$V_{cb}$ and LFUV from exclusive B decays

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Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement.
PDG AVERAGES

Long standing tension between exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$!

However:

Current PDG review (Oct. 2017):

$|V_{cb}| = (4.1 \pm 0.9) \times 10^{-3}$ (excl.)

$|V_{cb}| = (4.2 \pm 0.8) \times 10^{-3}$ (incl.)
NEW PHYSICS?

Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data. For a recent analysis see Jung & Straub 1801.01112
The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT) $V_{cb}$ plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[ 1 + O(\lambda^2) \right]$$

where it often dominates the theoretical uncertainty. $\frac{V_{ub}}{V_{cb}}$ constrains directly the UT

Our inability to determine precisely $V_{cb}$ hampers significantly NP searches
VIOLATION OF LFU WITH TAUS

\[ R \left( D^{(*)} \right) = \frac{\mathcal{B} (B \to D^{(*)} \tau \nu_\tau)}{\mathcal{B} (B \to D^{(*)} \ell \nu_\ell)} \]

\( \Delta \chi^2 = 1.0 \) contours

- LHCb15
- LHCb18
- Belle12
- Average
- Belle19
- Belle15

**Average of SM predictions**
- \( R(D) = 0.299 \pm 0.003 \)
- \( R(D^*) = 0.258 \pm 0.005 \)

**World Average**
- \( R(D) = 0.340 \pm 0.027 \pm 0.013 \)
- \( R(D^*) = 0.295 \pm 0.011 \pm 0.008 \)
- \( \rho = -0.38 \)
- \( P(\chi^2) = 27\% \)
INCLUSIVE $V_{cb}$

- Based on an OPE: rate and moments given by double exp in $\alpha_s, 1/m$
- B meson matrix elements of local operators parametrise non-pert physics. Perturbative Wilson coefficients at $O(\alpha_s^2, \alpha_s/m^2, 1/m^5)$
- Reliability of the method depends on control of higher orders.
- Theoretical uncertainties important: 3 loops width feasible needed for 1% error; QED corrections. Lattice can have an impact too (Hashimoto & Melis, Simula, PG)
- Experimental moments highly correlated, needs new analyses @ Belle II
- **Most recent global fit** (kin scheme) includes $m_{b,c}$ determinations, estimates of higher powers, provides a consistent picture, various cross-checks

$$|V_{cb}| = 42.00(64) \times 10^{-3}$$

Healy, Turzcyk, PG 1606.06174
EXCLUSIVE DECAYS

There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

\[
\langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = \left[ (p + k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] f_+^{B \to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0^{B \to D}(q^2)
\]

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp…
MODEL INDEPENDENT FF PARAMETRIZATION

crossing + analitycity

physical semileptonic region
\[ m_c^2 \leq q^2 \leq (m_B - m_D)^2 \]

2-point correlator cuts
\[ q^2 \geq (m_B + m_D)^2 \]

\[ \text{Im} \quad q \quad \propto \quad |f_i(q^2)|^2 \quad < \sum_{n} \text{cuts}(X_n) \]

poles at \( q^2 = m_{cB}^2 \) etc

\[ \sum_{n} X_n = c + b \quad \text{+ pert corr + condensates} \]

using quark-hadron duality + dispersion relations
UNITARITY CONSTRAINTS

\[
z = \frac{\sqrt{1 + w} - \sqrt{2}}{\sqrt{1 + w} + \sqrt{2}} \quad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}} \quad 0 < z < 0.056
\]

\[
f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z) \phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n
\]

Blaschke factors remove poles below threshold

Phase space factors

Fast converging expansion

**Truncated at order N**

\[
\sum_{n=0}^{N} (a_n^i)^2 < 1
\]

**Weak Unitarity Constraints**

Assuming saturation by single hadron channel

Where should one truncate the series? Where adding more terms becomes irrelevant
HQS breaking in FF relations

**HQET:**

\[ F_i(w) = \xi(w) \left[ 1 + c_b^i \frac{\alpha_s}{\pi} + c_b^i \epsilon_b + c_c^i \epsilon_c + \ldots \right] \quad \epsilon_{b,c} = \frac{\Lambda}{2m_{b,c}} \]

\( c_{b,c} \) can be computed using subleading IW functions from QCD sumrules

Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

**RATIOS**

\[ \frac{F_j(w)}{V_1(w)} = A_j \left[ 1 + B_j w_1 + C_j w_1^2 + D_j w_1^3 + \ldots \right] \quad w_1 = w - 1 \]

Roughly \( \epsilon_c \sim 0.25 \), \( \epsilon_c^2 \sim 0.06 \sim \epsilon_b \sim \frac{\alpha_s}{\pi} \) but coefficients??

