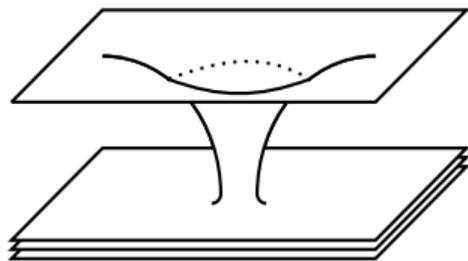


D3-brane solitons and black holes

Ronnie Rodgers



Utrecht University

Introduction

$\mathcal{N} = 4$ SYM admits BPS, spherically symmetric, dyonic solitons [[Schwarz 1405.7444](#)]

Dual to D3-branes in $AdS_5 \times S^5$

Black hole-like properties?

Introduction

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Dual to D3-branes in $AdS_5 \times S^5$

Black hole-like properties?

Based on:

- Prem Kumar, Andy O'Bannon, Anton Pribytok, R. R., 2011.13859
- Adam Chalabi, Prem Kumar, Andy O'Bannon, Anton Pribytok, R. R., Jacopo Sisti, 2011.13859

Related work:

- Nick Evans, Andy O'Bannon, R. R., 1912.09417
- Prem Kumar, Dorian Silvani, 1611.06033 and 1711.01554

Supersymmetric D3-brane solutions

$AdS_5 \times S^5$:

$$ds^2 = \frac{r^2}{L^2} (-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} (dr^2 + r^2 d\Omega_5^2)$$

Probe D3-brane embedding:

	t	x^1	x^2	x^3	r	S^5
D3	\times	\times	\times	\times	$r(\vec{x})$	\cdot

Look for solutions of

$$S_{D3} = -T_{D3} \int dt d^3x \sqrt{-\det(g + F)} + T_{D3} \int C_4$$

Supersymmetric D3-brane solutions

Preserved supersymmetries $\Gamma_\kappa \varepsilon = \varepsilon$

$$\Gamma_\kappa = \frac{1}{\sqrt{-\det(g + F)}} \sum_n \frac{(-1)^n}{2^n n!} \gamma^{a_1 b_1 \dots a_n b_n} F_{a_1 b_1} \dots F_{a_n b_n} (\sigma_3)^n i\sigma_2 \otimes \gamma_{0123}$$

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For $F_{0i} = \cos \chi \partial_i r$ and $\frac{1}{2} \eta_{il} \epsilon^{ljk} F_{jk} = \sin \chi \partial_i r$:

$$i\sigma_2 \otimes \Gamma_{0123} \varepsilon = \varepsilon, \quad \sigma_1 \otimes (-\cos \chi \Gamma_{1234} + \sin \chi \Gamma_{04}) \varepsilon = \varepsilon.$$

D3-brane equations of motion: $\partial_i \partial_i r = 0$

[Gauntlett, Koehl, Mateos, Townsend, Zamaklar, hep-th/9903156;

de Mello Koch, Paulin-Campbell, Rodrigues, hep-th/9903207; Horoku, Kaneko, hep-th/9908154]

Supersymmetric D3-brane solutions

$$r = r_0 + \sum_n \frac{L^2 \kappa_n}{|\vec{x} - \vec{x}_n|},$$

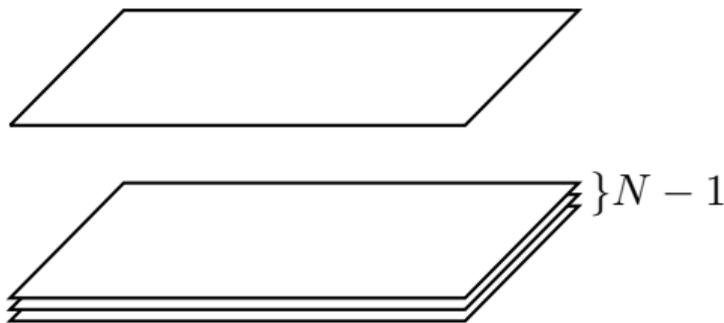
$$\vec{E} = \cos \chi \vec{\nabla} r, \quad \vec{B} = \sin \chi \vec{\nabla} r.$$

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$r = r_0$, $SU(N) \rightarrow SU(N-1) \times U(1)$:



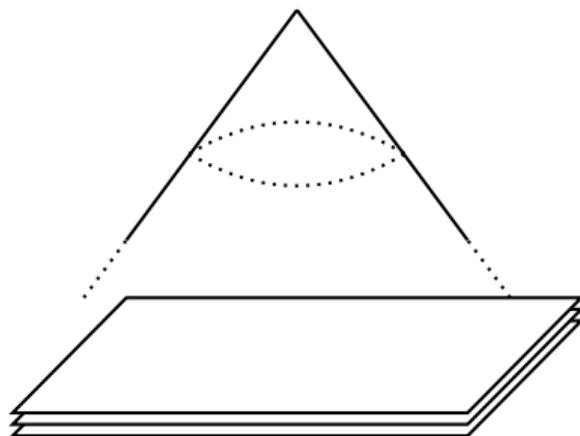
[Klebanov, Witten, hep-th/9905104]

