$K \rightarrow \pi \pi$ decays from lattice QCD

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Outline of Talk

1. Directly computing $K \rightarrow \pi \pi$ decay amplitudes
2. Evaluation of $A_2$
3. Evaluation of $A_0$
4. Conclusions
1. Directly computing $K \rightarrow \pi\pi$ decay amplitudes

- $K \rightarrow \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.

- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.

\[
I=2 \langle \pi\pi | H_W | K^0 \rangle = A_2 e^{i\delta_2}, \quad I=0 \langle \pi\pi | H_W | K^0 \rangle = A_0 e^{i\delta_0}.
\]

- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule ($\text{Re} A_0/\text{Re} A_2 \simeq 22.5$) and an understanding of the experimental value of $\varepsilon'/\varepsilon$, the parameter which was the first experimental evidence of direct CP-violation.
CP-violation in $K \to \pi\pi$ decays

- CP-violating experimental amplitudes:

\[
\eta_{+-} = \frac{\langle \pi^+\pi^- | H_W | K_L \rangle}{\langle \pi^+\pi^- | H_W | K_S \rangle} = \epsilon + \epsilon'
\]
\[
\eta_{00} = \frac{\langle \pi^0\pi^0 | H_W | K_L \rangle}{\langle \pi^0\pi^0 | H_W | K_S \rangle} = \epsilon - 2\epsilon'
\]
\[
\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = \frac{1}{6} \left( 1 - \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \right)
\]

- Theoretically (without isospin breaking corrections),

\[
\epsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left( \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right)
\]

where $\omega = \text{Re} A_2 / \text{Re} A_0 \approx 1/22$.

- Indirect CP-violation: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$
- Direct CP-violation: $\text{Re}(\epsilon'/\epsilon) = (16.6 \pm 2.3) \times 10^{-4}$
The effective $\Delta S = 1$ Hamiltonian can be written in the standard form:

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \{ z_i(\mu) + \tau y_i(\mu) \} Q_i(\mu),$$

where

- $G_F$ and $V_{ij}$ are the Fermi Constant and CKM matrix elements respectively;
- $\tau$ is the ratio of CKM matrix elements

$$\tau = - \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \simeq (1.543 + 0.635i) \times 10^{-3};$$

(to be refined)

- $Q_i(\mu)$ are four-quark operators defined at the renormalisation scale $\mu$ with Wilson Coefficients $z_i(\mu)$ and $y_i(\mu)$. 
Introduction to $K \rightarrow \pi\pi$ decays (cont.)

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left\{ z_i(\mu) + \tau y_i(\mu) \right\} Q_i(\mu)$$

- Role of lattice computations is to evaluate the hadronic matrix elements $\langle \pi\pi|Q_i(\mu)|K\rangle$.

- We can evaluate these matrix elements entirely non-perturbatively, but only in renormalisation schemes which can be defined beyond perturbation theory e.g. RI-(S)MoM.

- $p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$.

- Renormalisation condition chosen such that a suitable trace of this vertex = tree-level value.

- This does not include schemes based on dimensional regularisation, such as $\overline{\text{MS}}$.

- Since the Wilson coefficients were calculated using the $\overline{\text{MS}}$ scheme, there is necessarily an additional perturbative matching calculation from e.g. RI-(S)MoM $\rightarrow$ $\overline{\text{MS}}$.

- The uncertainties on the Wilson coefficients and also on the SM parameters are significant in the determination of $\epsilon'/\epsilon$. 

References for RBC-UKQCD calculations of $K \rightarrow \pi\pi$ decays

1. $A_0$ and $A_2$ amplitudes with unphysical quark masses and with the pions at rest.
   “$K$ to $\pi\pi$ decay amplitudes from lattice QCD,”
   “Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation”

2. $A_2$ at physical kinematics and a single coarse lattice spacing.
   “The $K \rightarrow (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,”
   “Lattice determination of the $K \rightarrow (\pi\pi)_{I=2}$ Decay Amplitude $A_2$”
   “Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,”
3 \( A_2 \) at physical kinematics on two finer lattices \( \Rightarrow \) continuum limit taken.

“\( K \to \pi \pi \ \Delta I = 3/2 \) decay amplitude in the continuum limit,”


4 \( A_0 \) at physical kinematics and a single coarse lattice spacing.

“Standard-model prediction for direct CP violation in \( K \to \pi \pi \) decay,”


See also: “Calculation of \( \epsilon'/\epsilon \) on the lattice" C.Kelly et al. PoS FPCP2016 (2017) 017

5 “An Improved standard model calculation of direct CP-violation on the lattice,”
RBC-UKQCD Collaborations, arXiv:20??..?????
The Maiani-Testa Theorem

\[ \vec{p}_K = 0 \]

\[ t_K \]

\[ t_H \]

\[ \vec{p}_\pi = 0 \]

\[ t_\pi, \vec{p}_\pi = \vec{q} \]

\[ t_\pi, \vec{p}_\pi = -\vec{q} \]

- \( K \to \pi\pi \) correlation function is dominated by the lightest state, i.e. the state with two-pions at rest (or the vacuum for \( I = 0 \)).

