[2105.08207]

Half-Wormhole in SYK w/ one time point

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(IAS)

CERN, Gauge/Gravity Duality.

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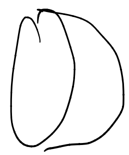
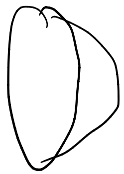
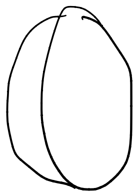
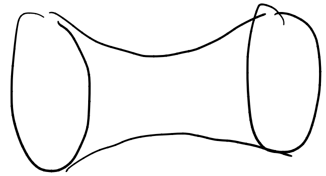
$Z(\beta) = \mathrm{Tr}\, e^{-\beta H} =$ β 

$Z(\beta_1)\, Z(\beta_2) =$ β_1   β_2 $+$ β_1  β_2

$\Rightarrow$ Factorization problem

Maldacena, Maoz

No sharp paradox :

- In UV-complete (by string theory) examples wormhole saddles are subdominant
  or
  unstable

- In ad hoc low energy models can be dominant

Maldacena, Maoz ; Marolf, Santos

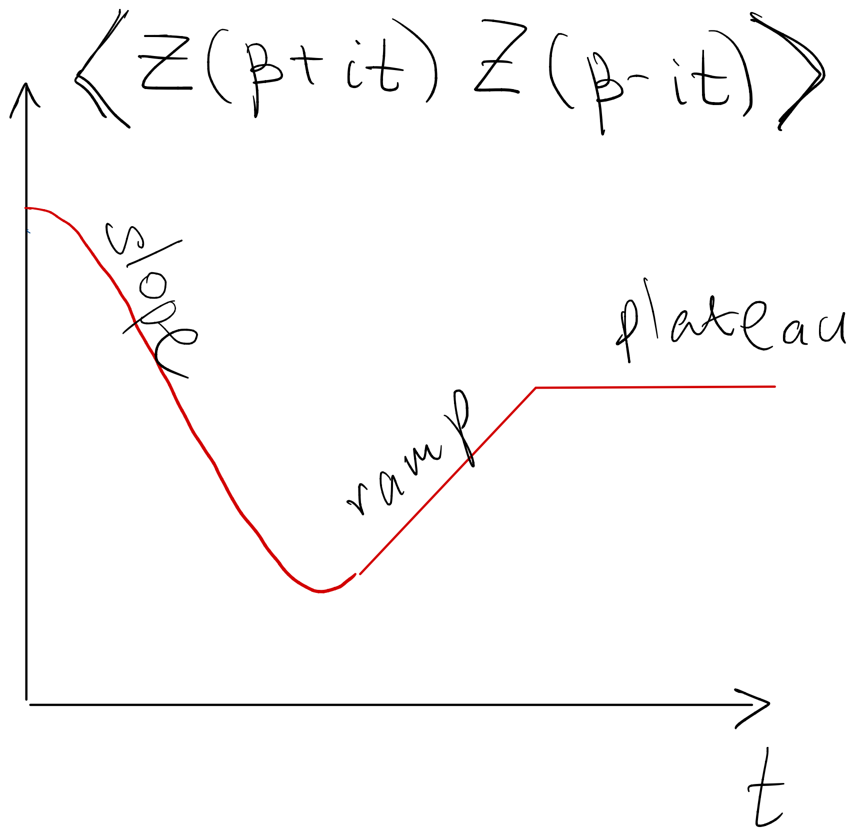
Wormholes are good

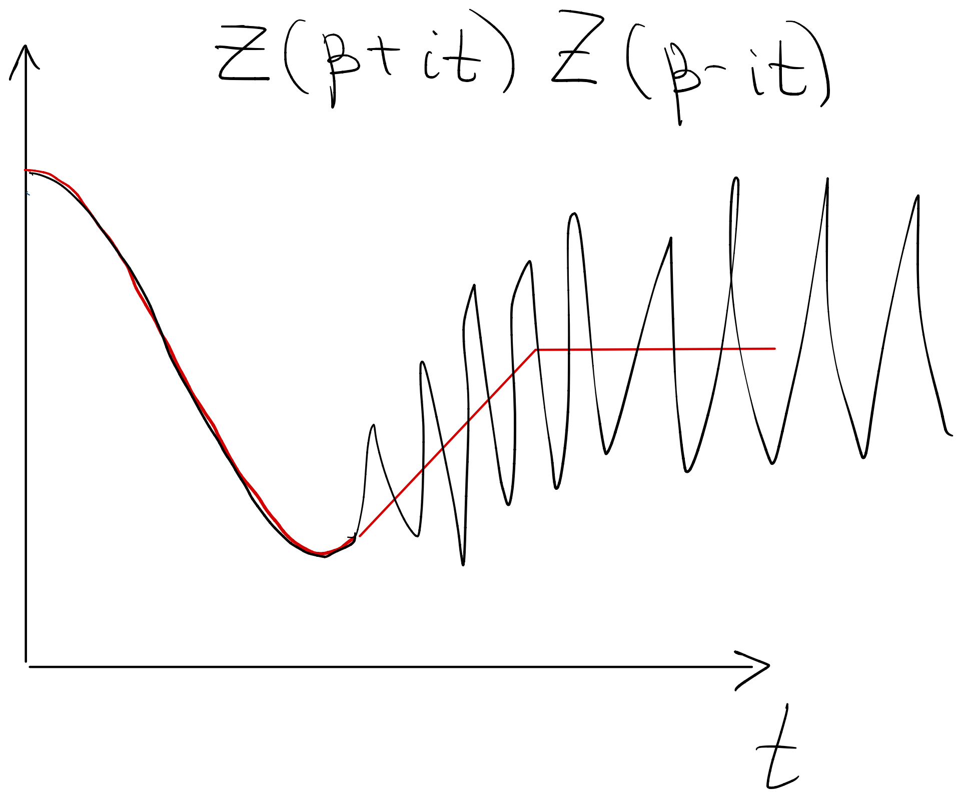
Explain long-time bulk physics:

- Spectral form factor — Saad, Shenker, Stanford; Cotler, Jensen; ....
- Correlators — Saad; Blommaert; ... (see talk by Sarosi)