Supersymmetric D3-brane solutions

$$r = r_0 + \sum_n \frac{L^2 \kappa_n}{|\vec{x} - \vec{x}_n|},$$

$$\vec{E} = \cos \chi \vec{\nabla} r, \quad \vec{B} = \sin \chi \vec{\nabla} r.$$

$$r = \frac{L^2 \kappa}{|\vec{x}|}, \text{ Wilson line:}$$



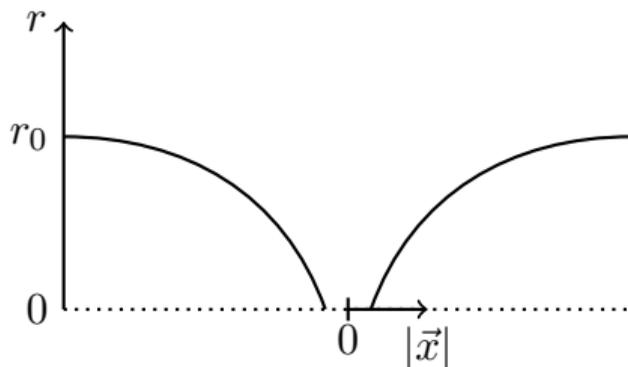
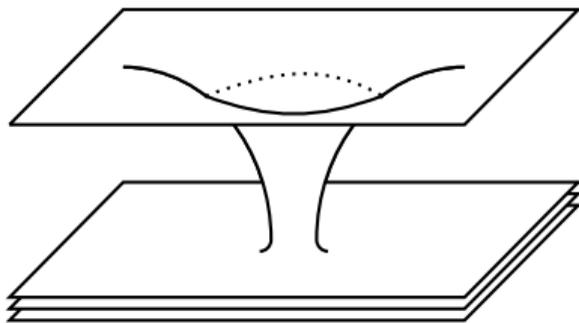
[Drukker, Fiol, hep-th/0501109]

Supersymmetric D3-brane solutions

$$r = r_0 + \sum_n \frac{L^2 \kappa_n}{|\vec{x} - \vec{x}_n|},$$

$$\vec{E} = \cos \chi \vec{\nabla} r, \quad \vec{B} = \sin \chi \vec{\nabla} r.$$

$$r = r_0 - \frac{L^2 \kappa}{|\vec{x}|}, \text{ soliton:}$$



Dyonic solitons [Schwarz 1405.7444]

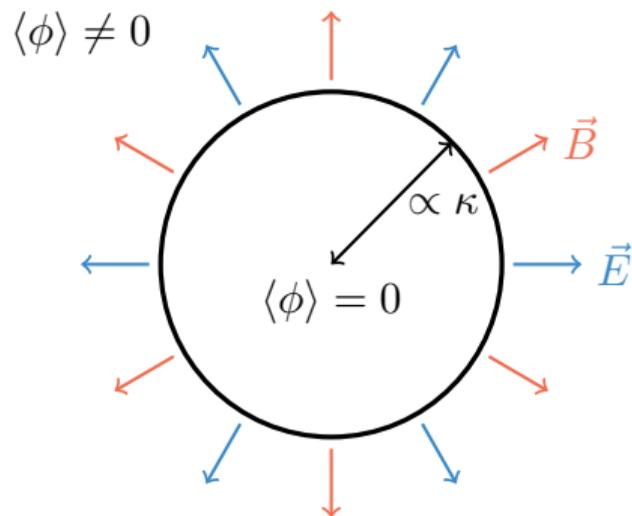
$$r = r_0 - \frac{L^2 \kappa}{|\vec{x}|}$$

Charges

$$p = \frac{4\kappa N}{\sqrt{\lambda}} \cos \chi, \quad q = \frac{\kappa \sqrt{\lambda}}{\pi} \sin \chi.$$

Mass

$$M = \frac{r_0}{L^2} \sqrt{\frac{\lambda}{4\pi^2} p^2 + \frac{4N^2}{\lambda} q^2}$$



Dyonic solitons [Schwarz 1405.7444]

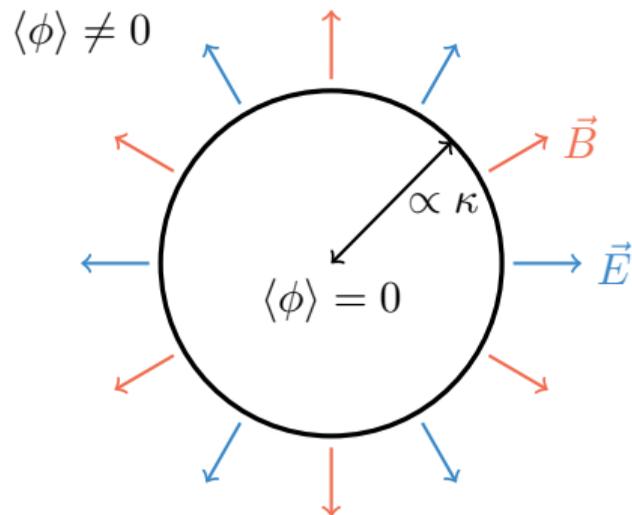
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Mass, charge \propto radius

Similar objects

Magnetic bags — multi-magnetic monopoles in $SU(2)$ YM + adjoint scalar:

- q -monopoles distributed on shell, thickness $\sim 1/M_W$, radius q/M_W .
- Scalar = 0 inside bag, $\propto \phi_0 - \kappa/|\vec{x}|$ outside.
- Charge, mass \propto radius.