Maiani and Testa, PL B245 (1990) 585

\[ C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots \]

- Solution 1: Study an excited state.
  Lellouch and Lüscher, hep-lat/0003023

- Solution 2: Introduce suitable boundary conditions such that the \( \pi\pi \) ground state is \( |\pi(\vec{q})\pi(-\vec{q})\rangle \).
  RBC-UKQCD, C.h.Kim hep-lat/0311003

For \( B \)-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.
Imagine now that we chosen the boundary conditions so that the ground state is \( |\pi(q)\pi(-q)\rangle \).

- In a finite volume each component of \( \vec{q} \) is quantised, with allowed values separated by \( 2\pi/L \).
- Thus in order to obtain the physical value of \( |\vec{q}| \) the volume must be chosen appropriately.
- Moreover, the \( J = 0 \), \( I = 0 \) and \( I = 2 \) channels are attractive and repulsive respectively and so the two cases must be studied on lattices of different volumes.
2. Evaluation of $A_2$

For $A_2$, there is no vacuum subtraction and we can use the Wigner-Eckart theorem to write

$$\langle (\pi\pi)^{I_3=1} | Q^{\Delta I=3/2}_{\Delta I_3=1/2,i} | K^+ \rangle = \frac{3}{2} \langle (\pi\pi)^{I_3=2} | Q^{\Delta I=3/2}_{\Delta I_3=3/2,i} | K^+ \rangle,$$

and impose anti-periodic conditions on the d-quark in one or more directions.

If we impose the anti-periodic boundary conditions in all 3 directions then the ground state is

$$\left| \pi \left( \frac{\pi}{L}, \frac{\pi}{L}, \frac{\pi}{L} \right) \pi \left( \frac{-\pi}{L}, \frac{-\pi}{L}, \frac{-\pi}{L} \right) \right>.$$

With an appropriate choice of $L$ and the number of directions, we can arrange that $E_{\pi\pi} = m_K$.

Isospin breaking by the boundary conditions is harmless here.
Finite Volume Effects

- These are obtained using the Poisson summation formula:
  \[
  \frac{1}{L} \sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} f(p^2) + \sum_{n \neq 0} \int_{-\infty}^{\infty} \frac{dp}{2\pi} f(p^2) e^{inpL},
  \]

- For single-hadron states the finite-volume corrections decrease exponentially with the volume \(\propto e^{-m \pi L}\). For multi-hadron states, the finite-volume corrections generally fall as powers of the volume.

- For two-hadron states, there is a huge literature following the seminal work by Lüscher and the effects are generally understood.
  - The \(K \rightarrow \pi\pi\) amplitudes are obtained from finite-volume matrix elements by the Lellouch-Lüscher factor which contains the derivative of the phase-shift. L.Lellouch & M.Lüscher, hep-lat/0003023; C.J.D.Lin, G.Martinelli, CTS & M.Testa, hep-lat/0104006; C.h.Kim, CTS & S.R.Sharpe, hep-lat/0507006 · · ·
  - More recently we have also determined the finite-volume corrections for \(\Delta m_K = m_{K_L} - m_{K_S}\). N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170

- For three-hadron states, there has been a major pioneering effort by Hansen and Sharpe leading to much theoretical clarification.
  see e.g. M.Hansen & S.Sharpe, arXiv:1901.00483
### Systematic error budget in our calculation of $A_2$

<table>
<thead>
<tr>
<th>Source</th>
<th>Re$A_2$</th>
<th>Im$A_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPR (nonperturbative)</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>NPR (perturbative)</td>
<td>2.9%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Finite volume corrections</td>
<td>2.4%</td>
<td>2.6%</td>
</tr>
<tr>
<td>Unphysical kinematics</td>
<td>4.5%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Wilson coefficients</td>
<td>6.8%</td>
<td>10%</td>
</tr>
<tr>
<td>Derivative of the phase shift</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>9%</td>
<td>12%</td>
</tr>
</tbody>
</table>

- **Wilson Coefficients** and **NPR(perturbative)** errors are not from our lattice calculation.
- Step-scaling can be used to increase the scale at which the matching is performed.
The amplitude $A_2$ is considerably simpler to evaluate than $A_0$.

Our first results for $A_2$ at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%.

