Remarks on Color Confinement

Igor R. Klebanov

Princeton University

Talk at CERN Workshop
Gauge-Gravity Duality 2021

July 26, 2021

Based in part on the recent paper

Ross Dempsey, IRK, Silviu Pufu, “Exact Symmetries and Threshold States in Two-Dimensional Models for QCD,” arXiv:2101.05432

Introduction

- The problem of Color Confinement in QCD is among the deepest in modern Theoretical Physics.
- While there has been great progress in Lattice Gauge Theory, there is no quantitative analytic understanding yet of the mass gap and confinement, even in the large N pure glue theory in 2+1 or 3+1 dimensions.
- At the same time, there have been tantalizing experimental discoveries of exotic hadronic states that put new focus on the physics of strong interactions.
- For example, the charmonium state X(3872) has mass that is very close to the sum of D-meson and D*-meson masses. X appears to be a "molecular" threshold state.

Clay Millenium Problems

Yang-Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Status: Unsolved

STRINGS MICHIGAN THEORY

July 10-15, 2000 University of Michigan Ann Arbor

"Millennium Madness"

Physics Problems for the Next Millennium

The best 10 problems were selected at the end of the conference by a selection panel consisting of:

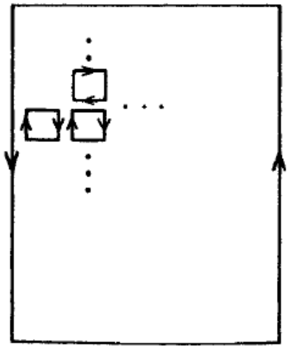
- Michael Duff (University of Michigan)
- David Gross (Institute for Theoretical Physics, Santa Barbara)
- Edward Witten (Caltech & Institute for Advanced Studies)

10. Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
Igor Klebanov, Princeton University
Oyvind Tafjord, McGill University

Lattice SU(N) Gauge Theory

- The gauge field kinetic term is encoded in the plaquette terms.

$$S = -(1/2g^2) \sum_{n,\,\mu\nu} \mathrm{tr}\, U_\mu(n) U_\nu(n+\mu)\, U_{-\mu}(n+\mu+\nu) U_{-\nu}(n+\nu) + \mathrm{h.c.}$$



- In the strong coupling expansion where these terms are treated as perturbations, the Area Law of the Wilson loop is obvious.

$$\left\langle \prod_C \exp[iB_\mu(n)] \right\rangle \approx \exp[-F(g^{-2})A]$$

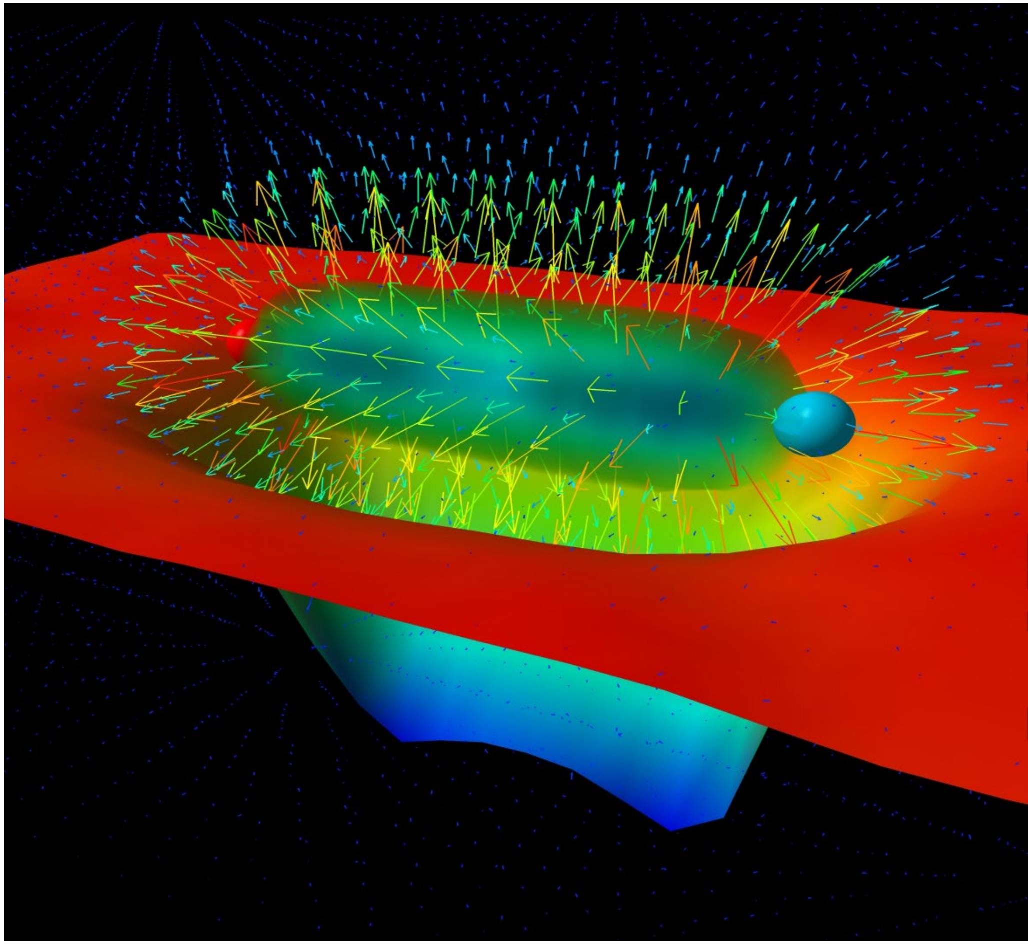
- To obtain the continuum limit, one needs to interpolate to the weak coupling limit on lattice scale due to Asymptotic Freedom

$$g^2(a) = \frac{g_0^2}{1 + (Cg_0^2/2\pi)\ln(a_0/a)}$$

- Can color confinement disappear in this limit? Numerical simulations strongly suggest that the answer is “No.” Lattice sizes up to ~100^4 now.