[Bolognesi, hep-th/0512133; Bolognesi, 1005.4642; Manton, 1111.2934]

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[Bolognesi, hep-th/0512133; Bolognesi, 1005.4642; Manton, 1111.2934]

Dyonic BPS solitons in $\mathcal{N} = 2$ $SU(2)$ SYM

- Shell of charge
- Scalar = 0 inside shell, $\propto \phi_0 - \kappa/|\vec{x}|$ outside

[Popescu, Shapere, hep-th/0102169]

Quasinormal modes

2011.13859

Quasinormal modes

Linearised fluctuations, frequency ω

Bosonic fluctuations: ϕ , Z^A , a_α , a_i

Spherical harmonic decomposition: [\[Faraggi, Pando-Zayas, 1101.5145\]](#)

$$\phi_{lm}(t, r, \theta, \varphi) = e^{-i\omega t} \sum_{l=0}^{\infty} \sum_{m=-l}^l \phi_{lm}(r) Y^{lm}(\theta, \varphi)$$

$$a_i^{lm}(t, r, \theta, \varphi) = e^{-i\omega t} \sum_{l=1}^{\infty} \sum_{m=-l}^l b_{lm}(r) \hat{Y}_i^{lm}(\theta, \varphi)$$

$$\hat{Y}_i^{lm} = \frac{1}{\sqrt{l(l+1)}} \epsilon_i^j \partial_j Y^{lm}$$

Quasinormal modes: boundary conditions

$$Z_{lm}''(r) + \left[\omega^2 \left(\frac{1}{r^4} + \frac{\kappa^2}{(1-r)^4} \right) - \frac{l(l+1)}{(1-r)^2} \right] Z_{lm}(r) = 0$$

(From now on $r_0 = 1$)

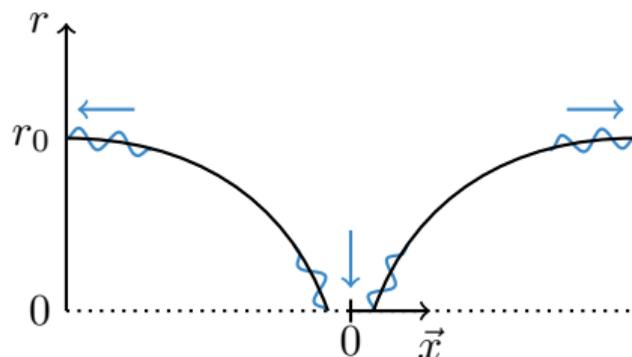
$r \rightarrow 0$:

$$Z_{lm} \sim r e^{\pm i\omega/r},$$

$r \rightarrow 1$:

$$Z_{lm} \sim (1-r) e^{\pm i\kappa\omega/(1-r)}$$

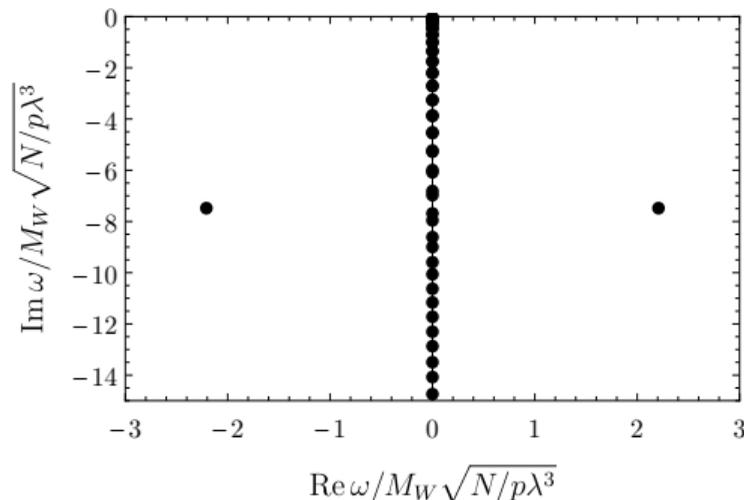
Choose + signs



Quasinormal modes: $l = 0$

QNMs have $\omega \propto \kappa^{-1/2}$:

$$u = \log [\sqrt{\kappa r}(1-r)] \quad \Rightarrow \quad \tilde{Z}''(u) - \left[\frac{1}{4} - 2\kappa\omega^2 \cosh(2u) \right] \tilde{Z}(u) = 0$$
$$\tilde{Z} = \left(e^{u/2} + \sqrt{\kappa} e^{-u/2} \right) Z_{00}$$

