V_{cb} and LFUV from exclusive B decays

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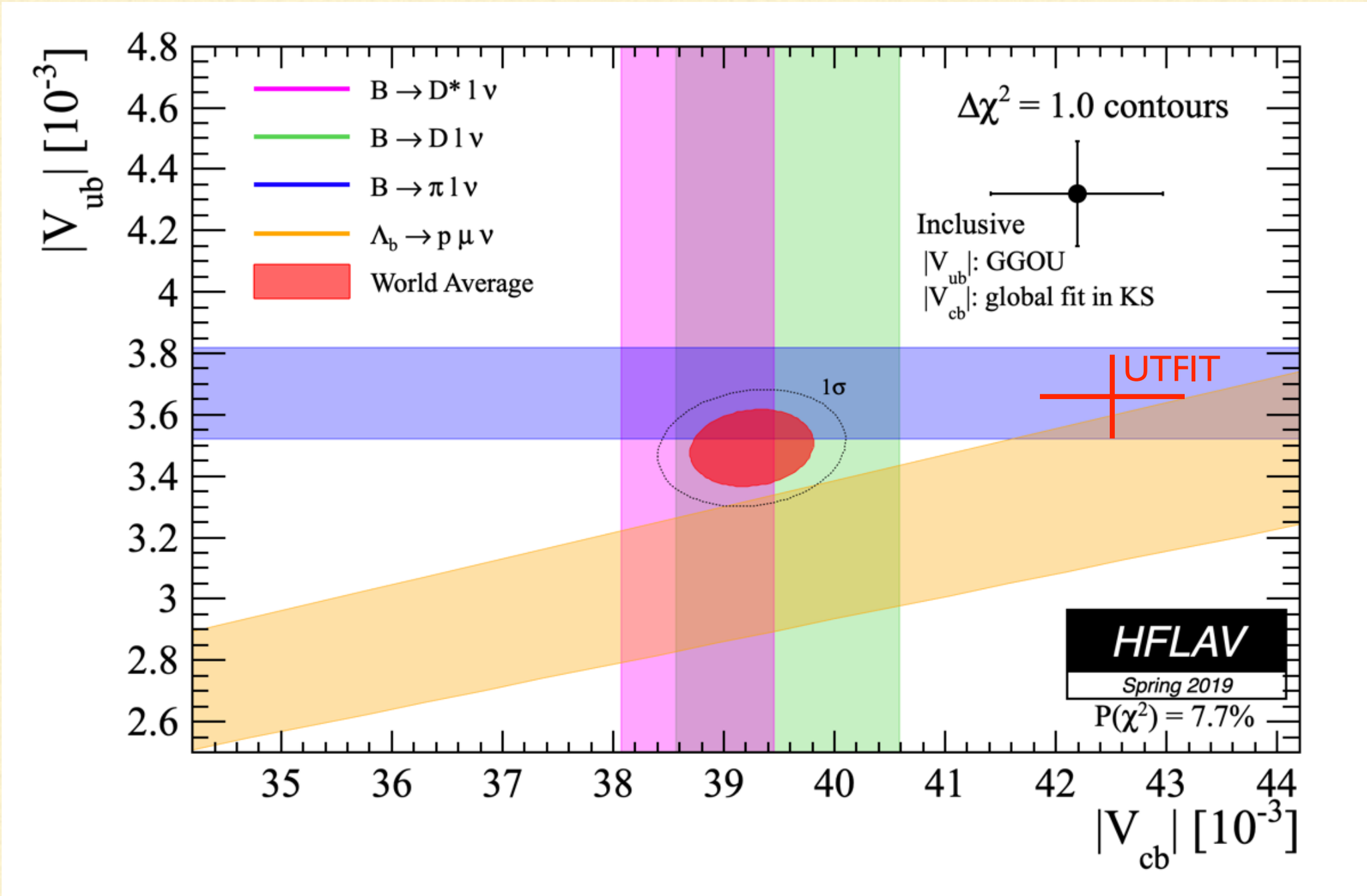
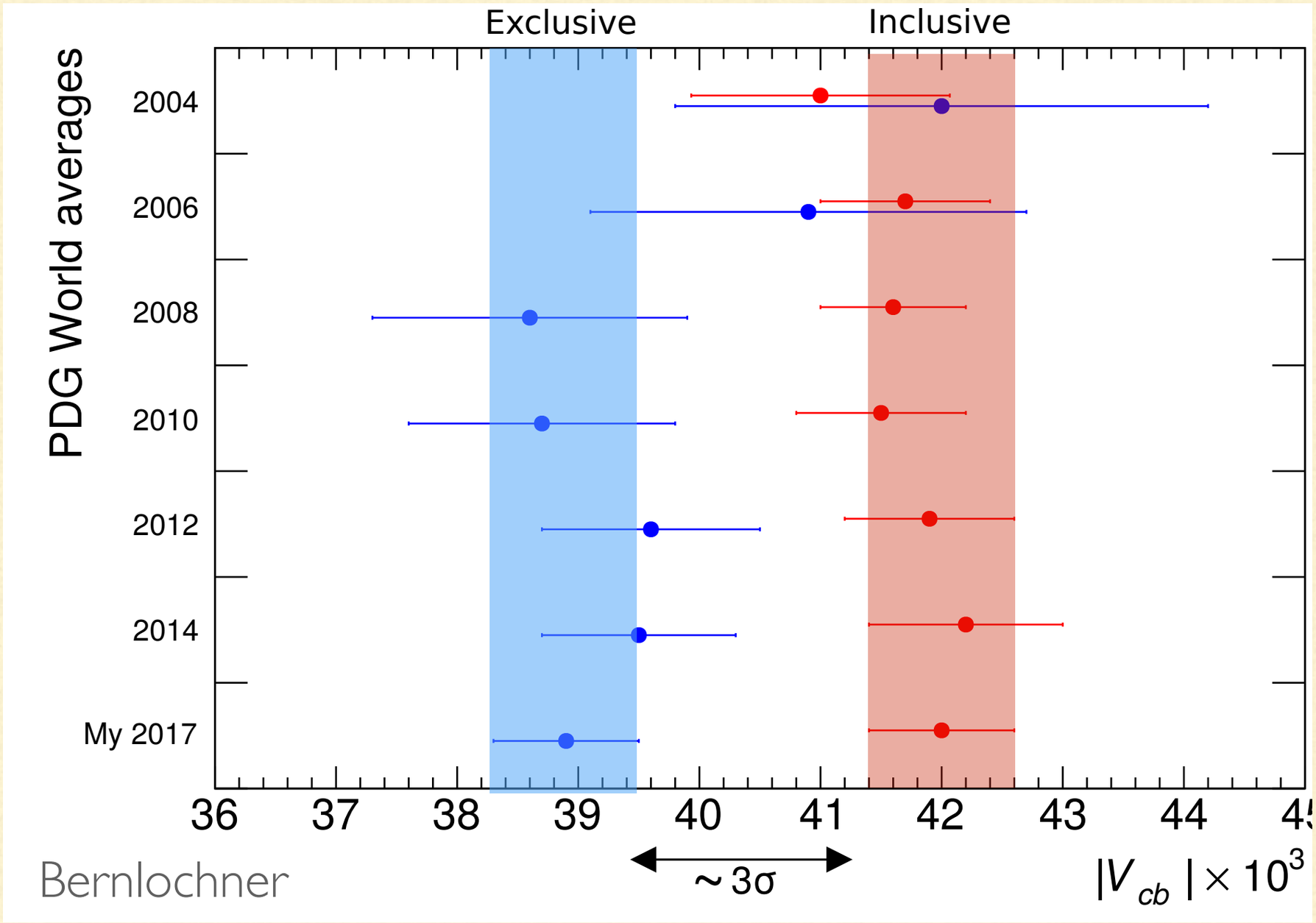
CERN Theory Institute, 6 february 2020

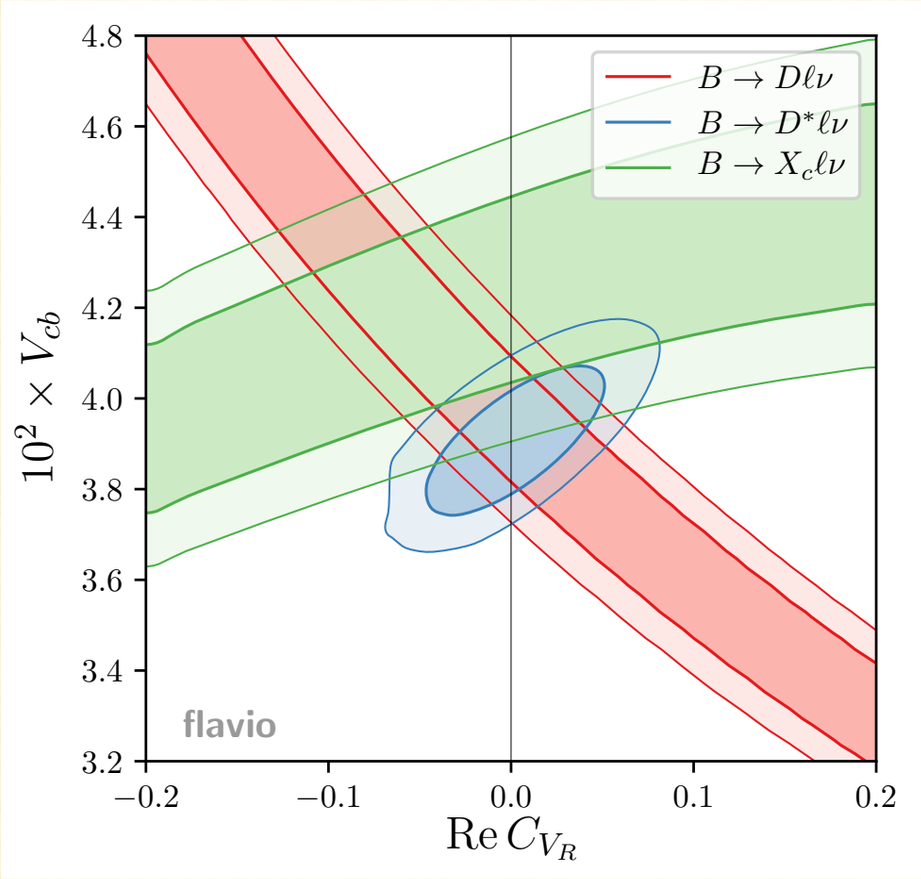


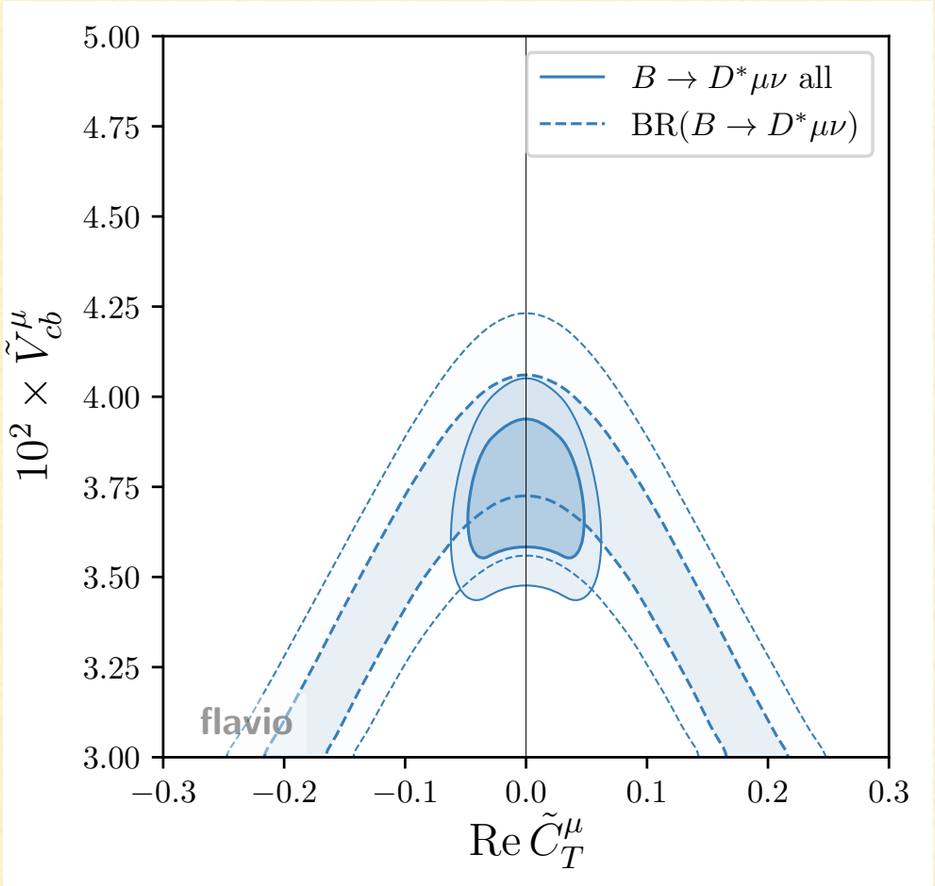
Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement

PDG AVERAGES



NEW PHYSICS?





Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data. For a recent analysis see Jung & Straub 1801.01112

The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

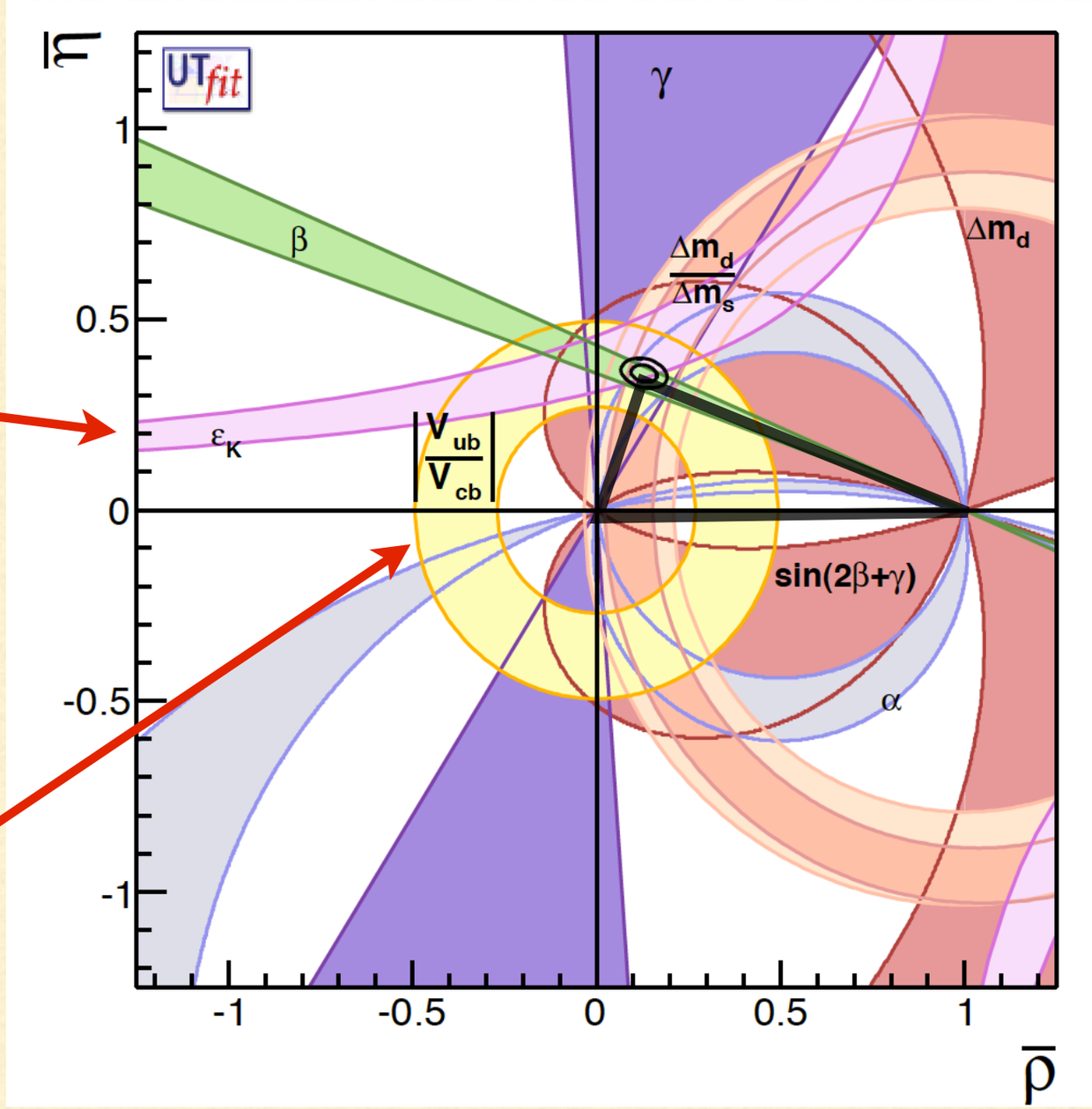
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