QCD and Strings

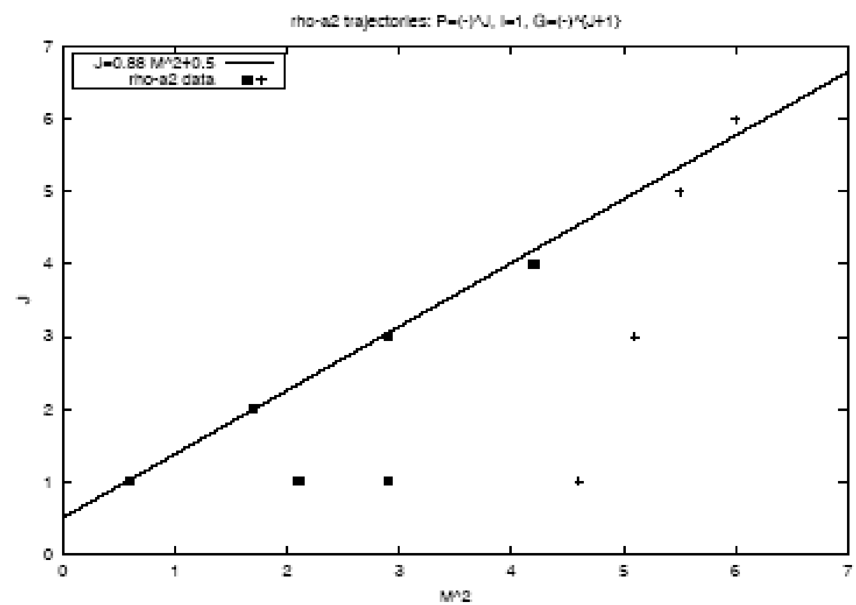
- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is described by the Nambu-Goto area action with corrections.
- So, strings have been observed in numerical simulations of Yang-Mills theory!



Open String Picture of Mesons

$$J = \alpha' m^2 + \alpha(0)$$

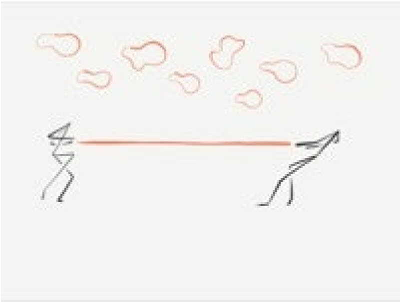
- Mesons are identified with excitations (rotational and vibrational) of a relativistic string of energy density ~ 1 GeV/fm, which is around 1.6 kiloJoules/cm.
- Regge trajectory starting with J=1 ρ meson.



Large N

- Large N gauge theory should define string theory with $g_{st} \sim 1/N$. ‘t Hooft
- For example, SU(N) gauge theory in d=4 should define a stable, non-supersymmetric theory of closed strings (glueballs) and open strings (mesons).
- Adding maximal supersymmetry makes this theory conformal. Then we know what the string theory is: type IIB in $AdS_5 \times S^5$. Maldacena; Gubser, IRK, Polyakov; Witten
- In this case, no color confinement, but there are still strings!
- It is possible to find more general backgrounds that describe confining theories. Witten; Polchinski, Strassler; IRK, Strassler; …

- KITP Program “Confinement, Flux Tubes, and large N,” is planned for January 3 – February 11, 2022.



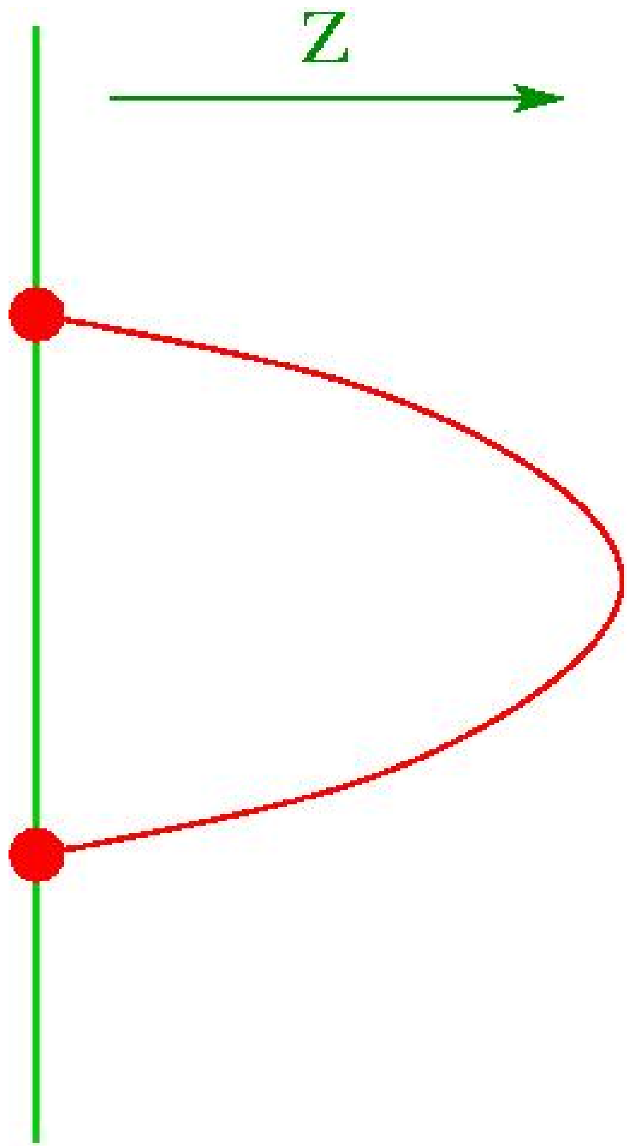
- Coordinators: Sergei Dubovsky, Monica Guica, IRK, Pedro Vieira.
- Scientific Advisors: David Gross, Michael Teper.

The quark anti-quark potential

- What has gauge/string duality taught us?