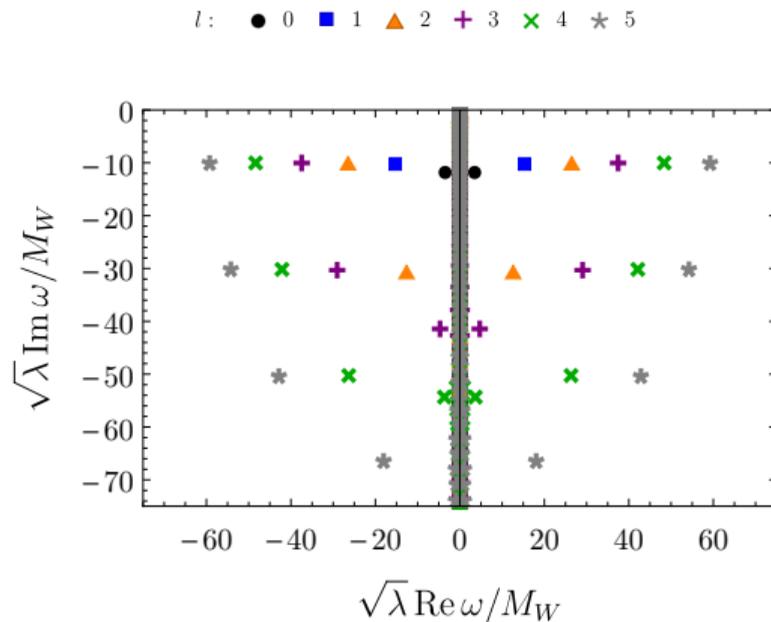


1 pair of modes + branch cut

QNMs computed with Leaver's matrix method

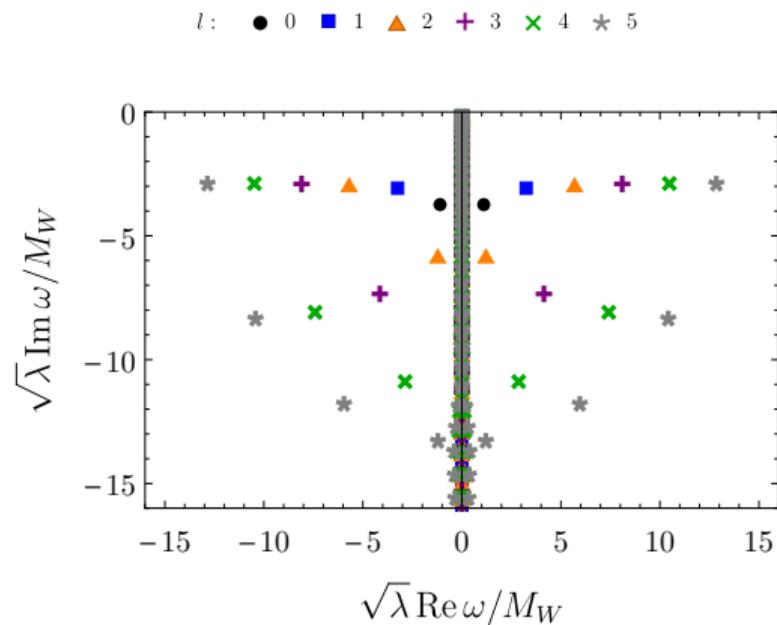
[Leaver, 1990]