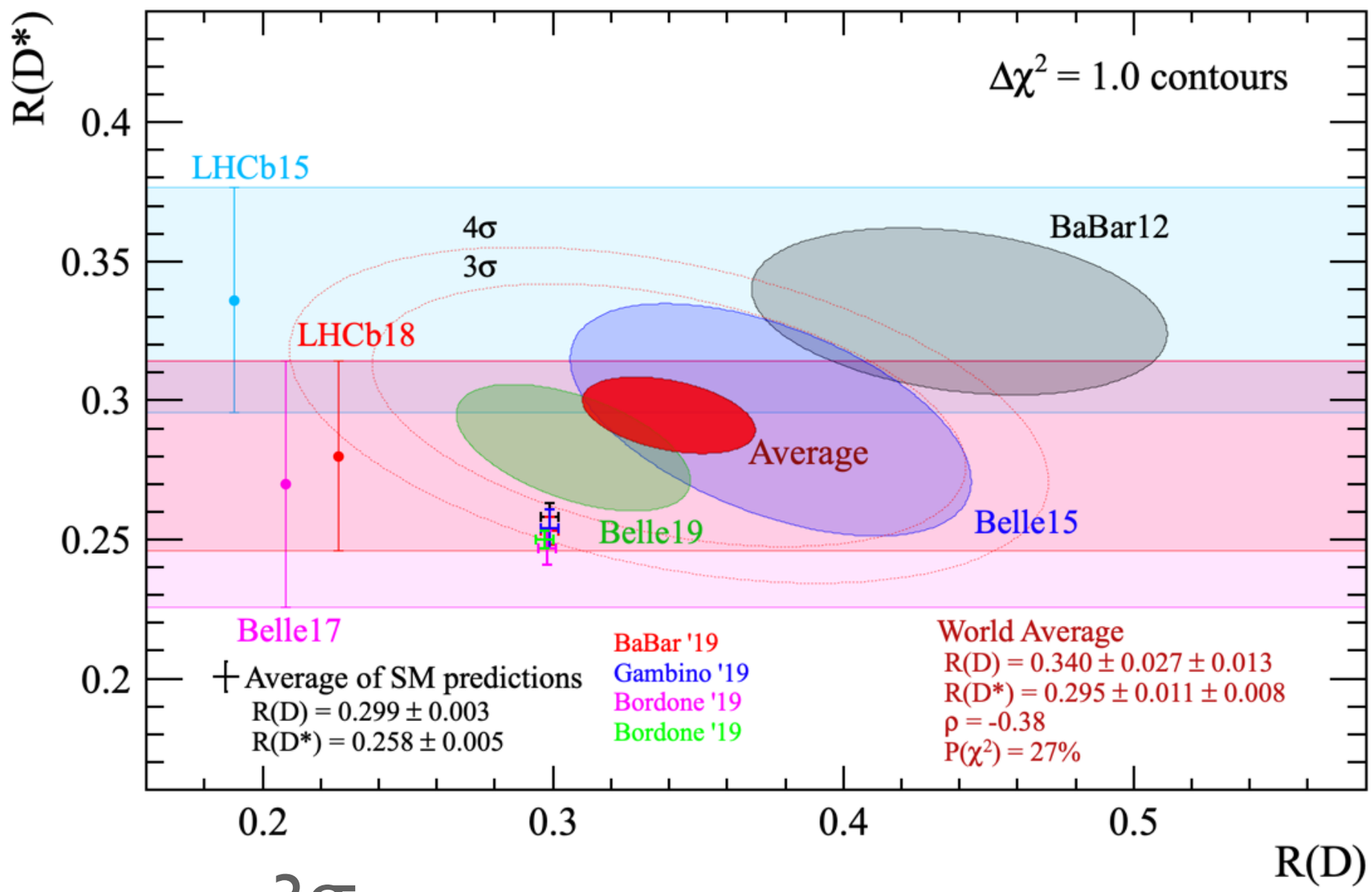
where it often dominates the theoretical uncertainty.
V_{ub}/V_{cb} constrains directly the UT



Our inability to determine precisely V_{cb} hampers significantly NP searches

VIOLATION OF LFU WITH TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



~3σ

M. Rotondo, Beauty 2019

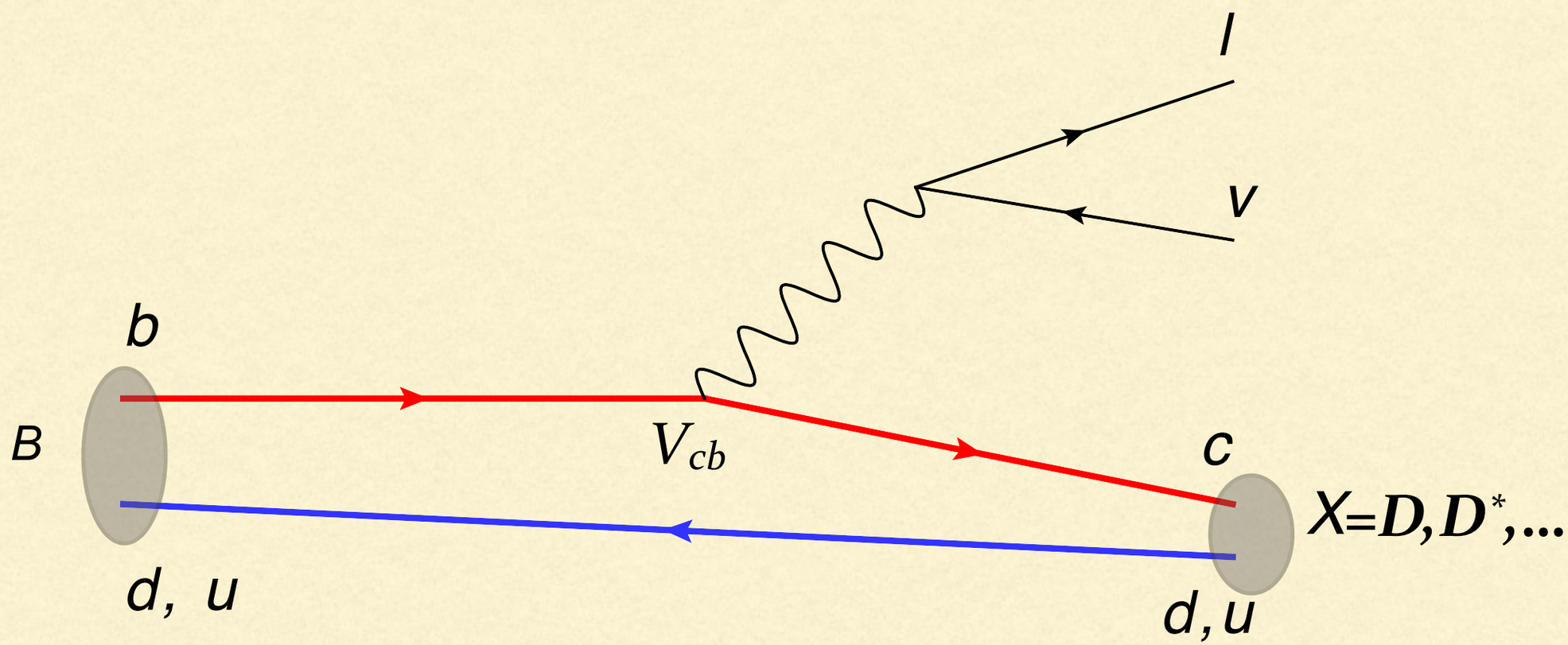
INCLUSIVE V_{cb}

- Based on an OPE: rate and moments given by double exp in α_s, 1/m
- B meson matrix elements of local operators parametrise non-pert physics. Perturbative Wilson coefficients at $O(\alpha_s^2, \alpha_s/m^2, 1/m^5)$
- Reliability of the method depends on control of higher orders.
- Theoretical uncertainties important: 3 loops width feasible needed for 1% error, QED corrections. Lattice can have an impact too (Hashimoto & Melis, Simula, PG)
- Experimental moments highly correlated, needs new analyses @ Belle II
- **Most recent global fit** (kin scheme) includes $m_{b,c}$ determinations, estimates of higher powers, provides a consistent picture, various cross-checks

$$|V_{cb}| = 42.00(64) \times 10^{-3}$$

Healy, Turzcyk, PG 1606.06174

EXCLUSIVE DECAYS



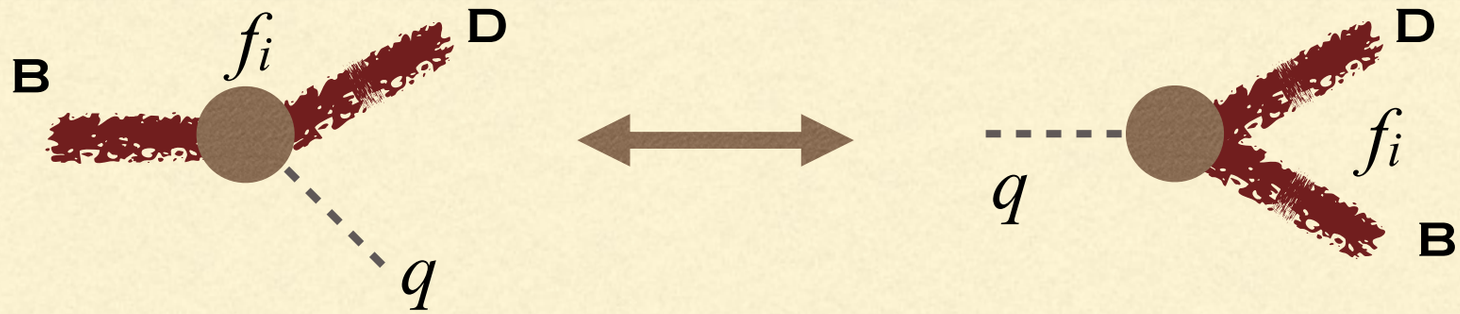
There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

$$\langle D(k)|\,\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2}q^\mu\right] f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2}q^\mu f_0^{B\to D}(q^2)$$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp…

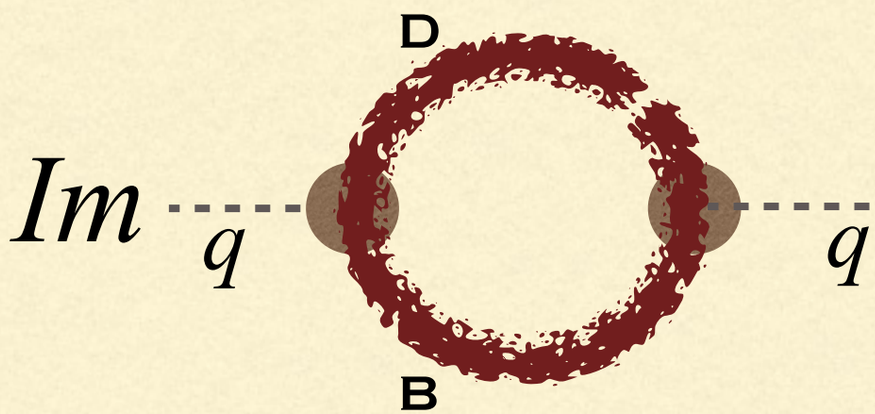
MODEL INDEPENDENT FF PARAMETRIZATION

crossing + analitycity



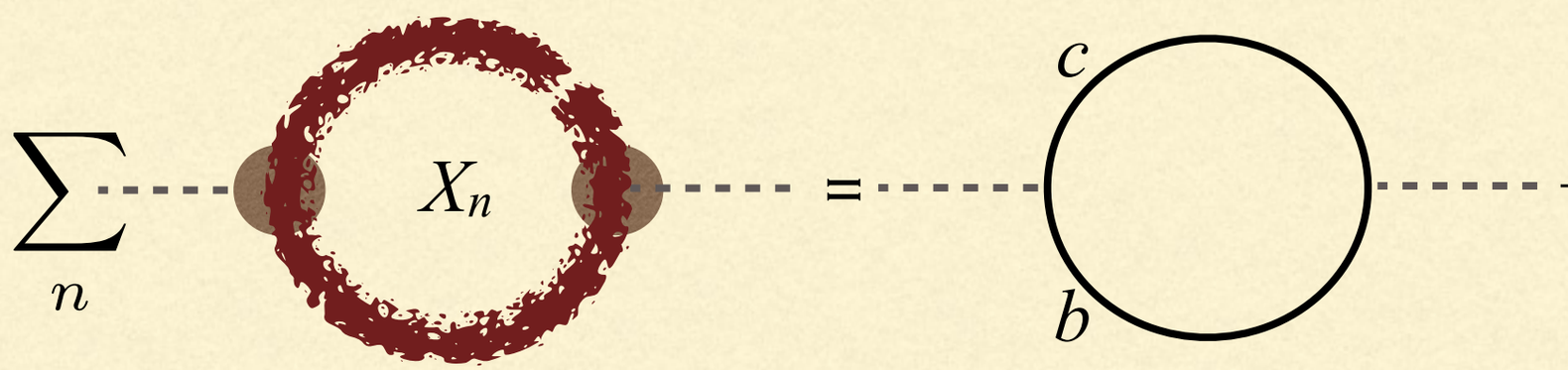
physical semileptonic region
$m_\ell^2 \leq q^2 \leq (m_B - m_D)^2$

2-point correlator cuts
$q^2 \geq (m_B + m_D)^2$



$$\propto |f_i(q^2)|^2 < \sum_n \text{cuts}(X_n)$$

poles at $q^2 = m^2_{Bc}$ etc



+ pert corr + condensates

using quark-hadron duality + dispersion relations

UNITARITY CONSTRAINTS

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \qquad 0 < z < 0.056$$

$$f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n$$

BGL Boyd Grinstein Lebed 1997

Blaschke factors remove poles below threshold

phase space factors

fast converging expansion

truncated at order N

$$\sum_{n=0}^{N} (a_n^i)^2 < 1$$

WEAK UNITARITY CONSTRAINTS

assuming saturation by single hadron channel

Where should one truncate the series? Where adding more terms becomes irrelevant

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \dots\right] \qquad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$c_{b,c}$ can be computed using subleading IW functions from QCD sumrules

Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

RATIOS $\frac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \dots\right] \qquad w_1 = w - 1$

Roughly $\epsilon_c \sim 0.25, \qquad \epsilon_c^2 \sim 0.06 \sim \epsilon_b \sim \frac{\alpha_s}{\pi}$ but coefficients??

In a few cases we can compare these ratios with recent lattice results:
there are 5-13% differences, always > NLO correction. For ex.:

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET\,@NLO} = 0.966(28)$$

Looking at NLO HQET corrections, NNLO can be sizeable, naturally O(10-20)%

STRONG UNITARITY CONSTRAINTS

Information on other channels with same quantum numbers makes the bounds tighter. HQS implies that all $B^{(*)} \to D^{(*)}$ ff either vanish or are prop to the Isgur-Wise function: any ff F_j can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\mathrm{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the a_i space for F_i in case of S, P, V, A currents

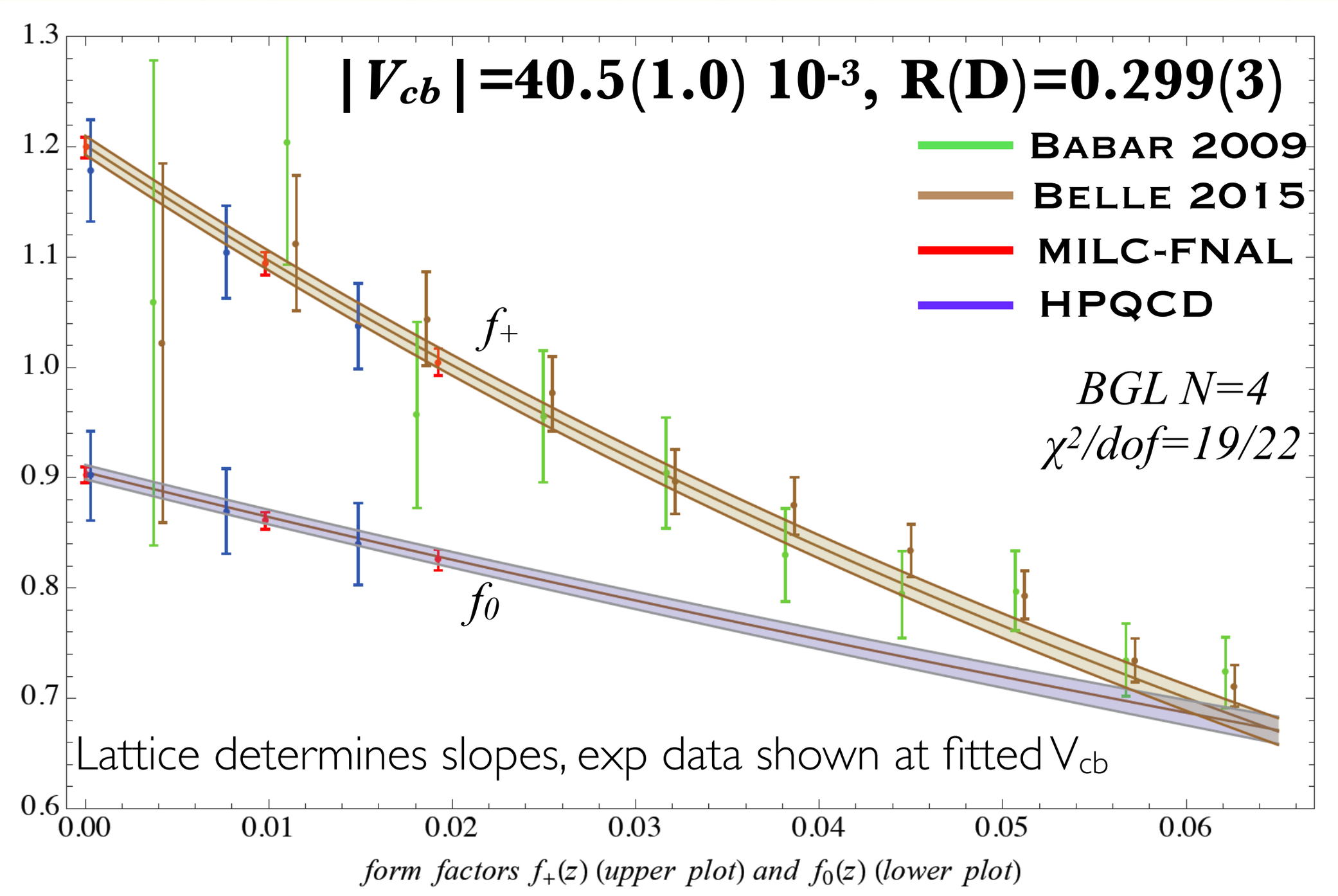
Caprini Lellouch Neubert (CLN, 1998) exploit NLO HQET relations between form factors + QCD sum rules to **reduce parameters** for ffs "up to 2% uncertainty", ***never included in exp analysis***. The *practical version of CLN* is

$$h_{A1}(z) = h_{A1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2,$$

only 2+2 parameters! but uncertainty? bias?