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



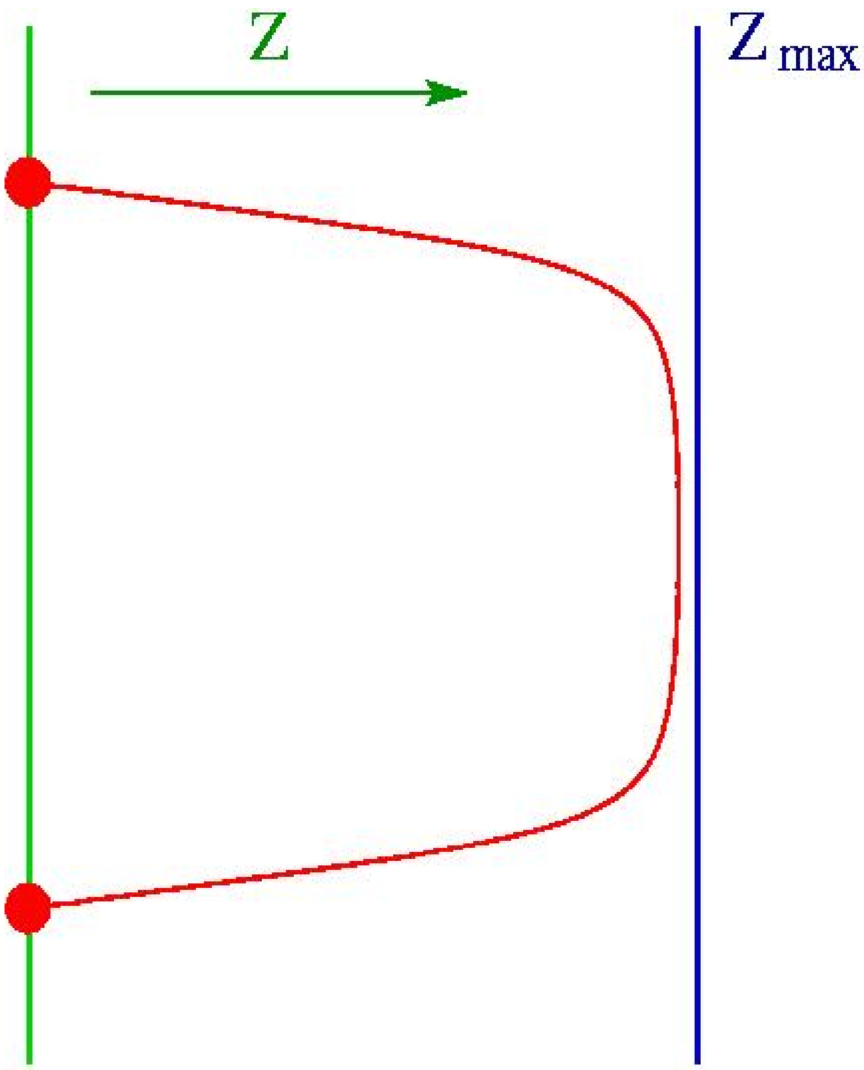
Confining = Fundamental

- The quark anti-quark potential is linear at large distances but nearly Coulombic at small distances.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

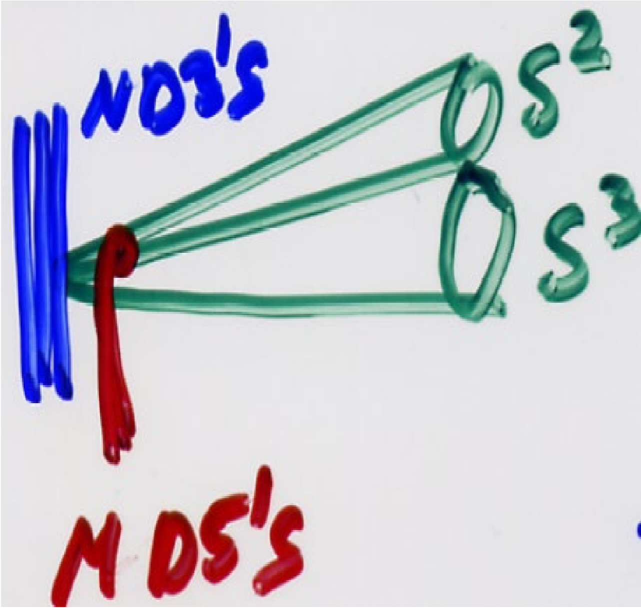
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold IRK, Strassler



$$ds^2_{10} = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds^2_6$$

- ds^2_6 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

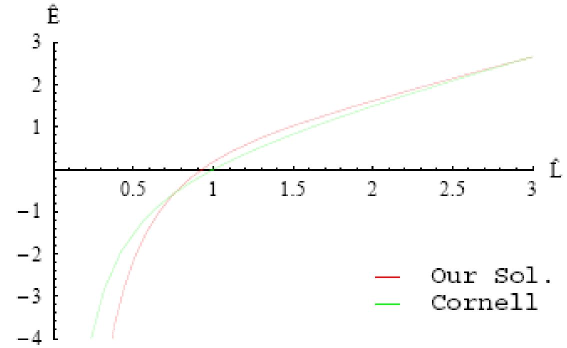
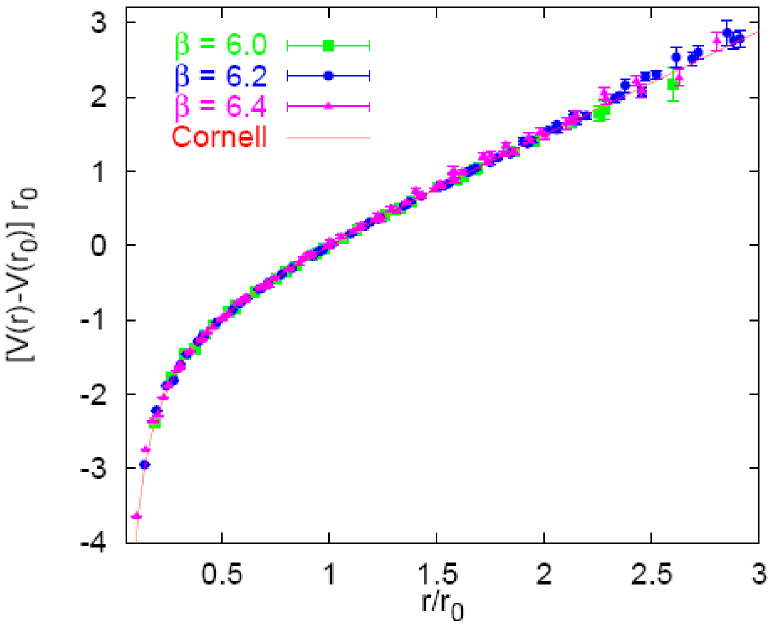


Figure 11: Comparison to the Cornell model



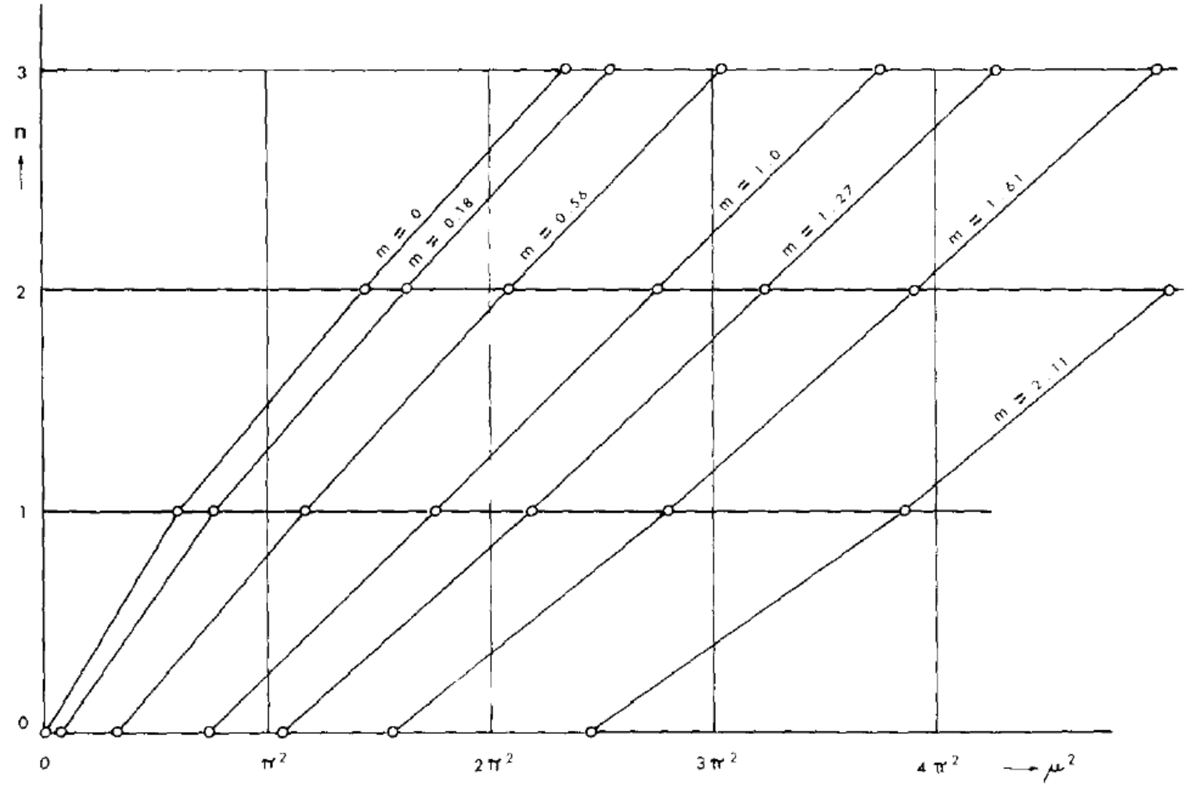
- Normal modes of the warped throat correspond to glueball-like bound states in the gauge theory.
- Their spectra have been calculated using standard methods of (super)gravity.
- Incorporates Dimensional Transmutation.