Quasinormal modes: $l \neq 0$



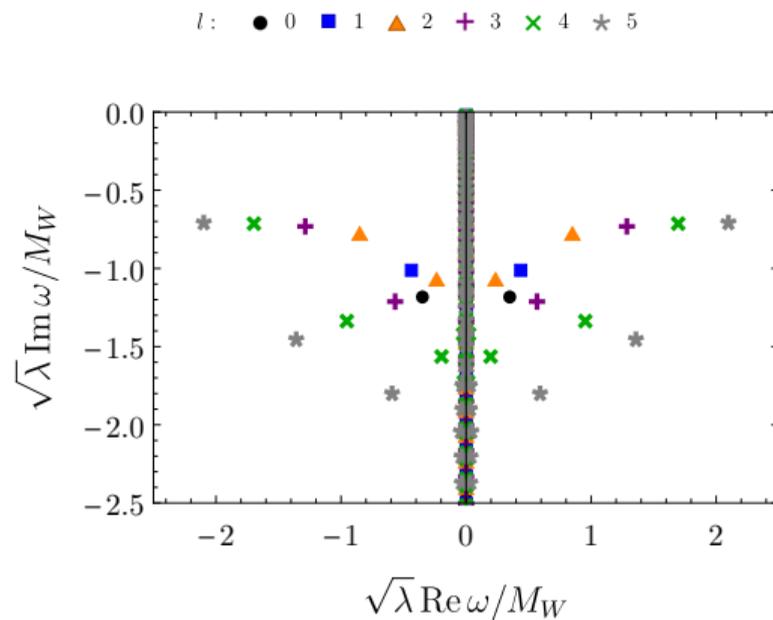
$$\kappa = 1/10$$

Quasinormal modes: $l \neq 0$



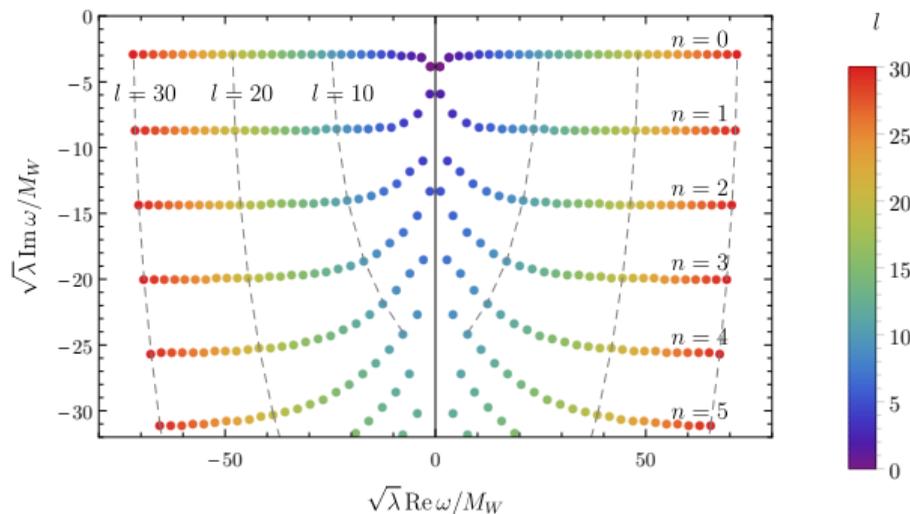
$$\kappa = 1$$

Quasinormal modes: $l \neq 0$



$$\kappa = 10$$

Quasinormal modes: $l \neq 0$

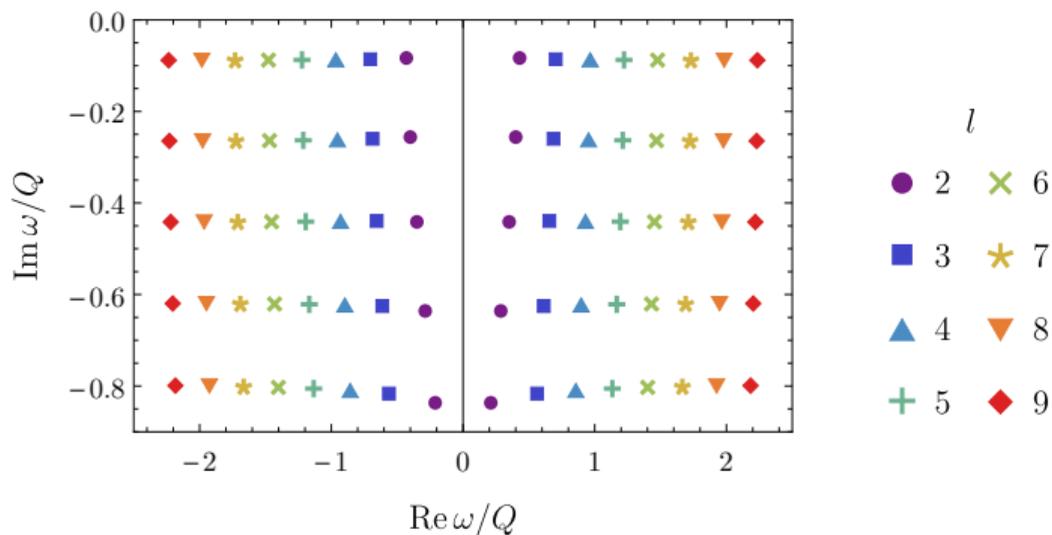


$$\kappa = 1$$

WKB ($l \gg 1$):

$$\omega \approx \left[\pm 2.35 l - 5.74 i \left(n + \frac{1}{2} \right) \right] \frac{M_W}{\sqrt{\lambda}},$$

Extremal Reissner-Nordström [Kokkotas, Schutz, 1988]



WKB ($l \gg 1$):

$$\omega \approx \left[\pm \frac{l}{4} - \frac{i}{4\sqrt{2}} \left(n + \frac{1}{2} \right) \right] Q,$$

Entropy

2011.13859

Entropy

Charge $\kappa \propto$ radius

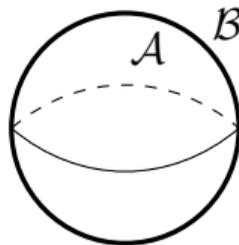
Entropy $\propto \kappa^2$? [Schwarz, 1405.7444]

Entanglement entropy:

$$\mathcal{H} = \mathcal{A} \otimes \mathcal{B}$$

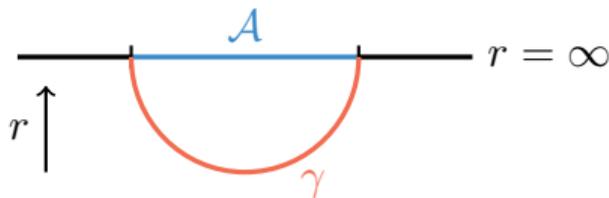
$$\rho_{\mathcal{A}} = \text{tr}_{\mathcal{B}} \rho$$

$$S_{\mathcal{A}} = -\text{tr}_{\mathcal{A}} (\rho_{\mathcal{A}} \log \rho_{\mathcal{A}})$$



What happens when $\mathcal{A} \supset$ soliton?

Entanglement entropy in holography



$$S_{\mathcal{A}} = \frac{\text{Area}[\gamma]}{4G_{\text{N}}}$$

[Ryu, Takayanagi, hep-th/0603001]

How to compute for probe branes?

$$I = I_{\text{bulk}} + I_{\text{brane}}, \quad I_{\text{brane}} \propto \epsilon \ll 1,$$

$$S_{\mathcal{A}} = \underbrace{S_{\mathcal{A}}^{(0)}}_{\mathcal{O}(\epsilon^0)} + \underbrace{S_{\mathcal{A}}^{(1)}}_{\mathcal{O}(\epsilon)} + \dots$$

EE and probe branes [Karch, Uhlemann, 1402.4497]

Spherical \mathcal{A} , map AdS to hyperbolic slicing: [Casini, Huerta, Myers, 1102.0440]

$$ds^2 = L^2 \left[\frac{d\zeta^2}{f(\zeta)} + f(\zeta)d\tau^2 + \zeta^2 du^2 + \zeta^2 \sinh^2 u d\Omega_2^2 \right]$$