- Page curve — Penington, Shenker, Stanford, Yang; Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini

(see talk by Karch)

What is the mechanism for getting a factorized answer?





$$Z(\beta+it)Z(\beta-it) \approx \underbrace{\langle Z(\beta+it)Z(\beta-it)\rangle}_{\text{wormhole}} + \text{oscillatory tems}$$

SYK w/ one time point

Saad, Shenker, Stanford, Yao; BM

$$Z = \int d^N\psi \, \exp\left\{ i^{q/2} \sum_{1 \le a_1 < \dots < a_q \le N} J_{a_1 \dots a_q} \psi_{a_1 \dots a_q} \right\}$$

$$\left(\psi_{a_1 \dots a_q} \equiv \psi_{a_1} \dots \psi_{a_q} \right)$$

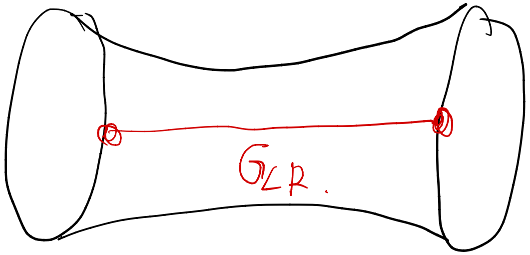
$$\langle J_{a_1 \dots a_q} J_{b_1 \dots b_q} \rangle = J^2 \delta_{a_1 b_1} \dots \delta_{a_q b_q}$$

$$J^2 = \frac{(q-1)!}{N^{q-1}}$$

- Study $Z_L Z_R$ w/ fixed couplings

- Introduce collective fields G_{LR}, Σ_{LR}

$$G_{LR}, \Sigma_{LR} \neq 0 \rightsquigarrow \text{wormholes}$$

Z_L  Z_R.

$$G_{LR} = \frac{1}{N} \sum \psi_i^L \psi_i^R$$

Saad
Shenker
Stanford
Yao.