LATTICE + EXP BGL FIT for $B \to D l \nu$

Bigi, PG 1606.08030



R(D) 1.3σ from exp

FLAG has very similar results

CLN cannot fit both ff missing higher orders!

$|V_{cb}|$ from $B \to D^* l \nu$ *new HFLAV (2019)*

LQCD provided only light lepton FF at zero recoil, w=1, where rate vanishes.
Experimental results must therefore be **extrapolated to zero-recoil**

Exp error only ~1.1% $\mathcal{F}(1)\eta_{ew}|V_{cb}| = 35.27(38) \times 10^{-3}$

Beware: HFLAV extrapolate with CLN (w/o error), χ^2/dof of=42.3/23!

Two unquenched lattice calculations

$\mathcal{F}(1) = 0.906(13)$

Bailey et al 1403.0635 (FNAL/MILC)

$\mathcal{F}(1) = 0.895(26)$

Harrison et al 1711.11013 (HPQCD)

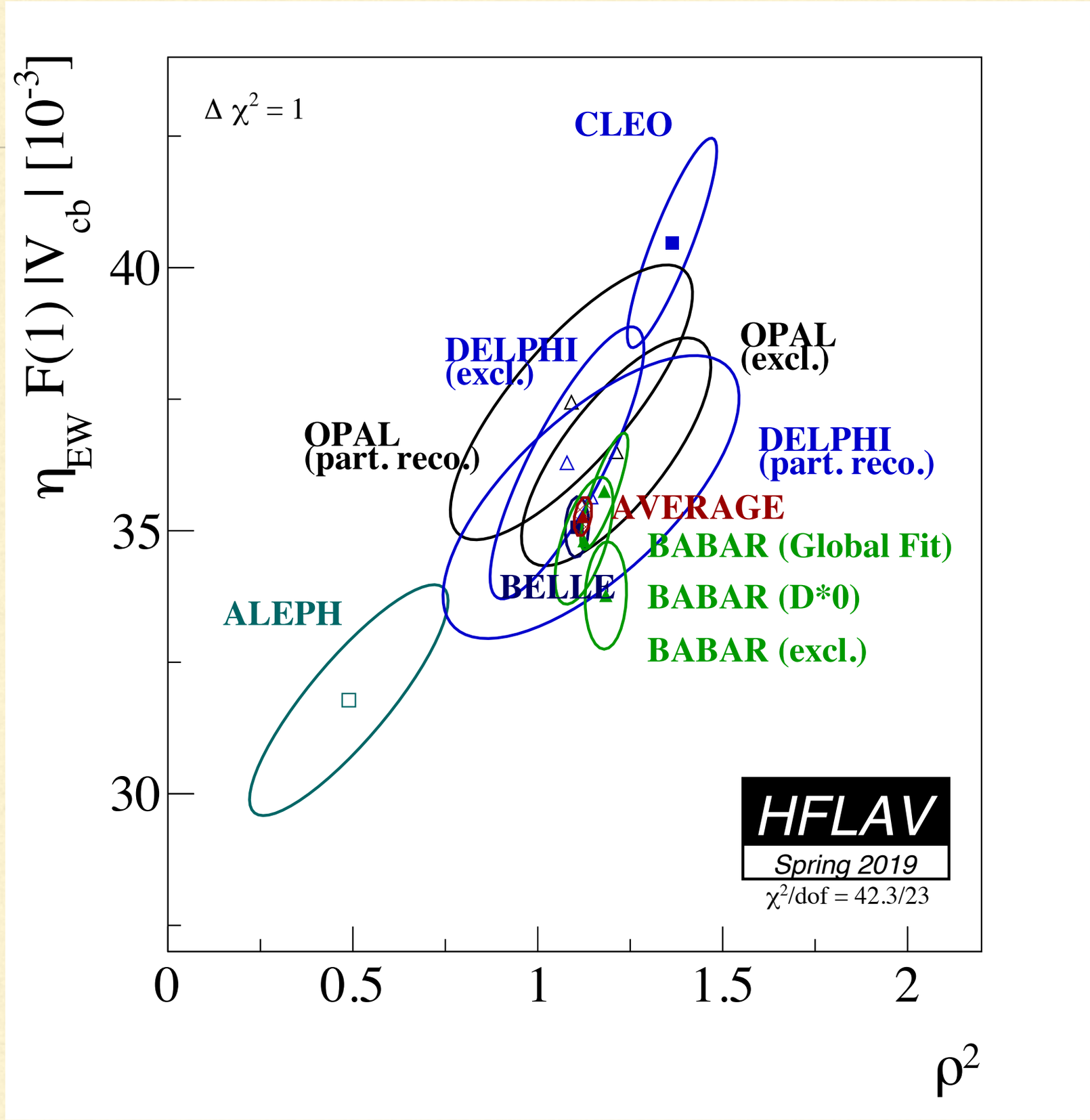
Using their average 0.904(12):

$|V_{cb}| = 38.76(69)\ 10^{-3}$

~ 3.4σ or ~ 8% from inclusive determination

Heavy quark sum rules F(1)< 0.925 and estimate of inelastic contribution F(1)≈0.86

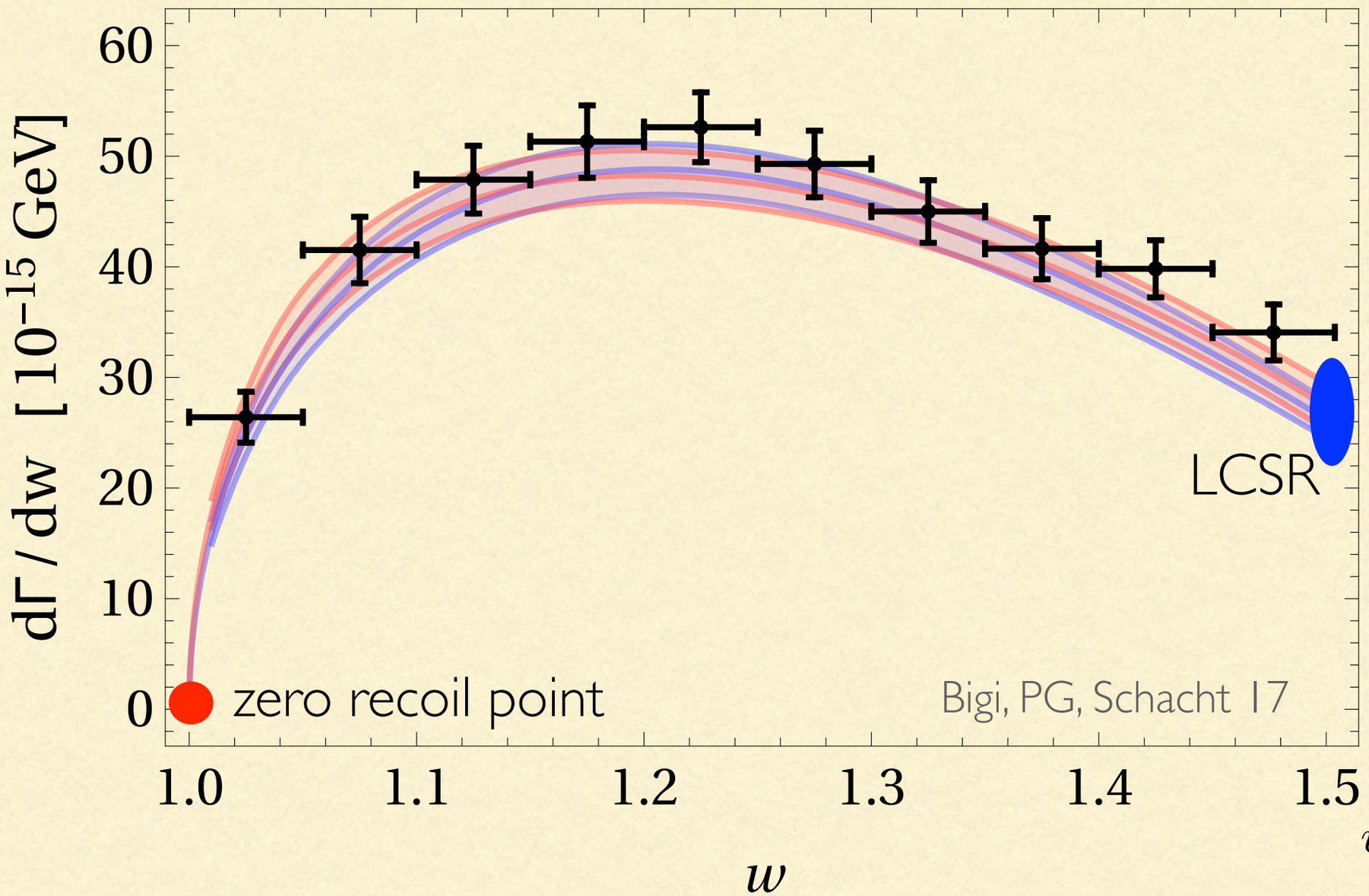
Mannel, Uraltsev, PG, 2012

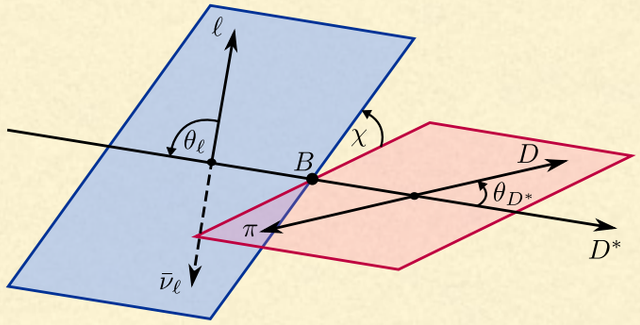

η_{EW} F(1) $|V_{cb}|$ [10^{-3}]
$\Delta\chi^2 = 1$
CLEO
DELPHI (excl.)
OPAL (excl.)
OPAL (part. reco.)
DELPHI (part. reco.)
AVERAGE
BABAR (Global Fit)
BELLE
BABAR (D*0)
BABAR (excl.)
ALEPH
HFLAV
Spring 2019
χ^2/dof = 42.3/23
40
35
30
0
0.5
1
1.5
2
ρ^2

2017 tagged Belle analysis (preliminary)

1702.01521

w and angular deconvoluted distributions (independent of parameterization).
All previous analyses are CLN based.





CLN + LCSR

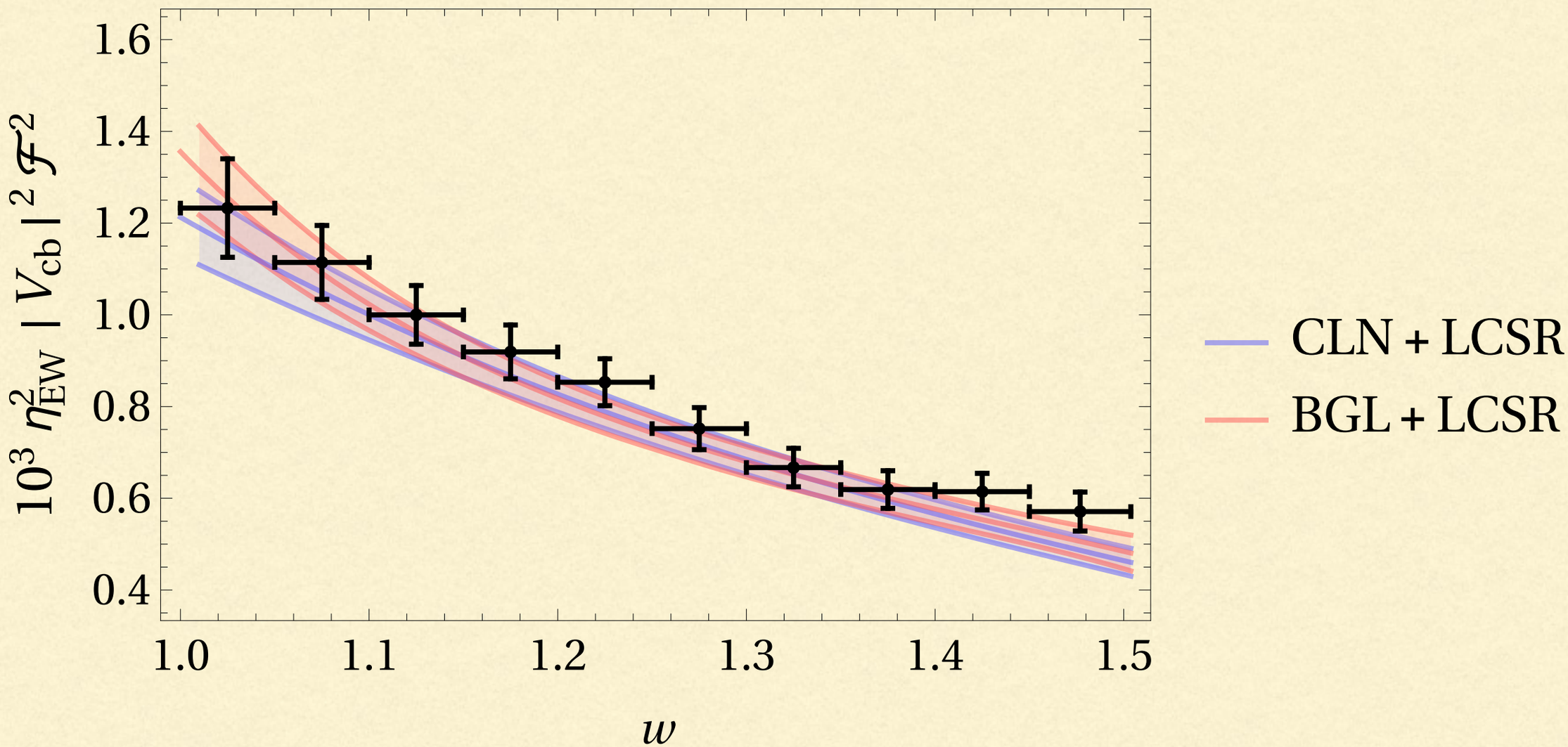
BGL + LCSR

see also
Kobach & Grinstein

$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

Bands show two parametrizations both fitting data well, with 6% different V_{cb}

EXTRAPOLATING TO ZERO RECOIL



+30 additional angular bins

Updating Strong Unitarity Bounds

Fit to new Belle's data + total branching ratio (world average) in 1707.09509
with UPDATED strong unit. bounds (including uncertainties & LQCD inputs)