- Perhaps the gauge/string duality has provided us with a "physicist's proof of confinement" in some exotic gauge theories like the one described by the warped deformed conifold.
- Yet, we still don't have a quantitative handle on the Asymptotically Free theories in 3+1 dimensions.
- As a modest step, "drop back" to 1+1 dimensional gauge theories, which can hopefully provide some intuition about aspects of the higher dimensional dynamics.

The ‘t Hooft Model

- 2d SU(N) gauge theory coupled to N_f fermions in the fundamental representation.
- Exactly solvable in the large N limit using the light-cone gauge: $A_- = 0$
- Find a single Regge trajectory of mesons whose masses are obtained by solving an integral equation.

$$\mu^2 \varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right) \varphi(x) - \mathrm{P}\int_0^1 \frac{\varphi(y)}{(y-x)^2}\, \mathrm{d}y$$



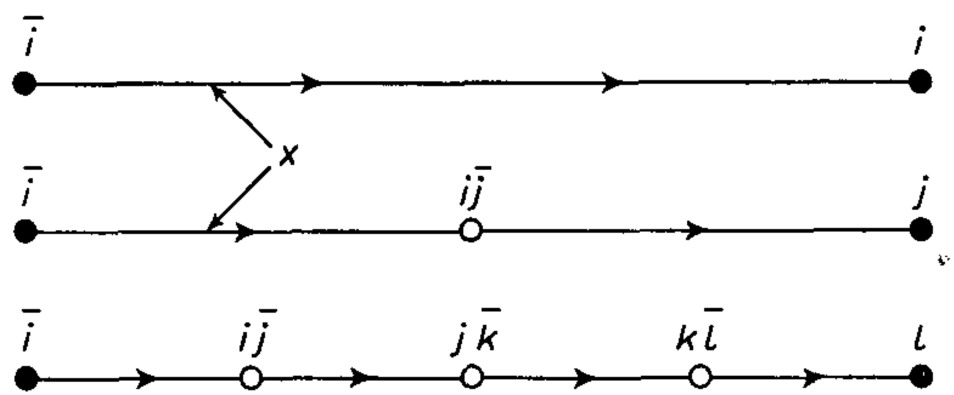
- Meson trajectories for different quark masses from ‘t Hooft’s 1974 paper.
- This beautiful toy model does not have any local adjoint degrees of freedom, which are crucial for higher-dimensional QCD.

2D QCD with Adjoint Matter

- Not exactly solvable at large N, but numerically tractable using Discretized Light-Cone Quantization (DLCQ). Dalley, IRK
- The model with an adjoint Majorana fermion (a toy gluino) has particularly nice properties. The mass is protected against renormalization by a discrete chiral Z_2 symmetry

$$S_{\rm f} = \int d^2x \, {\rm Tr} \left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

- That this SU(N) model has interesting topological structure has been known since the work of Witten in 1979.
- It has a discrete analogue of the 4-d theta angle, k=0,1, … N-1, where k is the number of probe quarks at spatial infinity and corresponding anti-quarks at minus infinity.



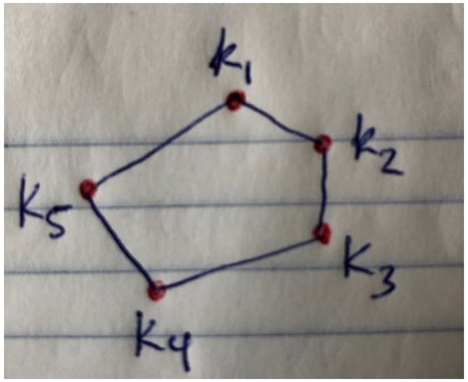
Mass Gap but No Confinement

- For the m=0 theory, the infrared dynamics is governed by the gauged WZW (coset) model

$$\frac{\mathrm{SO}(N^2-1)_1}{\mathrm{SU}(N)_N}$$

- This model has vanishing central charge, which proves that the lightest bound state is massive.
- The model is in a "gapped topological phase."
- The Wilson loop in the fundamental representation does not exhibit the area law. Gross, IRK, Matytsin, Smilga; Komargodski, Ohmori, Roumpedakis, Seifnashri; Dempsey, IRK, Pufu

- In the large N limit we can focus on the string-like single trace gluinoball states



$$|\Phi_{\rm b}(P^+)\rangle = \sum_{j=1}^{\infty} \int_0^{P^+} dk_1 \dots dk_{2j}\, \delta\Big(\sum_{i=1}^{2j} k_i - P^+\Big)$$
$$f_{2j}(k_1, k_2, \dots, k_{2j}) N^{-j}\, {\rm Tr}\, [b^\dagger(k_1) \dots b^\dagger(k_{2j})] |0\rangle$$

- Z_2 symmetry implements string orientation reversal

$$\mathcal{C} b^\dagger_{ij}(k) \mathcal{C}^{-1} = b^\dagger_{ji}(k)$$

- Remarkably, for m=0 some of these states appear to be "threshold bound states" of other states.
  Gross, Hashimoto, IRK
- This is due to the current algebra module structure of the m=0 theory. Kutasov, Schwimmer; Dempsey, IRK, Pufu

