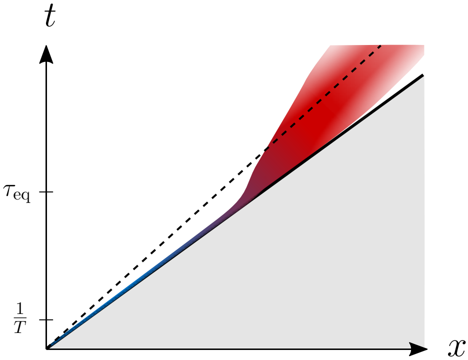
Thermalization of 2d QFTs

Luca Delacrétaz | University of Chicago



Gauge/Gravity Duality 2021 at CERN – July 27, 2021

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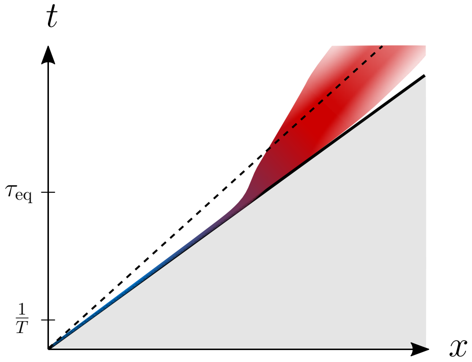

arXiv:2105.02229 with Liam Fitzpatrick, Ami Katz, and Matt Walters

Universal results away from famous lampposts of weak coupling, large N, SUSY, integrability

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A fruitful strategy has been to prove bounds on quantities of interest:

- Lieb-Robinson: $v_{\rm ent} \leq v_{\rm LR}$ Lieb Robinson '72, Bravyi Hastings Verstraete '06
- Positivity bounds in EFT Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06
- $c_{\rm UV} > c_{\rm IR}$ in 2d, $F_{\rm UV} > F_{\rm IR}$ in 3d, $a_{\rm UV} > a_{\rm IR}$ in 4d
  Zamolodchikov '86, Casini Huerta '12, Komargodski Schwimmer '11
- ANEC Faulkner Leigh Parrikar Wang '16, Hartman Kundu Tajdini '16
- Lyapunov exponent $\lambda_L \geq 2\pi T$ Maldacena Shenker Stanford '15

These bounds are obtained from general principles, e.g.: causality, unitarity, locality

We'd like to apply these principles to transport

$$\tau_{\mathrm{eq}} \gtrsim \frac{1}{T}$$

Sachdev '99, Zaanen '04

$$\frac{\eta}{s} \gtrsim 1$$

Kovtun Son Starinets '04

$$D \gtrsim v^2 \tau$$

Hartnoll '15

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Certain existing results along this line

Causality $\Rightarrow D \lesssim c^2 \tau_{\text{eq}}$ Hartman Hartnoll Mahajan '17

ANEC $\Rightarrow \tau_{\text{eq}} \gtrsim \frac{1}{T}\frac{1}{s_o} f(\eta/s)$ LVD Hartman Hartnoll Lewkowycz '18

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This talk:

Causality and hydrodynamics imply that all (1+1)d QFTs satisfy

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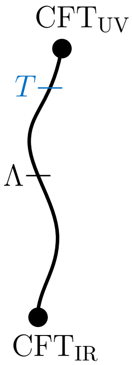

The bound becomes much stronger at high and low temperatures:

High T: $\mathrm{QFT} = \mathrm{CFT}_{\mathrm{UV}} + \lambda \int d^2x\, \mathcal{O}$ $\quad \Delta < 2$

$$\tau_{\text{eq}} \gtrsim \frac{1}{T} \frac{1}{c_{\mathrm{UV}}} \left(\frac{T}{\Lambda}\right)^{2(2-\Delta)}$$

$T \gg \Lambda$

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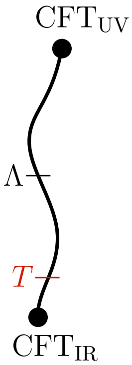

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Low T: $\mathrm{QFT} = \mathrm{CFT_{IR}} + \lambda \int d^2x\, \mathcal{O} + \cdots$

$$\tau_{\mathrm{eq}} \gtrsim \frac{1}{T} \frac{1}{c_{\mathrm{IR}}} \left(\frac{\Lambda}{T} \right)^{2(\Delta-2)}$$

What makes this result possible?