In a few cases we can compare these ratios with recent lattice results: there are 5-13% differences, always > NLO correction. For ex.:

\[ \left. \frac{A_1(1)}{V_1(1)} \right|_{\text{LQCD}} = 0.857(15), \quad \left. \frac{A_1(1)}{V_1(1)} \right|_{\text{HQET @NLO}} = 0.966(28) \]

Looking at NLO HQET corrections, NNLO can be sizeable, naturally O(10-20)\%
STRONG UNITARITY CONSTRAINTS

Information on other channels with same quantum numbers makes the bounds tighter. HQS implies that all $B(\rightarrow D f)$ ff either vanish or are prop to the Isgur-Wise function: any ff $F_j$ can be expressed as

$$F_j(z) = \left( \frac{F_j}{F_i} \right)_{\text{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the $a_i$ space for $F_i$ in case of S, P, V, A currents.

Caprini Lellouch Neubert (CLN, 1998) exploit NLO HQET relations between form factors + QCD sum rules to reduce parameters for ffs “up to 2% uncertainty”, never included in exp analysis. The practical version of CLN is

$$h_{A1}(z) = h_{A1}(1) \left[ 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3 \right]$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

only 2+2 parameters! but uncertainty? bias?
$|V_{cb}| = 40.5(1.0) \times 10^{-3}$, $R(D) = 0.299(3)$

- Babar 2009
- Belle 2015
- MILC-FNAL
- HPQCD

$BGL N=4$

$\chi^2/dof = 19/22$

Lattice determines slopes, exp data shown at fitted $V_{cb}$

- $R(D)$ is 1.3\sigma from exp
- FLAG has very similar results
- CLN cannot fit both ff due to missing higher orders!
\[ |V_{cb}| \text{ from } B \to D^* \ell \nu \text{ new HFLAV (2019)} \]

LQCD provided only light lepton FF at zero recoil, \( w=1 \), where rate vanishes. Experimental results must therefore be extrapolated to zero-recoil

**Exp error only \( \sim 1.1\% \)**  
\[ F(1) \eta_{ew} |V_{cb}| = 35.27(38) \times 10^{-3} \]

Beware: HFLAV extrapolate with CLN (w/o error), \( \chi^2/\text{dof of}=42.3/23! \)

Two unquenched lattice calculations

\[ F(1) = 0.906(13) \quad \text{Bailey et al 1403.0635 (FNAL/MILC)} \]

\[ F(1) = 0.895(26) \quad \text{Harrison et al 1711.11013 (HPQCD)} \]

Using their average 0.904(12):

\[ |V_{cb}| = 38.76(69) \times 10^{-3} \]

\( \sim 3.4\sigma \) or \( \sim 8\% \) from inclusive determination

**Heavy quark sum rules**  
\( F(1) < 0.925 \) and estimate of inelastic contribution \( F(1) \approx 0.86 \)  
Mannel, Uraltsev, PG, 2012
2017 tagged Belle analysis (preliminary)

w and angular deconvoluted distributions (independent of parameterization). All previous analyses are CLN based.

Bands show two parametrizations both fitting data well, with 6% different $V_{cb}$.

$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$

\[ w \]

zero recoil point

Bigi, PG, Schacht 17

see also Kobach & Grinstein
EXTRAPOLATING TO ZERO RECOIL

FIG. 1. Comparison of fit results with different parametrizations.

\[ 10^3 m_{EW}^2 |V_{cb}|^2 \frac{F^2}{w} \]

\( w \)

+30 additional angular bins

[References]


Updating Strong Unitarity Bounds

Fit to new Belle’s data + total branching ratio (world average) in 1707.09509 with UPDATED strong unit. bounds (including uncertainties & LQCD inputs)

for reference CLN fit: $|V_{cb}| = 0.0392(12)$

<table>
<thead>
<tr>
<th>BGL Fit:</th>
<th>Data + lattice</th>
<th>Data + lattice + LCSR</th>
<th>Data + lattice</th>
<th>Data + lattice + LCSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>unitarity</td>
<td>weak</td>
<td>weak</td>
<td>strong</td>
<td>strong</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>28.2/33</td>
<td>32.0/36</td>
<td>29.6/33</td>
<td>33.1/36</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$</td>
<td>0.0424 (18)</td>
<td>0.0413 (14)</td>
</tr>
</tbody>
</table>

