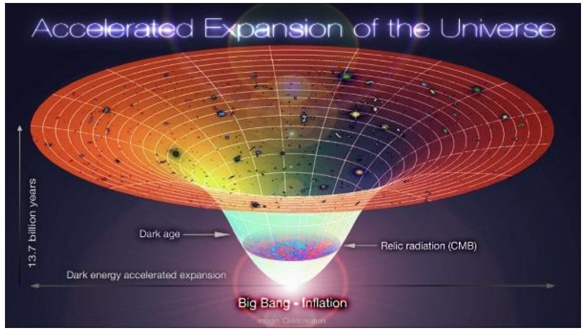

# HYDRODYNAMISATION AND ENTANGLEMENT ENTROPY IN EXPANDING SPACETIMES

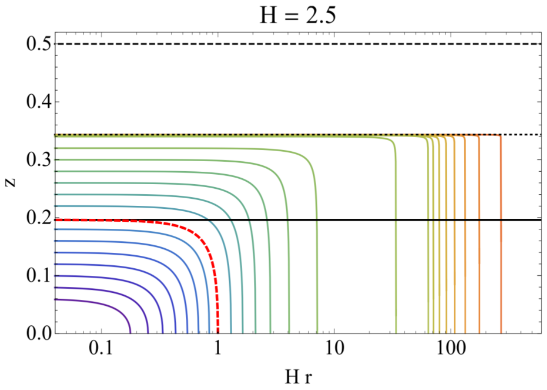
## HOLOGRAPHIC QFTS ON CURVED BACKGROUNDS





With Jorge Casalderrey-Solana, Christian Ecker and David Mateos


Reference: 2011.08194 (JHEP), to appear





**Wilke van der Schee**


Gauge/Gravity Duality 2021
CERN (virtual), 26 July 2021

# OUTLINE

**Set-up, hydrodynamics and empty de Sitter space-time**

- Non-conformal model
- Expansion driven decay towards empty de Sitter

**Entanglement in de Sitter space-time**

- Event and apparent horizons
- From boundary cosmological horizon to entanglement horizon/shadow

**Recent results / outlook on backreacted de Sitter (semi-classical)**

# NON-CONFORMAL MODEL ON DE SITTER$_4$

**De Sitter is conformally flat: almost trivial for CFT**

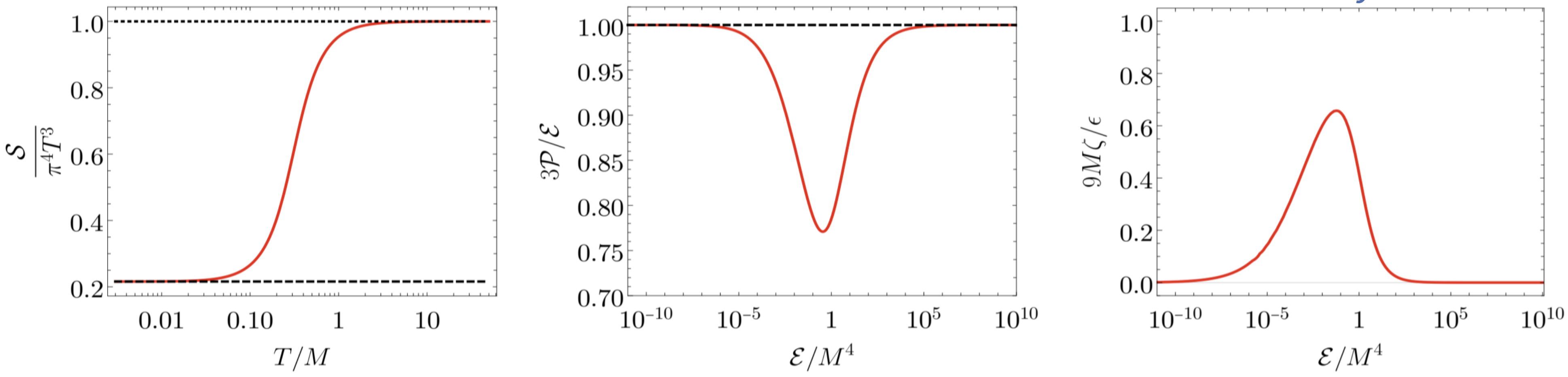
- Break scale invariance by V(Φ) with source M=1:

$$S = \frac{2}{8\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{4}R[g] - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$L^2V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

- Leads to non-trivial EOS and bulk viscosity (no shear considered):