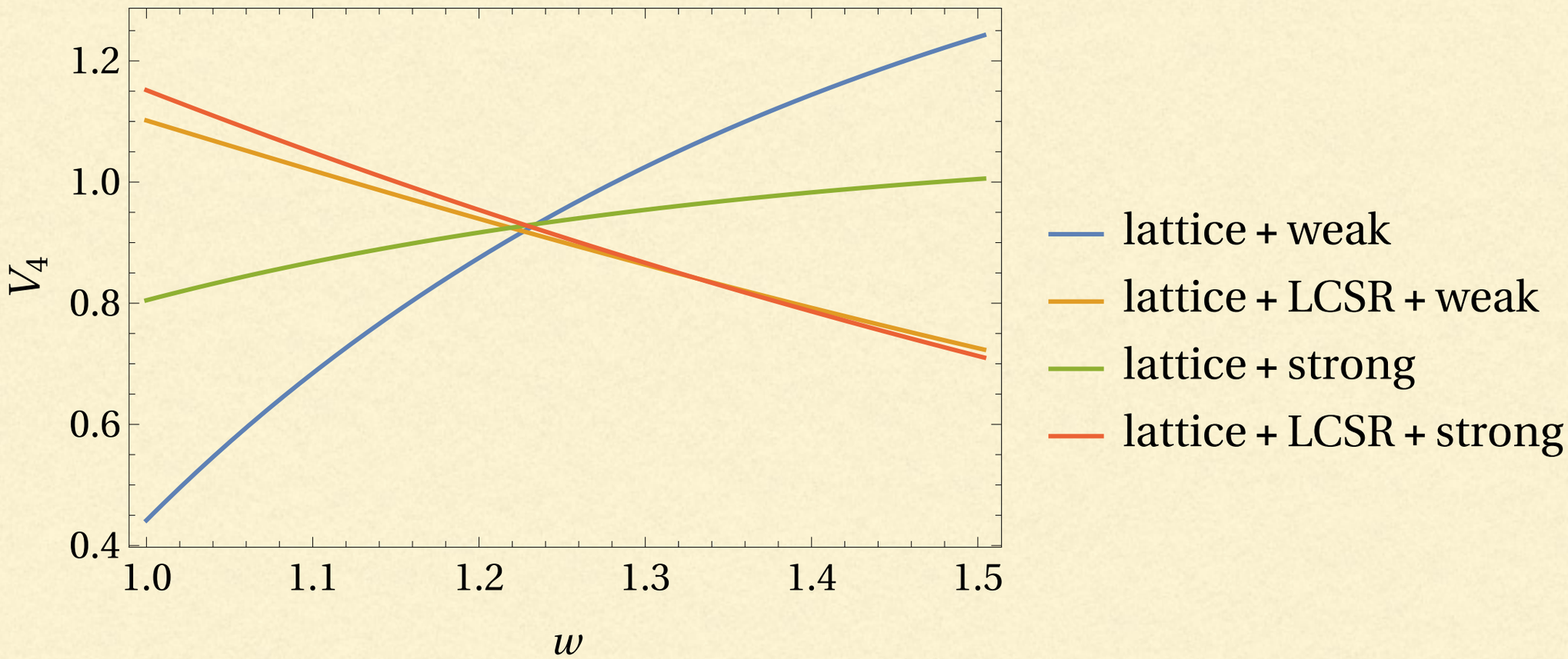
for reference CLN fit: **$|V_{cb}|$=0.0392(12)**

BGL Fit:	Data + lattice	Data + lattice + LCSR	Data + lattice	Data + lattice + LCSR
unitarity	weak	weak	strong	strong
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$\lvert V_{cb}\rvert$	0.0424 (18)	0.0413 (14)	0.0415 (13)	0.0406 $(^{+12}_{-13})$

LCSR: Light Cone Sum Rule results from Faller, Khodjamirian, Klein, Mannel, 0809.0222

Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but a 3.5-5% difference persists

SOMETIMES UNITARITY IS NOT ENOUGH…



The BGL fit tends to prefer **unphysical solutions** for V_4 with a positive w-slope. Imposing LCSR at w_{max} cures the problem, but one could equally well ask for a negative slope at w=1 or use HQET ratios with appropriate uncertainty. They all tend to lower V_{cb} slightly.

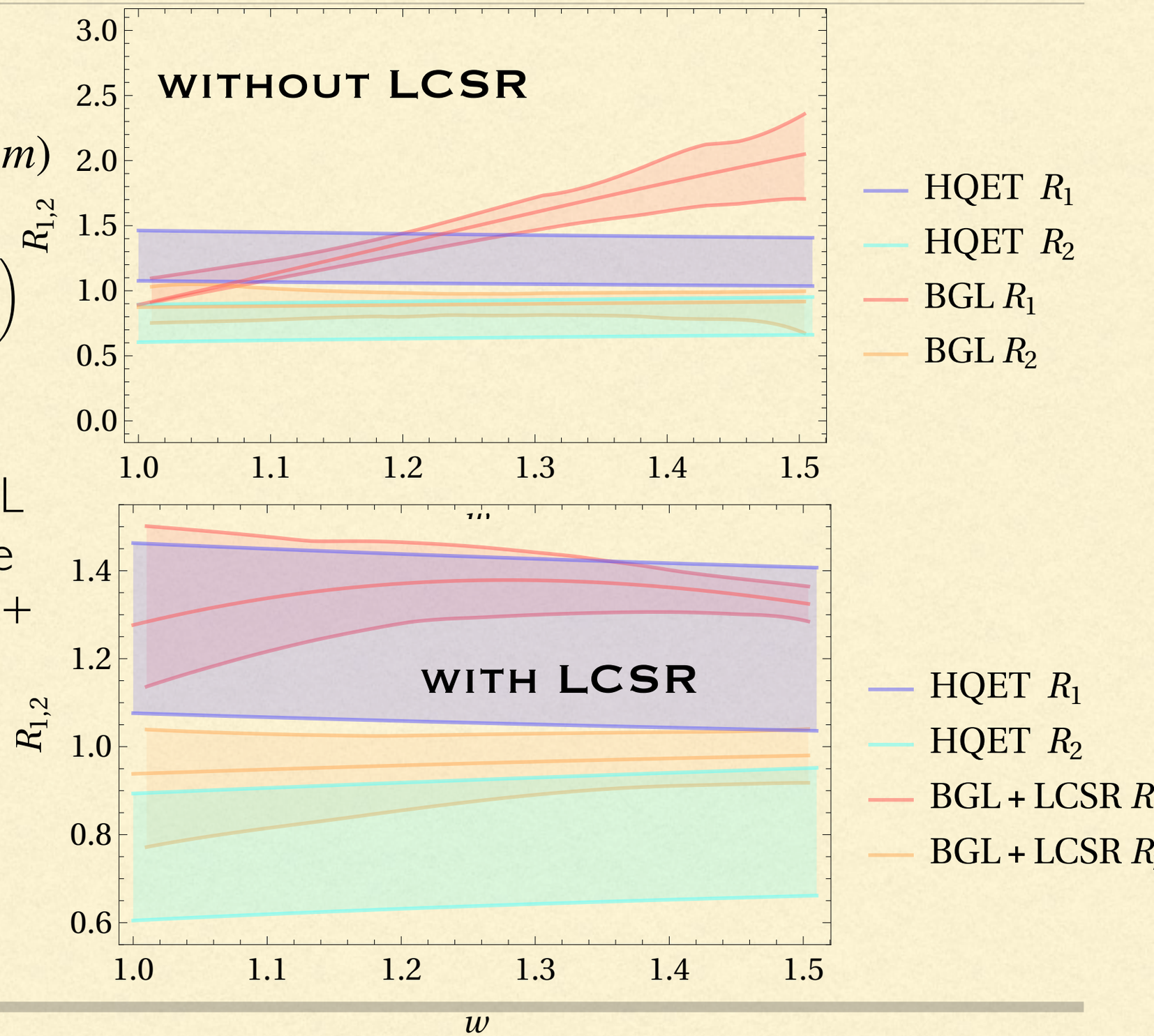
CONSISTENCY WITH HQET

$$R_1(w) = \frac{V_4(w)}{A_1(w)} = 1 + O(\alpha_s, 1/m)$$

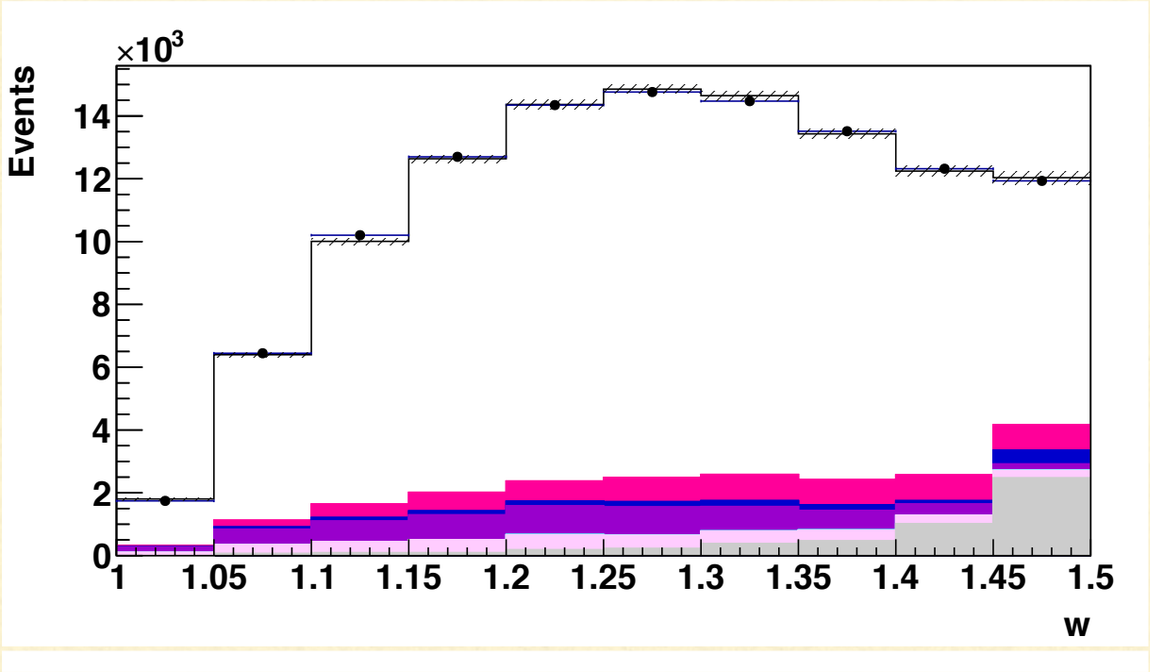
$$R_2(w) = \frac{w-r}{w-1}\left(1 - \frac{1-r}{w-r}\frac{A_5(w)}{A_1(w)}\right)$$

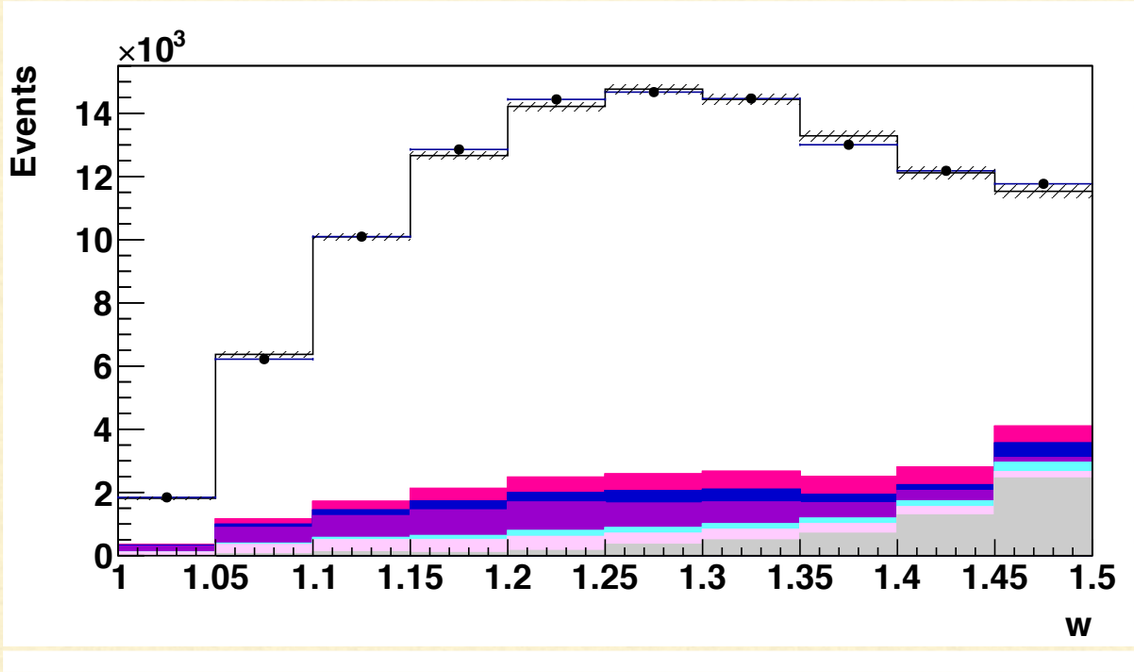
Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)

One can be consistent with HQET but still far from CLN!



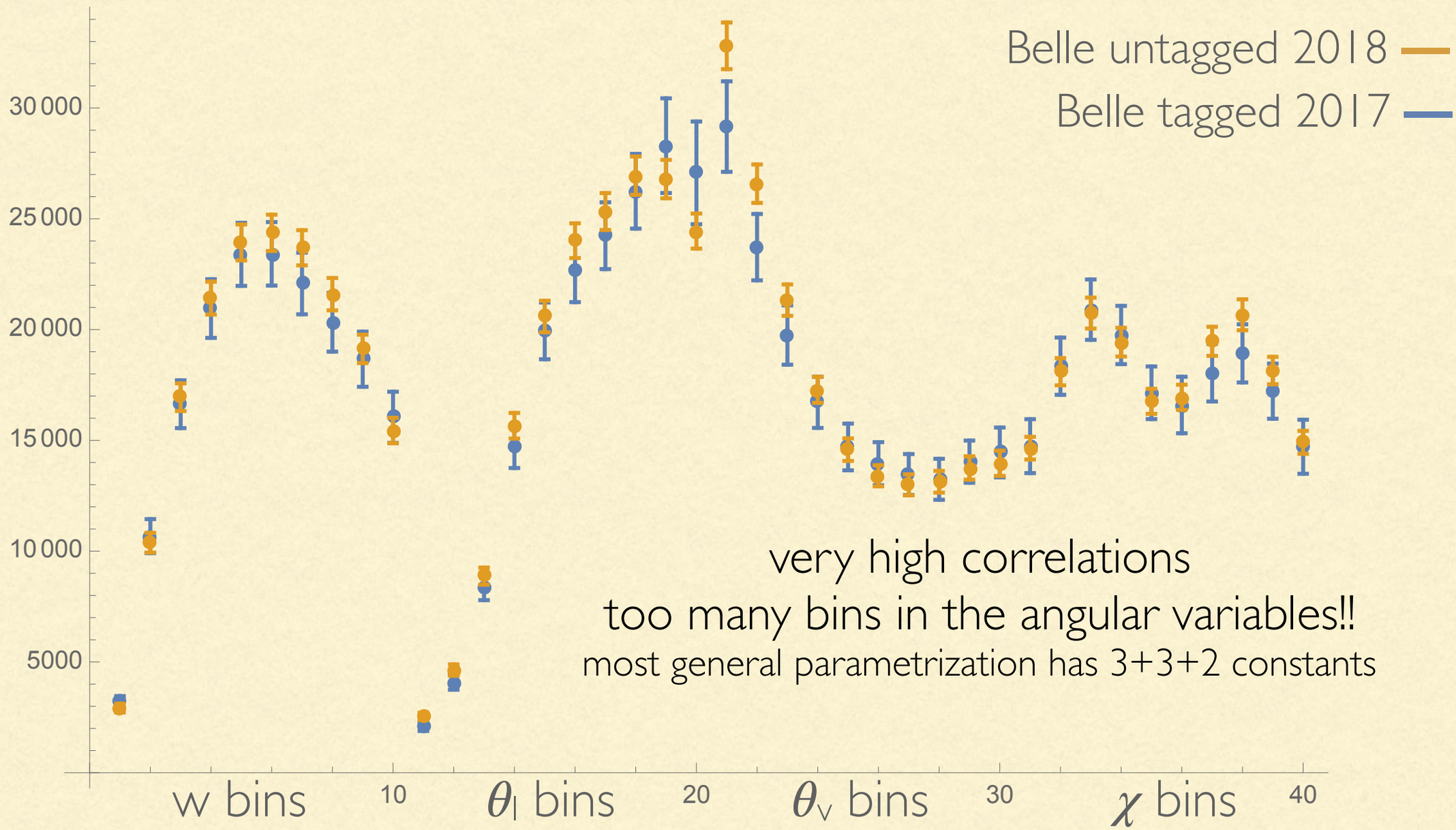
2018 UNTAGGED BELLE ANALYSIS 1809.03290v3





- Full Belle dataset, most precise study to date; provides data in a way that can be reanalysed with different assumptions. Central values obtained without systematics.
- CLN and BGL$^{(102)}$ analysis lead to very similar results, suggesting low **$|V_{cb}|$=38.4(0.9) 10^{-3}** .
- We used BGL$^{(222)}$ to fit the data, taking into account *D'Agostini effect* and got **$|V_{cb}|$=39.1(+1.5-1.3) 10^{-3}** 1905.08209

