Gauge/Gravity Duality 2021

Based on 2104.09529

$T\bar{T}$-deformed Fermionic Theories Revisited

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arXiv: 2104.09529

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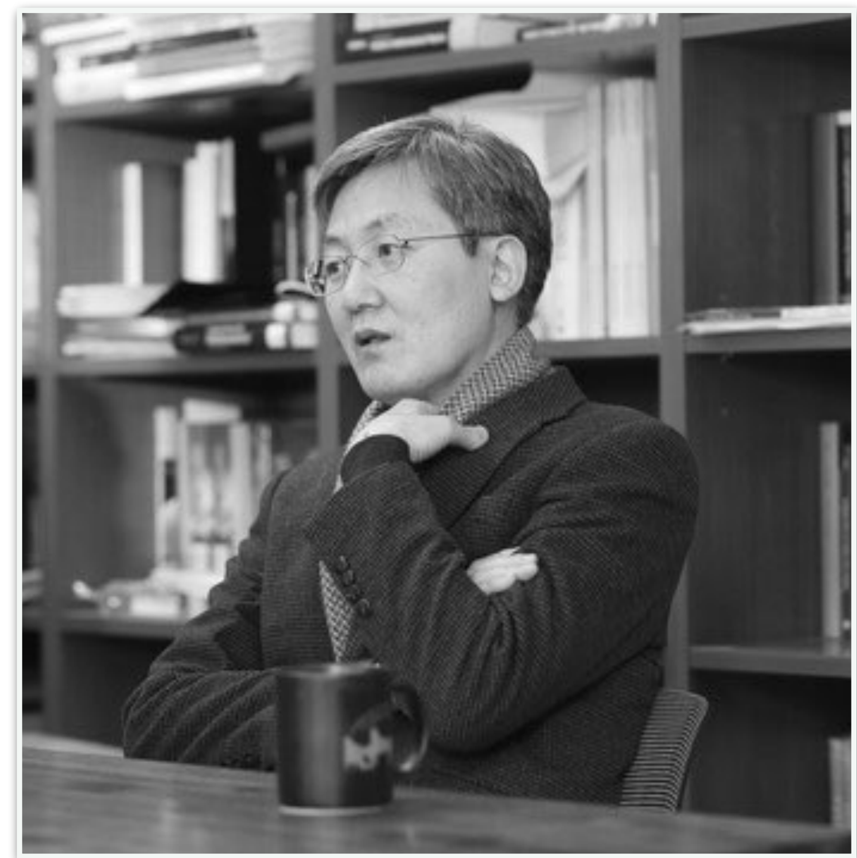