$$f(\zeta) = \zeta^2 - 1$$

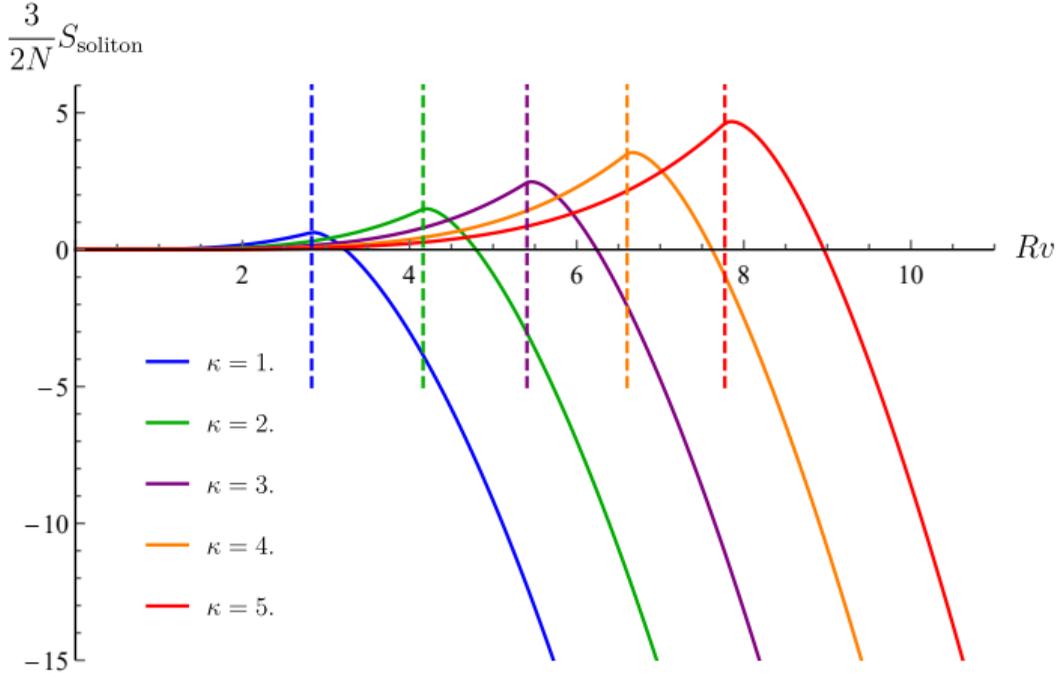
More general solution: $f(\zeta) = \zeta^2 - 1 - \frac{\zeta_h^4 - \zeta_h^2}{\zeta^2}$

$$S_{\mathcal{A}}^{(0)} = \left. \frac{\text{Area}[\text{horizon}]}{4G_N} \right|_{\zeta_h=1} \quad S_{\mathcal{A}}^{(1)} \sim \partial_{\zeta_h} I_{\text{brane}}^* \Big|_{\zeta_h=1}$$

Subtleties: boundary terms when brane hits ζ_h , counterterms.

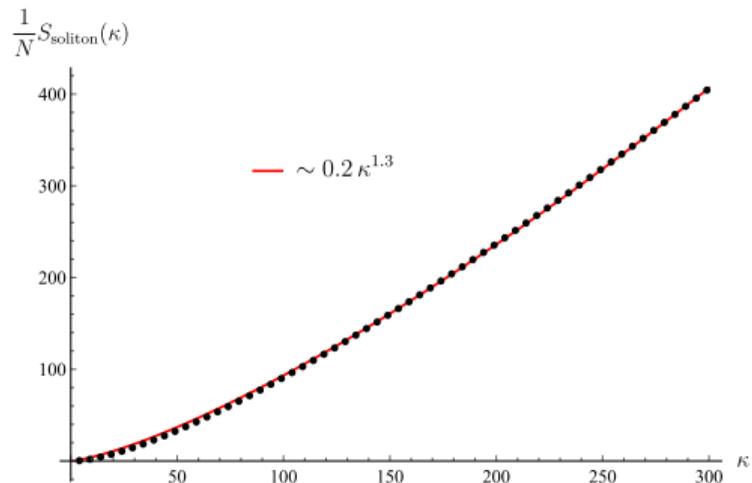
Generalised gravitational entropy [Lewkowycz, Maldacena, 1304.4926]

EE of soliton



EE of soliton

$$S_{\mathcal{A}}(\kappa) \propto \kappa^{1.3}$$



Conclusions

Summary and outlook

$\mathcal{N} = 4$ SYM admits some interesting spherically symmetric solitons

Quasinormal modes with some similarities to asymptotically flat black holes

Entanglement entropy does *not* scale with area, instead smaller power of radius

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Other ways to compute an entropy?

Field theory calculations?

M5-branes — black strings?

EE for other spherical shells?

Thank you!

Backup slides

Quasinormal modes: isospectrality

$q = 0$:

ϕ_{lm}, a_α^{lm} couple \rightarrow decoupled linear combinations Φ_1^{lm} and Φ_2^{lm}

$(Z_{lm}^A, b_{lm}, \Phi_1^{l-1,m}, \Phi_2^{l+1,m})$ *isospectral* — same QNMs

Prove by comparison of Schrödinger potentials

Short $l = 0$ multiplet $(Z_{00}^A, \Phi_2^{1,m})$

Multiplets look like those of $OSp(4^*|4)$ [Faraggi, Pando-Zayas, 1101.5145]

But this is *not* the supergroup

Schrödinger potentials

$$Z''(r) + P(r)Z'(r) + [\omega^2 Q(r) - R(r)] Z(r) = 0$$

Tortoise coordinate $\frac{dr_*}{dr} = \sqrt{Q(r)}$

New dependent variable $\psi(r_*) = e^{h(r_*)} y(r_*)$, where $\frac{dh(r_*)}{dr_*} = \frac{Q'(r) + 2P(r)Q(r)}{4Q(r)^{3/2}}$

EOM becomes:

$$\frac{d^2\psi(r_*)}{dr_*^2} + [\omega^2 - V(r_*)] \psi(r_*) = 0$$

Schrödinger potential:

$$V(r_*) = \frac{R(r)}{Q(r)} + \frac{Q''(r)}{4Q(r)^2} - \frac{5Q'(r)^2}{16Q(r)^3} + \frac{2P'(r) + P(r)^2}{4Q(r)}$$