CONSISTENCY OF 2017 AND 2018 DATASETS



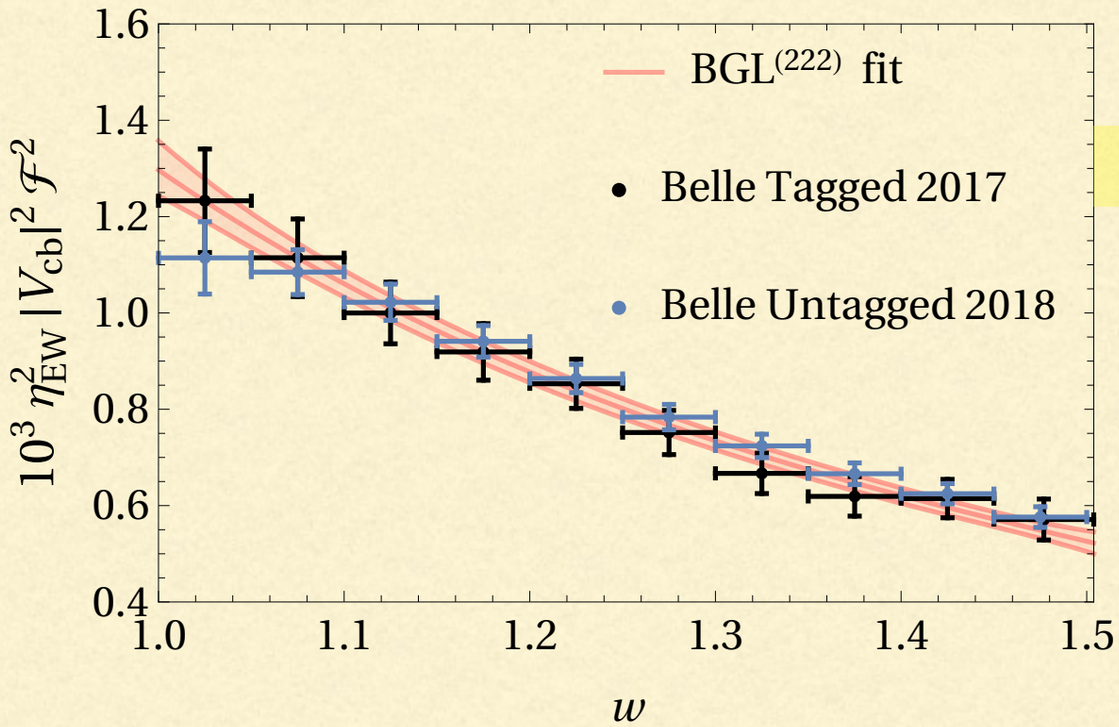
very high correlations

too many bins in the angular variables!!

most general parametrization has 3+3+2 constants

A GLOBAL FIT TO 2017 & 2018 DATA

Jung, Schacht, PG 1905.08209



$\mathrm{BGL}^{(222)}$	Data + lattice (weak)	Data + lattice (strong)
χ^2/dof	80.1/72	80.1/72
$\lvert V_{cb}\rvert 10^3$	$39.6\left(^{+1.1}_{-1.0}\right)$	$39.6\left(^{+1.1}_{-1.0}\right)$
a_0^f	0.01221(16)	0.01221(16)
a_1^f	$0.006\left(^{+32}_{-45}\right)$	$0.006\left(^{+20}_{-32}\right)$
a_2^f	$-0.2\left(^{+12}_{-8}\right)$	$-0.2\left(^{+7}_{-3}\right)$
$a_1^{\mathcal{F}_1}$	$0.0042\left(^{+22}_{-22}\right)$	$0.0042\left(^{+19}_{-22}\right)$
$a_2^{\mathcal{F}_1}$	$-0.069\left(^{+41}_{-37}\right)$	$-0.068\left(^{+41}_{-30}\right)$
a_0^g	$0.024\left(^{+21}_{-9}\right)$	$0.024\left(^{+12}_{-4}\right)$
a_1^g	$0.05\left(^{+39}_{-72}\right)$	$0.05\left(^{+21}_{-41}\right)$
a_2^g	$1.0\left(^{+0}_{-20}\right)$	$0.9\left(^{+0}_{-18}\right)$

- No parametrization dependence (CLN and BGL give ~same central value)
- About 1σ higher than HFLAV, larger uncertainty on firmer ground, p-value ~24% ***1.9σ from inclusive, consistent with B to Dlv***
- We truncate the BGL series when additional terms do not change the fit (no overfitting!). Fit stable. Strong constraints irrelevant.

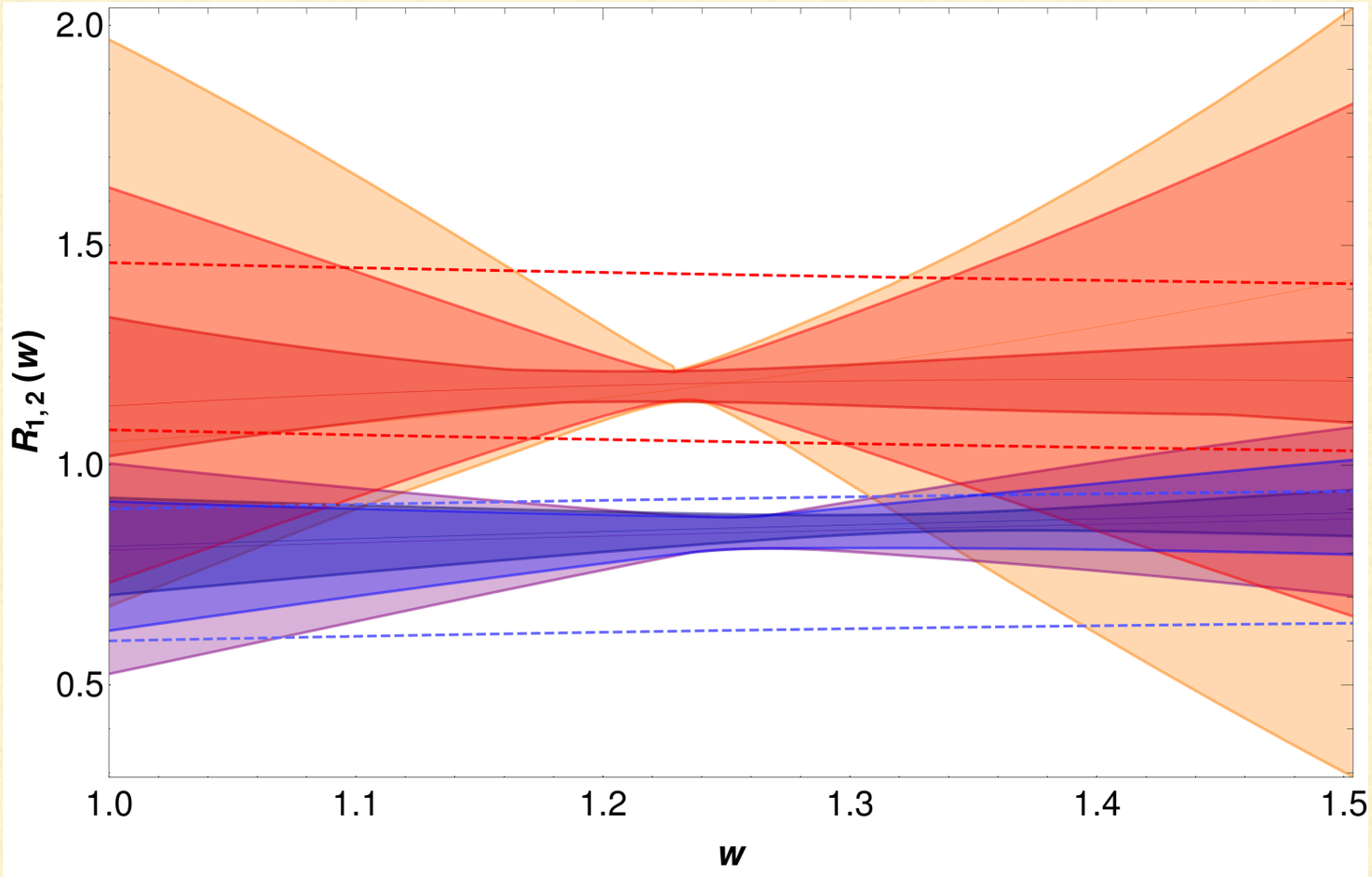
COMPARISON WITH HQET+QCDSR

1905.08209

Narrower bands correspond to adding strong unitarity and LCSRs

Comparison of $R_{1,2}$ from BGL fit to 2017+2018 data vs NLO HQET+QCDSR predictions (with parametric + 15% th uncertainty)

Very good agreement in both cases.

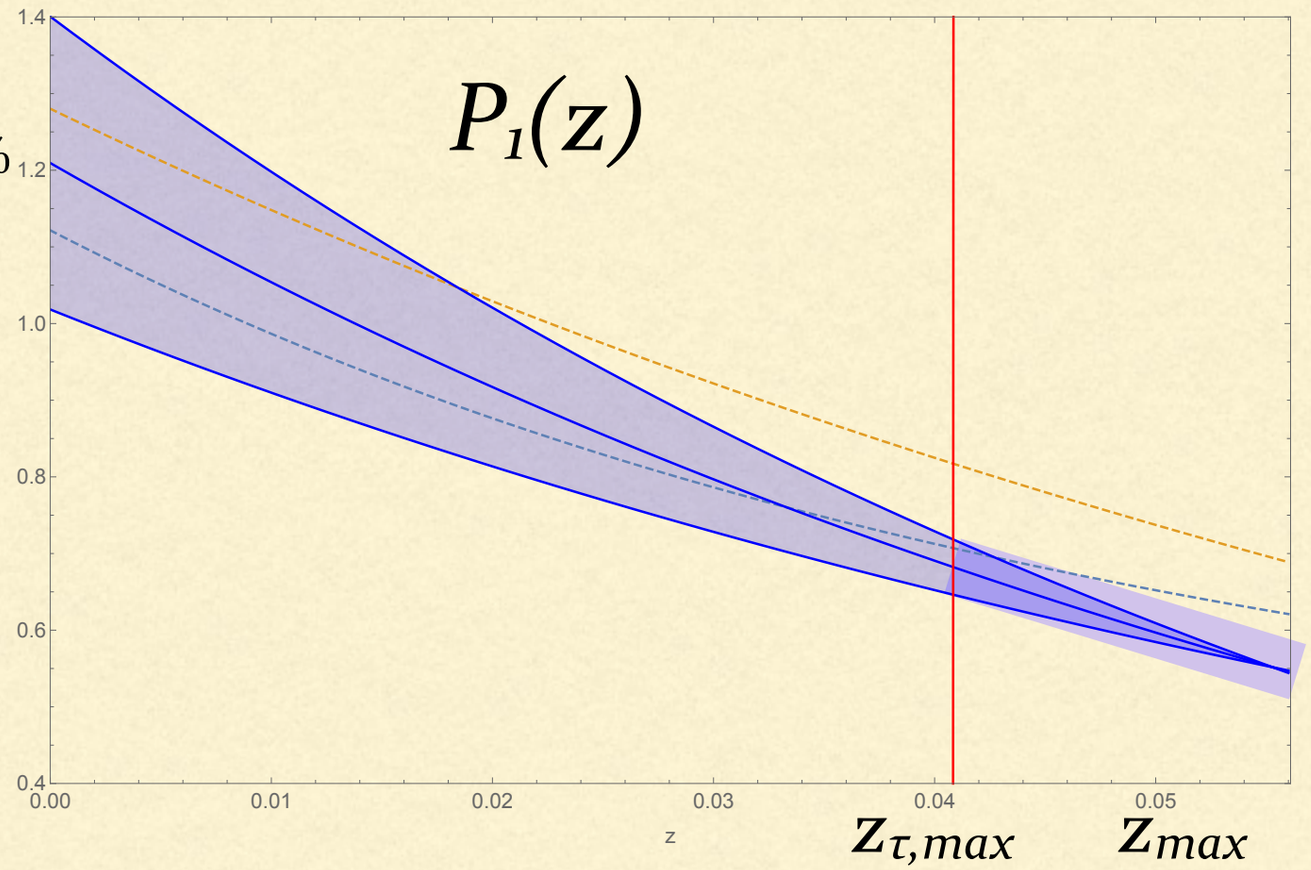


SEMITAUONIC DECAYS

1905.08209

Decays with tau require pseudoscalar FF unconstrained from fit, no lattice calculation yet. We use kinematic constraint at $q^2=0$ and HQET with conservative uncertainty.

NLO HQET+QCDSR ±15%



normalized to V_1
normalized to A_1
$A_5(w_{max})$

Endpoint constraint

$$P_1(w_{max}) = A_5(w_{max})$$

$$R(D^*) = 0.254^{+0.007}_{-0.006}\,, \qquad 2.8\sigma \text{ from exp}$$

$$P_\tau = -0.476^{+0.037}_{-0.034}\,, \qquad 1.4\sigma \text{ from exp}$$

$$F_L^{D^*} = 0.476^{+0.015}_{-0.014}\,,$$

COMPARISON WITH OTHER APPROACHES

Ref.	$R(D)$	Exp. deviation
[21]	0.299(3)	2.4σ
[32]	0.299(3)	2.4σ
[34]	0.302(3)	2.3σ
[37]	0.297(3)	2.4σ

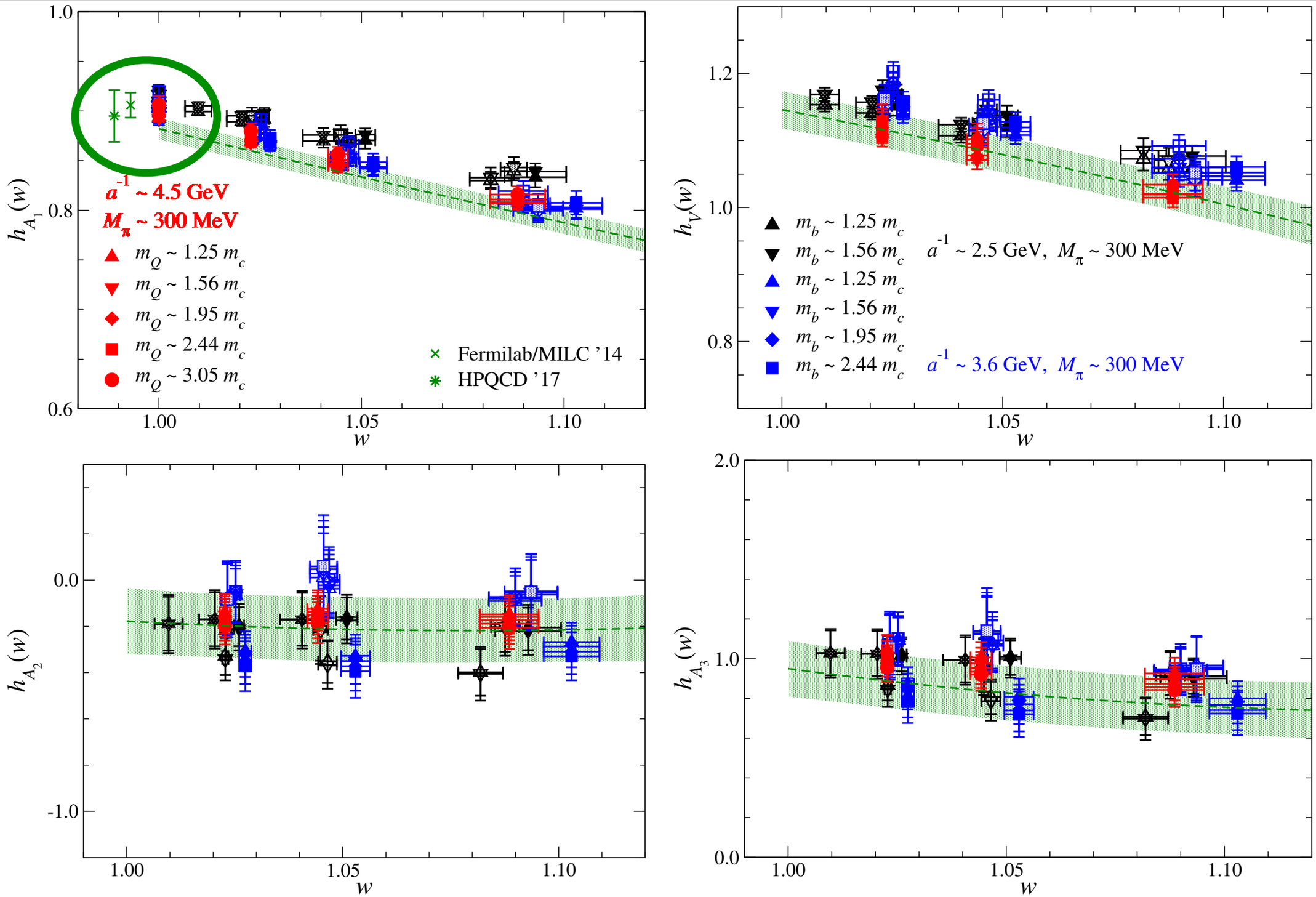
Ref.	$R(D^*)$	Exp. deviation
[32]	0.257(3)	3.3σ
[13,36]	$0.254\,\binom{7}{6}$	3.2σ
[34]	0.257(5)	3.1σ
[37]	0.250(3)	3.7σ

Table 2.3 Recent theory predictions for $R(D^{(*)})$. The deviations are calculated from the HFLAV summer 2018 updates $R(D)^{\text{exp}} = 0.407(39)(24)$ [18,38–40] and $R(D^*)^{\text{exp}} = 0.306(13)(7)$ [18,38–46], respectively. For older predictions see Refs. [47–49]. Table adapted and extended from Ref. [50].