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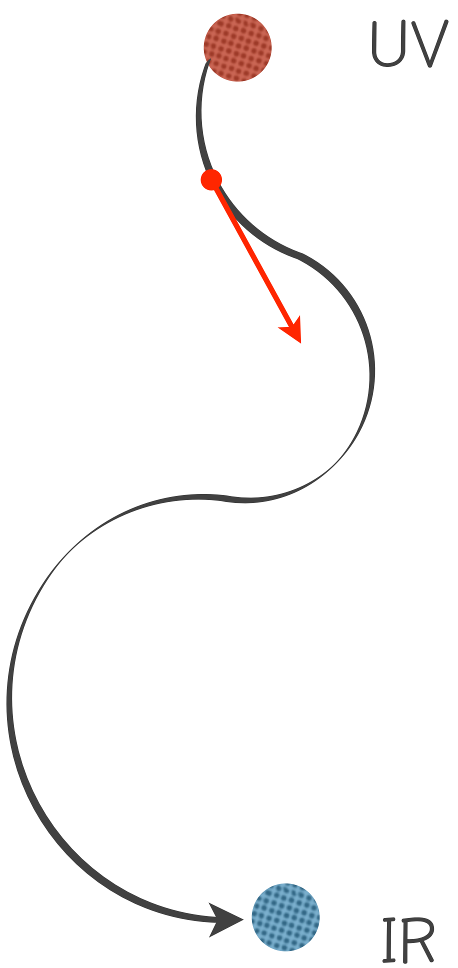
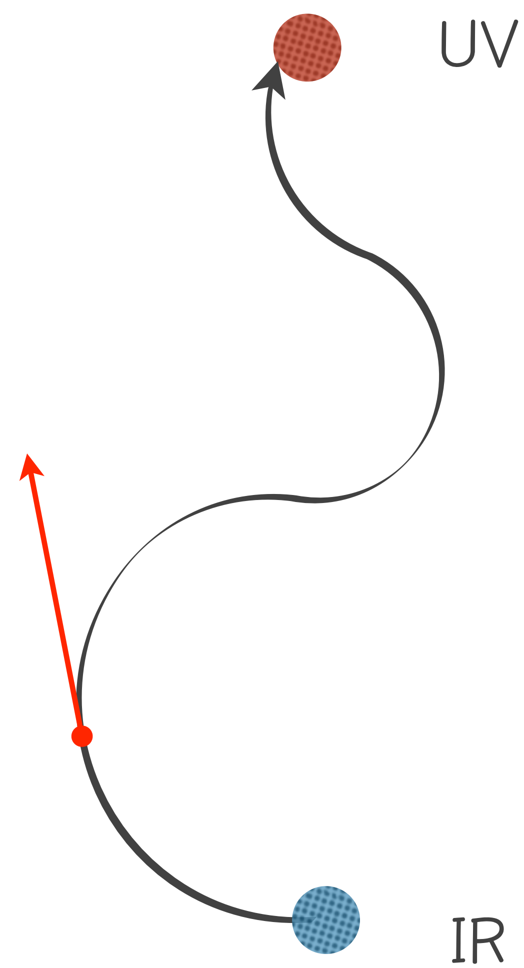
Irrelevant Deformation



Relevant Deformation



Contents

Short Review: $T\bar{T}$ Deformation

$T\bar{T}$ Deformation

- Deformation by (determinant of) energy momentum tensor (EMT) of 2D QFT
  - ✓ $\partial_\lambda \mathscr{L}_\lambda = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$
  - ✓ Irrelevant deformation
  - ✓ λ : deformation parameter of length-squared dimension
  - ✓ EMT can be evaluated in "reasonable" QFT: Universal
- Deformed spectrum: universal
  - ✓ $E_n(L,\lambda) = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}E_n+\frac{4\lambda^2}{L^2}P_n^2}-1\right]$ $\qquad P_n(L,\lambda) = P_n$
  - ✓ E_n , P_n : energy, momentum of undeformed theory
  - ✓ can be derived from factorization formula by Zamolodchikov
    $\langle n|T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}T_{\bar{z}z}|n\rangle = \langle n|T_{zz}|n\rangle\langle n|T_{\bar{z}\bar{z}} - \langle n|T_{z\bar{z}}|n\rangle\langle n|T_{\bar{z}z}|n\rangle$

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Deformed Lagrangian

- Flow equation $\partial_\lambda \mathscr{L} = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$
  - ✓ EMT on RHS: Derivative of deformed Lagrangian w.r.t. field
  - ✓ This leads to differential equation of the Lagrangian
  - ✓ Can be solved perturbatively in principle. Mostly, we can find exact solutions.
  - ✓ Initial condition: $\mathscr{L}[\lambda = 0] = \mathscr{L}_{\text{undeformed}}$
  - ✓ e.g. Deformation of free scalar field: $\mathscr{L} = -\frac{1}{2\lambda}\left[\sqrt{1 + 2\lambda(-\dot{\phi}^2 + \phi'^2)} - 1\right]$
- This is related to Nambu-Goto action for 3D target space with "static gauge"
  - ✓ usually difficult to quantize the NG action
- Relation to dynamical coordinate transformation and 2D gravity

Quantization of $T\bar{T}$ Deformed Theory and Deformed Spectrum

Question

- How can you reproduce the deformed spectrum from the deformed Lagrangian?