- Near-luminality of sound:

  $c_s^2 = \frac{\partial P}{\partial \varepsilon} \xrightarrow{\text{CFT}} \frac{1}{d-1}$

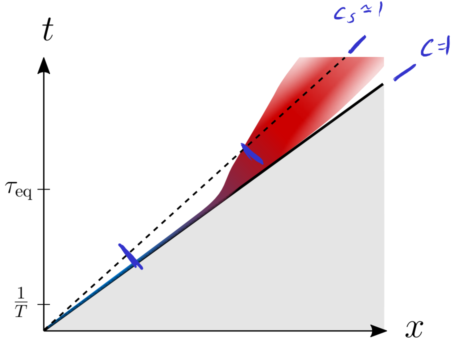
  Causality leaves little room

  for hydrodynamics in $d = 2$!

- Large hydro fluctuations in $d = 2$:

  $\omega = c_s k - iDk^2 \to c_s k - i\mathcal{D}k^{3/2}$

  $\mathcal{D}$ is fixed by thermodynamics (much simpler object than D!)



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The equilibrium thermodynamics of a relativistic QFT is fixed by the equation of state $P(T)$, μ, ...

$$\langle T_{\mu\nu}\rangle_\beta = (\varepsilon + P)\,\delta^0_\mu \delta^0_\nu + P\eta_{\mu\nu}\,,$$

$$\varepsilon + P = sT \qquad P(T)$$
$$dP = s dT \qquad \varepsilon(T)$$
$$s(T)$$

I like to work with the dimensionless entropy density $s_o(T) \equiv \dfrac{s(T)}{T^{d-1}}$

CFT: $\langle T_\mu{}^\mu\rangle = 0 \Rightarrow \varepsilon = P$

$$1 = \frac{d\varepsilon}{dP} = \frac{T ds}{s dT} = \frac{d\log s}{d\log T} \Rightarrow s = \frac{\pi}{3} c\, T$$

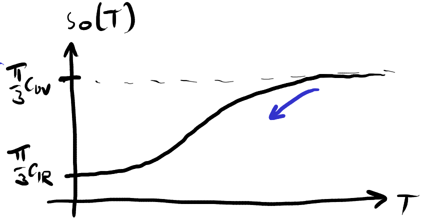
$$s_0 = \frac{\pi}{3} c$$

Causality: $c_s^2 \leq 1$

$$s = s_0 T$$

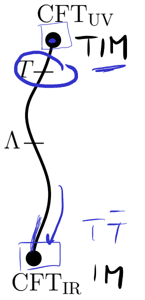
$$1 \leq \frac{1}{c_s^2} = \frac{d\varepsilon}{dP} = \frac{d\log s}{d\log T} = 1 + \frac{d\log s_0}{d\log T}$$

$$s_0'(T) \geq 0$$



Integrable flow

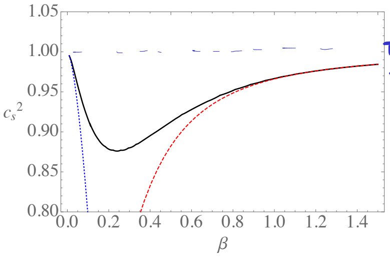
$$S_{\text{QFT}} = S_{\text{CFT}} + \lambda \int d^2x\, \mathcal{O}$$

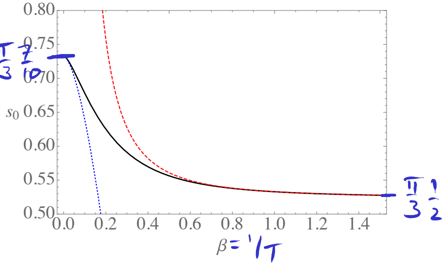


with CFT = Tricritical Ising model ($c_{\text{UV}} = \frac{7}{10}$)