LCSR: Light Cone Sum Rule results from Faller, Khodjamirian, Klein, Mannel, 0809.0222

Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but a 3.5-5% difference persists
The BGL fit tends to prefer **unphysical solutions** for $V_4$ with a positive $w$-slope. Imposing LCSR at $w_{\text{max}}$ cures the problem, but one could equally well ask for a negative slope at $w=1$ or use HQET ratios with appropriate uncertainty. They all tend to lower $V_{cb}$ slightly.
CONSISTENCY WITH HQET

\[ R_1(w) = \frac{V_4(w)}{A_1(w)} = 1 + O(\alpha_s, 1/m) \]
\[ R_2(w) = \frac{w - r}{w - 1} \left( 1 - \frac{1 - r}{w - r} A_5(w) \right) \]

Comparison of \( R_{1,2} \) from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)

One can be consistent with HQET but still far from CLN!
Full Belle dataset, most precise study to date; provides data in a way that can be reanalysed with different assumptions. Central values obtained without systematics.

CLN and BGL\(^{(102)}\) analysis lead to very similar results, suggesting low

\[ |V_{cb}| = 38.4(0.9) \times 10^{-3} \]

We used BGL\(^{(222)}\) to fit the data, taking into account D'Agostini effect and got

\[ |V_{cb}| = 39.1(+1.5-1.3) \times 10^{-3} \]
CONSISTENCY OF 2017 AND 2018 DATASETS

very high correlations

too many bins in the angular variables!!

most general parametrization has 3+3+2 constants
A GLOBAL FIT TO 2017 & 2018 DATA

Jung, Schacht, PG 1905.08209

<p>|</p>
<table>
<thead>
<tr>
<th>BGL(^{(222)})</th>
<th>Data + lattice (weak)</th>
<th>Data + lattice (strong)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\chi^2/\text{dof})</td>
<td>80.1/72</td>
<td>80.1/72</td>
</tr>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>10^3)</td>
</tr>
<tr>
<td>(a_0)</td>
<td>0.01221(16)</td>
<td>0.01221(16)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>0.006(32)</td>
<td>0.006(20)</td>
</tr>
<tr>
<td>(a_2)</td>
<td>(-0.2(12))</td>
<td>(-0.2(7))</td>
</tr>
<tr>
<td>(a_{1}^{F})</td>
<td>0.0042(22)</td>
<td>0.0042(19)</td>
</tr>
<tr>
<td>(a_{2}^{F})</td>
<td>(-0.069(14))</td>
<td>(-0.068(13))</td>
</tr>
<tr>
<td>(a_{0}^{g})</td>
<td>0.024(21)</td>
<td>0.024(12)</td>
</tr>
<tr>
<td>(a_{1}^{g})</td>
<td>0.05(39)</td>
<td>0.05(21)</td>
</tr>
<tr>
<td>(a_{2}^{g})</td>
<td>1.0(20)</td>
<td>0.9(18)</td>
</tr>
</tbody>
</table>

- No parametrization dependence (CLN and BGL give ~same central value)
- About 1\(\sigma\) higher than HFLAV, larger uncertainty on firmer ground, p-value ~24% 1.9\(\sigma\) from inclusive, consistent with B to Dlv
- We truncate the BGL series when additional terms do not change the fit (no overfitting!). Fit stable. Strong constraints irrelevant.
Comparison of $R_{1,2}$ from BGL fit to 2017+2018 data vs NLO HQET+QCDSR predictions (with parametric + 15% th uncertainty)

Very good agreement in both cases.

Narrower bands correspond to adding strong unitarity and LCSRs
Decays with tau require pseudoscalar FF unconstrained from fit, no lattice calculation yet. We use kinematic constraint at $q^2=0$ and HQET with conservative uncertainty.