Our latest results were obtained on two new ensembles, $48^3$ with $a \simeq 0.11$ fm and $64^3$ with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{align*}
\text{Re}(A_2) & = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8} \text{ GeV}. \\
\text{Im}(A_2) & = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13} \text{ GeV}.
\end{align*}$$

The experimentally measured value is $\text{Re}(A_2) = 1.479(4) \times 10^{-8} \text{ GeV}$.

Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of $A_2$ at physical kinematics can now be considered as standard.

We are not currently working towards improving this result.
Suppression of $A_2$ and the $\Delta I = 1/2$ Rule

- $\text{Re} A_2$ is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^i u^i)_L - (\bar{d}^i d^i)_L \right\} + (\bar{s}^i u^i)_L (\bar{u}^i d^i)_L$$

and two diagrams:

- $\text{Re} A_2$ is proportional to $C_1 + C_2$.
- The contribution to $\text{Re} A_0$ from $Q_2$ is proportional to $2C_1 - C_2$ and that from $Q_1$ is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \approx \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that $A_2$ is significantly suppressed!
- The strong suppression of $\text{Re} A_2$ is a major factor in the $\Delta I = 1/2$ rule.
Evidence for the Suppression of $\text{Re} A_2$

\[ C_{2,2}(\Delta, t) \times 10^9 \]

\[ t \quad 0 \ 5 \ 10 \ 15 \ 20 \ 25 \]

Physical Kinematics

- Notation $\textcircled{1} \equiv C_i$, $i = 1, 2$. 

Chris Sachrajda  
Low-Energy Precision Frontier, February 3rd 2020
3. Evaluation of $A_0$ and $\epsilon'/\epsilon$

- In 2015 RBC-UKQCD published our first result for $\epsilon'/\epsilon$ computed at physical quark masses and kinematics, albeit still with large relative errors:

  Z. Bai et al. (RBC-UKQCD), arXiv:1505.07863

\[
\left. \frac{\epsilon'}{\epsilon} \right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}
\]

to be compared with

\[
\left. \frac{\epsilon'}{\epsilon} \right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}.
\]

- Is this 2.1σ deviation real? ⇒ must reduce the uncertainties.
- The matrix elements themselves are calculated with a smaller relative error.

- This is by far the most complicated project that I have ever been involved with.

- Puzzle: For the $I = 0$ s-wave $\pi\pi$ phase shift we obtained $\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ$, to be compared with the dispersive results of 34°.

  G. Colangelo et al.
Overview of 2015 calculation

- $32^3 \times 64$ ensemble (Möbius DWF and Iwasaki + DSDR gauge action)
- $a^{-1} = 1.3784(68)$ GeV, $L = 4.53$ fm.
- G-parity boundary conditions in 3-directions
- 216 configurations
- Almost physical kinematics:
  - $m_\pi = 143.1(2.0)$ MeV
  - $m_K = 490.6(2.2)$ MeV
  - $E_{\pi\pi} = 498(11)$ MeV
Increase the statistics: 216 $\rightarrow$ 1438 configurations.
  - Reduce the statistical error;
  - Improved statistics allows for an in-depth study of the systematics.

Use an expanded set of operators to create the $\pi\pi$ state.

Improve the non-perturbative renormalisation, including step-scaling to match at a higher energy.

Significantly improve the analysis techniques. C.Kelly and T.Wang, arXiv:1911.04582

In addition there are improvements in non-lattice elements of the determination of $\epsilon'/\epsilon$. 
Increasing the statistics from 216 to 1438 configurations, the $\pi\pi$ correlation function is still well described by a single $\pi\pi$ state.

- It does not solve the $\delta_0$ puzzle however:

$$\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ \quad \rightarrow \quad \delta_0 = (19.1 \pm 2.5 \pm 1.2)^\circ \quad (\chi^2/\text{dof} = 1.6)$$
Adding more $\pi\pi$ interpolating operators

- The $\delta_0$-puzzle has been resolved by adding more interpolating operators for the $\pi\pi$ states.
- In particular the inclusion of a $\sigma$-like two-quark operator ($\bar{u}u + \bar{d}d$) has exposed a second state, e.g. for $t_f - t_i = 5$
  \[
  \begin{vmatrix}
  \langle \pi\pi(t_f)\pi\pi(t_i) \rangle & \langle \pi\pi(t_f)\sigma(t_i) \rangle \\
  \langle \sigma(t_f)\pi\pi(t_i) \rangle & \langle \sigma(t_f)\sigma(t_i) \rangle \\
  \end{vmatrix} = 0.439(50) \neq 0
  \]
- We have also included a third operator giving each pion a larger momentum $\pm (3, 1, 1)\pi/L$.
- At present we have only analysed 741 configurations with the additional operators. Remainder will be done in the future.
\[ \delta_0 = (31.7 \pm 0.6)^\circ \] from a fit in the range \( t = 5 - 15 \) (statistical error only).