*Entropy density* *Trace anomaly* *Bulk viscosity*



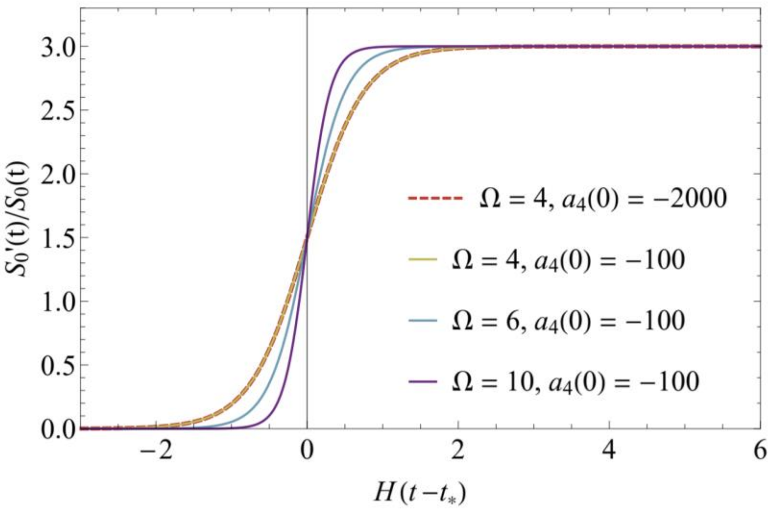
# HOW WE SET UP A STATE

**Non-trivial boundary metric:** $ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \, . \quad S_0(t) = e^{Ht}$

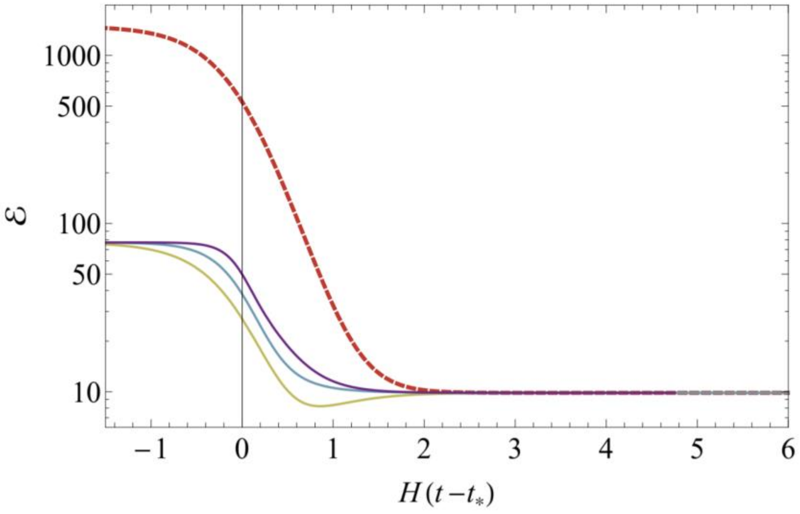
**Start with thermal (high-temperature) state in flat space**

- Quench system by suitable fast tanh to constant Hubble parameter
- Energy density decreases towards final `vacuum energy' (VE)
- Final (Bunch-Davis)-VE is ambiguous → chose scheme with zero VE

*Hubble parameter*




*Energy density*

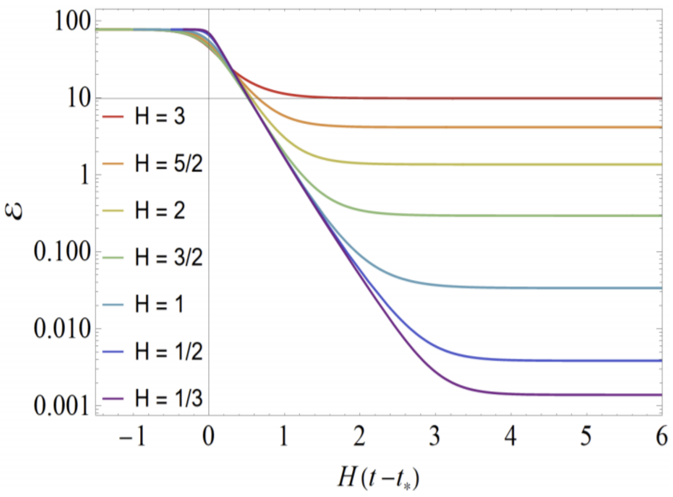


# TIME EVOLUTION OF THE PROTOCOL

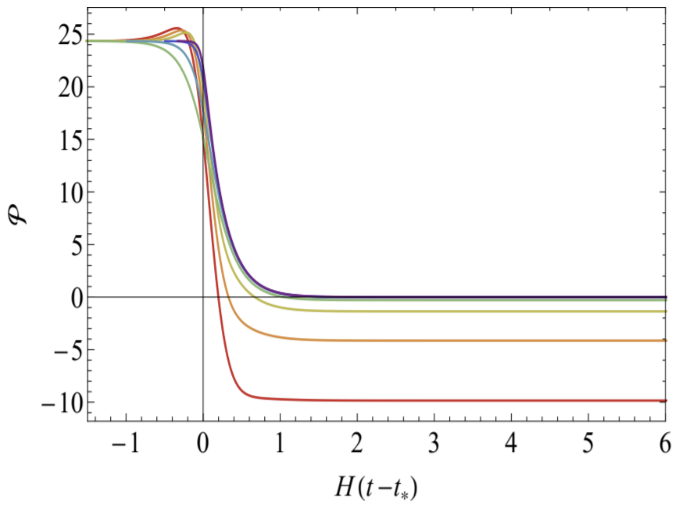
**Evolution of stress tensor for different Hubble constants**

- Energy density decreases towards VE (can be renormalised to zero)
- Pressure decreases, changes sign and becomes –VE
- Enthalpy is scheme independent, decays due to expansion

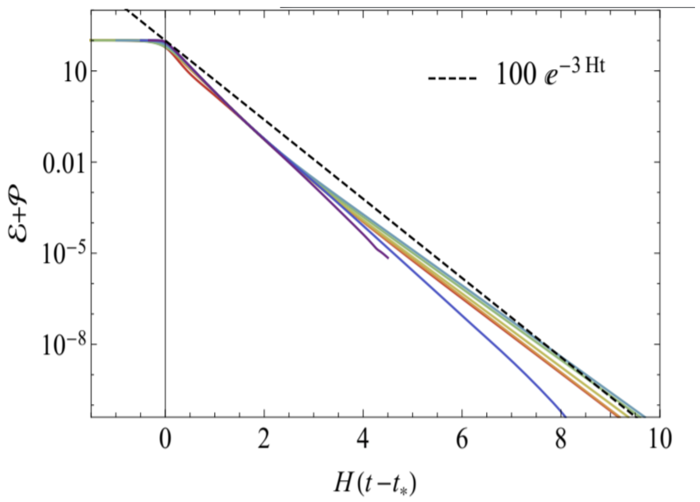
*Energy density*




*Pressure*




*Enthalpy*



# THE APPROACH TOWARDS HYDRODYNAMICS

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$$

**Comparing with the hydrodynamic constituent relations:**

$$T_{\perp}^{\mu\nu} = P(\varepsilon)\Delta^{\mu\nu} - \eta(\varepsilon)\sigma^{\mu\nu} - \zeta(\varepsilon)\Delta^{\mu\nu}(\nabla\cdot u)\,,$$
$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