$$\mathscr{L} = -\frac{1}{2\lambda}\left[\sqrt{1+2\lambda(-\dot{\phi}^2+\phi'^2)}-1\right]$$

$$E_n(L,\lambda) = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}E_n+\frac{4\lambda^2}{L^2}P_n^2}-1\right]$$

$$P_n(L,\lambda) = P_n$$

- Clever guess

$$\widetilde{H} = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}H_{(0)}+\frac{4\lambda^2}{L^2}P_{(0)}^2}-1\right]$$

$$H_{(0)} = H_+ + H_- \qquad P_{(0)} = H_+ - H_-$$

$$H_+ = \frac{\pi}{L}\sum_k \alpha_{-k}\alpha_k \qquad H_- = \frac{\pi}{L}\sum_k \alpha_{-k}\alpha_k$$

Free Fermion

- Free Fermion: $\mathscr{L}_0 = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-'$
- Conjugate momentum
  - ✓ $\pi_+ = \frac{\overleftarrow{\delta}\mathscr{L}}{\overleftarrow{\delta}\dot{\psi}_+} = \frac{i}{2}\psi_+$ and $\pi_- = \frac{\overleftarrow{\delta}\mathscr{L}}{\overleftarrow{\delta}\dot{\psi}_-} = \frac{i}{2}\psi_-$: no $\dot{\psi}_\pm$
  - ✓ forms the second class constraints: $\mathscr{C}_\pm = \pi_\pm - \frac{i}{2}\psi_\pm$
- Due to the 2nd class constraints $\mathscr{C}_\pm = \pi_\pm - \frac{i}{2}\psi_\pm$, we need to evaluate Dirac bracket.
  - ✓ $\{\mathscr{C}_\pm, \mathscr{C}_\pm\} = -i$ and $\{\mathscr{C}_+, \mathscr{C}_-\} = 0$
  - ✓ For example,
    $\{\psi_+(x_1), \psi_+(x_2)\}_D = 0 - \{\psi_+(x_1), \mathscr{C}_+\}\mathscr{M}^{-1}_{++}\{\mathscr{C}_+, \psi_+(x_2)\} = -i\delta(x_1 - x_2)$

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$T\bar{T}$ Deformation of Free Fermion

- For fermion case, the solution of flow equation is truncated.
  - ✓ Due to the fermi statistics, non-vanishing term is restricted. e.g. $\psi_+\partial_{++}\psi_+\psi_-\partial_=\psi_-$, $\psi_+\partial_=\psi_+\psi_-\partial_{++}\psi_-$
  - ✓ This is because Noether procedure does not produce higher derivative terms
- Deformed Lagrangian
  - ✓ $\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' + \frac{\lambda}{2}\left(-\psi_+\psi_+'\psi_-\dot{\psi}_- + \psi_+\dot{\psi}_+\psi_-\psi_-'\right)$
- Conjugate momentum
  - ✓ $\pi_+ = \frac{\overleftarrow{\delta}\mathscr{L}}{\overleftarrow{\delta}\dot{\psi}_+} = \frac{i}{2}\psi_+ + \frac{\lambda}{2}\psi_+\psi_-\psi_-'$ and $\pi_- = \frac{\overleftarrow{\delta}\mathscr{L}}{\overleftarrow{\delta}\dot{\psi}_-} = \frac{i}{2}\psi_- - \frac{\lambda}{2}\psi_+\psi_+'\psi_-$: Still no $\dot{\psi}_\pm$
  - ✓ forms the second class constraints:
    $\mathscr{C}_1 = \pi_+ - \frac{i}{2}\psi_+ - \frac{\lambda}{2}\psi_+\psi_-\psi_-'$, $\mathscr{C}_2 = \pi_- - \frac{i}{2}\psi_- + \frac{\lambda}{2}\psi_+\psi_+'\psi_-$

Dirac Bracket

- Dirac bracket of the deformed theory.

$$i\{\psi_+(x_1), \psi_+(x_2)\}_D = (1 + \lambda S_- + 2\lambda^2 S_+ S_-)\delta(x_1 - x_2)$$

$$i\{\psi_+(x_1), \psi_-(x_2)\}_D = -i\lambda(\psi'_+\psi_- + \psi_+\psi'_-)\delta(x_1 - x_2)$$

$$S_\pm = i\psi_\pm\psi'_\pm$$

- Hamiltonian of the deformed theory is
  - ✓ $H = \frac{i}{2}\int dx\left[\psi_+\psi'_+ - \psi_-\psi'_-\right]$: of the same form as free Hamiltonian
  - ✓ no explicit λ dependence
- Then, how does it produce the deformed spectrum?

