Hyperasymptotic approximation to the pole mass

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work in collaboration with
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  - The mixing between perturbative and NP effects may hinder estimating the real size of NP effects

General Expression

The perturbative part is calculated using the PV prescription, which is scale and scheme independent, within an hyperasymptotic expansion.

$$\mathrm{Obs}\left(\frac{Q}{\Lambda_{\mathrm{QCD}}}\right) = S_{\mathrm{PV}}(\alpha(Q)) + K_X^{(\mathrm{PV})}\alpha_X^{\gamma}(Q)\frac{\Lambda_X^d}{Q^d}\left(1+\mathcal{O}\left(\alpha_X(Q)\right)\right) + \mathcal{O}\left(\Lambda_X^{d'}/Q^{d'}\right) \quad (1)$$

- γ is the anomalous dimension of the local operator of dimension d,
- X represents the renormalization scheme ($X = \overline{MS}$, latt),
- $K_X^{(\mathrm{PV})}$ is the genuine NP contribution in OPE [PRD99,no.7,074019(2019)],
- The last term refers to higher order terms in the OPE ($d' > d > 0$).
- S_{PV} corresponds to the Principal Value (PV) of the inverse Borel transform.

General Expression

We have considered two alternatives that depend on the choice of truncation order of the perturbative sum,

1) N and $\mu \sim Q$ large but finite:

$$N = N_P \equiv |d| \frac{2\pi}{\beta_0 \alpha_X(\mu)} \left(1 - c\, \alpha_X(\mu)\right), \tag{2}$$

2) $N \to \infty$ and $\mu \to \infty$ in a correlated way:

$$N = N_A(\alpha) \equiv |d| \frac{2\pi}{\beta_0 \alpha_X(\mu)} \left(1 - c' \alpha_X(Q)\right), \tag{3}$$

where $c' > 0$, and $c \in \mathbb{R}$. d can be positive or negative

General Expression

In case 1), the general expresion for the PV part can be written as:

$$S_{\rm PV}(Q) = S_P + \sum_{\{|d|\}} S_{|d|} + \sum_{\{d>0\}} \Omega_d + \sum_{\{d<0\}} \Omega_d \ , \qquad (4)$$

with

$$S_P = \sum_{n=0}^{N_P(|d_{min}|)} p_n \alpha^{n+1}(\mu) \, , \qquad S_{|d|} = \sum_{n=N_P(|d|)+1}^{N_P(|d'|)} (p_n - p_n^{(as)}) \alpha^{n+1}(\mu) \, , \quad (5)$$

where $p_n^{(as)}$ reflects the asymptotic behavior associated to renormalons with dimensions $\leq |d|$, and $|d'| > |d|$ is next renormalon of the dimension of the closest renormalon to the origin in the Borel plane.

Heavy quark masses

$$m_{\mathrm{PV}}(\overline{m}) = m_P + \overline{m}\Omega_m + \sum_{n=N_P+1}^{2N_P} (r_n - r_n^{(as)})\alpha^{n+1}(\mu) + \overline{m}\Omega_2 + \overline{m}\Omega_{-2} + \cdots \, , \tag{6}$$

where $m_P \equiv \overline{m} + \sum_{n=0}^{N_P} r_n \alpha^{n+1}(\mu)$; the coefficients r_n are known up to $n = 3$ calculated through

$$m_{\mathrm{OS}}^{(N)} = \overline{m} + \sum_{n=0}^{N} r_n \alpha^{n+1}(\mu) \tag{7}$$

Therefore, m_P is nothing but the pole mass truncated to order $N = N_P$. And

$$r_n^{(\mathrm{as})}(\mu) = Z_m^X \mu \left(\frac{\beta_0}{2\pi}\right)^n \sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)} \tag{8}$$

General Expression

And $(u = \frac{\beta_0}{4\pi} t = \frac{d}{2})$,

$$\Omega_{\substack{IR \\ UV}} = \int_0^\infty dt\, e^{-t/\alpha_s(\mu)} \sum_{n=N_P+1}^{\infty} \frac{r_n^{(\mathrm{as})}}{n!} t^n$$

$$\approx \sqrt{\alpha(\mu)} K_X^{(P, \substack{IR \\ UV})} \left(\frac{\mu}{Q}\right)^{\substack{+ \\ -}|d|} e^{\frac{-2\pi|d|}{\beta_0 \alpha(\mu)}} \left(\frac{\beta_0 \alpha(\mu)}{4\pi}\right)^{-b'} \{1 + \mathcal{O}\left(\alpha(\mu)\right)\}.$$