**Symmetric set-up: only non-trivial part is bulk viscosity:**

$$\Delta\mathcal{P}^{\mathrm{hydro}}(t) \equiv \Delta p_{\mathrm{eq}}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2)\,,$$

**A subtlety: EOS and viscosity computed in flat space;**
**what is the energy density in de Sitter space?**

**We decided to fix renormalisation freedom so that late time solution has zero energy density**

(in any case: ambiguity is order H$^2$)
(also, note that scheme depends on H for our choice)

# THE APPROACH TOWARDS HYDRODYNAMICS

$$\Pi = -\zeta\left(\nabla\cdot u\right) + \zeta\tau_{\Pi} D\left(\nabla\cdot u\right) + \xi_1\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\left(\nabla\cdot u\right)^2 + \xi_3\Omega^{\mu\nu}\Omega_{\mu\nu} + \xi_4\nabla^{\perp}_{\mu}\ln s\nabla^{\mu}_{\perp}\ln s + \xi_5 R + \xi_6 u^{\alpha}u^{\beta}R_{\alpha\beta}\,.$$

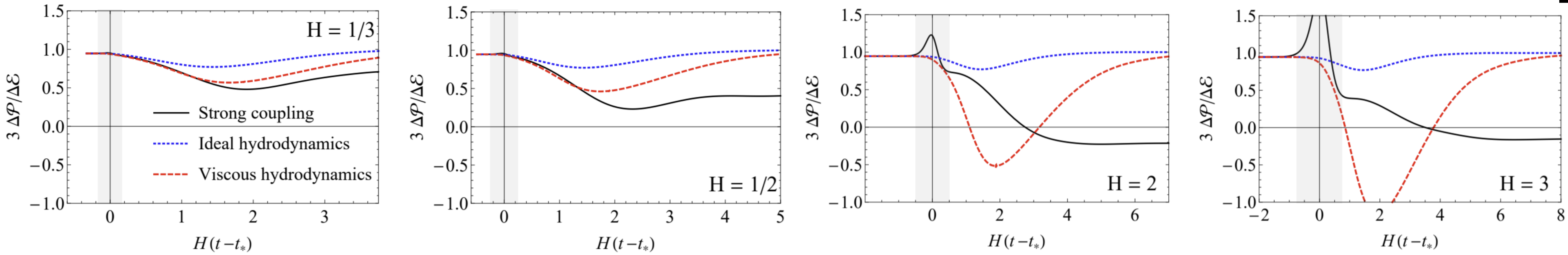
## Non-trivial hydrodynamic prediction

$$\Delta\mathcal{P}^{\rm hydro}(t) \equiv \Delta p_{\rm eq}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2)\,,$$

- Conjecture: ambiguities cancelled by higher order transport coefficient $\xi_5$

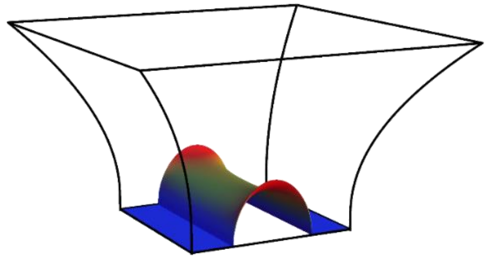
## Results

- Viscous hydro works for small H (gradients). Negative `EOS' for large H.




Paul Romatschke, Relativistic Viscous Fluid Dynamics and Non-Equilibrium Entropy (2009)