Result: 2 saddles at large N

$$\boxed{Z_L Z_R \simeq \langle Z_L Z_R \rangle + \Phi(0)}$$

"wormhole" "half-wormhole"

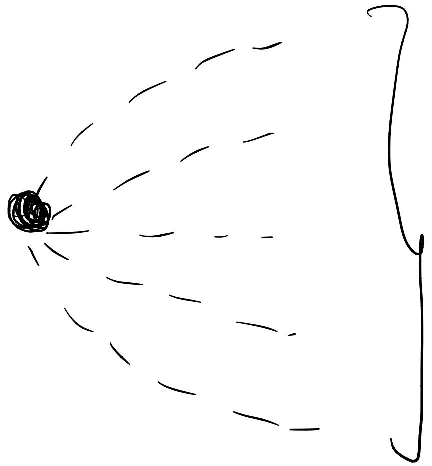
* depends strongly on exact couplings
* restores factorization

$\ast$

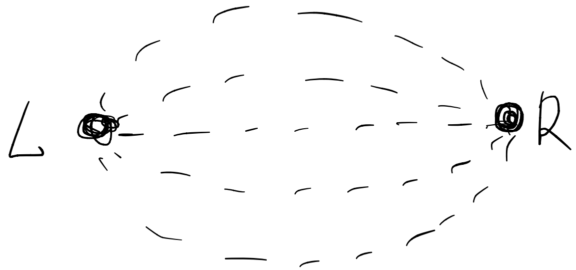
$$Z = \sum \operatorname{sgn}(a)\, J_{a_1 \dots a_q} \cdots J_{a_{N-(q-1)} \cdots a_N}$$

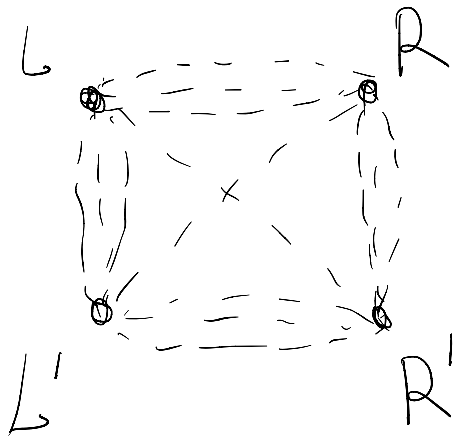
$$\equiv \mathrm{Pf}(J) \qquad \text{(Hyperpfaffian)}$$

$$=$$



N/q

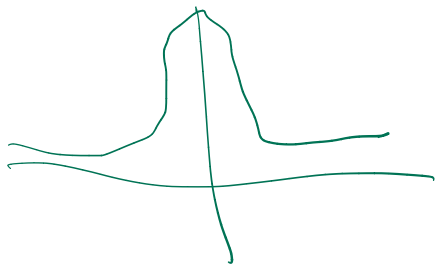
* $\langle Z^2 \rangle =$  $\simeq \sqrt{q}\, e^{-(1-\frac{1}{q})N} \left(J^2\right)^{N/q}$

* $\langle Z^4 \rangle =$  $\simeq 3 \langle Z^2 \rangle$

Collective fields G, Σ in non-averaged theory.