### Endpoint constraint

$$P_1(w_{\text{max}}) = A_5(w_{\text{max}})$$

### Results

- $R(D^*) = 0.254^{+0.007}_{-0.006}$, 2.8$\sigma$ from exp
- $P_\tau = -0.476^{+0.037}_{-0.034}$, 1.4$\sigma$ from exp
- $F_L^{D^*} = 0.476^{+0.015}_{-0.014}$

**Normalized to $V_i$**

- NLO HQET+QCDSR ±15%

**Normalized to $A_1$**

- Endpoint constraint

**Normalized to $V_i$**

- NLO HQET+QCDSR ±15%

**Normalized to $A_1$**

- Endpoint constraint
COMPARISON WITH OTHER APPROACHES

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$R(D)$</th>
<th>Exp. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[21]</td>
<td>0.299(3)</td>
<td>2.4σ</td>
</tr>
<tr>
<td>[32]</td>
<td>0.299(3)</td>
<td>2.4σ</td>
</tr>
<tr>
<td>[34]</td>
<td>0.302(3)</td>
<td>2.3σ</td>
</tr>
<tr>
<td>[37]</td>
<td>0.297(3)</td>
<td>2.4σ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$R(D^*)$</th>
<th>Exp. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[32]</td>
<td>0.257(3)</td>
<td>3.3σ</td>
</tr>
<tr>
<td>[13,36]</td>
<td>0.254 $^{7}_{6}$</td>
<td>3.2σ</td>
</tr>
<tr>
<td>[34]</td>
<td>0.257(5)</td>
<td>3.1σ</td>
</tr>
<tr>
<td>[37]</td>
<td>0.250(3)</td>
<td>3.7σ</td>
</tr>
</tbody>
</table>

Table 2.3 Recent theory predictions for $R(D^{(*)})$. The deviations are calculated from the HFLAV summer 2018 updates $R(D)^{\text{exp}} = 0.407(39)(24)$ [18,38–40] and $R(D^*)^{\text{exp}} = 0.306(13)(7)$ [18,38–46], respectively. For older predictions see Refs. [47–49]. Table adapted and extended from Ref. [50].

S.Schacht, MITP report 2002.xxxxx
On the other hand, there has been no large difference in $M$ between $\pi^+$ and $\pi^0$, and
$\langle \pi^0 \rangle \approx 1.25$.

Also, $h_A(w)$ is consistent with the results of Ref. [2].
JLQCD results for $R_{1,2}$ are consistent with our latest fit and with NLO HQET+QCDSR, but confirm large higher orders in $h_{A1}/f_+$.
BLINDED FNAL-MILC RESULTS

Unquenched calculation of all $B \to D^*$ form factors at small recoil

Blinding is introduced as a global normalisation factor close to 1, which multiplies all form factors in the same way.

Discrepancies with JLQCD, but further changes should expected in the final results

<table>
<thead>
<tr>
<th>Source</th>
<th>$h_V$ (%)</th>
<th>$h_{A_1}$ (%)</th>
<th>$h_{A_2}$ (%)</th>
<th>$h_{A_3}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>1.1</td>
<td>0.4</td>
<td>4.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Isospin effects</td>
<td>0.0</td>
<td>0.0</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$\chi^2$/PT/cont. extrapolation</td>
<td>1.9</td>
<td>0.7</td>
<td>6.3</td>
<td>2.9</td>
</tr>
<tr>
<td>Matching</td>
<td>1.5</td>
<td>0.4</td>
<td>0.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Heavy quark discretization*</td>
<td>2.5</td>
<td>1.2</td>
<td>9.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

*Preliminary estimate, analysis in progress

A.Vaquero, BNL 9/2019
THE IMPORTANCE OF THE SLOPE

Blinding affects only marginally the slope of the ff $F$, which is key to $V_{cb}$.

Plot suggests large and well determined slope, $dF/dw|_{w=1}$.

If it were -1.40(7) the fit could still accommodate a high $V_{cb}$.

Here we use new improved LCSR results by Gubernari, Kokulu, van Dyk, 1811.00983 that improve upon 0809.0222.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>$10^3 V_{cb}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>40.8(0.8)</td>
<td>84.5/73</td>
</tr>
<tr>
<td>slope+LCSR</td>
<td>40.8(0.8)</td>
<td>88.0/76</td>
</tr>
</tbody>
</table>
Reanalysis of tagged $B^0$ and $B^+$ data, unbinned 4 dimensional fit
with simplified BGL and CLN
About 6000 events
No data provided yet

No clear BGL$^{111}$/CLN difference

$V_{cb} = 0.0384(9)$
Can HQET relations be further exploited, for instance including dominant NNLO terms $O(1/m_c^2)$?

$$F_i(w) = \xi(w) \left[ 1 + c^i_{\alpha_s}(w) \frac{\alpha_s}{\pi} + c^i_b(w) \epsilon_b + c^i_c(w) \epsilon_c + d^i \epsilon_c^2 \right] \quad \epsilon_{b,c} = \frac{\Lambda}{2m_{b,c}}, \quad \alpha_s \sim \epsilon_b \sim \epsilon_c^2$$

6 subsubleading IW functions, expanded around $w=1$. To fix them and for NP analyses we need additional info from LQCD or LCSR.