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2$$

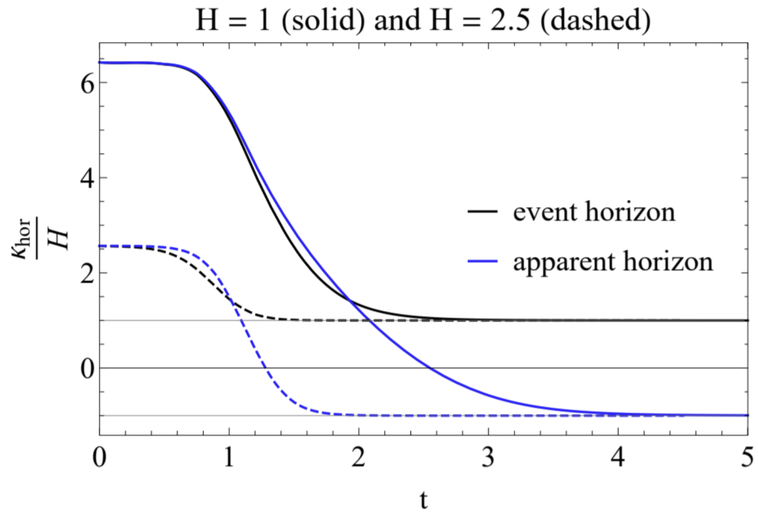


# BLACK HOLE THERMODYNAMICS

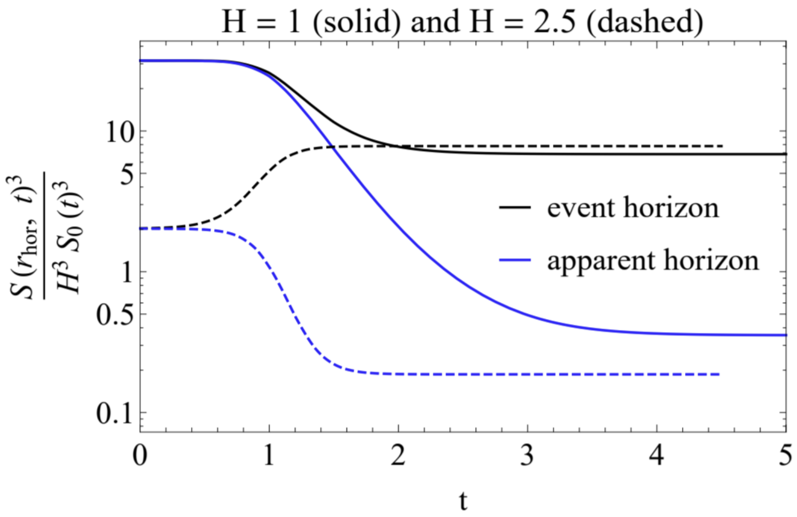
**Keep track of bulk event and apparent horizons (EF coordinates)**

- Dynamical setting: horizons not coincide at late times:
- Surface gravities can be shown analytically: $\kappa_{\mathrm{EH}} = -\kappa_{\mathrm{AH}} = H$
  EH confirms Hawking's temperature in de Sitter
- Area density apparent horizon vanishes for conformal theory

*Surface gravity*



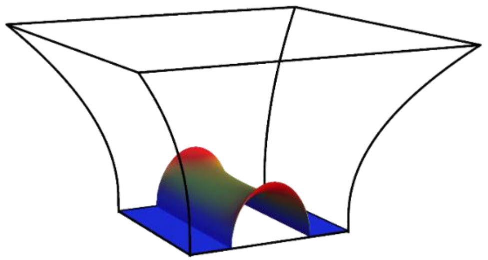

*Area densities*




Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
Alex Buchel, Entanglement entropy of N = 2* de Sitter vacuum (2019)

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2$$

# BLACK HOLE THERMODYNAMICS

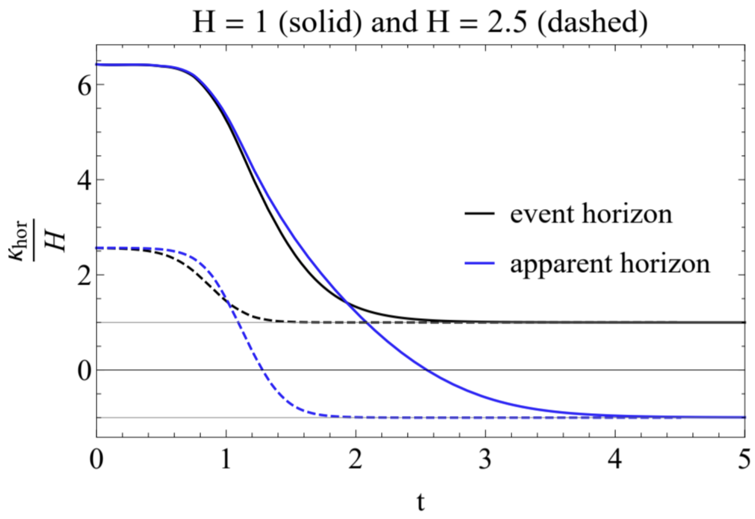


**Several interpretational issues**

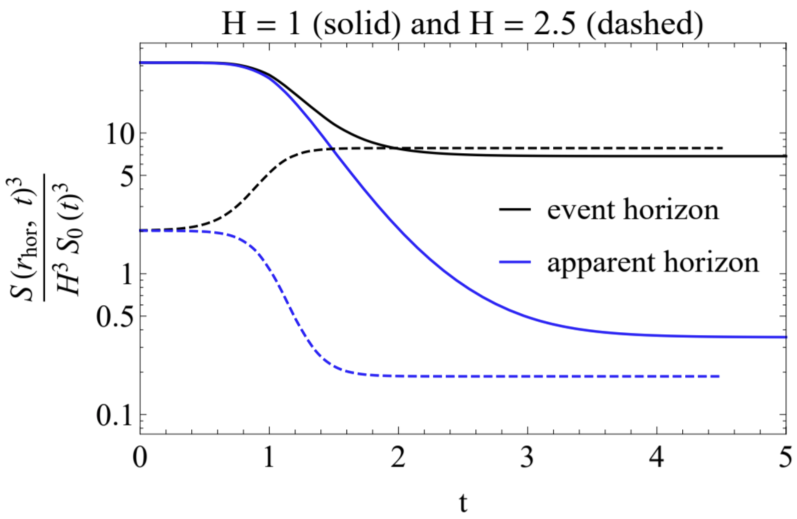
- Expanding space: mapping boundary to bulk horizon not clear
- Apparent horizon: time slicing dependent
- In general: no volume law entropy density expected