DLCQ

- Make one of the light-cone directions compact. Brodsky, Hornbostel, Pauli
- Anti-periodic boundary conditions

$$\psi_{ij}(x^-) = -\psi_{ij}(x^- + 2\pi L) \qquad P^+ = K/(2L)$$

- K is an integer.

$$\psi_{ij}(x) = \frac{1}{\sqrt{2\pi L}} \sum_{\text{odd } n>0} \left(B_{ij}(n) e^{-in\frac{x}{2L}} + B^\dagger_{ji}(n) e^{in\frac{x}{2L}} \right)$$

- Single-trace gluinoball states

$$\frac{1}{N^{p/2}} \operatorname{tr}\left(B^\dagger(n_1)\cdots B^\dagger(n_p)\right)|0\rangle \qquad \textstyle\sum_{i=1}^p n_i = K$$

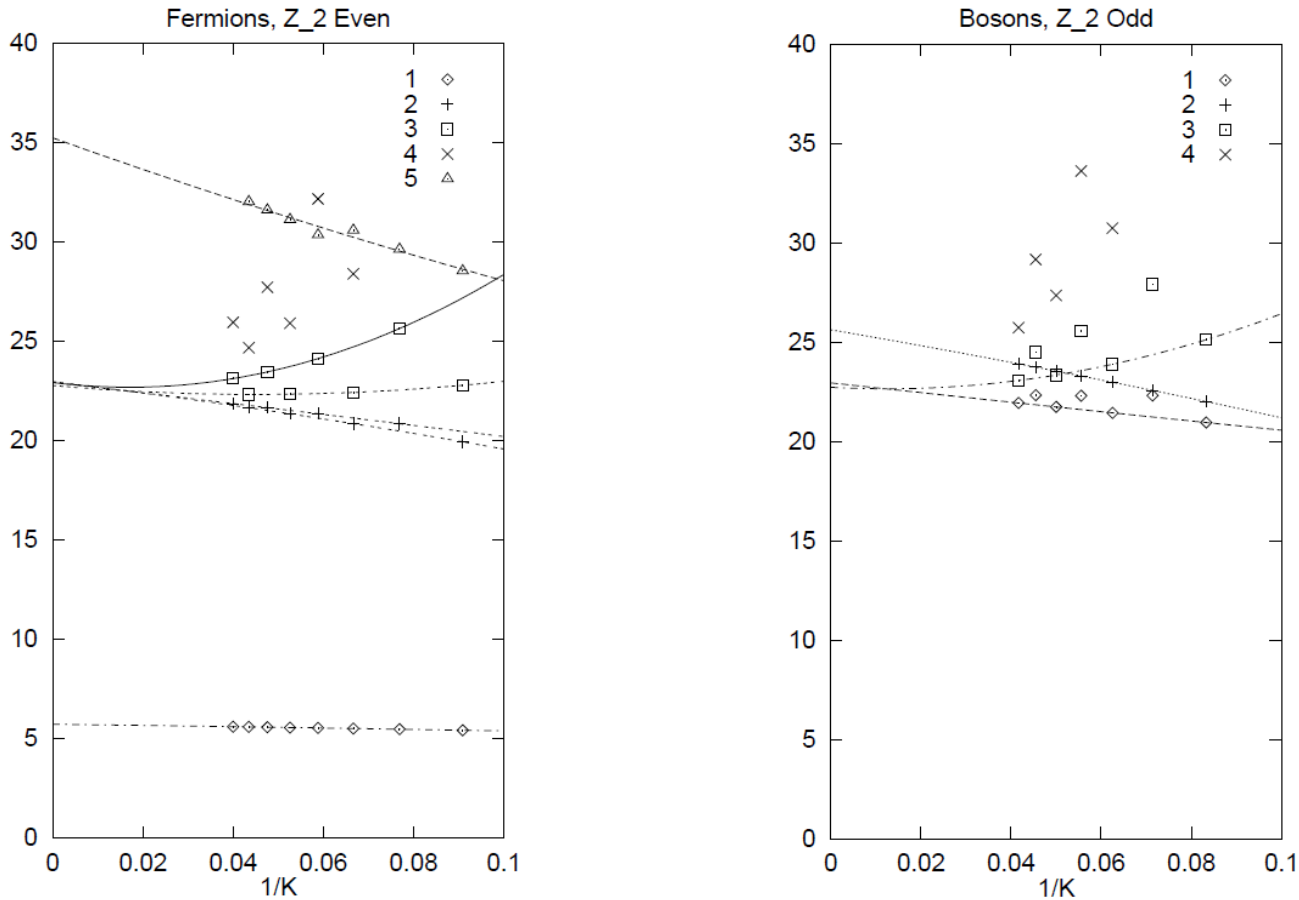
- For even K, these closed string-like states are bosons. For odd K they are fermions.
- In the large N limit, the light-cone Hamiltonian takes single-trace states into other single-trace states.
- We need to carry out "Exact Diagonalization"

K	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Dim	18	28	40	58	93	141	210	318	492	762	1169	1791	2786	4338	6712

- The continuum limit is that of large K, and the highest values we have reached are

K	35	36	37	38	39	40	41
Dim	5.9×10^5	9.3×10^5	1.5×10^6	2.3×10^6	3.6×10^6	5.7×10^6	9.0×10^6

$5.72 g^2 N/\pi$



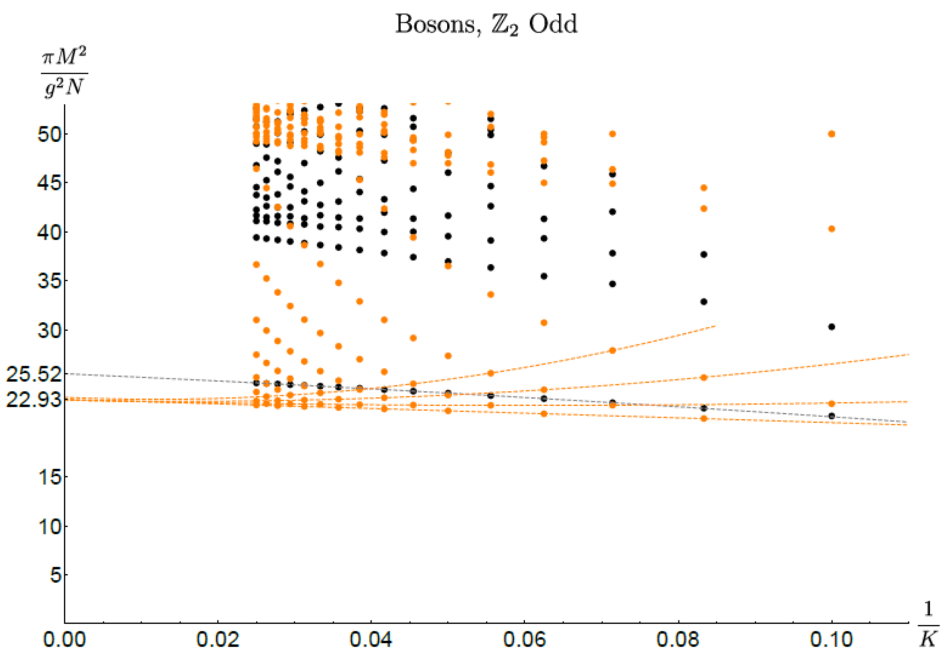
$$M^2 = K\left(\frac{M_{F1}^2(n)}{n} + \frac{M_{F1}^2(K-n)}{(K-n)}\right)$$