Transformation

* Goal: Finding a transformation $\psi_{\pm,k} = F_{\pm,k}[b,\bar{b}]$ from free fermi oscillator $b_k, \bar{b}_k$ to $\psi_{\pm,k}$ such that

$$H[\psi_+,\psi_-] = \frac{\pi}{L}\sum_k (-k\psi_{+,-k}\psi_{+,k} + k\psi_{-,-k}\psi_{-,k})$$
$$P[\psi_+,\psi_-] = \frac{\pi}{L}\sum_k (-k\psi_{+,-k}\psi_{+,k} - k\psi_{-,-k}\psi_{-,k})$$
$$\psi_\pm(x) = \frac{1}{\sqrt{L}}\sum_k \psi_{\pm,k} e^{\frac{2\pi i k x}{L}}$$

↔

$$\widetilde{H}[b,\bar{b}] = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}(H_+ + H_-) + \frac{4\lambda^2}{L^2}(H_+ - H_-)^2} - 1\right]$$
$$\widetilde{P}[b,\bar{b}] = H_+ - H_-$$
$$H_+ = -\frac{\pi}{L}\sum_k k b_{-k} b_k \qquad H_- = \frac{\pi}{L}\sum_k k\bar{b}_{-k}\bar{b}_k$$

$$i\{\psi_{+,k},\psi_{+,q}\}_{Dirac} = \delta_{k+q,0} + \frac{\lambda}{L^2}S_{-,k+q} + \frac{2\lambda^2}{L^4}(S_+S_-)_{k+q}$$
$$i\{\psi_{-,k},\psi_{-,q}\}_{Dirac} = \delta_{k+q,0} - \frac{\lambda}{L^2}S_{+,k+q} + \frac{2\lambda^2}{L^4}(S_+S_-)_{k+q}$$
$$i\{\psi_{+,k},\psi_{-,q}\}_{Dirac} = -\frac{\lambda}{L^2}K_{k+q}$$
$$S_\pm = i\psi_\pm\psi'_\pm$$
$$K = i(\psi'_+\psi_- + \psi_+\psi'_-)$$

↔

$$i\{b_k,b_q\}_{Dirac} = i\{\bar{b}_k,\bar{b}_q\}_{Dirac} = \delta_{k+q,0}$$
$$i\{b_k,\bar{b}_q\}_{Dirac} = 0$$

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Perturbative Solution

- Solution:

$$\psi_{+,k} = b_k + \frac{\lambda}{L^2}\psi^{(1)}_{+,k}[b,\bar{b}] + \cdots,$$
$$\psi_{-,k} = \bar{b}_k + \frac{\lambda}{L^2}\bar{\psi}^{(1)}_{-,k}[b,\bar{b}] + \cdots$$

✓ $$\psi^{(1)}_{+,k} = 2\pi \sum_{\substack{r,s \\ r+s\neq 0}} \frac{(k-r-s)s}{r+s} b_{k-r-s} : \bar{b}_r \bar{b}_s : - \pi b_k \sum_r r : \bar{b}_{-r}\bar{b}_r :$$

✓ $$\psi^{(1)}_{-,k} = -2\pi \sum_{\substack{r,s \\ r+s\neq 0}} \frac{(k-r-s)s}{r+s} : b_r b_s : \bar{b}_{k-r-s} + \pi \sum_r r : b_{-r} b_r : \bar{b}_k$$

- Non-local
- At quantum level, it is confirmed up to order $\mathcal{O}(\lambda)$