S.Schacht, MITP report 2002.xxxxx

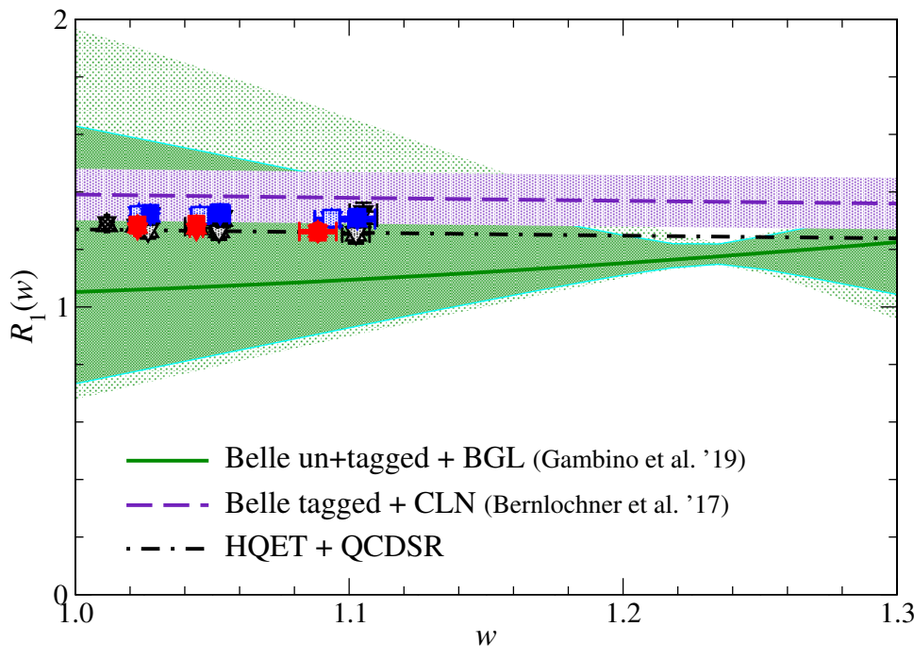
PRELIMINARY JLQCD RESULTS

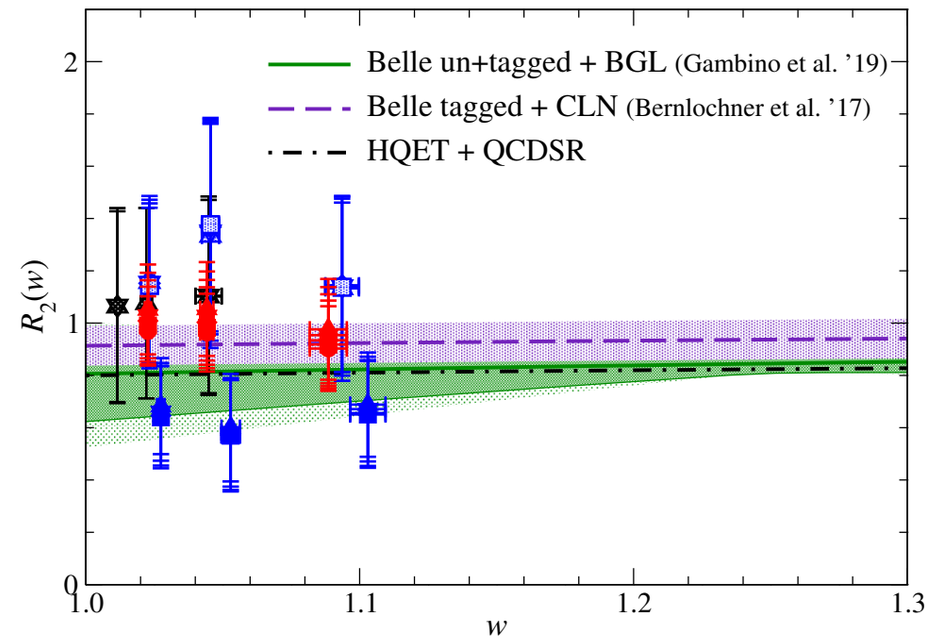
1912.11770



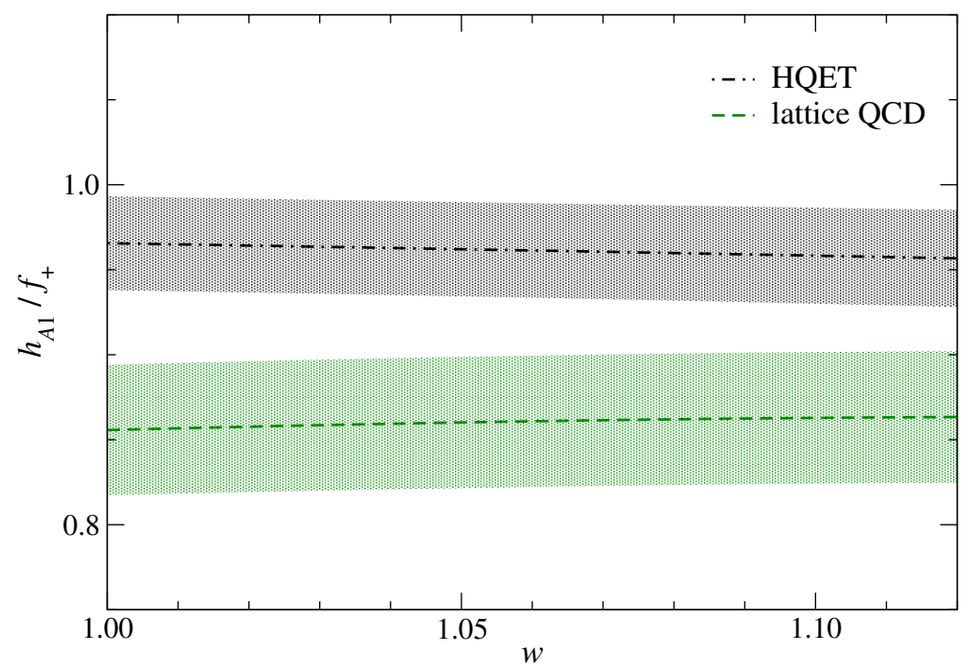
PRELIMINARY JLQCD RESULTS

1912.11770





JLQCD results for $R_{1,2}$ are consistent with our latest fit and with NLO HQET+QCDSR, but confirm large higher orders in h_{A1}/f_+



BLINDED FNAL-MILC RESULTS

Unquenched calculation of all B→D* form factors at small recoil

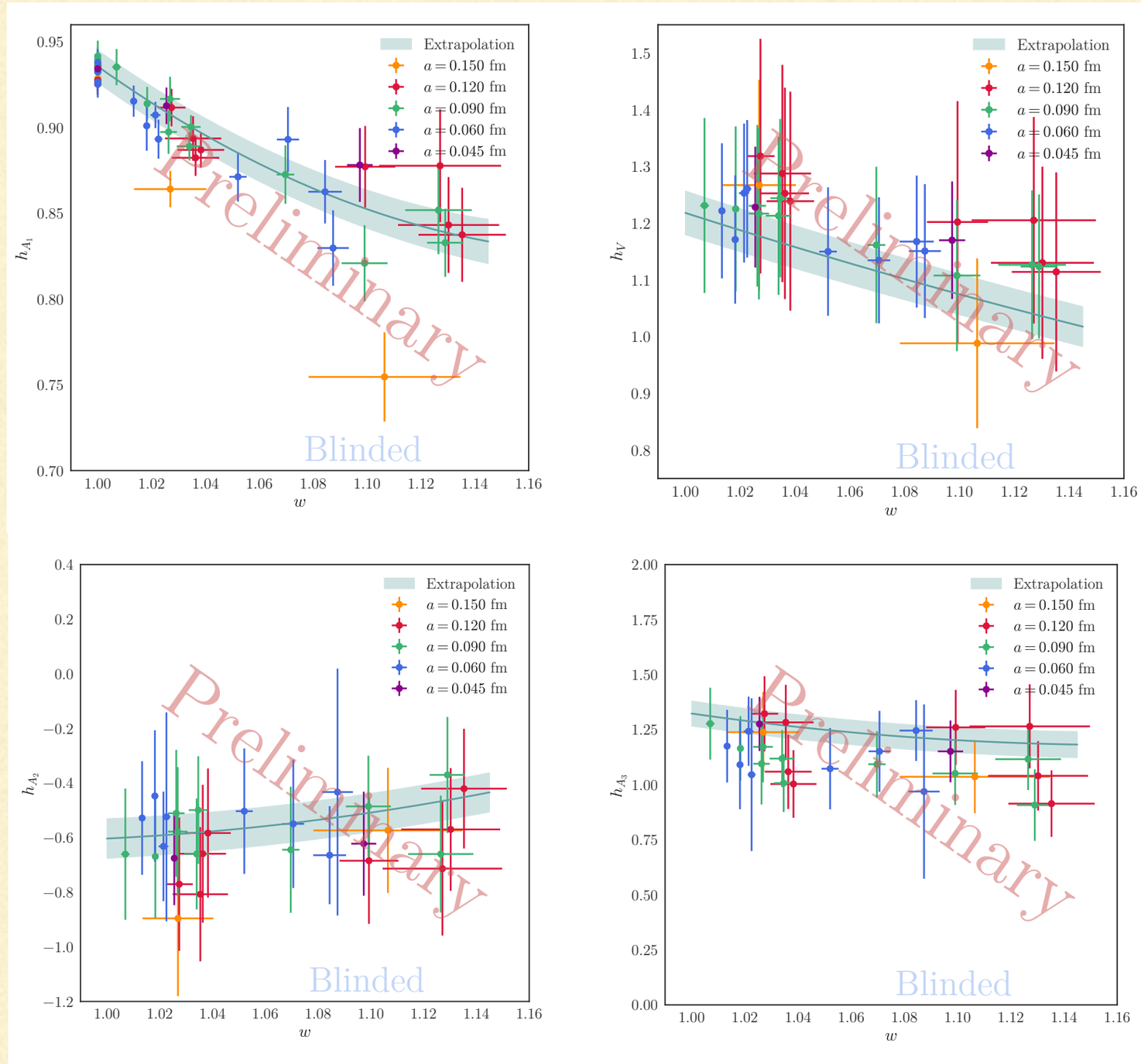
Blinding is introduced as a global normalisation factor close to 1, which multiplies all form factors in the same way.

Discrepancies with JLQCD, but further changes should expected in the final results

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Statistics	1.1	0.4	4.9	1.9
Isospin effects	0.0	0.0	0.6	0.3
χPT/cont. extrapolation	**1.9**	**0.7**	**6.3**	**2.9**
Matching	*1.5*	*0.4*	*0.1*	*1.5*
*Heavy quark discretization**	*2.5*	*1.2*	*9.0*	*6.0*

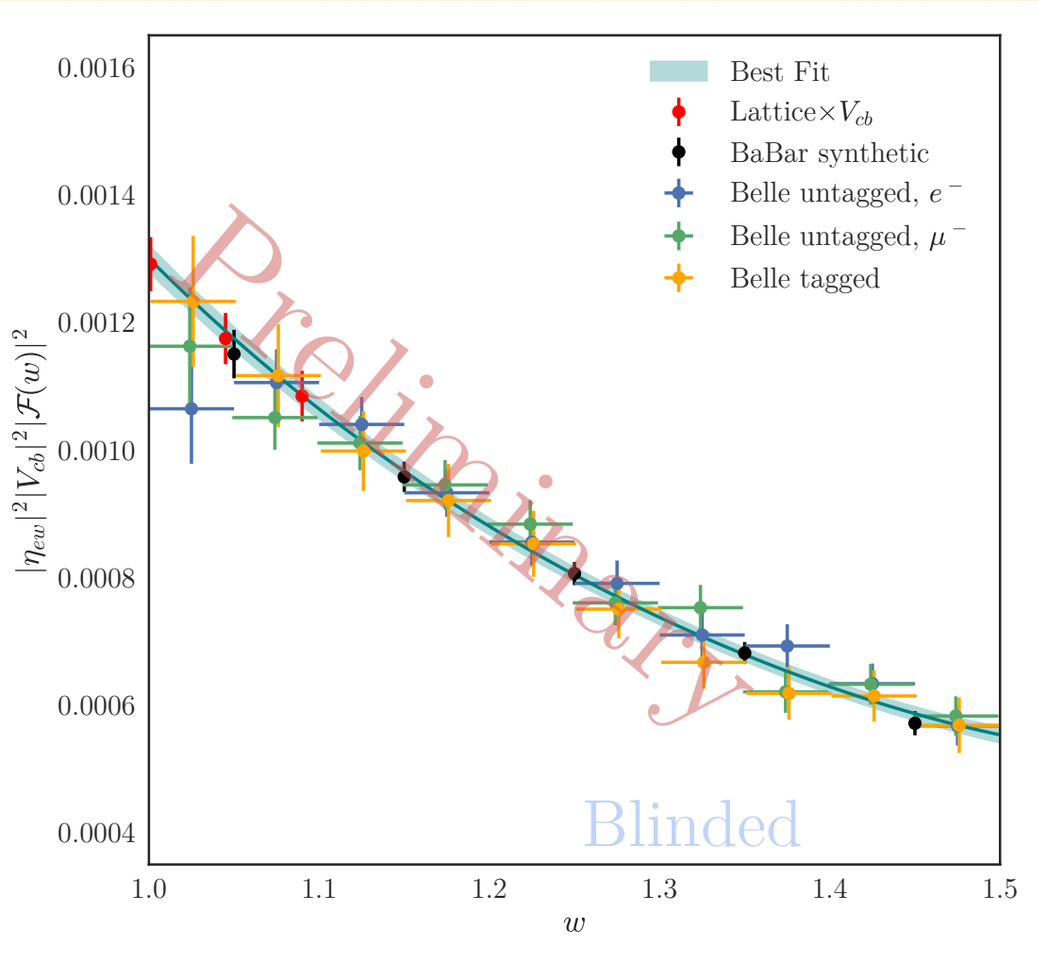
Errors at $w = 1.10$

***Preliminary estimate, analysis in progress**



A.Vaquero, BNL 9/2019

THE IMPORTANCE OF THE SLOPE



Blinding affects only marginally the slope of the ff F, which is key to V_{cb}.

Plot suggests large and well determined slope, $dF/dw|_{w=1}$.