Bordone, Jung, Van Dyk, building on 1703.05330 (Bernlochner et al) and 1707.09977 (Jaiswal et al) use available LQCD and LCSR calculations and Belle $B\to D^{(*)}\ell\nu$ data using BGL with strong unitarity bounds.

$$V_{cb}=40.3(0.8)10^{-3}$$
$$R(D)=0.297(3)$$
$$R(D^*)=0.250(3)$$
**B_s DECAYS BEYOND THE SU(3) LIMIT**

B_s decays can be studied at LHCb. There are a few lattice results, most recently by HPQCD 1906.00701. LCSRs are available for all FFs.

There is no evidence for SU(3)_F breaking at present.

Assuming only the subsubleading IW is SU(3)_F symmetric, the strong unitarity bounds become slightly more constraining.

\[ V_{cb} = 40.0(0.7) \times 10^{-3} \]
\[ R(D) = 0.298(3) \]
\[ R(D^*) = 0.250(3) \]
**V_{cb} FROM B_s DECAYS**

Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{(*)-} \mu^+\nu_\mu$ decays

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \to D_s^- \mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^- \mu^+\nu_\mu)},$$

$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \to D_s^{*-} \mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^{*-} \mu^+\nu_\mu)}$$

$|V_{cb}|_{\text{CLN}} = (41.4 \pm 0.6 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3},$

$|V_{cb}|_{\text{BGL}} = (42.3 \pm 0.8 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3},$

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL$_{222}$
OTHER DECAY MODES

LHCb can access $|V_{ub}/V_{cb}|$ from $\Lambda_b$ and $B_s$ decays.

$B_s \to K\ell\nu$

- HPQCD, RBC-UKQCD, ALPHA
  - Bouchard et al. PRD90(2014)054506
  - Flynn et al. PRD91(2015)074510
  - Bahr et al. PLB757(2016)473
- New 2019: Fermilab/MILC

O. Witzel, Beauty 2019
CONCLUSIONS

- Despite conflicting results, revisiting the exclusive $b \to c$ decays was important: uncertainties were underestimated and results possibly biased. The practical CLN must be abandoned.

- Several lattice coll. are computing all necessary FFs, in parallel with Belle-II improved measurements (also for the inclusive moments). LHCb have shown they can contribute too.

- Inclusive/Exclusive tension remains, but weaker. Hopefully, it will disappear. LQCD will decide the fate of the $V_{cb}$ puzzle.

- Semitauonic anomaly persists and the SM predictions are robust.

- Experiments should provide results in a model independent way: unfolded spectra or alternative but equivalent information. Theoretical prejudice is transient by definition and should never be hardwired into precision measurements.

- Something is moving also for $V_{ub}$. New promising possibilities at Belle-II.
BACKUP MATERIAL
it is numerically very small, as the main e
con the form factors employed to predict the yields, but
effect, see Table I. There is a residual ambiguity depending
deed we observe a significant shift in
atic uncertainty as a fraction of the predicted yield. In-
which however can be avoided by expressing the system-
the systematic uncertainty as a fraction of the yield in
relative uncertainties. It is well-known that computing
the systematic uncertainties, which Ref. [24] provides as
slightly higher than in the 2017 analysis.

exceeding 0.94 and that the correlations are generally
exceeding 0.94 and that the correlations are generally
increase the central value of
considerable impact on the result of the fits, and it gen-
the inclusion of systematic errors and correlations has a
form constrained fits subject to Eq. (5).

separately. Notice that, unless specified, we always per-
plies to fits performed on the electron and muon data
with
The uncertainty in an uncontrolled way.
This is di
erent from adopting gaussian priors for each