**Resolution** → entanglement entropy is well defined

*Surface gravity*




*Area densities*




Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
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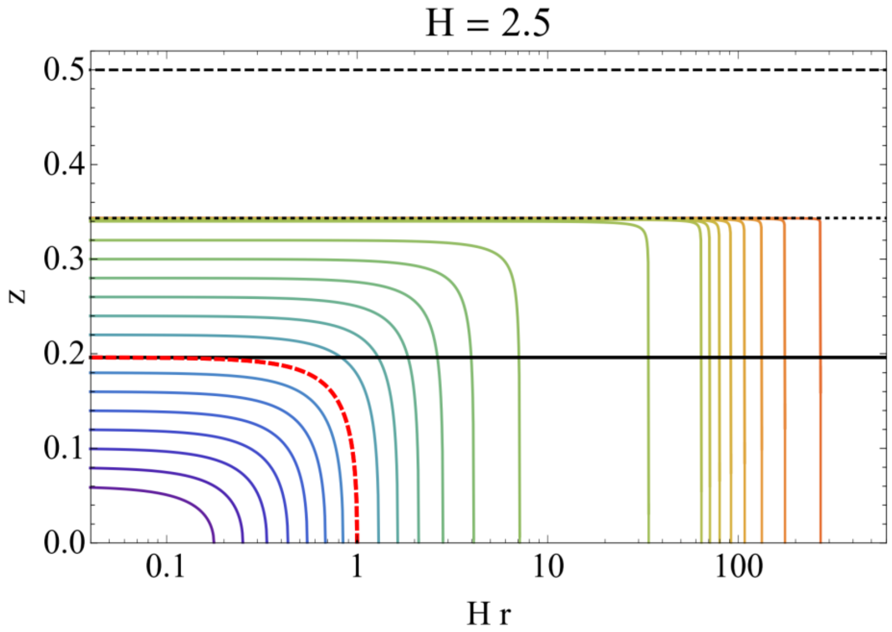
# ENTANGLEMENT IN DE SITTER

Extremal surfaces dual to spherical entangling regions:

- Large entangling regions probe beyond event horizon
- A new `entanglement horizon’ forms, between AH and EH, with zero surface gravity

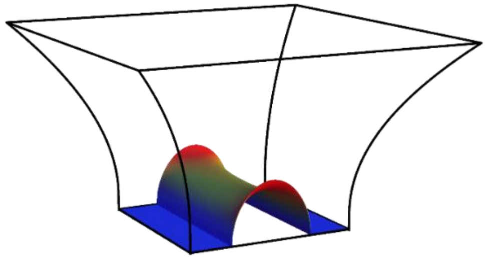



Apparent horizon: $T = -\frac{H}{2\pi}$

Entanglement horizon: $T = 0$

Event horizon: $T = \frac{H}{2\pi}$

Boundary



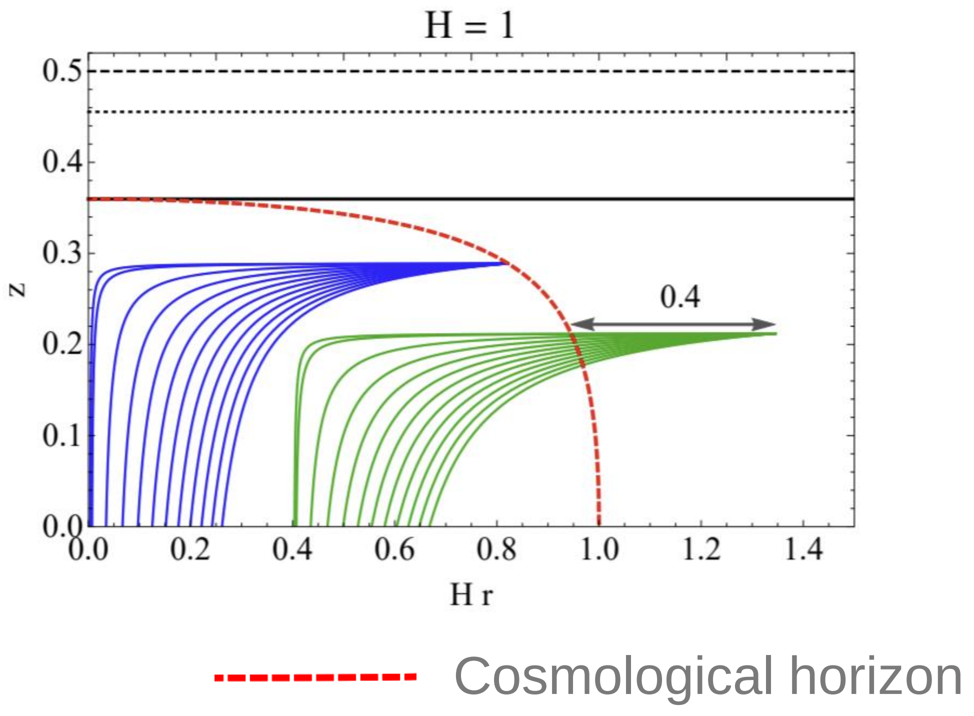
Cosmological horizon

Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

# ENTANGLEMENT IN DE SITTER

Extremal surface dual to cosmological horizon:

- Separates points in the bulk from which light can reach the (boundary) origin
- *Boundary cosmological horizon → full bulk cosmological horizon*