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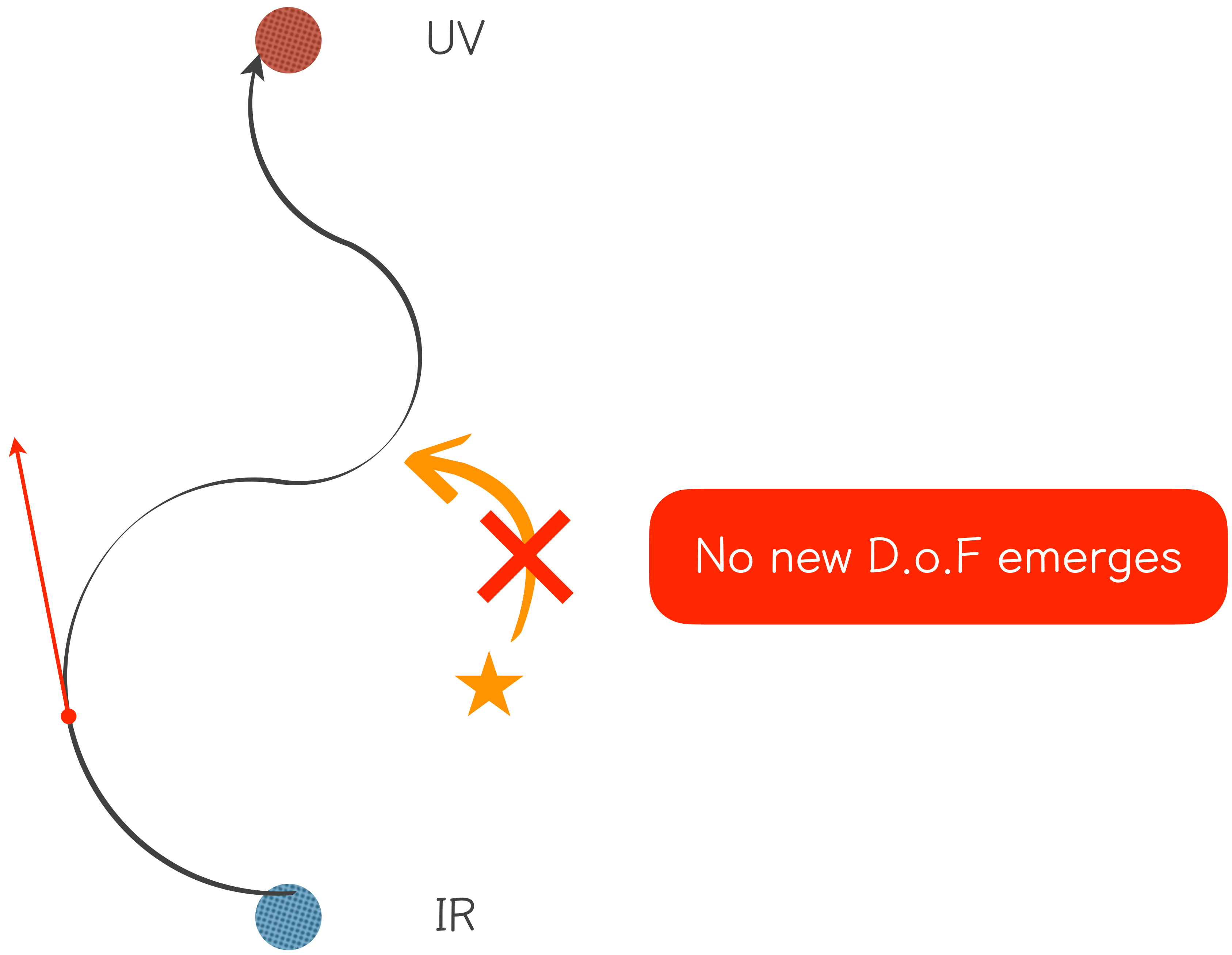
Comparison

- Fermion case
  - ✓ Deformed Hamiltonian does not have explicit λ dependence
  - ✓ The algebra of phase space variables is changed.
  - ✓ Not canonical transformation.
  - ✓ This generates λ dependence for the spectrum
- Scalar field case
  - ✓ Deformed Hamiltonian has λ dependence
  - ✓ The algebra of phase space variables is not changed.
    : canonical transformation

