contributions at NNLO have been calculated and a “new NNLO calculation of the non-EW-penguin part of the weak Hamiltonian does not move the central value", M.Cerdà-Sevilla at Kaon 2019, reporting on work with M.Gorbahn, S.Jäger and A.Kokulu

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- Last Friday, Guido Martinelli presented our theoretical framework and results for IB corrections to leptonic decays and the status of the development of the corresponding framework for semileptonic decays.
- The extension of this framework to $K \to \pi\pi$ decays is considerably more complicated.
  - Some first steps, towards including electromagnetism in $K \to \pi\pi$ decays we taken by N.Christ and X.Feng. arXiv:1711.09339
- At the current stage of precision we are not concerned with including $O(1\%)$ corrections.
  - However, because of the $\Delta I = \frac{1}{2}$ rule, the corrections are expected to be amplified.

- Recently a detailed updated study of isospin corrections was presented in the framework of ChPT and the large N_C approximation.

  V.Cirigliano, H.Gisbert, A.Pich, A.Rodriguez-Sanchez, arXiv:1911.01359

  The authors write the formula for ϵ' in the form:

$$\epsilon' = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left[\frac{\mathrm{Im}\,A_2^{\mathrm{ewp}}}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\right],$$

  where $\omega_+ = \mathrm{Re}\,A_2^+/\mathrm{Re}\,A_0$ and A_2^+ is A_2 obtained from the physical decay $K^+ \to \pi^+\pi^0$ at NLO

$$\hat{\Omega}_{\mathrm{eff}} = \left(17.0^{+9.1}_{-9.0}\right) \times 10^{-2}\,.$$

  (At LO the corresponding number is 19.5 ± 3.9.)

- This is a very significant effect, certainly requiring further investigation.

4. $K \to \pi\pi$-decays - Conclusions

- The calculation of $A_0(K \to \pi\pi)$ and ϵ'/ϵ will be very substantially improved over our 2015 result.
  - Statistical improvement: 216 $\to$ 741 configurations (and $\to$ 1438 configurations in the future).
  - 3 $\pi\pi$ interpolating operators used to separate the ground and excited states.
  - Significantly improved analysis techniques to quantify the effects of autocorrelations and to obtain correct p-values (blocked jacknife errors, inclusion of fluctuations in covariance matrix etc.).
- The δ_0 puzzle now appears to be solved:

$$\delta_0(m_K) = 31.7(6)^\circ\,.$$

- Draft of paper is in preparation.

Results will be published “soon".

- Improved calculation of the Wilson Coefficients relevant for $\mathrm{Im}A_0/\mathrm{Re}A_0$ almost complete. M.Cerdà-Sevilla, M.Gorbahn, S.Jäger and A.Kokulu
  - Matching of the matrix elements renormalised in the RI-SMoM scheme to $\overline{\mathrm{MS}}$ still only known at one-loop. C.Sturm & C.Lehner, arXiv:1104.4948