Relevant deformation $\mathcal{O} = \epsilon'$ ($\Delta = \frac{6}{5}$) triggers RG flow to Ising model ($c_{\text{IR}} = \frac{1}{2}$)

Full RG flow is integrable Zamolodchikov '91





At high/low T, $s_o(T)$ can be accessed with conf. perturbation theory

Ludwig Cardy '87

$$S_{\text{QFT}} = S_{\text{CFT}} + \lambda \int d^2x\, \mathcal{O}$$

$$P = \frac{1}{\beta V} \log Z = \frac{\pi}{6} c_{UV} T^2 - \underbrace{\lambda \langle \mathcal{O} \rangle_\beta}_{=0} + \frac{\lambda^2}{2} \int d^2x\, \underbrace{\langle \mathcal{O}(x)\, \mathcal{O} \rangle_\beta}_{\left|\frac{\pi T}{\sinh \frac{\pi}{\beta}(x - i\tau)}\right|^{2\Delta}} + \dots$$

$$s = \frac{dP}{dT}$$

$$s_o(T) = \frac{\pi}{3} c_{\text{UV}} \left[1 - \alpha_\Delta \left(\frac{\lambda}{T^{2-\Delta}} \right)^2 + \cdots \right] \qquad \text{with } \alpha_\Delta = \frac{\Gamma(2-\Delta)\Gamma(\frac{\Delta}{2})^2}{\Gamma(\Delta)\Gamma(1-\frac{\Delta}{2})^2} \geq 0$$

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Non-integrable QFTs at $T > 0$ are described by hydrodynamics at late times

$$t \gtrsim \tau_{\mathrm{eq}}$$

At weak coupling $\tau_{\mathrm{eq}} \gg \frac{1}{T}$; at strong coupling one expects $\tau_{\mathrm{eq}} \sim \frac{1}{T}$

Conjectured bound $\tau_{\mathrm{eq}} \gtrsim \frac{1}{T}$ Sachdev '99, Zaanen '04

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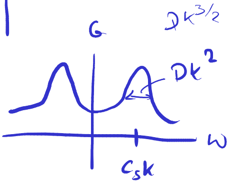
A linearized hydrodynamic treatment produces Kadanoff Martin '63

$$G^R_{\mathcal{T}_{00}\mathcal{T}_{00}}(\omega, k) \simeq \chi_{\varepsilon\varepsilon} \frac{c_s^2 k^2}{c_s^2 k^2 - \omega^2 - iD\omega k^2} \, ,$$



with $\chi_{\varepsilon\varepsilon} = T\partial_T \varepsilon$. Wightman function:

$$\begin{aligned} \langle \mathcal{T}_{00}\mathcal{T}_{00}\rangle(\omega, k) &\simeq 2T\chi_{\varepsilon\varepsilon} \frac{D c_s^2 k^4}{(\omega^2 - c_s^2 k^2)^2 + (D\omega k^2)^2} \\ &\simeq \frac{T\chi_{\varepsilon\varepsilon}}{Dk^2}\left[g_{\rm diff}\left(\frac{\omega + c_s k}{\frac{1}{2}Dk^2}\right) + g_{\rm diff}\left(\frac{\omega - c_s k}{\frac{1}{2}Dk^2}\right)\right] \end{aligned}$$

$$g_{\rm diff}(x) = \frac{1}{1+x^2}$$

BUT: hydrodynamic fluctuations are large in $d = 2$

Forster Nelson Stephen '77, Narayan Ramaswamy '02, Spohn '14

Loop correction to sound attenuation is $\quad d \geq 3 \quad d = 2$

$$D(\omega) = D + |\omega|^{\frac{d-3}{2}} + \cdots$$

$\langle T T \rangle =$ 

Perturbative expansion $\delta T + \delta T^2 + \cdots$ is uncontrolled

Diffusive universality replaced by KPZ universality

$$\langle \mathcal{T}_{00} \mathcal{T}_{00} \rangle(\omega, k) \simeq \frac{T\chi_{\varepsilon\varepsilon}}{\mathcal{D}k^{3/2}} \left[g_{\rm KPZ}\left(\frac{\omega + c_s k}{\mathcal{D}k^{3/2}}\right) + g_{\rm KPZ}\left(\frac{\omega - c_s k}{\mathcal{D}k^{3/2}}\right)\right]$$