A Caveat in $T\bar{T}$ Deformation of Fermionic Theories

Another Property of $T\overline{T}$ Deformation



Energy-momentum Tensor

- ✳ Flow equation for $T\overline{T}$ deformation : $\partial_\lambda \mathscr{L} = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$

- ✳ Two ways to calculate energy-momentum tensor

  - ✓ Noether procedure: not always symmetric (e.g. fermion)

$$T^{\mu}{}_{\nu} = \frac{\delta \mathscr{L}}{\delta \partial_\mu \phi}\partial_\nu \phi - \delta^{\mu}_{\nu}\mathscr{L}$$

  - ✓ Metric variation: symmetric by construction

$$T^{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}}$$

- ✳ They can be related by improvement term

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$T\bar{T}$ Deformation of Free Fermion

- From Noether Energy-momentum tensor

$$\mathcal{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' + \frac{\lambda}{2}\left(-\psi_+\psi_+'\psi_-\dot{\psi}_- + \psi_+\dot{\psi}_+\psi_-\psi_-'\right)$$

- From Symmetric Energy-momentum tensor

$$\mathcal{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' + \frac{3\lambda}{8}\left(-\psi_+\psi_+'\psi_-\dot{\psi}_- + \psi_+\dot{\psi}_+\psi_-\psi_-'\right)$$

$$-\frac{\lambda}{8}\psi_+\dot{\psi}_+\psi_-\dot{\psi}_- + \frac{\lambda}{8}\psi_+\psi_+'\psi_-\psi_-'$$

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Emergent D.o.F.

- Free fermion: $\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-'$
  - ✓ Conjugate momentum

$$\pi_+ = \frac{\delta S}{\delta \dot{\psi}_+} = \frac{i}{2}\psi_+$$

$$\pi_- = \frac{\delta S}{\delta \dot{\psi}_-} = \frac{i}{2}\psi_-$$

2nd class constraints

- $T\overline{T}$ deformation of free fermion:

$$\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' \boxed{- \frac{\lambda}{8}\psi_+\dot{\psi}_+\psi_-\dot{\psi}_-} + \cdots$$

- Conjugate momentum

$$\pi_+ = \frac{\delta S}{\delta \dot{\psi}_+} = \frac{i}{2}\psi_+ \boxed{- \frac{\lambda}{8}\psi_+\psi_-\dot{\psi}_-}$$

- $$\pi_- = \frac{\delta S}{\delta \dot{\psi}_-} = \frac{i}{2}\psi_- \boxed{+ \frac{\lambda}{8}\psi_+\psi_-\dot{\psi}_+}$$

Not constraints any more

Toy Model for Negative Norm State

- Quantum mechanical toy model

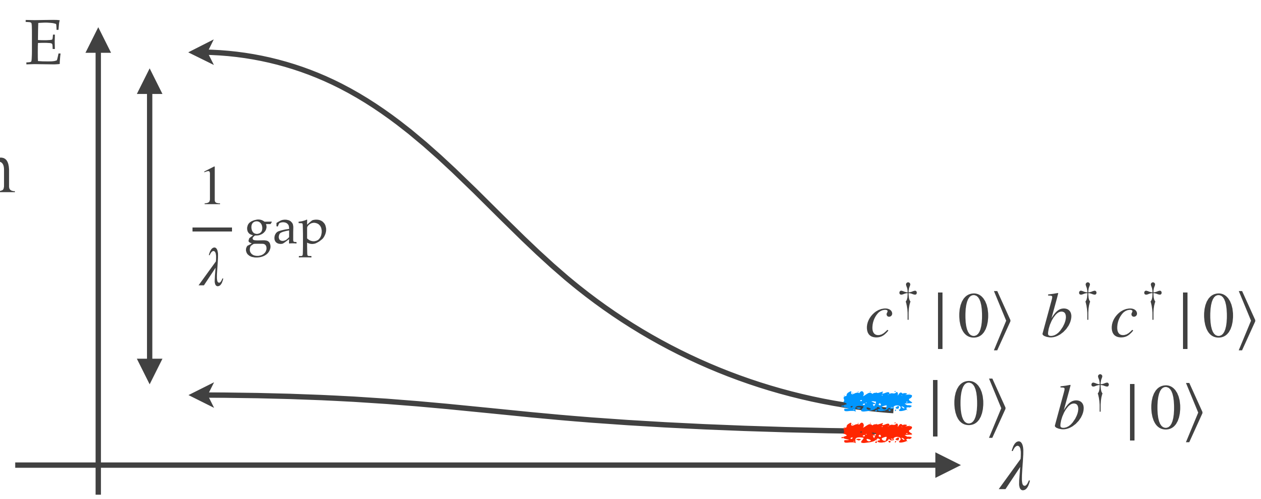
$$L = \frac{i}{2}\bar{\psi}\dot{\psi} - \frac{i}{2}\dot{\bar{\psi}}\psi + m\bar{\psi}\psi - \lambda\dot{\bar{\psi}}\dot{\psi}$$