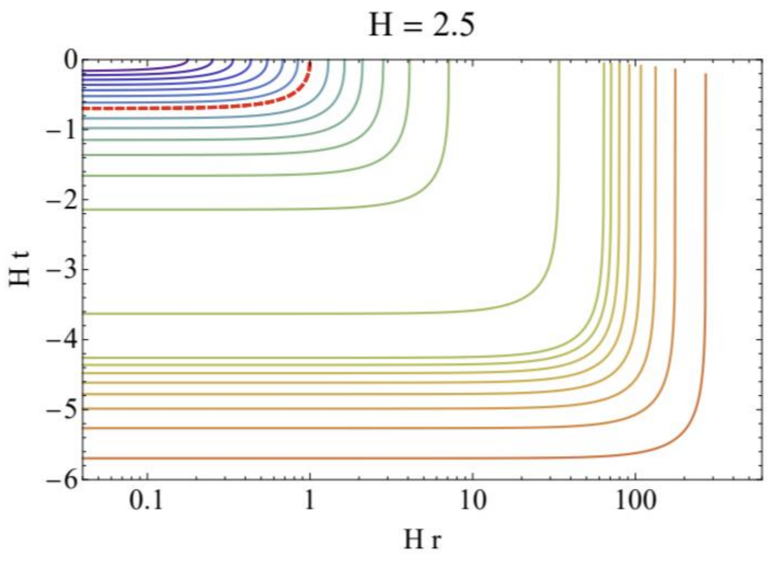


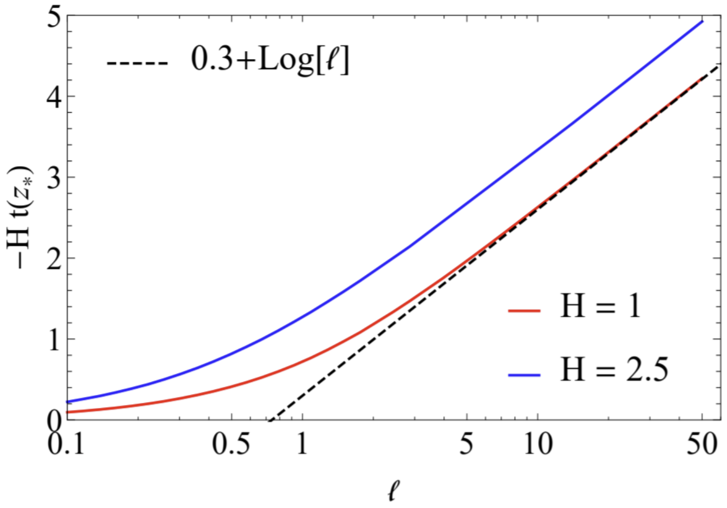

Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

# ENTANGLEMENT IN DE SITTER

**Extremal surfaces go backward in time**

- Time at the deepest point grows exactly as log(l) for large l
- *Implies that `entanglement horizon' contribution has a constant instead of volume law contribution*
- Standard `area law' divergence still applies




Cosmological horizon

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

# OUTLOOK: BOUNDARY GRAVITY

**Future work: study dynamics including semi-classical gravity:**

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = 8\pi G_{N,4} \langle T_{\mu\nu} \rangle$$

- Stress-tensor includes possible cosmological constant
- NB: renormalisation counterterms are now physical
- We treat the boundary Newton constant as a (small) parameter

**Dynamics of scale factor $S_0(t)$ is now consequence of Friedmann equations**

**How to initialise the dynamics**

- Start with thermal Minkowski profile and small boundary $G_{N,4}$

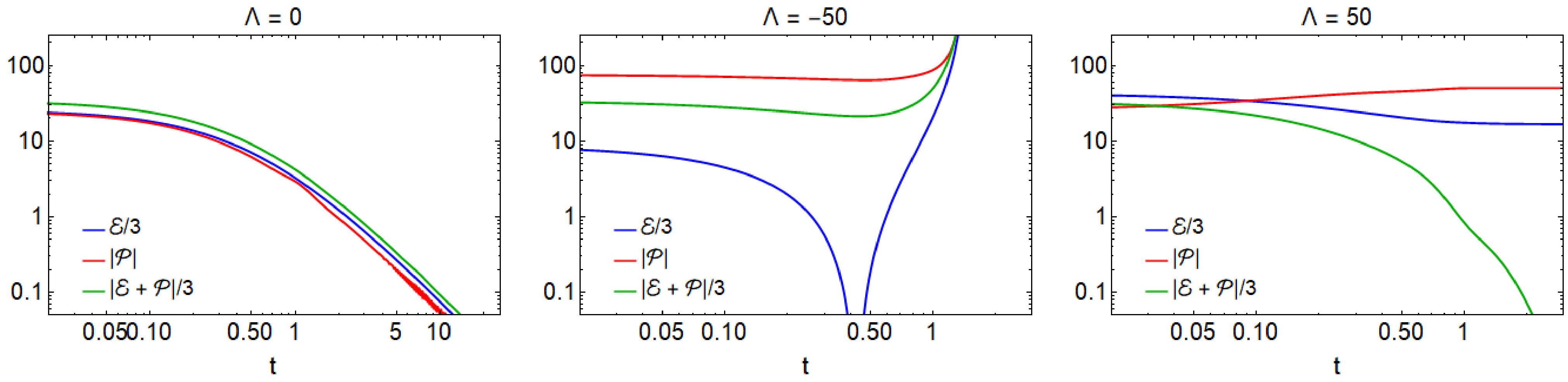


Jewel Ghosh, Elias Kiritsis, Francesco Nitti and Lukas Witkowski, Back-reaction in de Sitter QFTs: holography, gravitational DBI action and f(R) gravity (2020)
Paul Chesler and Abraham Loeb, Holographic duality and mode stability of de Sitter space in semiclassical gravity (2020)

