Five-loop QCD effects to the relation between the pole and running heavy quark masses within different estimating procedures and probe of extra theoretical uncertainties

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Outline

- $\overline{\text{MS}}$-on-shell heavy quark mass relation: the known four-loop results
- Motivation to study the contributions of the 5-th order: progress in their consideration starting from 2019 (previous status was summarized by A. S. Kronfeld at QWG-2019 in Torino)
- Approaches to probe the unknown higher-order PT effects
  - The renormalon-based analysis
  - The Naive-Nonabelianization procedure
  - The effective charges approach
- Consideration of the asymptotic structure of the $\overline{\text{MS}}$-on-shell mass PT relation for the heavy c, b and t-quarks
- Conclusion

Talk is based on the papers of [Eur.Phys.J.C 80 (2020) 12, 1160, Kataev and Molokoedov]; [JETP Lett. 108 (2018) 12, 777-782, Kataev and Molokoedov]; [EPJ Web Conf. 191 (2018) 04005, Contribution to: Quarks 2018, Kataev and Molokoedov].

$\overline{\text{MS}}$-on-shell heavy quark mass relation

The pole mass M_q of the *heavy* quark is defined as a pole of the renormalized quark propagator on the mass shell (ON-SHELL). It can be related to the unrenormalized bare mass $m_{0,q}$ via

$$m_{0,q} = Z_m^{\text{OS}} M_q,$$

where the mass renormalization constant Z_m^{OS} is expressed through the one-particle irreducible self-energy quark operator at the Minkowskian region $p^2 = M_q^2$.

The running mass $\overline{m}_q(\mu^2)$ may be defined (also for light quarks) both in the Euclidean and in the Minkowskian domain within the $\overline{\text{MS}}$ subtraction scheme of UV divergences. Its relation with the bare mass:

$$m_{0,q} = Z_m^{\overline{\text{MS}}}\, \overline{m}_q(\mu^2)\,,$$

$$\frac{\overline{m}_q(\tilde{\mu}^2)}{\overline{m}_q(\mu^2)} = \exp\left(\int\limits_{\alpha_s(\mu^2)}^{\alpha_s(\tilde{\mu}^2)} dx\, \frac{\gamma_m(x)}{\beta(x)}\right)$$

$$\begin{cases} \beta(\alpha_s) = \mu^2 \dfrac{\partial a_s}{\partial \mu^2} = -\sum\limits_{i\geq 0} \beta_i \left(\dfrac{\alpha_s}{\pi}\right)^{i+2} & \text{— } \beta\text{-function (known up to 5-loop level)} \\ \gamma_m(\alpha_s) = \mu^2 \dfrac{\partial}{\partial \mu^2} \ln \overline{m}_q(\mu^2) = -\sum\limits_{i\geq 0} \gamma_i \left(\dfrac{\alpha_s}{\pi}\right)^{i+1} & \text{— anomalous mass dimension} \end{cases}$$

'17, Baikov, Chetyrkin, Kühn; Herzog et al.

'14, Baikov, Chetyrkin, Kühn; '17, Luthe et al.

$$a_s = \alpha_s/\pi.$$

$\overline{\text{MS}}$-on-shell heavy quark mass relation: the known four-loop results

$$\frac{M_q}{\overline{m}_q(\overline{m}_q^2)} = \frac{Z_m^{\overline{\text{MS}}}}{Z_m^{\text{OS}}} = 1 + \sum_{k\geq 1} t_k^M a_s^k(\overline{m}_q^2)\ ,$$

For the case of $SU(3)$ gauge group with n_l massless flavors (in loops) and single heavy quark:

$$t_1^M = \frac{4}{3} \quad \textit{('81, Tarrach)}$$

$$t_2^M = \frac{307}{32} - \frac{\zeta_3}{6} + \boxed{\frac{\pi^2}{3} + \frac{\pi^2 \ln 2}{9}} - \left(\frac{71}{144} + \boxed{\frac{\pi^2}{18}}\right) n_l \quad \textit{('90, Gray, Broadhurst et al.)}$$

$$t_3^M = \frac{8481925}{93312} + \frac{58}{27}\zeta_3 + \boxed{\frac{652841}{38880}\pi^2 - \frac{575}{162}\pi^2 \ln 2 - \frac{22}{81}\pi^2 \ln^2 2 - \frac{1439}{432}\pi^2\zeta_3 - \frac{695}{7776}\pi^4}$$
$$+ \frac{1975}{216}\zeta_5 - \frac{55}{162}\ln^4 2 - \frac{220}{27}\text{Li}_4\left(\frac{1}{2}\right) + \left(-\frac{231847}{23328} - \frac{241}{72}\zeta_3 + \frac{\ln^4 2}{81} + \frac{8}{27}\text{Li}_4\left(\frac{1}{2}\right)\right.$$
$$\left.\boxed{-\frac{991}{648}\pi^2 - \frac{11}{81}\pi^2 \ln 2 + \frac{2}{81}\pi^2 \ln^2 2 + \frac{61}{1944}\pi^4}\right) n_l + \left(\frac{2353}{23328} + \frac{7}{54}\zeta_3 + \boxed{\frac{13}{324}\pi^2}\right) n_l^2 \quad \textit{'00, Melnikov, Ritbergen}$$