Hello!

- Phase space: $\psi\,,\bar{\psi}\,,\pi\,,\bar{\pi}$ → no constraints → $b\,,b^{\dagger}\,,c\,,c^{\dagger}$

$$\{b, b^{\dagger}\} = 1$$
$$\{c, c^{\dagger}\} = -1$$

- Spectrum



- Negative norm: $\langle 0|cc^{\dagger}|0\rangle = -1$

  ✓ Non-unitary???

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Recovery of Unitarity

- Define J operator: unitary and Hermitian

$$J \equiv 1 + 2c^\dagger c \qquad \begin{aligned} JcJ &= -c \\ Jc^\dagger J &= -c^\dagger \end{aligned} \qquad \begin{aligned} JbJ &= b \\ Jb^\dagger J &= b^\dagger \end{aligned}$$

- Define J-inner product

$$\langle \mathcal{O} \rangle_J \equiv \langle J \mathcal{O} \rangle$$

- Positive-definite norm: $\langle cc^\dagger \rangle_J = 1$

Negative Norm or Non-Hermiticity

✳ In $T\overline{T}$ deformation of fermion, one can define J operator (in large λ limit).

$$H = \int dx \left[\lambda\psi_+\psi_+'\psi_-\psi_-' + 3(\pi_- - \frac{i}{2}\psi_-)\psi_-' - 3(\pi_+ - \frac{i}{2}\psi_+)\psi_+' + \frac{i}{2}\psi_+\psi_+' - \frac{i}{2}\psi_-\psi_-'\right] + \mathcal{O}(\lambda^{-1})$$

New J-inner product ➡ New J-Hermitian

H and P cannot be J-Hermitian at the same time!!

- ➡ $|E,p\rangle$ is not orthogonal
- ➡ Formula for deformed spectrum is not valid

$$E_n(L,\lambda) = \frac{L}{2\lambda}\left[\sqrt{1 + \frac{4\lambda}{L}E_n + \frac{4\lambda^2}{L^2}P_n^2} - 1\right]$$

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$T\bar{T}$ Deformation of Free Fermion

✳ From Noether Energy-momentum tensor

$$\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' + \frac{\lambda}{2}\left(-\psi_+\psi_+'\psi_-\dot{\psi}_- + \psi_+\dot{\psi}_+\psi_-\psi_-'\right)$$

“Good”

✳ From Symmetric Energy-momentum tensor

$$\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-' + \frac{3\lambda}{8}\left(-\psi_+\psi_+'\psi_-\dot{\psi}_- + \psi_+\dot{\psi}_+\psi_-\psi_-'\right)$$

$$-\frac{\lambda}{8}\psi_+\dot{\psi}_+\psi_-\dot{\psi}_- + \frac{\lambda}{8}\psi_+\psi_+'\psi_-\psi_-'$$

“Bad”

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$T\bar{T}$ Deformation of SUSY Model