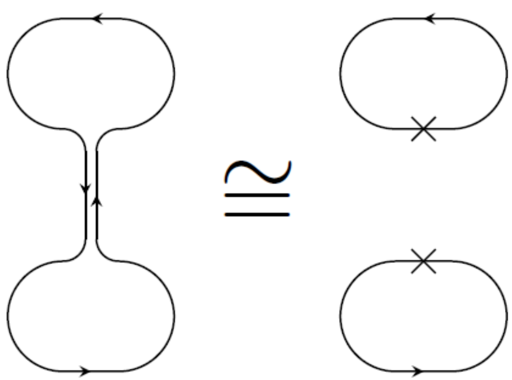
- First observation of threshold states. The threshold is at 4 times the lightest mass-squared.

Gross, Hashimoto, IRK (1997)

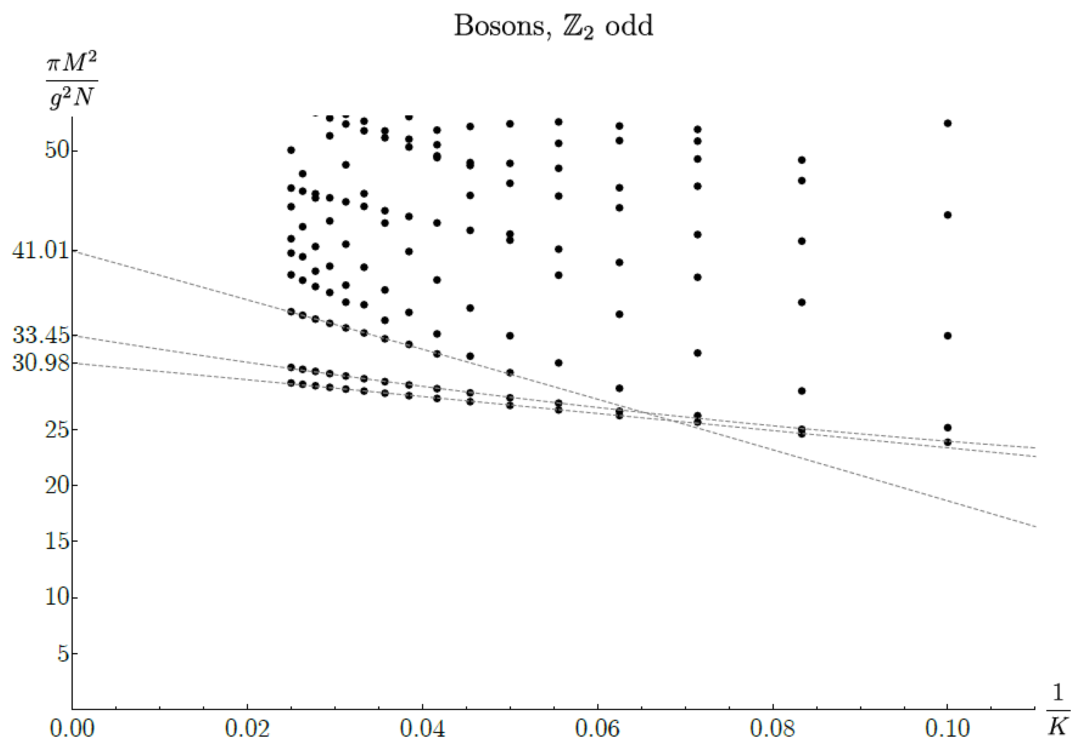
Exact Degeneracies

- In the work with Ross Dempsey and Silviu Pufu we obtained a better understanding of the exactly degenerate states marked with orange dots.





- Appearance of the continuous spectrum of single-trace states suggests that the massless adjoint model is not confining.
- Spectrum becomes discrete for m>0



Kac-Moody Algebra

$$[J_{ij}(n), J_{kl}(m)] = \delta_{kj} J_{il}(n+m) - \delta_{il} J_{kj}(n+m) + k_{\mathrm{KM}} \frac{n\, \delta_{n+m,0}}{2} \left(\delta_{il}\delta_{kj} - \frac{1}{N} \delta_{ij}\delta_{kl} \right)$$

- The Hamiltonian of massless models

$$P^{-} = \frac{g^2 L}{\pi} \sum_{\text{even } n \neq 0} \frac{\mathrm{tr}\,[J(-n)J(n)]}{n^2} = \frac{2g^2 L}{\pi} \sum_{\text{even } n > 0} \frac{\mathrm{tr}\,[J(-n)J(n)]}{n^2}$$

- Diagonalize within different current blocks
  Kutasov, Schwimmer

$$J_{i_1 j_1}(-n_1) J_{i_2 j_2}(-n_2) \cdots J_{i_p j_p}(-n_p) \left|\chi\right\rangle_I$$

- The primaries for the massless adjoint model are

$$\begin{aligned}
\mathfrak{n}=0: &\qquad |0\rangle\,, \\
\mathfrak{n}=1: &\qquad B^\dagger_{ji}(1)|0\rangle\,, \\
\mathfrak{n}=2: &\qquad \left(B^\dagger_{ji}(1)B^\dagger_{lk}(1)-\frac{1}{N}\delta_{kj}J_{il}(-2)+\frac{1}{N}\delta_{il}J_{kj}(-2)\right)|0\rangle \\
\mathfrak{n}=3: &\qquad \left(B^\dagger_{ji}(1)B^\dagger_{lk}(1)B^\dagger_{nm}(1)-\text{traces}\right)|0\rangle\,,
\end{aligned}$$

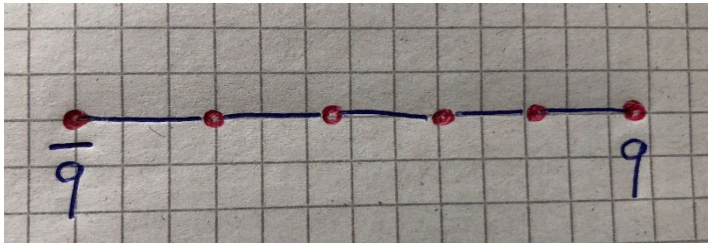
- After acting with raising operators, the descendants must be SU(N) singlets.
- This structure explains some exact degeneracies seen in the numerical diagonalizations. The P^- eigenvalues for n>1 sectors are sums of those in the n=1 sector.