Ω_d is the terminant [R.B.Dingle,1973] of the perturbation series associated to the singularity located at $u = d/2$ in the Borel plane. For the case of IR (UV) renormalons $d > 0$ ($d < 0$) the general analytic expression of Ω_d can be found in [Phys.Rev.D99(2019)7,074019 (Phys.Rev.D101(2020)3,034002)].

Large β_0: $b = 0$

$$
\begin{aligned}
B[m_{\rm PV}^{(\beta_0)} - \overline{m}](u) \;=\; & \overline{m}\frac{C_F}{4\pi}\left[\left(\frac{\overline{m}^2}{\mu^2}\right)^{-u} e^{-c_{\overline{MS}}u} 6(1-u)\frac{\Gamma(u)\Gamma(1-2u)}{\Gamma(3-u)} \right. \\
& \left. -\frac{3}{u} + R(u)\right] \qquad (9)
\end{aligned}
$$

where $u = \frac{\beta_0}{4\pi}t$, and

$$
R(u) = \sum_{n=1}^{\infty} \frac{1}{(n!)^2}\frac{d^n}{dz^n}G(z)\left|_{z=0} u^{n-1}\right. = -\frac{5}{2} + \frac{35}{24}u + \mathcal{O}(u^2) \qquad (10)
$$

$$
G(u) = -\frac{1}{3}(3+2u)\frac{\Gamma(4+2u)}{\Gamma(1-u)\Gamma^2(2+u)\Gamma(3+u)} \,. \qquad (11)
$$

Large β_0: Method (1)

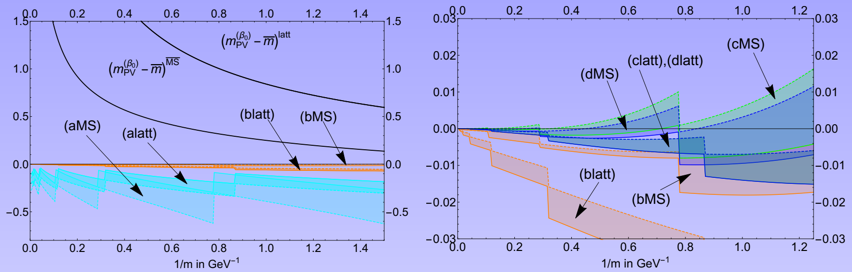

Figure: Comparison of lattice and $\overline{MS}$ scheme results for $n_f = 3$ in method 1).
Left panel: We plot $m_{\rm PV}$ and the differences: (a) $m_{\rm PV} - m_P$, and (b) $m_{\rm PV} - m_P - \overline{m}\Omega_m$ in the lattice and $\overline{MS}$ scheme with $n_f = 3$ light flavours.
Right panel: as in the left panel, including (c) $m_{\rm PV} - m_P - \overline{m}\Omega_m - \sum_{n=N_P+1}^{2N_P}(r_n - r_n^{(\rm as)})\alpha^{n+1}$, and (d) $m_{\rm PV} - m_P - \overline{m}\Omega_m - \sum_{n=N_P+1}^{2N_P}(r_n - r_n^{(\rm as)})\alpha^{n+1} - \overline{m}\Omega_{-2}$