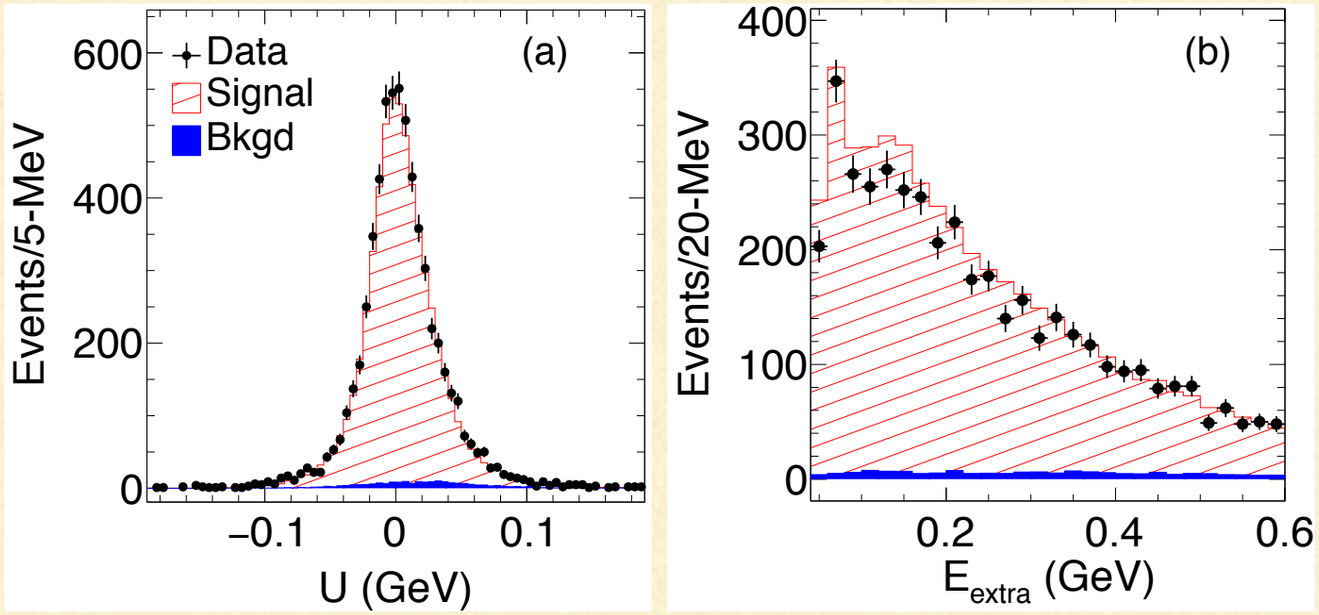
If it were -1.40(7) the fit could still accomodate a high V_{cb}

Here we use **new improved LCSR** results by Gubernari, Kokulu, van Dyk, 1811.00983 that improve upon 0809.0222

Constraints	$10^3\ V_{cb}$	χ^2
slope	40.8(0.8)	84.5/73
slope+LCSR	40.8(0.8)	88.0/76

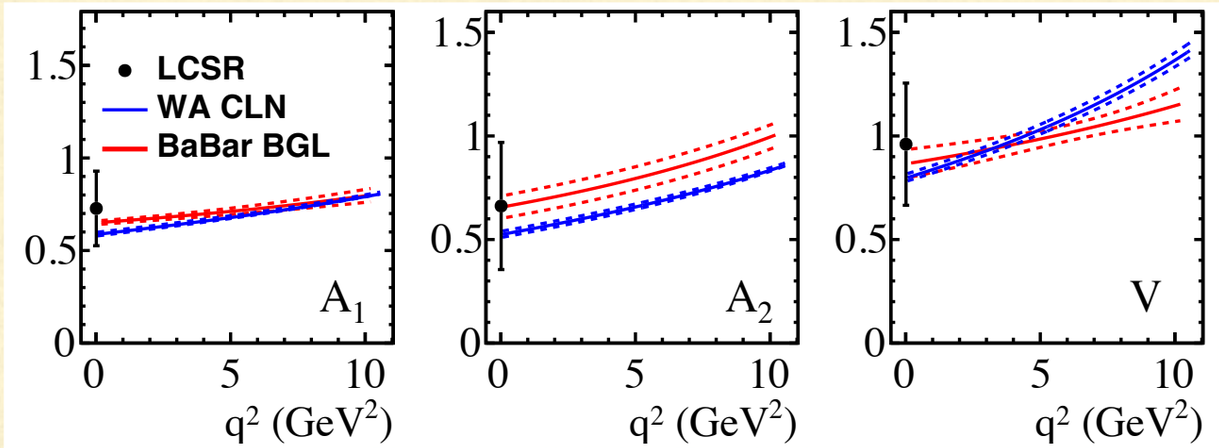
BABAR REANALYSIS 1903.10002

Reanalysis of tagged B^0 and B^+ data, unbinned 4 dimensional fit with simplified BGL and CLN
About 6000 events
No data provided yet



No clear BGL(III)/CLN difference

V_{cb}=0.0384(9)



BGL form factors compared with CLN HFLAV

EXPLOITING HQET & LCSRS 1908.09398

Can HQET relations be further exploited, for instance including dominant NNLO terms $O(1/m_c^2)$?

$$F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}(w)\frac{\alpha_s}{\pi} + c^i_b(w)\epsilon_b + c^i_c(w)\epsilon_c + d^i\epsilon_c^2\right] \qquad \epsilon_{b,c} = \bar{\Lambda}/2m_{b,c}, \qquad \alpha_s \sim \epsilon_b \sim \epsilon_c^2$$

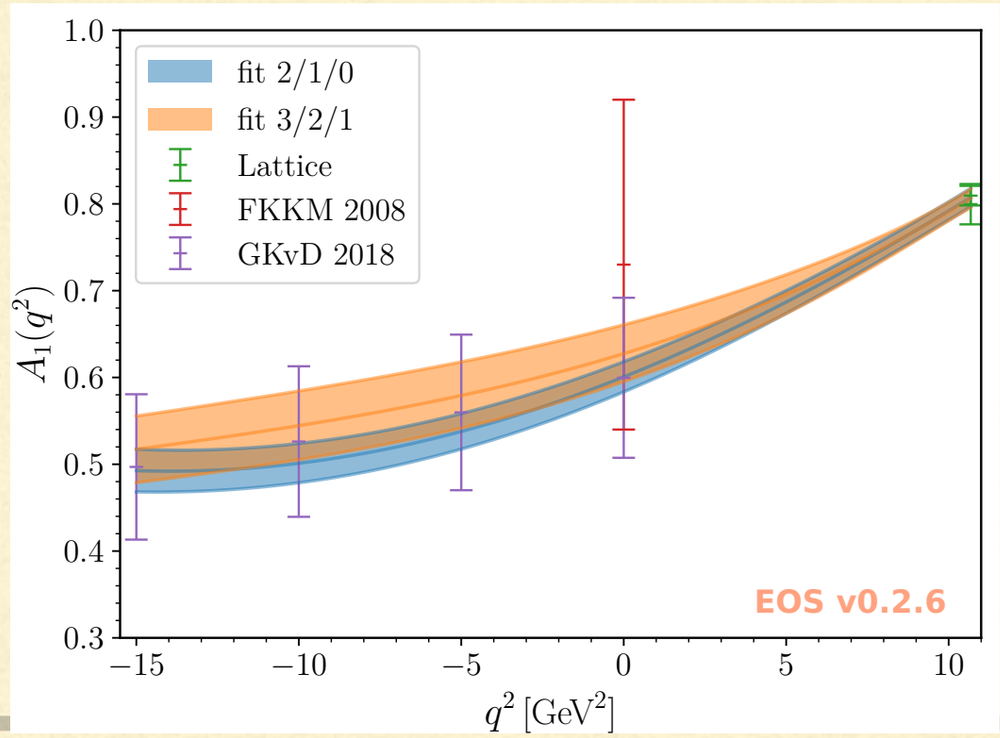
6 subsubleading IW functions, expanded around w=1. To fix them and for NP analyses we need additional info from LQCD or LCSR.

Bordone, Jung, Van Dyk, building on 1703.05330 (Bernlochner et al) and 1707.09977 (Jaiswal et al) use available LQCD and LCSR calculations and Belle B->D(*)lv data using BGL with *strong unitarity bounds*

V_{cb}=40.3(0.8)10^{-3}

R(D)=0.297(3)

R(D*)=0.250(3)



B_s DECAYS BEYOND THE SU(3) LIMIT

Bordone, Gubernari, Jung, Van Dyk 1912.09335

B_s decays can be studied at LHCb. There are a few lattice results, most recently by HPQCD 1906.00701 LCSRs are available for all FFs.

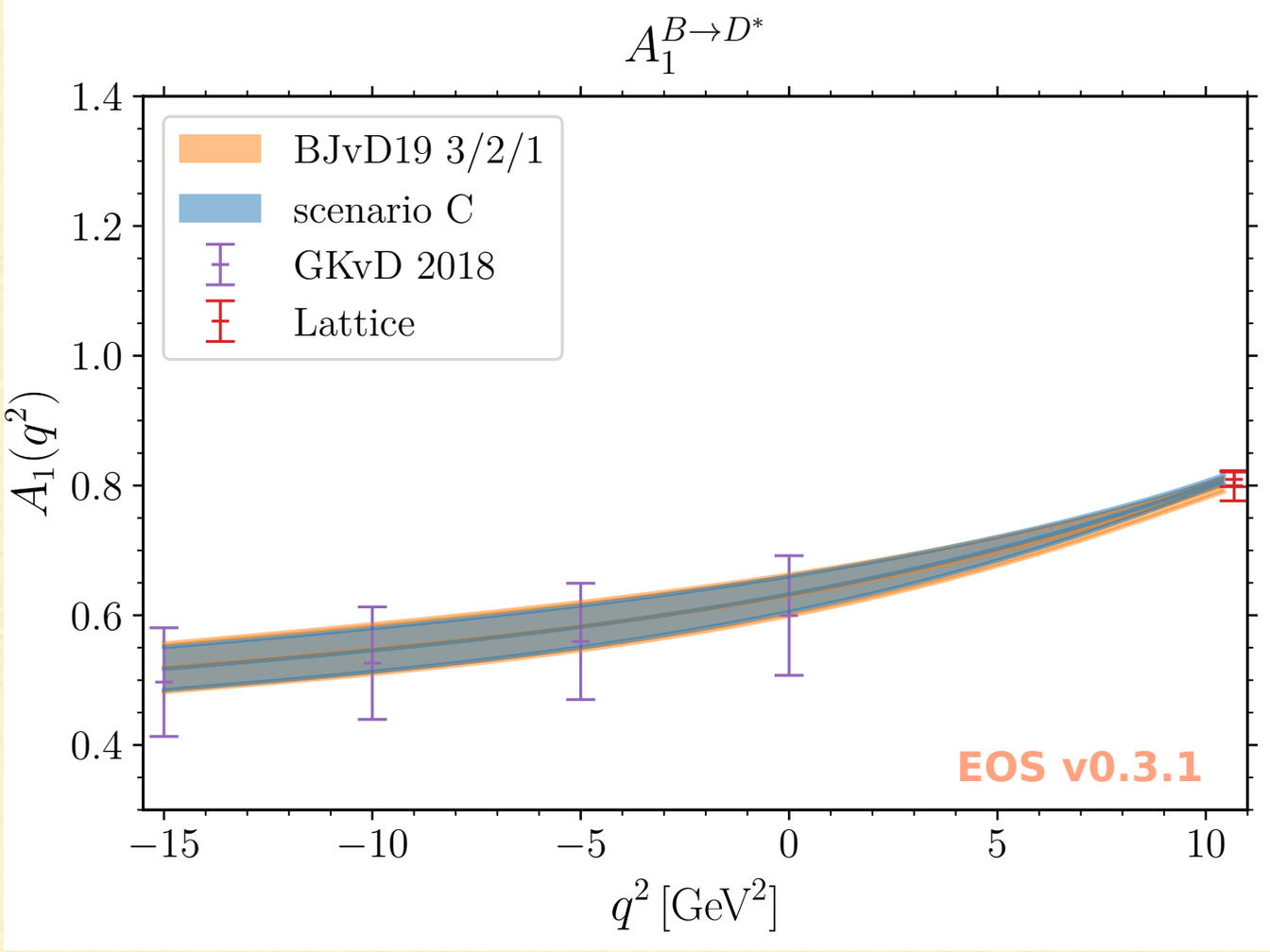
There is no evidence for $SU(3)_F$ breaking at present.

Assuming *only* the subsubleading IW is $SU(3)_F$ symmetric, the strong unitarity bounds become slightly more constraining.

V_{cb}=40.0(0.7)10^{-3}

R(D)=0.298(3)

R(D*)=0.250(3)



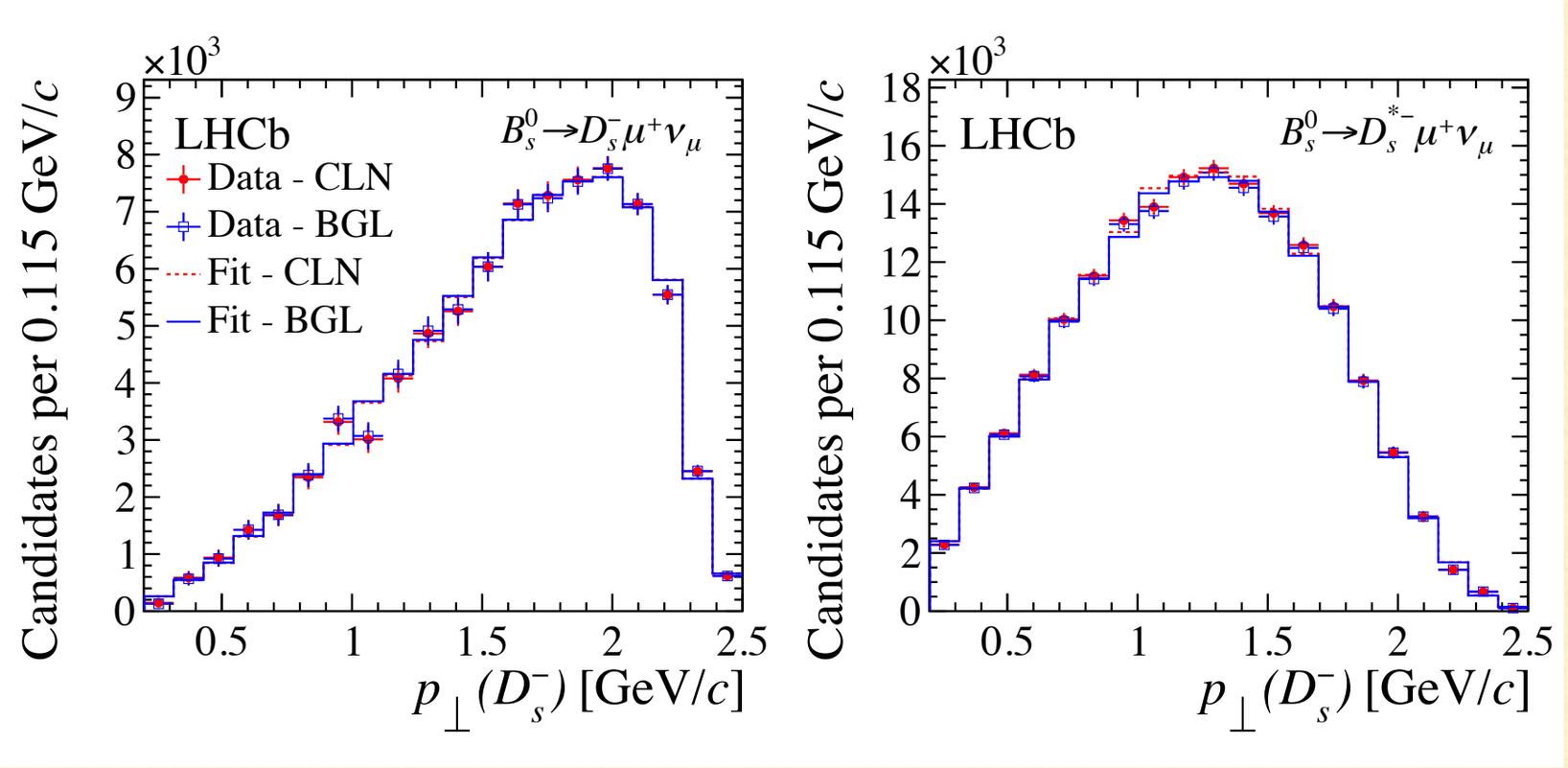
V_{cb} FROM B_s DECAYS

2001.03225

Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ decays

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^-\mu^+\nu_\mu)},$$

$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \to D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)}$$



$$|V_{cb}|_{\rm CLN} = (41.4 \pm 0.6\,({\rm stat}) \pm 0.9\,({\rm syst}) \pm 1.2\,({\rm ext})) \times 10^{-3}\,,$$
$$|V_{cb}|_{\rm BGL} = (42.3 \pm 0.8\,({\rm stat}) \pm 0.9\,({\rm syst}) \pm 1.2\,({\rm ext})) \times 10^{-3}\,,$$

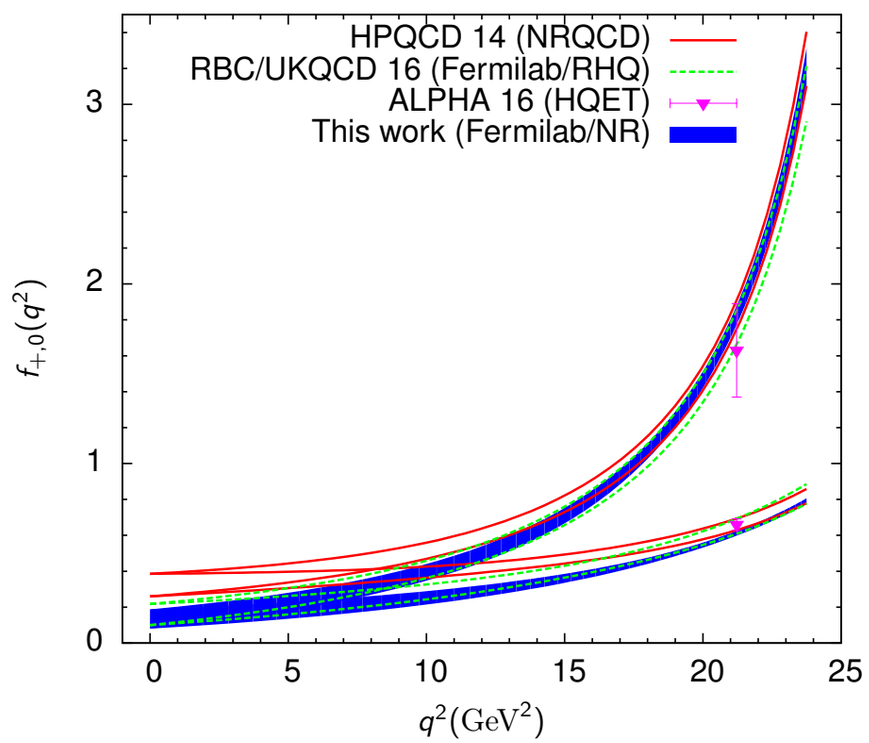
Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL_{222}