An important point concerns the implementation of
the D’Agostini effect into account (see

\begin{table}
\begin{center}
\begin{tabular}{cccc}
\hline
\hline
data & fit & par & $\chi^2$/dof & $|V_{cb}|10^3$
\hline
2018 & stat & BGL$^{(102)}$ & 53.0/35 & 38.8 ± 0.6
2018 & stat & CLN & 56.6/36 & 39.2 ± 0.6
2018 & naive & BGL$^{(102)}$ & 32.6/35 & 39.7 ± 0.9
2018 & naive & CLN & 32.4/36 & 39.7 ± 0.9
2018 & BGL$^{(102)}$ & 32.5/35 & 40.3 ± 0.9
2018 & CLN & 32.4/36 & 40.3 ± 0.9
2018 & stat & BGL$^{(222)}$ & 47.7/32 & 37.6$^{+1.0}_{-0.9}$
2018 & naive & BGL$^{(222)}$ & 31.2/32 & 38.6$^{+1.5}_{-1.4}$
2018 & BGL$^{(222)}$ & 31.2/32 & 39.1$^{+1.5}_{-1.3}$
2017/18, slope & BGL$^{(222)}$ & 84.5/73 & 40.8 ± 0.8
2017/18, LCSR & BGL$^{(222)}$ & 80.5/75 & 39.5 ± 1.0
2017/18, LCSR, slope & BGL$^{(222)}$ & 88.0/76 & 40.8 ± 0.8
\hline
\end{tabular}
\end{center}
\end{table}

TABLE I. Results of various fits to the $B \to D^*\ell\nu$ data. The $|V_{cb}|$ error always includes the lattice uncertainty. In the second column \textit{stat} stands for ”only statistical errors”, \textit{naive} stands for ”systematic errors as fractions of the yield”, in all other cases we take the D’Agostini effect into account (see text). All fits are with weak unitarity constraints.
In this paper we have studied the impact of a new conclusion about 50\% larger. Including in the fit the previous results obtained with weak unitarity constraints and no quoted lattice QCD and experimental results. We show here a table summarizing the results obtained in Refs. [3, 6, 13] and this work, based on the LCSR input only.

### FIG. 4. Summary of $V_{cb}$

<table>
<thead>
<tr>
<th>Decay</th>
<th>Data Source</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow X_c$</td>
<td>BarBar/Belle'04–'10, [3]</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D$</td>
<td>BaBar'09+Belle'16, [4–6]</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D^*$</td>
<td>Belle'17, [2,13,18]</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D^*$</td>
<td>Belle'18, [2,18] + this work</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D^*$</td>
<td>Belle'17'18, [2,18] + this work</td>
<td></td>
</tr>
</tbody>
</table>

We also show that higher values of $f_{B}$ are indeed compatible with the available data. Indeed, preliminary results of lattice calculations suggest a slope of the relative freedom around 1.9% higher than reported there, with an uncertainty of 2.1%. However, it is lattice QCD that will decide the eventual fate of the puzzle.
Meson masses from ETMC

\[ M_{HQ} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \ldots \]

- On the lattice one can compute mesons for arbitrary quark masses
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, \(a=0.62-0.89\) fm, \(m_\pi=210-450\) MeV, heavy masses from \(m_c\) to \(3m_c\), ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1 GeV, good sensitivity up to 1/m^3 corrections
- Results consistent with s.l. fits

See also Kronfeld & Simone hep-ph/0006345, 1802.04248

Melis, Simula, PG 1704.06105
### INCLUSIVE $V_{ub}$

<table>
<thead>
<tr>
<th>scheme</th>
<th>Input parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref.</td>
<td>BLNP</td>
</tr>
<tr>
<td>$m_b$ (GeV)</td>
<td>$4.582 \pm 0.026$</td>
</tr>
<tr>
<td>$\mu^2$ (GeV$^2$)</td>
<td>$0.145^{+0.091}_{-0.097}$</td>
</tr>
</tbody>
</table>