with $\mathcal{D}$ fixed by the equation of state

$D \sim P''(T)$

Contents

Causality bound on hydrodynamics

Diffusion cannot emerge too soon, or it would be acausal

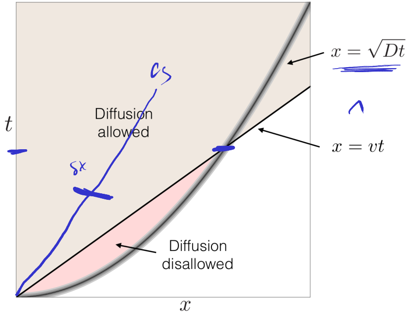
$$\tau_{\mathrm{eq}} \gtrsim \frac{D}{c^2}$$

Geroch '95, Kostadt Liu '01

Baier Romatschke Son Starinets Stephanov '07

Hartman Hartnoll Mahajan '17

τ_{eq}



For collective excitations with dispersion relation $\omega = c_s k - i\mathcal{D}k^z$, correlators are peaked at $x = c_s t$ with width $\delta x \sim (\mathcal{D}t)^{1/z}$ $< (c - c_s)t$

If $z > 1$, such a mode cannot emerge too soon

$$\tau_{\mathrm{eq}} \gtrsim \frac{\mathcal{D}^{1/(z-1)}}{(c-c_s)^{z/(z-1)}}$$

KPZ: $z = 3/2$ $\qquad \tau_{eq} \gtrsim \frac{D^2}{(1-c_s)^3}$.

One finds that any 2d QFT must satisfy

$$\tau_{\text{eq}} \gtrsim \boxed{\frac{1}{T}}\,\frac{1}{s_o}\underbrace{\left[\frac{1}{2}\frac{1}{(1-c_s)^3}\left(\frac{d\log c_s}{d\log s}+1-c_s^2\right)^2\right]}_{\text{large @ high/low } T.}$$

"Planckian"

weak @ large N

At high T, using conformal PT, this becomes

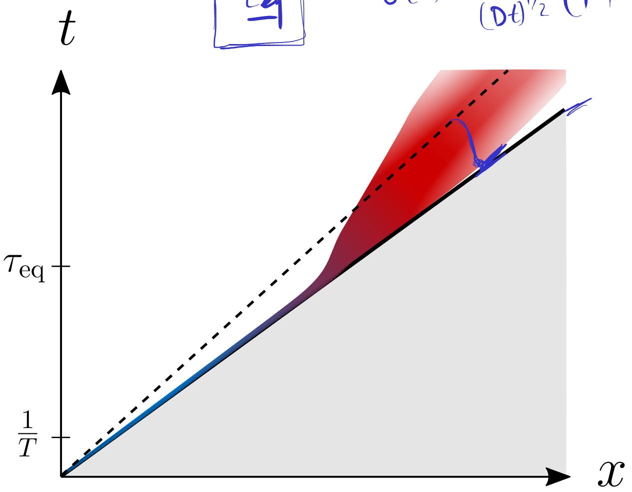
$$\underline{\tau_{\text{eq}} \gtrsim \frac{1}{T}\frac{1}{c_{\text{UV}}}\left(\frac{T}{\Lambda}\right)^{2(2-\Delta)}}$$

$T \gg \Lambda$.

2d QFTs typically thermalize slowly

$\boxed{\tau_{eq}}$

$$G(t) \sim \frac{1}{(Dt)^{1/2}}\left(1 + \frac{\tau_{eq}}{t} + \dots\right)$$



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Bonus topics

- **Higher dimensions**
  No causality tension for $d > 2$. One can in fact show that $c_s^2 \to \frac{1}{d-1}$ can be approached from above or below as $T \to \infty$.