- Super-current $\mathscr{T}_{\mu\alpha}$ containing energy momentum tensor

$$\mathscr{T}_{++\,+} = S_{++\,+} + \theta^{+}T_{++\,++} + \theta^{-}Z_{++} - \theta^{+}\theta^{-}\partial_{++}S_{-\,++}$$

- Deformed $\mathscr{N} = (1,1)$ SUSY model in the superspace

$$\int d^2\theta\,\left(\mathscr{T}_{++\,+}\mathscr{T}_{=\,-} + \mathscr{T}_{++\,-}\mathscr{T}_{=\,+}\right)$$

evaluated by Noether procedure

- Emergent of extra degrees of freedom

$$\mathscr{L}_\lambda = \mathscr{L}_0 + \lambda\mathscr{L}_1 + \lambda^2\mathscr{L}_2$$

$$\psi_+\dot{\psi}_+\psi_-\dot{\psi}_-$$

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Summary of Other Results

Topics of Our Paper

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Spectrum from string

Relation to string actions

Hamiltonian Analysis

emergent negative norm state

N=(1,1) SUSY

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Supercharges and Fermi Global Charge for (non-SUSY) $T\bar{T}$ deformation of Free $\mathcal{N} = (1,1)$ SUSY Model

- Supercharges

$$Q^1_+ = \int dx\, \psi_+(\pi + \phi') \qquad Q^1_- = \int dx\, \psi_-(\pi - \phi')$$

$$\dot{\phi} = \frac{\pi\sqrt{1 + 2\lambda\phi'^2}}{\sqrt{1 + 2\lambda\pi^2}} + \cdots \neq \pi$$

- Global symmetry

Shift scalar field $\quad \phi(x) \longrightarrow \phi(x) + a \quad \Rightarrow \quad \mathbb{P}^2 = \frac{2\pi}{L}\int dx\, \pi$

Shift fermion $\quad \psi_\pm(x) \longrightarrow \psi_\pm(x) + \eta_\pm \quad \Rightarrow \quad Q^2_\pm = -\frac{8\pi i}{L}\int dx\, \pi_\pm$

$a\,,\, \eta_\pm$: constants

SUSY and Global Symmetry Algebra

SUSY

$$\{Q^1_\pm, Q^1_\pm\}_D = -2i(H \pm P)$$

$$\{Q^1_+, Q^1_-\}_D = 0$$

Global

$$\{Q^2_\pm, Q^2_\pm\}_D = -\frac{16\pi^2 i}{L} - \frac{16\pi^2 i\lambda}{L^2}(H \mp P)$$

$$\{Q^2_+, Q^2_-\}_D = 0$$

$$\{Q^1_\pm, Q^2_\pm\}_D = -2i\left(\mathbb{P}^2 \pm \frac{4\pi^2}{L^2}\mathbb{W}^2\right)$$

$$\{Q^1_\pm, Q^2_\mp\}_D = 0$$

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3D Target Space SUSY

- $\mathcal{N} = 2$ SUSY of 3D target space

$Q^1_\pm$: $\mathcal{N} = (1,1)$ supercharge

$Q^2_\pm$: fermionic global charge

$$\{Q^\alpha_a, Q^\beta_b\}_D = -2i\delta_{ab}(\Gamma^\mu C)^{\alpha\beta}\mathbb{P}_\mu - \frac{2i}{2\pi\ell_s^2}\sigma^3_{ab}A^{\alpha\beta}$$

topological charge from WZ term

$$\Gamma^\mu C\mathbb{P}_\mu = \begin{pmatrix} H & \mathbb{P}^2 \\ \mathbb{P}^2 & \frac{8\pi^2}{L} + 2\Lambda H \end{pmatrix} \qquad A = \Gamma_\mu C \oint d\sigma\, \partial_\sigma X^\mu = \frac{L^2}{4\pi^2}\begin{pmatrix} P & \frac{4\pi^2}{L^2}\mathbb{W}^2 \\ \frac{4\pi^2}{L^2}\mathbb{W}^2 & -2\Lambda P \end{pmatrix}$$

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Thank You