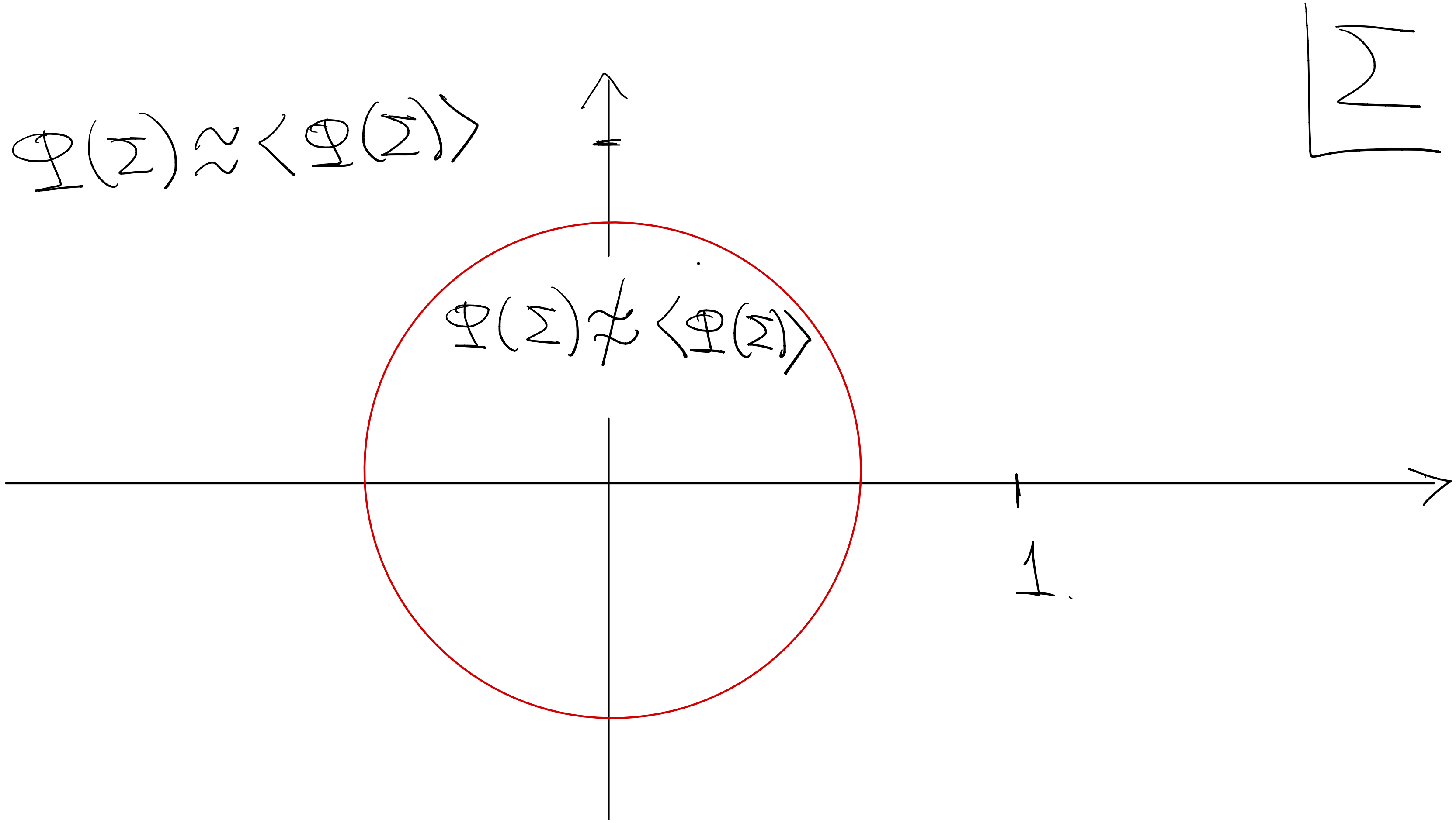
$$\boxed{Z_L Z_R = \int d\Sigma_{LR}\, \Psi(\Sigma)\, \Phi(\Sigma)}$$

$$\Psi(\Sigma) = \int dG_{LR}\, e^{N\left(-\Sigma G + \frac{1}{q} G^q\right)}$$



$$\Phi(\Sigma) = \int d\psi^{L,R} \exp\Big\{ \Sigma\, \psi^L_a \psi^R_a + i^{q/2} J_{a_1 \ldots a_q} \left(\psi^L_{a_1 \ldots a_q} + \psi^R_{a_1 \ldots a_q} \right) - i^q J^2 \psi^L_{a_1 \ldots a_q} \psi^R_{a_1 \ldots a_q} \Big\}$$

Is $\Phi(\Sigma)$ self-averaging?



$$Z^2 = \left(\int_{SA} + \int_{NSA} \right) \underline{\Psi}(\Sigma)\, \underline{\Phi}(\Sigma)$$