- **Finite density**
  Generalization to QFTs with global symmetries, even with $\mu > 0$

- **'Swampland' bounds**
  Causality $\Rightarrow$ CFT + $\lambda \int T\bar{T}$ + ... cannot be UV completed if $\lambda < 0$

- **Large c and holography**
  $\boxed{\tau_{eq} \gtrsim \frac{D}{(1-c_s)^2}}$ $\quad c_s \to 1$
  Different bounds when $c \gg 1$ – hydrodynamic fluctuations are suppressed, but causality still leads to strong constraint
  $D \sim 1/T$ $\quad \boxed{\tau_{eq} \sim 1/T}$
  $D \ll 1/T$

- **A special case: free scalar $+\phi^4$**
  Thermal physics described by strongly coupled zero-mode living in 0+1d ('Linde problem')

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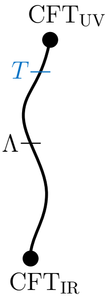
EFT BOUND

At high T we found: $s_o(T) = \frac{\pi}{3} c_{\mathrm{UV}} \left[1 - \alpha_\Delta \frac{\lambda^2}{T^{2(2-\Delta)}} + \cdots \right]$

with $\alpha_\Delta = \frac{\Gamma(2-\Delta)\Gamma(\frac{\Delta}{2})^2}{\Gamma(\Delta)\Gamma(1-\frac{\Delta}{2})^2} \geq 0$ (for $\Delta \leq 2$).



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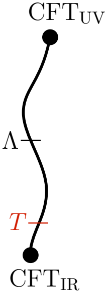

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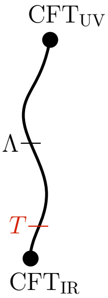
At low T: $\mathrm{QFT} = \mathrm{CFT}_{\mathrm{IR}} + \lambda \int \mathcal{O} + \cdots$

Conformal perturbation theory gives the same result

$$P = \frac{\pi}{6} c_{\mathrm{IR}} T^2 - \lambda \langle \mathcal{O} \rangle_\beta + \frac{\lambda^2}{2} \int d^2x \, \langle \mathcal{O}(x)\mathcal{O} \rangle_\beta + \cdots ,$$

with now $\Delta \geq 2$.

Positive correction to $s_o(T)$ if $3 \geq \Delta \geq 2$, but not for $\Delta > 3$!



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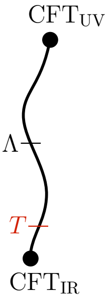

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Positive correction to $s_o(T)$ if $3 \geq \Delta \geq 2$, but not for $\Delta > 3$!

$\mathcal{T}\overline{\mathcal{T}}$ saves the day: $\langle \mathcal{T}\overline{\mathcal{T}} \rangle_\beta = c_{\rm IR}^2 T^4$

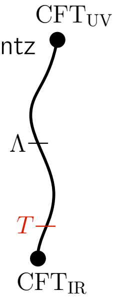
$$s_o(T) = \frac{\pi}{3} c_{\rm IR} \left[1 + \lambda_{\mathcal{T}\overline{\mathcal{T}}} c_{\rm IR} T^2 + \cdots \right] \qquad \text{causality} \Leftrightarrow \lambda_{\mathcal{T}\overline{\mathcal{T}}} > 0$$

Zamolodchikov '04, Cardy '15

Bottomline:

Subluminality of sound implies that any* 2d IR CFT with a Lorentz invariant UV completion must have $\lambda_{\mathcal{T}\overline{\mathcal{T}}} > 0$

($*$ with no irrelevant scalar $2 \leq \Delta \leq 3$)



IR consistency

Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06 $\partial\phi^4$

Camanho Edelstein Maldacena Zhiboedov '14

Also: Aoki, Bellazzini, Caron-Huot, de Rham, Hinterbichler, Huang, Joyce, Khoury, Kundu, Melville, Noumi, Pajer, Penedones, Remmen, Riva, Serra, Sgarlata, Shiu, Tolley, Trodden, Zhang, Zhou, (+ many more...)

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IR consistency

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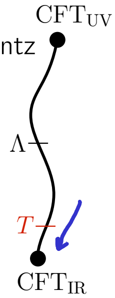
Here: the IR theory can be strongly interacting!

(Most) 2d CFTs are 'at the edge' of allowed theories

Is that also the case for higher dimensional CFTs?



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