A New 2D Model for Mesons

- If we add N_f fundamental Dirac fermions to the adjoint Majorana, we find a model which contains both gluinoballs and mesons: Dempsey, IRK, Pufu

$$S = \int d^2x \left[\mathrm{tr}\left(-\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\overline{\Psi}\not{D}\Psi - \frac{m_{\mathrm{adj}}}{2}\overline{\Psi}\Psi \right) + i\sum_{\alpha=1}^{N_f} \left(\overline{q}_\alpha \not{D} q_\alpha - m_{\mathrm{fund}}\overline{q}_\alpha q_\alpha \right) \right]$$

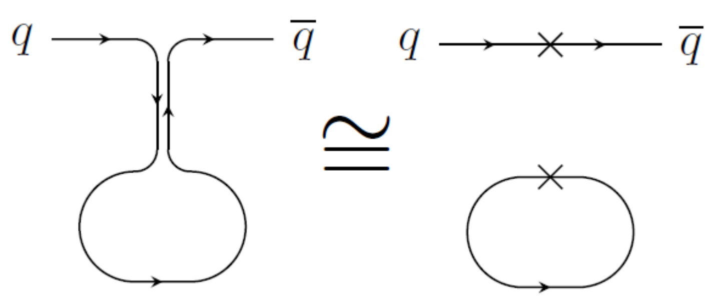
- The mesons are more complicated than in the ‘t Hooft model, since they also contain the adjoint quanta. There are now multiple Regge trajectories of mesons, which can be bosonic or fermionic.

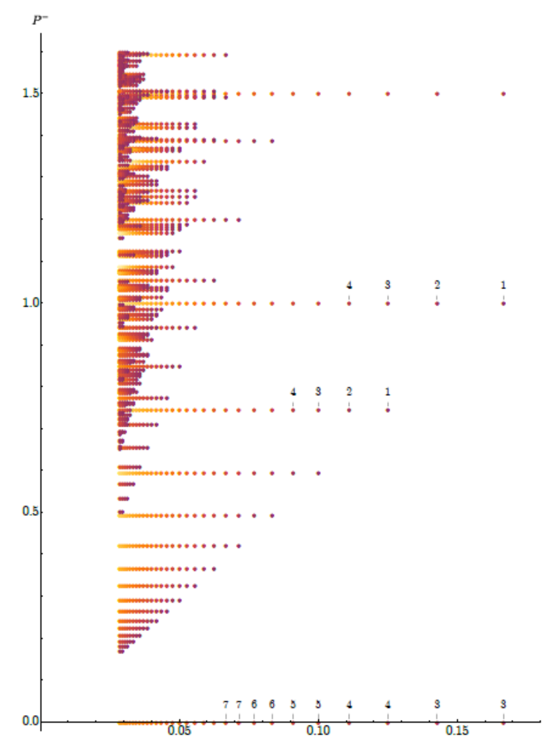


- The large N meson light-cone wave functions

$$|\{g\}_{\alpha\beta}; P^+\rangle = \frac{(P^+)^{(n-1)/2}}{N^{(n-1)/2}} \sum_n \int_0^1 dx_1 \; \cdots \; dx_n \, \delta\left(\sum_{i=1}^n x_i - 1\right)$$
$$\times \, g_n(x_1, \ldots, x_n) c^\dagger_\alpha(k_1) b^\dagger(k_2) \cdots b^\dagger(k_{n-1}) d^\dagger_\beta(k_n) \, |0\rangle$$

- The mesons exhibit interesting patterns of DLCQ degeneracies.