Wormhole saddle

$$Z^2 \supset \int_{SA} d\Sigma \; \Psi(\Sigma) \cdot \Sigma^N$$

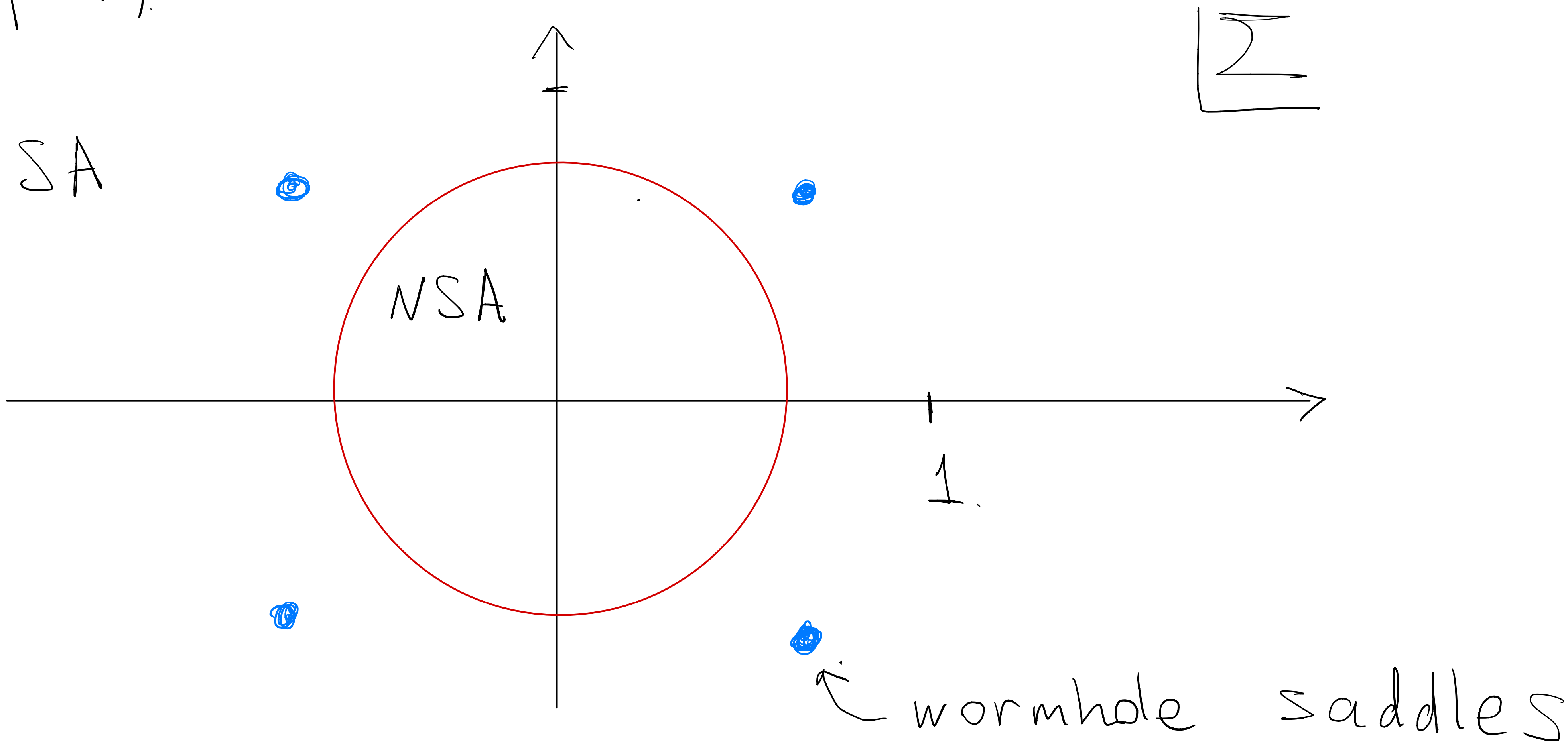
$$= \int_{SA} d\Sigma \, dG \; \exp\left\{ N\left(-\Sigma G + \frac{1}{q} G^q + \log \Sigma \right) \right\}$$

$$\Sigma: \quad = G^{q-1}$$

$$\Sigma: \quad = \frac{1}{G}$$

$$\Rightarrow \quad \boxed{\begin{aligned} G^q &= 1 \\ \Sigma &= \frac{1}{G} \end{aligned}}$$

$(q=4)$.