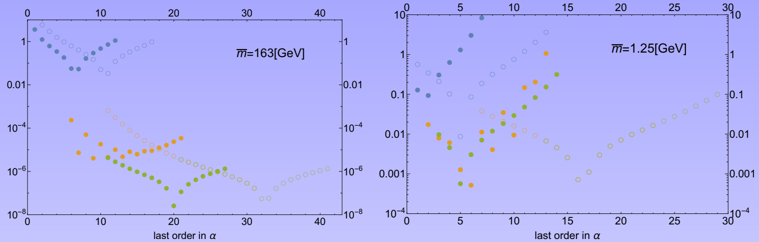

Figure: $|m_{\rm PV} - m_{\rm PV}^{\rm Hyperas.}|$ for $\overline{m} = 163$ GeV (left panel) and $\overline{m} = 1.25$ GeV (right panel). Blue Points are $|m_{\rm PV} - m_N|$. Orange points are $|m_{\rm PV} - m_P - \overline{m}\Omega_m - \sum_{n=N_P+1}^{N}(r_n - r_n^{\rm (as)})\alpha^{n+1}|$ with $c = 1.21/1.39$ and $c = 1.36/0.11$. Green points are $|m_{\rm PV} - m_P - \overline{m}\Omega_m - \sum_{n=N_P+1}^{2N_P}(r_n - r_n^{\rm (as)})\alpha^{n+1} - \bar{m}\Omega_{-2} - \sum_{n=2N_P+1}^{N}(r_n - r_n^{\rm (as)})\alpha^{n+1}|$, where in the last sum the two first renormalons are subtracted. Change of color correspond to the inclusion of Ω_m and Ω_{-2}. Full points have been computed in the $\overline{MS}$ scheme and empty points in the lattice scheme. We work with $n_f = 3$.

$\overline{\Lambda}_{\rm PV}$ from lattice

The energy of a meson made of static quark and a light valence quark is given by

$$E_{\rm MC}(a) = \delta m_{\rm latt}^{\rm PV} + \overline{\Lambda}_{\rm PV} + \mathcal{O}\left(a\Lambda_{\rm QCD}^2\right) \tag{12}$$

Then we can perform the hyperasymptotic expansion for method 1) $(c_n \equiv r_n/\nu)$

$$\overline{\Lambda}_{\rm PV}\left(n_f=0\right) = E_{\rm MC}(a) - \delta m_{\rm latt}^{P} - \frac{1}{a}\Omega_m - \sum_{N_P+1}^{N'=2N_P} \frac{1}{a}\left[c_n - c_n^{\rm (as)}\right]\alpha^{n+1} + \mathcal{O}(a) \tag{13}$$

with $\delta m_{\rm latt}^{P} = \sum_{n=0}^{N_P} \frac{1}{a} c_n \alpha^{n+1}$, where the coefficient c_n are known up to $n=19$; $E_{\rm MC}(a)$ is the Monte-Carlo lattice simulation [PRD51,5105(1995); NPPS42,385(1995); PRD54,3526(1996)], with the range of lattice spacing given by $1/a \sim 2.93 r_0^{-1} \div 9.74 r_0^{-1}$.

We can take only $N' = 2N_P$ terms due to our ignorance about the location of the next renormalon.

$\bar{\Lambda}_{\rm PV}$ from lattice

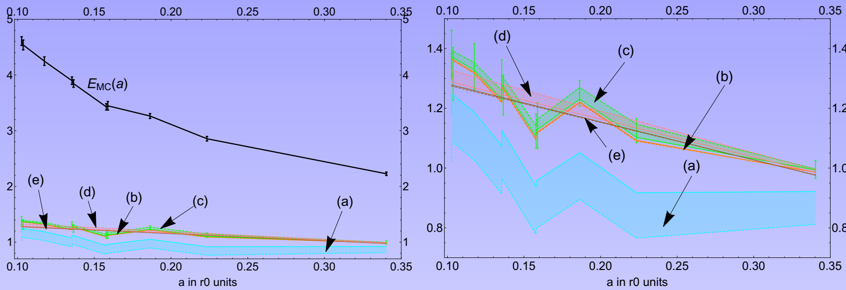

Figure: **Left panel**: The continuous lines are drawn to guide the eye. The other lines correspond to (13) truncated at different orders in the hyperasymptotic expansion. (a) $E_{\rm MC}(a) - \delta m_P(1/a)$, (b) $E_{\rm MC}(a) - \delta m_P(1/a) - \frac{1}{a}\Omega_m$, (c) $E_{\rm MC}(a) - \delta m_P(1/a) - \frac{1}{a}\Omega_m - \sum_{N_P+1}^{N'=2N_P} \frac{1}{a}[c_n - c_n^{(\rm as)}]\alpha^{n+1}$ (in this last case we include the error of the lattice points), (d) is the fit of the right-hand-side of (13) to $\bar{\Lambda}_{\rm PV}(n_f = 0) - Ka$. **Right panel**: As in the Left panel but in a smaller range. $r_0^{-1} \approx 400$ MeV.