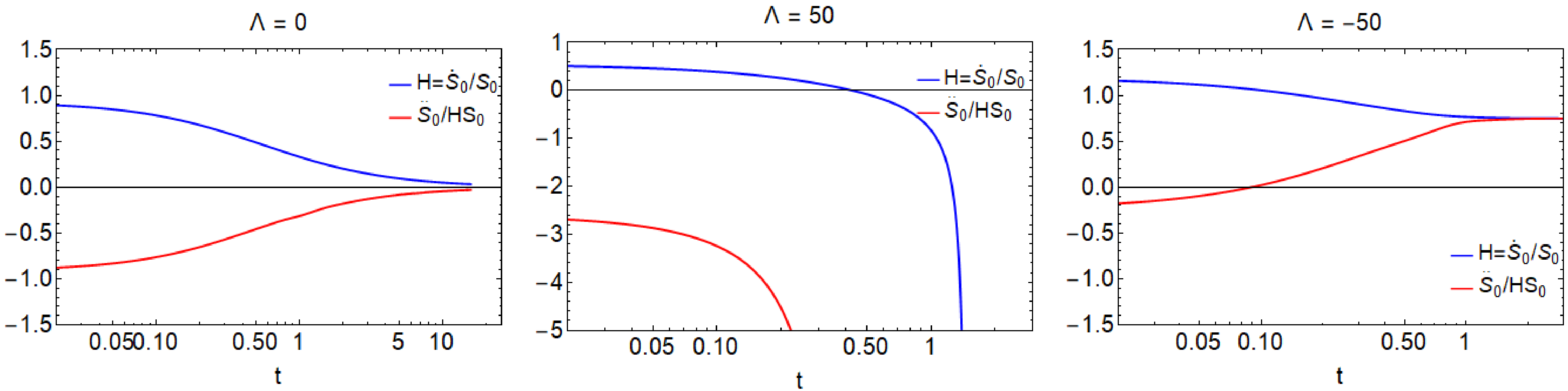
# BOUNDARY GRAVITY

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

**Stress-energy tensor for zero, positive and negative $\Lambda$**



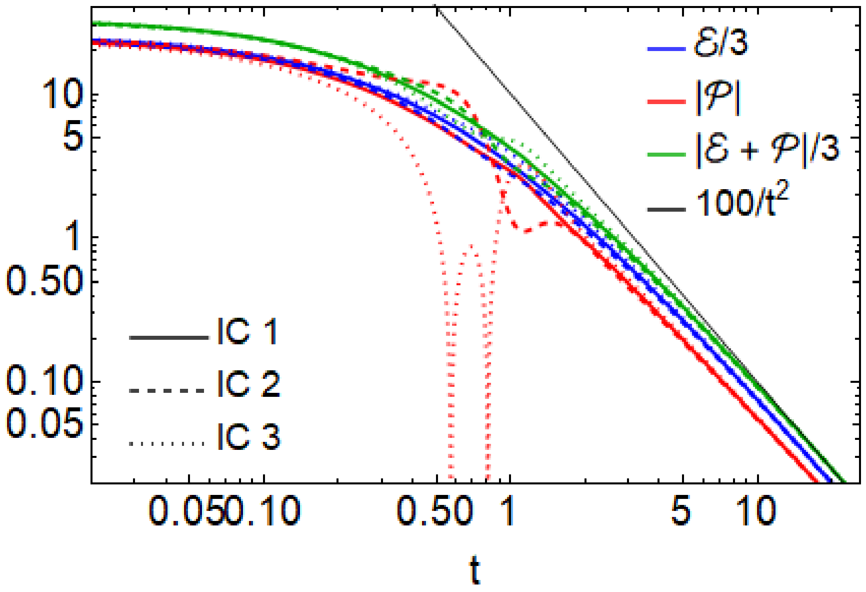

**Corresponding Hubble rates: approaching zero, negative infinity and a constant**

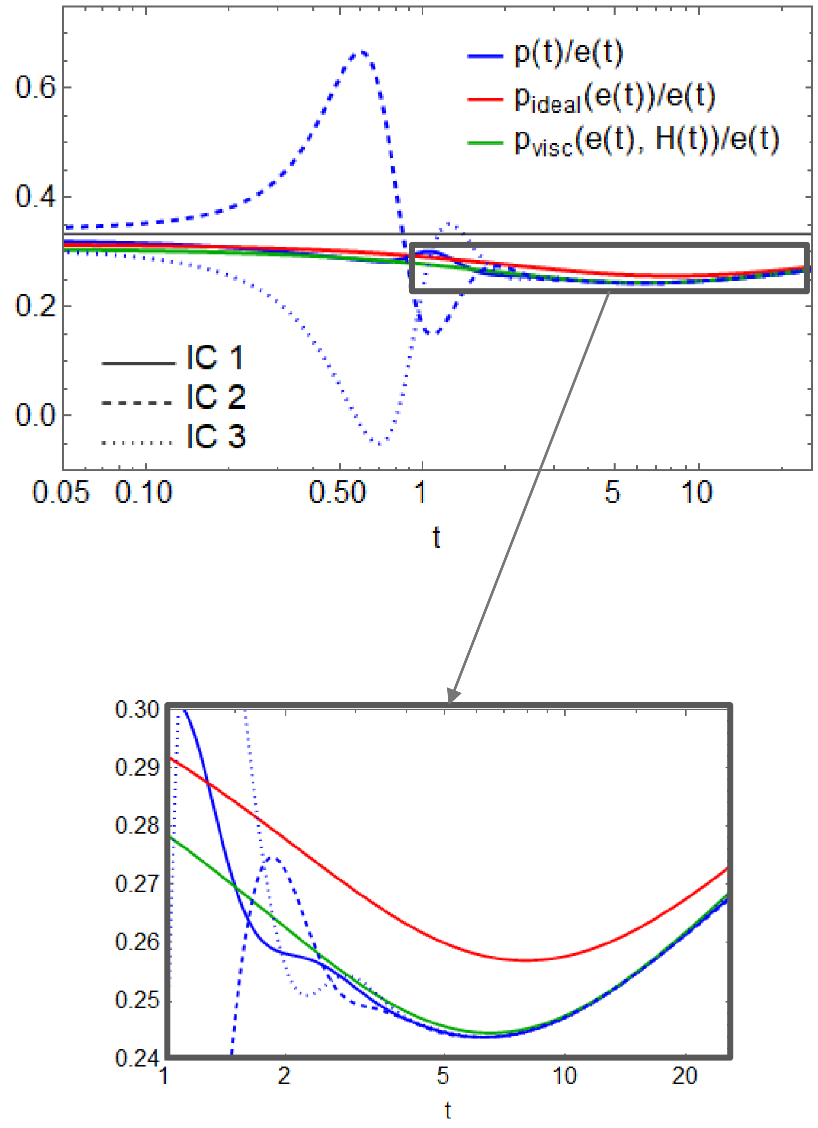


# HYDRODYNAMISATION AND  BOUNDARY GRAVITY

**Three different initial conditions for $\Lambda = 0$:**

**Stress-tensor, and hydrodynamisation:**
**all hydrodynamise within a time of ~1/T**



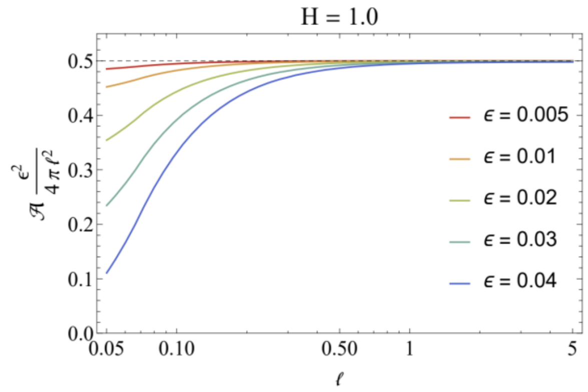



# DISCUSSION

**Hydrodynamics and Entanglement in de Sitter**

- Viscous hydrodynamics works for small gradients
- Event and apparent horizon differ: negative temperature AH (?)
- Extremal surfaces beyond cosmological horizon probe behind EH
- Extremal surface cosmological horizon extends into bulk as bulk CH

**Numerically hard to extract `interesting' piece of the EE: area HRT surface:**




**Phenomenologically not realistic: physics at a vacuum with $e < T^4$, with $T$ about $10^{-30}$ K..**

**Outlook: backreacted de Sitter. In principle nothing stops us from relaxing symmetries?**

# HOLOGRAPHIC RENORMALISATION

$$ds_b^2 = -dt^2 + S_0(t)^2 d\vec{x}^2 . \quad S_0(t) = e^{Ht}$$

$$ds^2 = -A(r,t)dt^2 + 2drdt + S(r,t)^2 d\vec{x}^2 , \quad \phi = \phi(r,t) ,$$

**Action needs (scheme-dependent) counter-terms:**

$$S_{\rm ct} = \frac{1}{8\pi G}\int d^4x\sqrt{-\gamma}\left[\left(-\frac{1}{8}R[\gamma] - \frac{3}{2} - \frac{1}{2}\phi^2\right) + \frac{1}{2}(\log\rho)\,\mathcal{A} + \left(\alpha\mathcal{A} + \beta\phi^4\right)\right],$$

$$\mathcal{A} = \mathcal{A}_g + \mathcal{A}_\phi , \qquad \mathcal{A}_g = \frac{1}{16}(R^{ij}R_{ij} - \frac{1}{3}R^2) , \qquad \mathcal{A}_\phi = -\frac{\phi^2}{12}R$$