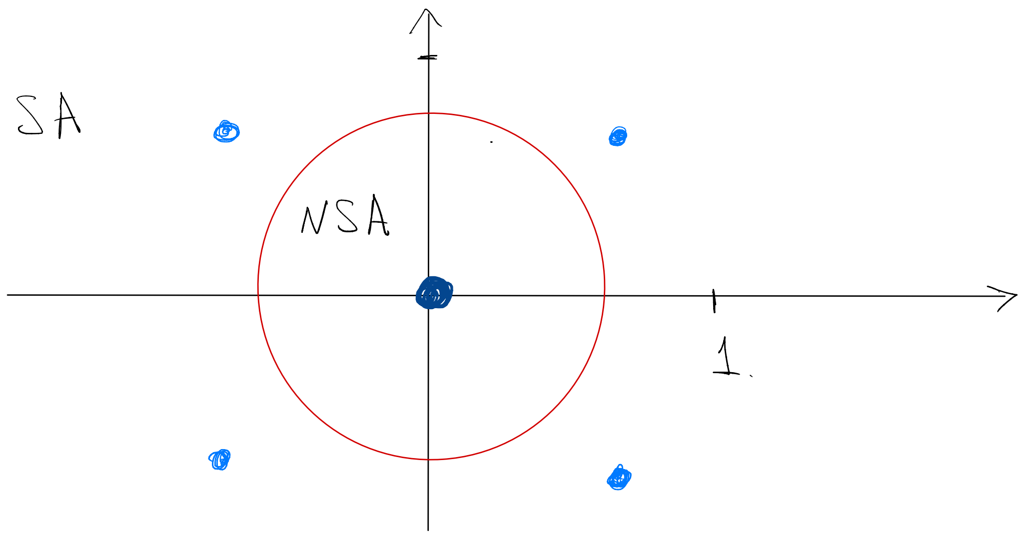


$\ast \quad Z^2 > \sqrt{q}\, e^{-\left(1-\frac{1}{q}\right)N} = \langle Z^2 \rangle$

Half-wormhole saddle

$$z^2 \supset \int_{NSA} d\Sigma \; \Psi(\Sigma)\, \Phi(\Sigma)$$

- $\Psi(\Sigma)$ – exp peaked near $\Sigma = 0$
- $\Phi(\Sigma) \approx \Phi(0)$ near $\Sigma = 0$



$$Z^2 \supset \Phi(0) \quad \text{from} \quad \text{NSA}$$

$$\boxed{Z^2 \approx \langle Z^2 \rangle + \Phi(0)}$$

$$\Phi(0) = \int d\psi^{L,R} \exp\left\{ i^{q/2} J_{a_1 \dots a_q} \left(\psi^L_{a_1 \dots a_q} + \psi^R_{a_1 \dots a_q} \right) - i^q J^2 \psi^L_{a_1 \dots a_q} \psi^R_{a_1 \dots a_q} \right\}$$

$$\left(A = \{a_1 < \dots < a_q\}\right)$$

$$\Phi(0) = \sum_{\substack{A_1 < \dots < A_{N/q} \\ B_1 < \dots < B_{N/q}}}' \operatorname{sgn}(A)\operatorname{sgn}(B) \; :J_{A_1} J_{B_1}: \dots :J_{A_{\frac{N}{q}}} J_{B_{\frac{N}{q}}}:$$

$$:J_A J_B: = J_A J_B - J^2 \delta_{AB}$$

$$\langle \Phi(0) \rangle = 0$$

Approximation is good for typical couplings.

$$\text{Error} = z^2 - \left(\langle z^2 \rangle + \Phi(0) \right)$$

$$\frac{\langle \text{Error}^2 \rangle}{\langle z^4 \rangle} = \frac{\#}{N^{q-2}} \qquad q > 2$$

Perturbation theory near half-wormhole to all orders

$$z^2 = \int d\Sigma \; \Psi(\Sigma)\, \Phi(\Sigma')$$

$$\Phi(\Sigma') = \sum_{k=0}^{N/q} \frac{\Sigma'^{kq}}{(kq)!}\, \Phi^{(kq)}(0)$$

$$\boxed{\begin{aligned} z^2 &= \sum_{k=0}^{N/q} \frac{1}{k!}\left(J^2\right)^k \left(-\frac{\partial}{\partial J^2}\right)^k \Phi(0) \\ &= \Phi(0) + \dots \end{aligned}}$$

Wormhole is a large fluctuation around half-wormhole

$$\frac{1}{(N/q)!}\left(J^2\right)^{N/q}\left(-\frac{\partial}{\partial J^2}\right)^{N/q}\Phi(0) = \langle z^2 \rangle$$