Schrödinger potentials

For us $r_* = c + \frac{\kappa r}{1-r} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{(1-r)^4}{\kappa^2 r^4}\right)$

$$V_{\Phi_1, l}(r_*) = \frac{l(l+1)r^4(1-r)^2}{f(r)} - \frac{[5 - 4(l+1)r]r^2(1-r)^6}{f^2(r)} + \frac{7r^2(1-r)^{10}}{f^3(r)}$$

$$V_{\Phi_2, l}(r_*) = \frac{l(l+1)r^4(1-r)^2}{f(r)} - \frac{(5 + 4lr)r^2(1-r)^6}{f^2(r)} + \frac{7r^2(1-r)^{10}}{f^3(r)}$$

$$V_{Z, l}(r_*) = \frac{l(l+1)r^4(1-r)^2}{f(r)} + \frac{5r^2(1-r)^6}{f^2(r)} - \frac{5r^2(1-r)^{10}}{f^3(r)}$$

where $f(r) = \kappa^2 r^4 + (1-r)^4$

Isospectrality

Two fluctuations ψ_{\pm}

$$\frac{d^2\psi_{\pm}(r_*)}{dr_*^2} + [\omega^2 - V_{\pm}(r_*)] \psi_{\pm}(r_*) = 0$$

$$V_{\pm}(r_*) = W^2(r_*) \mp \frac{dW(r_*)}{dr_*}$$

Given solution for ψ_{\pm} , can generate solution for ψ_{\mp} :

$$\psi_{\mp}(r_*) = \pm W(r_*)\psi_{\pm}(r_*) + \frac{dy_{\pm}(r_*)}{dr_*}$$

So ψ_{\pm} have same QNMs — *isospectral*

Isospectrality

We find

$$V_{Z,l}(r_*) = W_{1,l}^2(r_*) - \frac{dW_{1,l}(r_*)}{dr_*}, \quad V_{\Phi_1,l-1}(r_*) = W_{1,l}^2(r_*) + \frac{dW_{1,l}(r_*)}{dr_*},$$

$$V_{Z,l}(r_*) = W_{2,l}^2(r_*) - \frac{dW_{2,l}(r_*)}{dr_*}, \quad V_{\Phi_2,l+1}(r_*) = W_{2,l}^2(r_*) + \frac{dW_{2,l}(r_*)}{dr_*},$$

where

$$W_{1,l}(r_*) = \frac{lr^2(1-r)}{f(r)^{1/2}} + \frac{r(1-r)^5}{f(r)^{3/2}}, \quad W_{2,l}(r_*) = -\frac{(l+1)r^2(1-r)}{f(r)^{1/2}} + \frac{r(1-r)^5}{f(r)^{3/2}}.$$

$$\text{where } f(r) = \kappa^2 r^4 + (1-r)^4$$

$\Rightarrow Z_{lm}$, $\Phi_1^{l-1,m}$, and $\Phi_2^{l+1,m}$ have same QNMs

Generalised gravitational entropy

Euclidean signature $\tau \sim \tau + 2\pi$ — circles wind boundary of \mathcal{A}

Extend period to $\tau \sim \tau + 2\pi n$

$$S_{\mathcal{A}} = \lim_{n \rightarrow 1} (\partial_n - 1) I(n)$$

Reproduces Ryu-Takayanagi formula [Lewkowycz, Maldacena, 1304.4926]

For spherical entangling regions in CFTs

$$ds^2 = L^2 \left[\frac{d\zeta^2}{f(\zeta)} + f(\zeta) d\tau^2 + \zeta^2 du^2 + \zeta^2 \sinh^2 u d\Omega_{d-2} \right],$$

$$f(\zeta) = \zeta^2 - 1 - \frac{\zeta_h^d - \zeta_h^{d-2}}{\zeta^{d-2}}, \quad \zeta_h = \frac{\sqrt{1 + n^2 d(d-2)} + 1}{nd}.$$

EE and probe branes [Karch, Uhlemann, 1402.4497]

With brane: $I = I_{\text{bulk}}[\Phi] + I_{\text{brane}}[\Phi, X]$

Imagine solving EOM order-by-order in back-reaction:

$$\Phi = \Phi^{(0)} + \Phi^{(1)} + \dots$$

$$X = X^{(0)} + X^{(1)} + \dots$$

Entanglement entropy:

$$S_{\mathcal{A}} = S_{\mathcal{A}}^{(0)} + S_{\mathcal{A}}^{(1)} + \dots$$

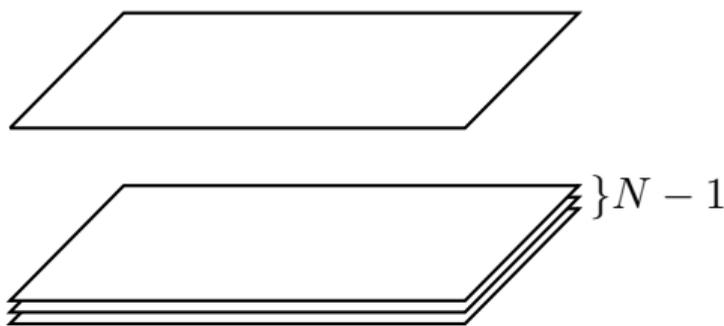
$$S_{\mathcal{A}}^{(0)} = \lim_{n \rightarrow 1} (\partial_n - 1) I_{\text{bulk}} \left(\Phi^{(0)} \right) = \frac{\text{Area}[\gamma^{(0)}]}{4G_{\text{N}}}$$