**Leads to an ambiguity in the stress-tensor:**

$$\begin{aligned}\mathcal{E}(t) = &-\frac{3a_{(4)}(t)}{4} - M\bar{\phi}_{(2)}(t) + \frac{3S_0'(t)^4}{16S_0(t)^4} + M^2\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{2S_0''(t)}{3S_0(t)}\right)\\ &- M^2\alpha\frac{S_0'(t)^2}{2S_0(t)^2} - M^4\left(\beta - \frac{7}{36}\right),\\ \mathcal{P}(t) = &-\frac{a_{(4)}(t)}{4} + \frac{1}{3}M\bar{\phi}_{(2)}(t) + \frac{S_0'(t)^2\left(S_0'(t)^2 - 4S_0(t)S_0''(t)\right)}{16S_0(t)^4}\\ &- \frac{M^2}{3}\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{13S_0''(t)}{12S_0(t)}\right) + M^2\alpha\left(\frac{S_0'(t)^2}{6S_0(t)^2} + \frac{S_0''(t)}{3S_0(t)}\right) + M^4\left(\beta - \frac{5}{108}\right)\end{aligned}$$

**$\alpha$ and $\beta$ encode scheme dependencies (cosmological constant); fixed such that late time solution has vanishing energy**

Ambiguities come in at order $H^2$

# BOUNDARY GRAVITY – CONSISTENT INITIAL CONDITIONS

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

**Near-boundary expansion metric:**

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{l^2}{r^2}\left(dr^2 + g_{ij}(x,r)dx^i dx^j\right), \text{ where}$$
$$g(x,r) = g_{(0)} + r^2 g_{(2)} + \cdots + r^d g_{(d)} + h_{(d)} r^d \log r^2 + \mathcal{O}(r^{d+1})$$

- Crucial subtlety: logarithmic terms h solely determined by source g(0)
- Consistent IC not much of a problem with known source (h(n) analytically known)

**Dynamics of scale factor** ***$S_0(t)$*** **is now consequence of Friedmann equations**

**Sufficiently smooth solution requires knowledge of sufficient # of log’s**

- Log’s depend on time derivatives of S0(t)
- Solution: extract $\partial_t^4 S_0(t)$ from near-boundary expansion of scalar
- Use time derivates to treat first few logs analytically

Sebastian de Haro, Kostas Skenderis and Sergey N. Solodukhin, Holographic Reconstruction of Spacetime and Renormalization in the AdS/CFT Correspondence (2001)