Eberhardt observed similar phenomena in tensionless string half-wormhole $\qquad\qquad\qquad$ wormhole.

$$Z^2 = \Phi_0 + \dots + \Phi_{N/q}$$

$$\frac{\langle \Phi_k^2 \rangle}{\langle Z^4 \rangle} \propto \frac{1}{N^{k(q-2)}}$$

$N \gg 1$

k - fixed.

$$\frac{\langle \Phi_{\frac{N}{q}-k}^2 \rangle}{\langle Z^4 \rangle} \propto \frac{1}{N^{k(q-2)}}$$

Open problems

* 1d gravity dual of the toy model?

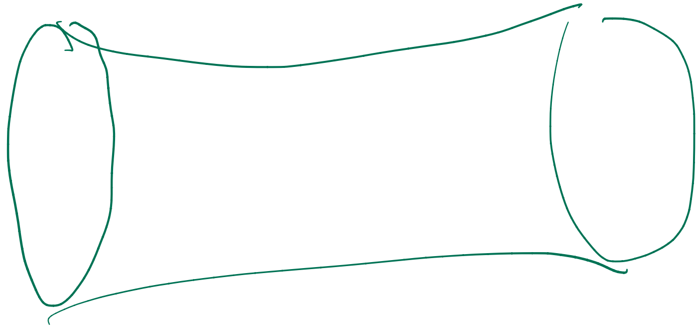
  Casali, Marolf, Maxfield, Rangamani

* HW in (brownian) SYK — Saad, Shenker, Stanford, Yao

  matrix models — Blommaert, Kruthoff

  JT

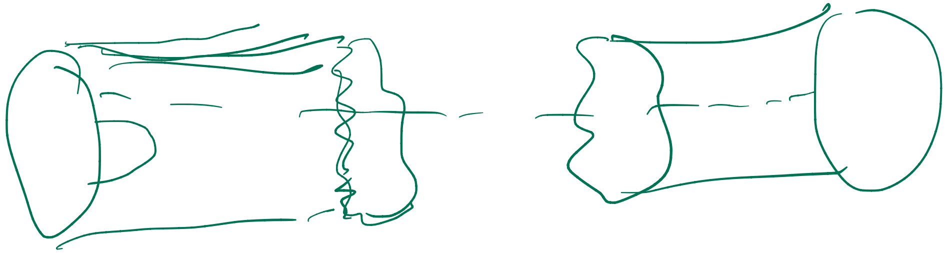
* HW, fixed couplings $\overset{?}{\rightsquigarrow}$ Black Hole interior — Chen, Maldacena



G_{LR}, G_{LL}, G_{RR}.

$G_{LR} = 0$, $\Sigma_{LR} = 0$

G_{LL}, G_{RR}



THANKS

FOR YOUR ATTENTION!

Why is $\Phi(0)$ a "half-wormhole"?

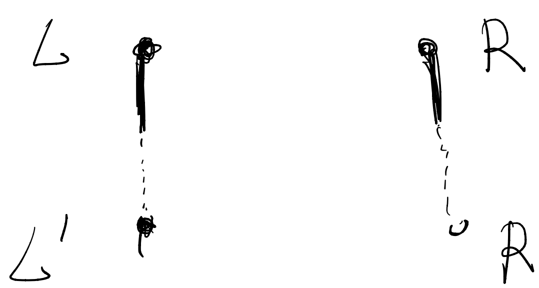
$$\Phi(0) \simeq \sqrt{\langle \Phi(0)^2 \rangle} \qquad \langle \Phi(\Sigma) \rangle = \Sigma^N$$

$$\langle \Phi(0)^2 \rangle = \int d\Sigma_\alpha \, dG_\alpha \times$$

$$\exp\left\{ N\left(\log\left(-\Sigma_{LL'}\Sigma_{RR'} + \Sigma_{LR'}\Sigma_{RL'}\right) - \underline{\Sigma_\alpha G_\alpha} + \underline{\frac{1}{q}} G_\alpha^q \right) \right\}$$

$$\alpha \in \{ LL', RR', LR', RL' \}$$

$$\langle \Phi(0)^2 \rangle \simeq$$



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