Recall that the fit from dispersion theory is about 34°.

The \( \pi \pi (3, 1, 1) \) operator turns out not to be very important.
We also evaluate the phase-shifts from the correlation functions at non-zero CoM momenta. (Components of momenta given in units of $\pi/L$.)

The increasing density of excited states makes it difficult to separate the states $\Rightarrow$ poor plateaus.

In the right-hand plot, only statistical errors are included and the curve comes from

Attempting to determine the phase-shifts with $\vec{p}_{\pi\pi} \neq 0$

We also evaluate the phase-shifts from the correlation functions at non-zero CoM momenta. (Components of momenta given in units of $\pi/L$.)

The increasing density of excited states makes it difficult to separate the states $\Rightarrow$ poor plateaus.

In the right-hand plot, the three points from right-to-left correspond to (0,0,0), (2,0,0) and (2,2,0) and the curve comes from

We are currently completing the analysis of the $\langle \pi\pi | Q_i | K \rangle$ matrix elements, the amplitude $A_0$ and $\epsilon'/\epsilon$.

The above is a sample plot for the matrix element of an unspecified (here) operator $Q$. $(t = t_{\pi\pi} - t_{\text{op}})$
### Estimates of Systematic Errors in 2015 Calculation

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<td>3%</td>
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Representative fractional systematic errors for contributions to $\text{Re} A_0$ and $\text{Im} A_0$
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- Systematic error associated with the one-loop truncation in the SMoM $\rightarrow \overline{MS}$ matching at $\mu = 1.53$ GeV is the largest contribution.

- We now use step-scaling to perform this matching at $\mu = 4.0$ GeV.
### Estimates of Systematic Errors in 2015 Calculation

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- The uncertainty on the Wilson coefficients is estimated by taking the difference between the NLO and LO contributions.

- Partial contributions at NNLO have been calculated and a “new NNLO calculation of the non-EW-penguin part of the weak Hamiltonian does not move the central value”, M.Cerdà-Sevilla at Kaon 2019, reporting on work with M.Gorbahn, S.Jäger and A.Kokulu.
Last Friday, Guido Martinelli presented our theoretical framework and results for IB corrections to leptonic decays and the status of the development of the corresponding framework for semileptonic decays.

The extension of this framework to $K \rightarrow \pi\pi$ decays is considerably more complicated.

- Some first steps, towards including electromagnetism in $K \rightarrow \pi\pi$ decays were taken by N.Christ and X.Feng.

At the current stage of precision we are not concerned with including $O(1\%)$ corrections.

- However, because of the $\Delta I = \frac{1}{2}$ rule, the corrections are expected to be amplified.
Recently a detailed updated study of isospin corrections was presented in the framework of ChPT and the large $N_C$ approximation.


The authors write the formula for $\epsilon'$ in the form:

$$\epsilon' = \frac{i\omega+e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left[ \frac{\text{Im}A_2^{\text{ewp}}}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \left( 1 - \hat{\Omega}_{\text{eff}} \right) \right],$$

where $\omega_+ = \text{Re}A_2^{+}/\text{Re}A_0$ and $A_2^{+}$ is $A_2$ obtained from the physical decay $K^+ \rightarrow \pi^+\pi^0$ at NLO

$$\hat{\Omega}_{\text{eff}} = \left( 17.0^{+9.1}_{-9.0} \right) \times 10^{-2}.$$  

(At LO the corresponding number is $19.5 \pm 3.9$.)

This is a very significant effect, certainly requiring further investigation.
4. $K \rightarrow \pi\pi$-decays - Conclusions

- The calculation of $A_0(K \rightarrow \pi\pi)$ and $\epsilon'/\epsilon$ will be very substantially improved over our 2015 result.
  - Statistical improvement: 216 $\rightarrow$ 741 configurations (and $\rightarrow$ 1438 configurations in the future).
  - 3 $\pi\pi$ interpolating operators used to separate the ground and excited states.
  - Significantly improved analysis techniques to quantify the effects of autocorrelations and to obtain correct $p$-values (blocked jacknife errors, inclusion of fluctuations in covariance matrix etc.).

- The $\delta_0$ puzzle now appears to be solved:
  \[
  \delta_0(m_K) = 31.7(6)^{\circ}.
  \]

- Draft of paper is in preparation.

  Results will be published “soon”.

- Improved calculation of the Wilson Coefficients relevant for $\text{Im}A_0/\text{Re}A_0$ almost complete.
  - M.Cerdà-Sevilla, M.Gorbahn, S.Jäger and A.Kokulu
  - Matching of the matrix elements renormalised in the RI-MSMoM scheme to $\overline{\text{MS}}$ still only known at one-loop.
    - C.Sturm & C.Lehner, arXiv:1104.4948