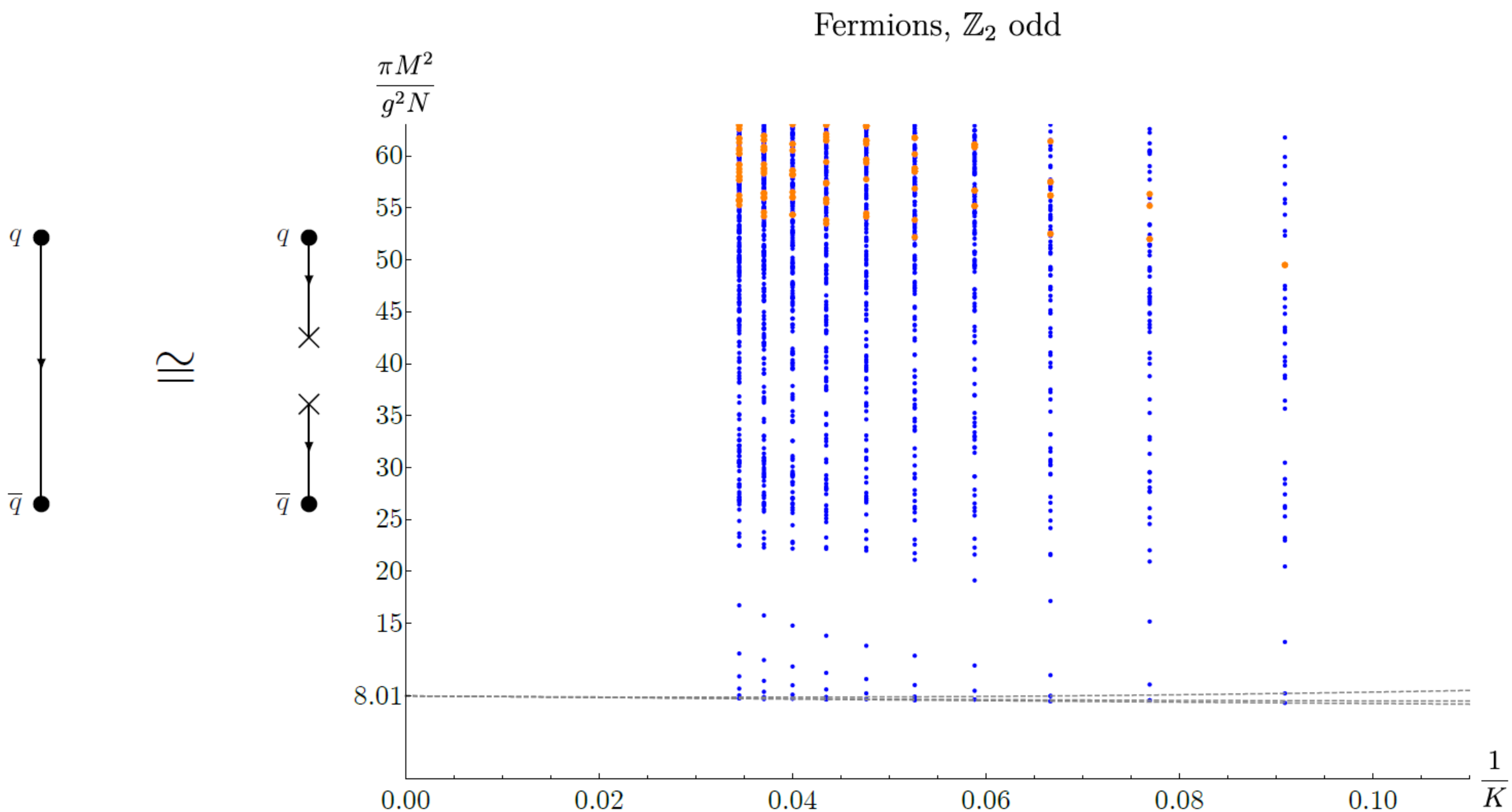




No Confinement

- The WZW effective action depends only on the Kac-Moody level. It is the same in the SU(N) theory coupled to a massless Majorana adjoint as in the theory coupled to N fundamental Dirac fermions: $k_{\mathrm{KM}} = N$
- This implies that the massive spectra are the same in these two models. Kutasov, Schwimmer
- This was also used as an argument for screening: it is obvious in the second model. Gross, IRK, Matytsin, Smilga

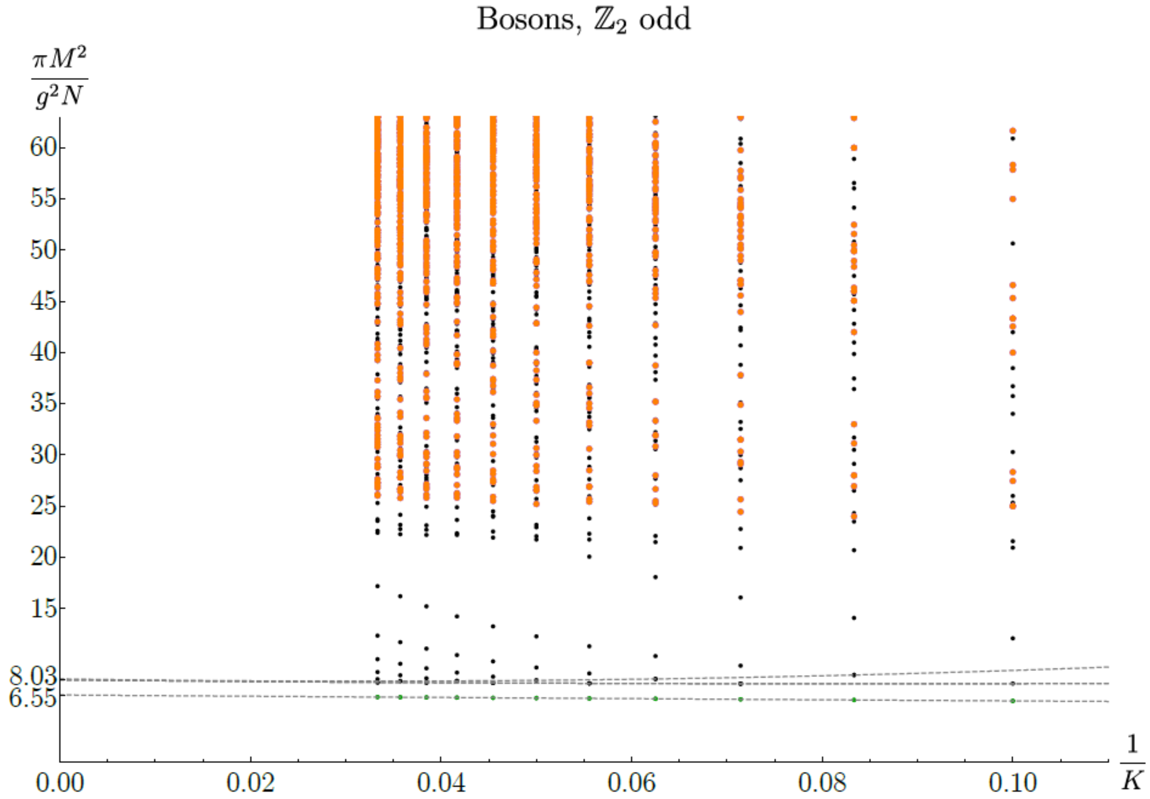
- This argument can be made quantitative by coupling both models to a massive quark.
- Some meson states in theory T (massive quark plus adjoints) are seen to “fall apart” into states in theory T’ (massive quark plus N massless ones).
- For fermionic mesons, the degeneracies are exact with pairs of states in theory T’ containing just one massive quark (the heavy-light states). Such mesons are shown with blue dots.



Fermions, $\mathbb{Z}_2$ odd
$\frac{\pi M^2}{g^2 N}$
$\frac{1}{K}$
q
$\bar{q}$
$\cong$
8.01
15
20
25
30
35
40
45
50
55
60
0.00
0.02
0.04
0.06
0.08
0.10

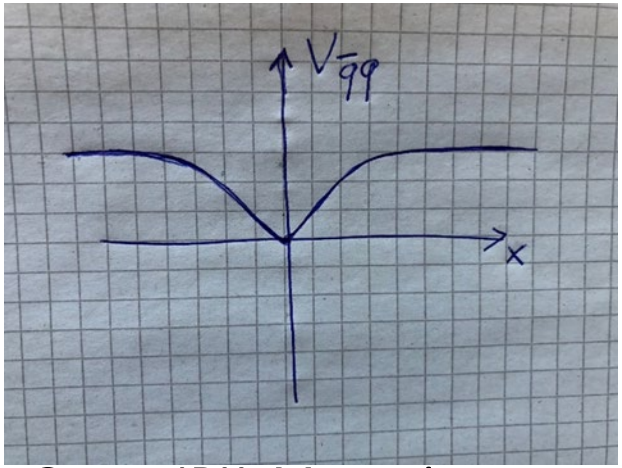
Bosonic Mesons

- Here the degeneracies are not exact, but appear in the continuum limit.
- There is also a bound state clearly seen below the threshold.



Liberation

- For heavy fundamentals and massless adjoints the spectrum of mesons is continuous above a certain threshold.
- This implies that the q-qbar potential flattens at infinity!



- The massless adjoints renormalize string tension to 0. Gross, IRK, Matytsin, Smilga; Komargodski, Ohmori, Roumpedakis, Seifnashri
- For a small mass, string tension becomes non-zero giving a model of weak confinement.

Discussion

- Don’t take confinement for granted!
- Even in 1+1 dimensions QCD strings can become tensionless.
- Proof of Color Confinement in 2+1 and 3+1 dimensions would be very important.
- Improve connections between QCD Strings and Fundamental Strings.
- Need new analytical insights and approximations to the spectra of hadrons and k-string tensions.