$\overline{\Lambda}_{\rm PV}$ from lattice

$$\overline{\Lambda}_{\rm PV} = 1.42 r_0^{-1} (\text{stat.})^{-0.01}_{+0.04} (c)^{+0.05}_{-0.05} (Z_m)^{+0.16}_{-0.16} \tag{14}$$

Errors:

- (stat.) refers to statistical errors of the fit.
- Modulus of the difference with the evaluation with a negative c with smallest possible modulus.
- Normalization $Z_m^{latt} = 17.9(1.0)$ [PoSLATTICE2013,371(2014)], correlated with the error of c_n.
- $\alpha(1/a)$ comes from a phenomenologicall fit [NPB622,328(2002)], which is negligible (error associated to the fit is $0.5 - 1\%$).

$\overline{\Lambda}_{pot}^{\rm PV}$ from potential

We now use the knowledge of lattice data of $E_{\Sigma_g^+}(r_0; a)$ [PRD46,2636(1992); PRD56,2566(1997)], that is the ground state energy of two static sources in the fundamental representation at a fixed distance r_0. Then we define the quantity [PRD69,094001(2004)]

$$\overline{\Lambda}_{pot}(a) \equiv \frac{E_{\Sigma_g^+}(r_0; a)}{2} + \Delta \tag{15}$$

where Δ is a normalization constant. Following method 1), we can expand $\overline{\Lambda}_{pot}(a)$ within the PV prescription as

$$\overline{\Lambda}_{pot}^{\rm PV} \equiv \frac{E_{\Sigma_g^+}(r_0 \ ; a)}{2} + \Delta - \delta m_{\rm latt}^{P} - \frac{1}{a}\Omega_m - \sum_{N_{P+1}}^{N'=2N_P} \frac{1}{a}\left[c_n - c_n^{(\rm as)}\right]\alpha^{n+1} + \mathcal{O}(1/a) \tag{16}$$

$\overline{\Lambda}^{PV}_{pot}$ from potential

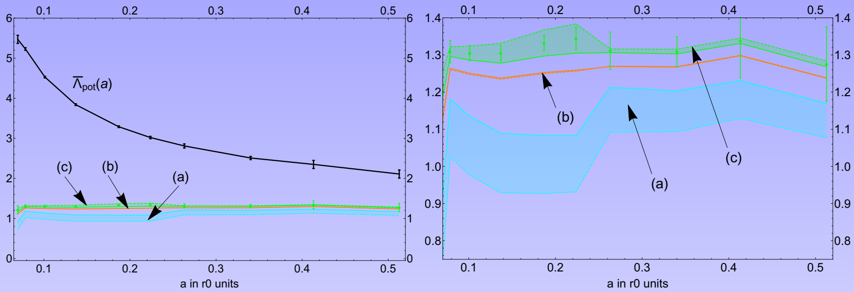

Figure: **Left panel**: The continuous lines are drawn to guide the eye. The other lines correspond to Eq.(16) truncated at different orders in the hyperasymptotic expansion: (a) $\bar{\Lambda}_{pot}(a) - \delta m_P(1/a)$, (b) $\bar{\Lambda}_{pot}(a) - \delta m_P(1/a) - \frac{1}{a}\Omega_m$, (c) $\bar{\Lambda}_{pot}(a) - \delta m_P(1/a) - \frac{1}{a}\Omega_m - \sum_{N_P+1}^{N'=2N_P} \frac{1}{a}[c_n - c_n^{(\mathrm{as})}]\alpha^{n+1}$ (in this last case we include the error of the lattice points). **Right panel**: As in the left panel but in a smaller range. $r_0^{-1} \approx 400$ MeV.