| Ref. | $|V_{ub}|$ values [$10^{-3}$] |
|------|-----------------------------|
| CLEO $E_e$ [564] | $4.22 \pm 0.49^{+0.29}_{-0.34}$ | $3.86 \pm 0.45^{+0.25}_{-0.27}$ | $4.23 \pm 0.49^{+0.22}_{-0.31}$ | $3.42 \pm 0.40^{+0.17}_{-0.17}$ | - |
| Belle $M_X, q^2$ [566] | $4.51 \pm 0.47^{+0.27}_{-0.29}$ | $4.43 \pm 0.47^{+0.19}_{-0.21}$ | $4.52 \pm 0.48^{+0.25}_{-0.28}$ | $3.93 \pm 0.41^{+0.18}_{-0.17}$ | $4.68 \pm 0.49^{+0.30}_{-0.30}$ |
| Belle $E_e$ [565] | $4.93 \pm 0.46^{+0.26}_{-0.29}$ | $4.82 \pm 0.45^{+0.23}_{-0.21}$ | $4.95 \pm 0.46^{+0.16}_{-0.21}$ | $4.48 \pm 0.42^{+0.20}_{-0.20}$ | - |
| **BABAR $E_e$ [560]** | $4.41 \pm 0.12^{+0.27}_{-0.27}$ | $3.85 \pm 0.11^{+0.08}_{-0.07}$ | $3.96 \pm 0.10^{+0.17}_{-0.17}$ | - | - |
| **BABAR $E_e, s_h^{max}$ [563]** | $4.71 \pm 0.32^{+0.33}_{-0.38}$ | $4.35 \pm 0.29^{+0.28}_{-0.30}$ | - | $3.81 \pm 0.19^{+0.19}_{-0.18}$ | - |
| Belle $p_\ell, (M_X, q^2)$ fit [554] | $4.50 \pm 0.27^{+0.20}_{-0.22}$ | $4.62 \pm 0.28^{+0.13}_{-0.13}$ | $4.62 \pm 0.28^{+0.09}_{-0.10}$ | $4.50 \pm 0.30^{+0.20}_{-0.20}$ | - |
| **BABAR $M_X$ [555]** | $4.24 \pm 0.19^{+0.25}_{-0.25}$ | $4.47 \pm 0.20^{+0.19}_{-0.24}$ | $4.30 \pm 0.20^{+0.20}_{-0.21}$ | $3.83 \pm 0.18^{+0.19}_{-0.19}$ | - |
| **BABAR $M_X$ [555]** | $4.03 \pm 0.22^{+0.22}_{-0.22}$ | $4.22 \pm 0.23^{+0.21}_{-0.27}$ | $4.10 \pm 0.23^{+0.16}_{-0.17}$ | $3.75 \pm 0.21^{+0.18}_{-0.18}$ | - |
| **BABAR $M_X, q^2$ [555]** | $4.32 \pm 0.23^{+0.26}_{-0.28}$ | $4.24 \pm 0.22^{+0.18}_{-0.21}$ | $4.33 \pm 0.23^{+0.24}_{-0.27}$ | $3.75 \pm 0.20^{+0.17}_{-0.17}$ | $4.50 \pm 0.24^{+0.29}_{-0.29}$ |
| **BABAR $P_+ [555]** | $4.09 \pm 0.25^{+0.25}_{-0.25}$ | $4.17 \pm 0.25^{+0.28}_{-0.27}$ | $4.25 \pm 0.26^{+0.26}_{-0.27}$ | $3.57 \pm 0.22^{+0.19}_{-0.18}$ | - |
| **BABAR $p_\ell^*, (M_X, q^2)$ fit [555]** | $4.33 \pm 0.24^{+0.19}_{-0.21}$ | $4.45 \pm 0.24^{+0.12}_{-0.13}$ | $4.44 \pm 0.24^{+0.09}_{-0.10}$ | $4.33 \pm 0.24^{+0.19}_{-0.19}$ | - |
| **BABAR $p_\ell^* [555]** | $4.34 \pm 0.27^{+0.20}_{-0.21}$ | $4.43 \pm 0.27^{+0.13}_{-0.13}$ | $4.43 \pm 0.27^{+0.09}_{-0.11}$ | $4.28 \pm 0.27^{+0.19}_{-0.19}$ | - |
| Belle $M_X, q^2$ [567] | - | - | - | - | $5.01 \pm 0.39^{+0.32}_{-0.32}$ |
| Average | $4.44^{+0.13+0.21}_{-0.14-0.22}$ | $3.99 \pm 0.10^{+0.09}_{-0.10}$ | $4.32 \pm 0.12^{+0.12}_{-0.13}$ | $3.99 \pm 0.13^{+0.18}_{-0.12}$ | $4.62 \pm 0.20^{+0.29}_{-0.29}$ |

Importance of the model used to simulate the signal