EE and probe branes [Karch, Uhlemann, 1402.4497]

$$S_{\mathcal{A}}^{(1)} = \lim_{n \rightarrow 1} \left[\int_{\zeta_h}^{\infty} d\zeta \int d^p y \frac{\delta \mathcal{L}_{\text{brane}}}{\delta \Phi} \partial_n \Phi + \int d^p y \frac{\delta \mathcal{L}_{\text{ct,brane}}}{\delta \Phi} \partial_n \Phi - \int d^p y \mathcal{L}_{\text{brane}}|_{\zeta=\zeta_h} \partial_n \zeta_h + \int d^p y N_\mu \Theta_{\text{brane}}^\mu|_{\zeta=\zeta_h} \right]_{2\pi} \Big|_{\Phi=\Phi^{(0)}, X=X^{(0)}}$$

- $[\cdot]_{2\pi}$ indicates τ to be integrated over range $[0, 2\pi]$
- $N_\mu =$ unit normal to $\zeta = \zeta_h$
- $\Theta_{\text{brane}}^\mu =$ total derivative terms from $\delta \mathcal{L}_{\text{brane}} / \delta X$
- No bulk terms at this order, since $\Phi^{(0)}$ extremises bulk action
- No $\delta \mathcal{L}_{\text{brane}} / \delta X$ terms due to brane equations of motion

Coulomb branch EE



Compute EE from probe brane or RT from formula &

$$ds^2 = H(\vec{y})^{-1/2} (-dt^2 + d\vec{x}^2) + H(\vec{y})^{1/2} d\vec{y}^2, \quad H(\vec{y}) = 1 + 4\pi g_s \alpha'^2 \sum_n \frac{N_n}{|\vec{y} - \vec{y}_n|^4}$$

$$S_{\mathcal{A}} = \begin{cases} 0, & vR < 1, \\ \frac{2}{3}N \left[3 \cosh^{-1}(vR) - \left(vR + \frac{2}{vR}\right) \sqrt{(vR)^2 - 1} \right], & vR > 1. \end{cases}$$

Soliton EE: large R

Coulomb branch, $vR \gg 1$:

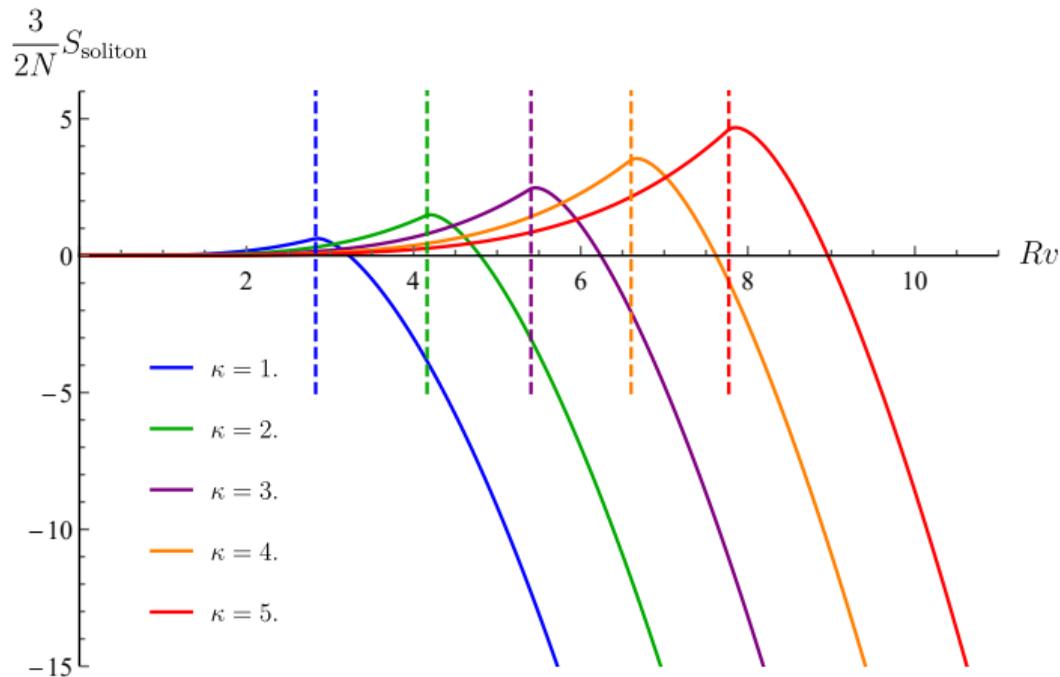
$$S_{\mathcal{A}}^{\text{Coulomb}} = \frac{2}{3}N \left[-(vR)^2 + 3 \log(vR) + 3 \log 2 + \frac{3}{2} \right] + \mathcal{O} \left(\frac{1}{(vR)^2} \right)$$

$$v \rightarrow v_{\text{eff}} = v - \kappa/R$$

$$S_{\mathcal{A}}^{\text{soliton}}(vR) = S_{\mathcal{A}}^{\text{Coulomb}}(v_{\text{eff}}R) + pS_{\mathcal{A}}^{\text{string}} + \mathcal{O} \left(\frac{1}{(vR)^2} \right)$$

$$S_{\mathcal{A}}^{\text{string}} = \sqrt{\lambda}/3 \text{ [Lewkowycz, Maldacena, 1312.5682]}$$

Radius of soliton



Brane hits ζ_h when $R \geq (\kappa^{2/3} + 1)^{3/2}$ (dashed lines)

Radius of soliton

$$S_{\mathcal{A}} \propto \kappa^{1.2}?$$