$\overline{\Lambda}_{\rm PV}$ from B meson

Within the HQEFT, the mass of a hadron containing a heavy quark can be represent as an expansion of the form (we have considered only the bottom quark, with $\overline{m}_b = 4.186$ GeV, and $m_B = 5.313$ GeV the spin-average value of B-meson)

$$M_B = m_{\rm PV} + \overline{\Lambda}_{\rm PV} + O\left(\frac{1}{m_{\rm PV}}\right) \tag{17}$$

following the method 1), we have

$$\overline{\Lambda}_{\rm PV} = M_{B/D} - m_{\rm P}\left(\overline{m}_{b/c}\right) - \overline{m}_{b/c}\Omega_m - \sum_{N_P+1}^{N'=2N_P}\left[r_n - r_n^{(\rm as)}\right]\alpha^{n+1} + \cdots \tag{18}$$

$\overline{\Lambda}_{\rm PV}$ from B meson

The central value is obtained taking $N_P = 3$ ($c = 0.3611$) (the last term in eq.(18) is set to zero, as we do not have more terms of the perturbative expansion).

$$\overline{\Lambda}_{\rm PV} = 477(\mu)^{-8}_{+17}\,(Z_m)^{+11}_{-12}\,(\alpha)^{-8}_{+9}\ \mathrm{MeV} \tag{19}$$

Errors

- The variation of μ was taken in the range $\mu \in (\overline{m}_b/2, 2\overline{m}_b)$.
- $Z_m^{\overline{MS}}(n_f = 3) = 0.5626(260)$ [JHEP1409,045(2014)].
- The variation of α is given by the "world average" value $\Lambda_{\overline{MS}}^{(n_f=3)} = 332 \pm 17$ MeV [PRD98.n°3,030001(2018)].
- The uncertainty related to the truncation N_P (from $N_P = 2$ to $N_P = 3$) is below 1 MeV.

Top Mass Uncertainty

We determine the precision of $\overline{m}_t$ if $m_{t,\mathrm{PV}}$ or $m_{t,P}$ is known (taking $\overline{m}_t = 163$ GeV as a reference) with nowadays knowledge of the pertubative expansion.
As $m_{\mathrm{PV}}(\overline{m}) - \overline{m}$ share the same renormalon, we can run the top mass in a renormalon free way until we reach a top mass low enough that we can use the hyperasymptotic expansion (scales smaller than charm quark mass).
The we defined

$$\begin{aligned} \mathcal{F}(\overline{m}, n_f) &\equiv \frac{d}{d\overline{m}}(m_{\mathrm{PV}}(\overline{m}) - \overline{m}) \simeq \frac{d}{d\overline{m}} \sum_{n=0} r_n^{(n_f)}(\overline{m}; \nu = \overline{m}) \alpha_{(n_f)}^{n+1}(\overline{m}) \\ &\equiv \sum_{n=1}^{N+1} f_n(\overline{m}) \left(\frac{\alpha_{(n_f)}(\overline{m})}{\pi} \right)^n . \end{aligned} \quad (20)$$

valid up to $N \sim 2N_P$ since $m\Omega_m$ and $r_n^{(as)}$ are $\overline{m}$-independent.

Top Mass Uncertainty

Considering heavy quarks with mases larger than $\Lambda_{\rm QCD}$, we have the general expression for PV-mass

$$m_{\rm PV}(\overline{m}) = \overline{m} + \sum_{n=0} r_n^{(n_f)}(\overline{m}; \nu = \overline{m})\alpha_{(n_f)}^{n+1}(\overline{m}) + \delta m_b^{(n_f)}(\overline{m}) + \delta m_c^{(n_f)}(\overline{m}) + \delta m_{bc}^{(n_f)}(\overline{m}) . \qquad (21)$$