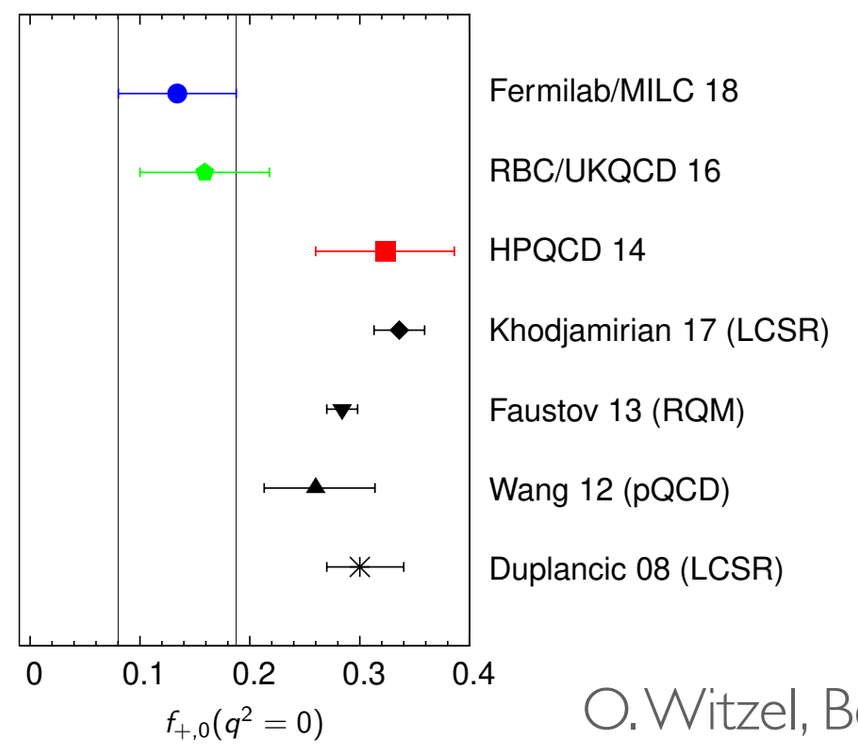
OTHER DECAY MODES

LHCb can access $|V_{ub}/V_{cb}|$ from Λ_b and B_s decays.

$B_s \to K\ell\nu$

- HPQCD, RBC-UKQCD, ALPHA
  [Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473]
- New 2019: Fermilab/MILC [Bazavov et al. PRD100(2019)034501]





O. Witzel, Beauty 2019

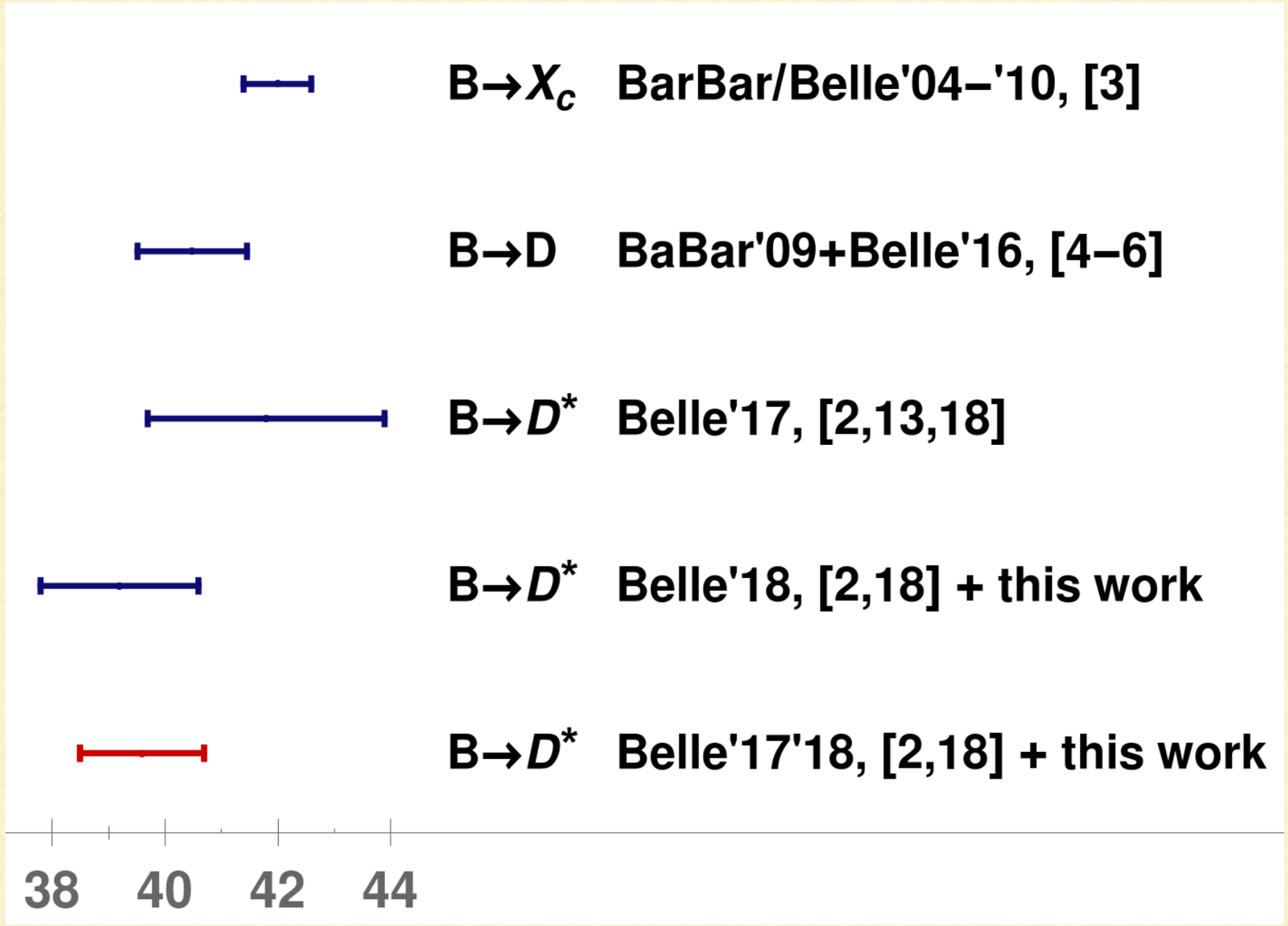
CONCLUSIONS

- Despite conflicting results, revisiting the exclusive *b* to *c* decays was ***important***: uncertainties were underestimated and results possibly biased. The *practical CLN* must be abandoned.
- Several lattice coll. are computing all necessary FFs, in parallel with Belle-II improved measurements (also for the inclusive moments). LHCb have shown they can contribute too.
- Inclusive/Exclusive tension remains, but weaker. Hopefully, it will disappear. LQCD will decide the fate of the V_{cb} puzzle.
- Semitauonic anomaly persists and the SM predictions are robust.
- Experiments should *provide results in a model independent way: unfolded spectra or alternative but equivalent information*. Theoretical prejudice is transient by definition and should never be hardwired into precision measurements.
- Something is moving also for V_{ub}. New promising possibilities at Belle-II.

BACKUP MATERIAL

data	fit	par	χ^2/dof	$\lvert V_{cb}\rvert 10^3$
2018	stat	$\mathrm{BGL}^{(102)}$	53.0/35	38.8 ± 0.6
2018	stat	CLN	56.6/36	39.2 ± 0.6
2018	naive	$\mathrm{BGL}^{(102)}$	32.6/35	39.7 ± 0.9
2018	naive	CLN	32.4/36	39.7 ± 0.9
2018		$\mathrm{BGL}^{(102)}$	32.5/35	40.3 ± 0.9
2018		CLN	32.4/36	40.3 ± 0.9
2018	stat	$\mathrm{BGL}^{(222)}$	47.7/32	$37.6^{+1.0}_{-0.9}$
2018	naive	$\mathrm{BGL}^{(222)}$	31.2/32	$38.6^{+1.5}_{-1.4}$
2018		$\mathrm{BGL}^{(222)}$	31.2/32	$39.1^{+1.5}_{-1.3}$
2017/18, slope		$\mathrm{BGL}^{(222)}$	84.5/73	40.8 ± 0.8
2017/18, LCSR		$\mathrm{BGL}^{(222)}$	80.5/75	39.5 ± 1.0
2017/18, LCSR, slope		$\mathrm{BGL}^{(222)}$	88.0/76	40.8 ± 0.8

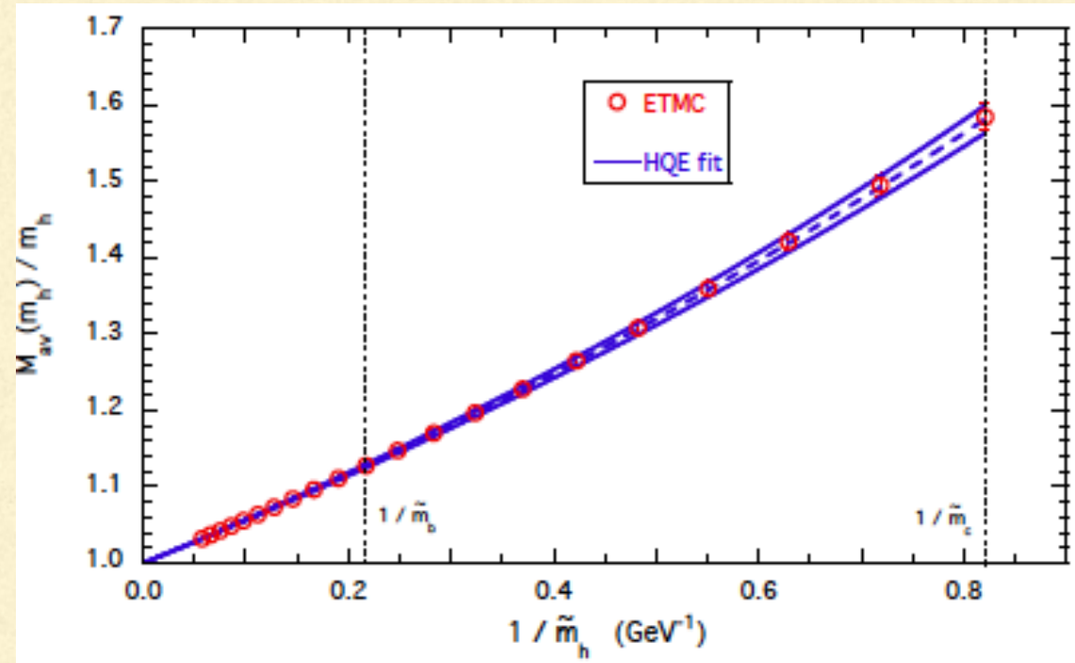
TABLE I. Results of various fits to the $B \to D^* \ell \nu$ data. The $|V_{cb}|$ error always includes the lattice uncertainty. In the second column *stat* stands for "only statistical errors", *naive* stands for "systematic errors as fractions of the yield", in all other cases we take the D'Agostini effect into account (see text). All fits are with weak unitarity constraints.


B→X_c BarBar/Belle'04–'10, [3]
B→D BaBar'09+Belle'16, [4–6]
B→D^* Belle'17, [2,13,18]
B→D^* Belle'18, [2,18] + this work
B→D^* Belle'17'18, [2,18] + this work
38
40
42
44

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \dots$$



- on the lattice one can compute mesons for arbitrary quark masses

  see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections
- Results consistent with s.l. fits

INCLUSIVE V_{ub}

HFLAV 1909.12524

	BLNP	DGE	GGOU	ADFR	BLL
Input parameters					
scheme	SF	$\overline{MS}$	kinetic	$\overline{MS}$	$1S$
Ref.	[572, 573]	Ref. [574]	see Sec. 6.2.2	Ref. [575]	Ref. [556]
m_b (GeV)	4.582 ± 0.026	4.188 ± 0.043	4.554 ± 0.018	4.188 ± 0.043	4.704 ± 0.029
μ_π^2 (GeV^2)	$0.145\ ^{+0.091}_{-0.097}$	-	0.414 ± 0.078	-	-
Ref.	$\lvert V_{ub}\rvert$ values $[10^{-3}]$				
CLEO E_e [564]	$4.22 \pm 0.49^{+0.29}_{-0.34}$	$3.86 \pm 0.45^{+0.25}_{-0.27}$	$4.23 \pm 0.49^{+0.22}_{-0.31}$	$3.42 \pm 0.40^{+0.17}_{-0.17}$	-
Belle M_X, q^2 [566]	$4.51 \pm 0.47^{+0.27}_{-0.29}$	$4.43 \pm 0.47^{+0.19}_{-0.21}$	$4.52 \pm 0.48^{+0.25}_{-0.28}$	$3.93 \pm 0.41^{+0.18}_{-0.17}$	$4.68 \pm 0.49^{+0.30}_{-0.30}$
Belle E_e [565]	$4.93 \pm 0.46^{+0.26}_{-0.29}$	$4.82 \pm 0.45^{+0.23}_{-0.23}$	$4.95 \pm 0.46^{+0.16}_{-0.21}$	$4.48 \pm 0.42^{+0.20}_{-0.20}$	-
BABAR E_e [560]	$4.41 \pm 0.12^{+0.27}_{-0.27}$	$3.85 \pm 0.11^{+0.08}_{-0.07}$	$3.96 \pm 0.10^{+0.17}_{-0.17}$	-	-
BABAR $E_e, s_{\rm h}^{\rm max}$ [563]	$4.71 \pm 0.32^{+0.33}_{-0.38}$	$4.35 \pm 0.29^{+0.28}_{-0.30}$	-	$3.81 \pm 0.19^{+0.19}_{-0.18}$	
Belle p_ℓ^*, (M_X, q^2) fit [554]	$4.50 \pm 0.27^{+0.20}_{-0.22}$	$4.62 \pm 0.28^{+0.13}_{-0.13}$	$4.62 \pm 0.28^{+0.09}_{-0.10}$	$4.50 \pm 0.30^{+0.20}_{-0.20}$	-
BABAR M_X [555]	$4.24 \pm 0.19^{+0.25}_{-0.25}$	$4.47 \pm 0.20^{+0.19}_{-0.24}$	$4.30 \pm 0.20^{+0.20}_{-0.21}$	$3.83 \pm 0.18^{+0.20}_{-0.19}$	-
BABAR M_X [555]	$4.03 \pm 0.22^{+0.22}_{-0.22}$	$4.22 \pm 0.23^{+0.21}_{-0.27}$	$4.10 \pm 0.23^{+0.16}_{-0.17}$	$3.75 \pm 0.21^{+0.18}_{-0.18}$	-
BABAR M_X, q^2 [555]	$4.32 \pm 0.23^{+0.26}_{-0.28}$	$4.24 \pm 0.22^{+0.18}_{-0.21}$	$4.33 \pm 0.23^{+0.24}_{-0.27}$	$3.75 \pm 0.20^{+0.17}_{-0.17}$	$4.50 \pm 0.24^{+0.29}_{-0.29}$
BABAR P_+ [555]	$4.09 \pm 0.25^{+0.25}_{-0.25}$	$4.17 \pm 0.25^{+0.28}_{-0.37}$	$4.25 \pm 0.26^{+0.26}_{-0.27}$	$3.57 \pm 0.22^{+0.19}_{-0.18}$	-
BABAR p_ℓ^*, (M_X, q^2) fit [555]	$4.33 \pm 0.24^{+0.19}_{-0.21}$	$4.45 \pm 0.24^{+0.12}_{-0.13}$	$4.44 \pm 0.24^{+0.09}_{-0.10}$	$4.33 \pm 0.24^{+0.19}_{-0.19}$	-
BABAR p_ℓ^* [555]	$4.34 \pm 0.27^{+0.20}_{-0.21}$	$4.43 \pm 0.27^{+0.13}_{-0.13}$	$4.43 \pm 0.27^{+0.09}_{-0.11}$	$4.28 \pm 0.27^{+0.19}_{-0.19}$	-
Belle M_X, q^2 [567]	-	-	-	-	$5.01 \pm 0.39^{+0.32}_{-0.32}$
Average	$4.44^{+0.13+0.21}_{-0.14-0.22}$	$3.99 \pm 0.10^{+0.09}_{-0.10}$	$4.32 \pm 0.12^{+0.12}_{-0.13}$	$3.99 \pm 0.13^{+0.18}_{-0.12}$	$4.62 \pm 0.20^{+0.29}_{-0.29}$

Importance of the model used to simulate the signal