Taking the decoupling of bottom and charm into account for lower values of $\overline{m}_t$ (absorved in $\delta m_{b/c/bc}^{(n_f)}$), we obtain

$$m_{\rm PV}(\overline{m}_t) = \overline{m}_t + \int_{\mu_b}^{\overline{m}_t} d\overline{m} \left(\mathcal{F}(\overline{m},5) + \frac{d}{d\overline{m}}(\delta m_b^{(5)}(\overline{m}) + \delta m_c^{(5)}(\overline{m}) + \delta m_{(bc)}^{(5)}(\overline{m})) \right) + \int_{\mu_c}^{\mu_b} d\overline{m} \left(\mathcal{F}(\overline{m},4) + \frac{d}{d\overline{m}}(\delta m_b^{(4)}(\overline{m}) + \delta m_c^{(4)}(\overline{m}) + \delta m_{(bc)}^{(4)}(\overline{m})) \right) + m_{\rm PV}(\mu_c) - \mu_c \ . \qquad (22)$$

with

$$m_{\mathrm{PV}}\left(\mu_c\right)=m_P\left(\mu_c\right)+\mu_c\Omega_m+\cdots+\delta m_b^{(3)}\left(\mu_c\right)+\delta m_c^{(3)}\left(\mu_c\right)+\delta m_{(bc)}^{(3)}\left(\mu_c\right) \tag{23}$$

We can see the decoupling scale for each heavy quark, and estimate the associated error.

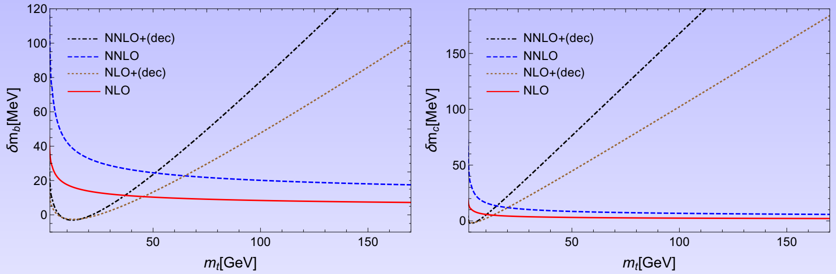

Figure: Plots of $\delta m_{b/c}$ as a function of $\overline{m}_t$.

Top Mass Uncertainty

Rewritten the pole mass as

$$m_{\mathrm{PV}}(\overline{m}_t) \equiv \overline{m}_t + \Delta\mathcal{F} + \Delta m_Q + m_{\mathrm{PV}}(\mu_c) - \mu_c \qquad (24)$$

we obtain for each contribution

$$\Delta\mathcal{F} = 9291(22)\ \mathrm{MeV} \qquad (25)$$

$$\Delta m_Q = -2.5|_{\mathcal{O}(\alpha^2)} + 0.8|_{\mathcal{O}(\alpha^3)} = -1.7\ \mathrm{MeV} \qquad (26)$$

$$m_{\mathrm{PV}}(5\mathrm{GeV}) = 5744(\mu)^{-7}_{+15}\,(Z_m)^{+9}_{-9}\ \mathrm{MeV} \qquad (27)$$

given

$$m_{t,\mathrm{PV}}(163\mathrm{MeV}) = 173.033(\alpha)^{+119}_{-123}\mathrm{GeV} \qquad (28)$$

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- The large β_0 was used as a toy-model observable. It works as expected (as in the static potential case).
- We see $u = 1/2$ IR renormalon and $u = -1$ UV renormalon. The latter heavily depends on the factorization scale μ used.
- We have performed determinations of $\bar{\Lambda}_{\rm PV}$ using quenched lattice QCD. For these observables we have perturbative expansions to high orders. This allows us to test the method and go beyond the superasymptotic and the leading term in the hyperasymptotic approximation. We observe $\mathcal{O}(a\Lambda_{\rm QCD}^2)$ corrections for the energy of B-meson like mass in the static approximation.

Conclusions

- We also determine $\bar{\Lambda}_{\mathrm{PV}}$ from the B meson mass assuming that the heavy quark mass is known. We determine the error associated to the incomplete knowledge of the perturbative expansion in determinations of the heavy quark mass.

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THANKS!

General Expression

And

$$\Omega_{\substack{IR\\UV}} = \int_0^\infty dt\, e^{-t/\alpha_s(\mu)} \sum_{n=N_P+1}^{\infty} \frac{r_n^{(\mathrm{as})}}{n!} t^n$$

$$\approx \sqrt{\alpha(\mu)} K_X^{(P,\substack{IR\\UV})} \left(\frac{\mu}{Q}\right)^{\substack{+\\-}|d|} e^{\frac{-2\pi|d|}{\beta_0\alpha(\mu)}} \left(\frac{\beta_0\alpha(\mu)}{4\pi}\right)^{-b'} \{1+$$

$$\bar{K}_{X,1}^{(P,\substack{IR\\UV})}\alpha(\mu) + \bar{K}_{X,2}^{(P,\substack{IR\\UV})}\alpha^2(\mu) + \mathcal{O}\left(\alpha^3(\mu)\right)\Big\}$$

$$\equiv \Delta\Omega_{\substack{IR\\UV}}(db) + c_1\Delta\Omega_{\substack{IR\\UV}}(db) + \omega_2\Delta\Omega_{\substack{IR\\UV}}(db) + \ldots$$

where

$$\Delta\Omega_{\substack{IR\\UV}}(db) = Z_{\mathcal{O}_d}^X \left(\frac{\mu}{Q}\right)^{\substack{+\\-}|d|} \frac{1}{\Gamma(1+b')} \left(\frac{\beta_0}{2\pi d}\right)^{(N_P+1)} \alpha_X^{(N_P+2)}(\mu) \times I_{\substack{IR\\UV}} \tag{29}$$

General Expression

$$I_{\substack{IR\\UV}} = \int_0^\infty dx \; x^{N_P+1+b'} \frac{e^{-x}}{1 \mp x^{\frac{\beta_0 \alpha_X(\mu)}{2\pi|d|}}} \tag{30}$$

In case 2), it is possible to show that:

$$S_{\mathrm{PV}}(Q) = S_A + \int_0^{\frac{4\pi}{\beta_0 \chi}} dt e^{-t/\alpha_X(\overline{m})} B[S_{\mathrm{PV}} - S_A](t)\,, \tag{31}$$

where

$$S_A = \sum_{n=0}^{N_A(|d_{min}|)} p_n \alpha^{n+1}(\mu)\,. \tag{32}$$

General Expression

$$m_{\mathrm{PV}} = m_A + K_X^{(A)} \Lambda_X + \mathcal{O}(\alpha \Lambda_X)\,, \tag{33}$$

where

$$m_A = \overline{m} + \lim_{\mu \to \infty;2)} \sum_{n=0}^{N_A} r_n \alpha^{n+1}(\mu) \tag{34}$$

and

$$K_X^{(A)} = \frac{2\pi}{\beta_0} Z_m^X \left(\frac{\beta_0}{4\pi}\right)^b \int_{-c',\mathrm{PV}}^{\infty} dx\, e^{\frac{-2\pi\, dx}{\beta_0}} \frac{1}{(-x)^{1+b}}\,. \tag{35}$$

It is also possible to show that

$$m_A = \overline{m} + \int_0^{\frac{4\pi}{\beta_0 \chi}} dt e^{-t/\alpha_X(\overline{m})} B[m_{\mathrm{PV}} - \overline{m}](t)\,. \tag{36}$$

Large β_0: comparing method (1) and (2)

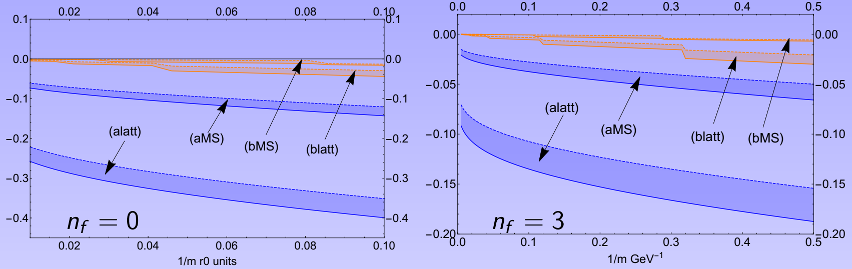

Figure: We plot (a) $m_{\rm PV} - m_A - K_X^{(A)}\Lambda_X$ for $n_f = 0$ (left panel) and $n_f = 3$ (right panel) in the lattice and $\overline{MS}$ scheme. For each case, we generate bands by computing m_A with $c' = 1$ and $c' = c'_{\rm min}$. We also compare with (b) $m_{PV} - m_P - \overline{m}\Omega_m$ obtained with method 1).