The coefficients t_k^M contain $\boxed{\pi^2 - \text{terms}}$ appearing upon calculation of Z_m^{OS} in Minkowskian region.

$$t_2^M = -1.0414 n_l + 13.443,$$
$$t_3^M = +0.6527 n_l^2 - 26.655 n_l + 190.60,$$
$$t_4^M = -0.6781 n_l^3 + 43.396 n_l^2 - 745.72 n_l + 3567.61,$$

('16, Marquard et al.), where the central values of n_l and n_l^0-dependent terms are consistent with a number of other works.

Up to 4 order the coefficients t_k^M have sign-alternating structure in n_l. What will happen next?

Input data PDG(20)

$\overline{m}_c(\overline{m}_c^2) = 1.27 \pm 0.02$ GeV,

$\overline{m}_b(\overline{m}_b^2) = 4.18^{+0.03}_{-0.02}$ GeV,

$\overline{m}_t(\overline{m}_t^2) = 164.3 \pm 0.6$ GeV

$\alpha_s(M_Z^2) = 0.1179$ at $M_Z = 91.1876$ GeV

$\Longrightarrow$

$\Lambda^{(n_l=3)}_{\overline{\mathrm{MS}}} = 289$ MeV, $\alpha_s(\overline{m}_c^2) = 0.3929$,

$\Lambda^{(n_l=4)}_{\overline{\mathrm{MS}}} = 207$ MeV, $\alpha_s(\overline{m}_b^2) = 0.2246$,

$\Lambda^{(n_l=5)}_{\overline{\mathrm{MS}}} = 88$ MeV, $\alpha_s(\overline{m}_t^2) = 0.1083$.

$$\frac{M_c}{1\ \mathrm{GeV}} \approx 1.270 + \overset{\alpha_s}{0.212} + \overset{\alpha_s^2}{0.205} + \overset{\alpha_s^3}{0.289} + \overset{\alpha_s^4}{0.529}\,,$$
$$\frac{M_b}{1\ \mathrm{GeV}} \approx 4.180 + 0.398 + 0.198 + 0.144 + 0.135\,,$$
$$\frac{M_t}{1\ \mathrm{GeV}} \approx 164.300 + 7.552 + 1.608 + 0.496 + 0.195\,.$$

Asymptotic structure of the $M_c/\overline{m}_c(\overline{m}_c^2)$ is observed in the 2-nd order of PT.
For normalization $\mu = 3$ GeV $\Rightarrow \overline{m}_c(3\ \mathrm{GeV}) = 0.988$ GeV, $\alpha_s(3\ \mathrm{GeV}) = 0.2529$,
$M_c/1\ \mathrm{GeV} \approx 0.988 + 0.247 + 0.162 + 0.136 + 0.141$, **then the asymptotics is seen in the 3-rd order.**

To probe the asymptotic behavior for the case of b-quark it is necessary to estimate the value of the $\mathcal{O}(\alpha_s^5)$-term.

For the t-quark the study of this problem is even more intriguing.

- The pole mass of c-quark is poorly defined already in the 2-nd (3-rd) order. Check: Whether the estimation of the 5-th order contribution will further support this conclusion?

- For the b-quark the magnitude of the 5-th order PT correction is important for understanding when the asymptotics will manifest.

- When will the asymptotic structure of the relation $M_t/\overline{m}_t(\overline{m}_t^2)$ reveal itself?

The renormalon-based asymptotic formula

At large $k \gg 1$ the behavior of the coefficients of the Borel image of the relation $M_q/\overline{m}_q(\overline{m}_q^2)$ defines the behavior of t_k^M:

$$t_k^{M,\, r-n} \xrightarrow{k\to\infty} \pi N_m (2\beta_0)^{k-1} \frac{\Gamma(k+b)}{\Gamma(1+b)} \left(1 + \frac{s_1}{k+b-1} + \frac{s_2}{(k+b-1)(k+b-2)} + \frac{s_3}{(k+b-1)(k+b-2)(k+b-3)} + \mathcal{O}\left(\frac{1}{k^4}\right)\right)$$

'99, Beneke; '01, Pineda; '17, Beneke et al.; '17, Komijani

The coefficients t_k^M grow factorially with increasing k. $b = \beta_1/2\beta_0^2$.

Using results of the explicit calculations of t_3^M and t_4^M, one can obtain *'14, Ayala et al.; '17, Beneke et al.*

n_l	3	4	5	6	7	8
N_m, $[\mathcal{O}(a_s^3)]$	0.572	0.552	0.524	0.482	0.408	0.272
N_m, $[\mathcal{O}(a_s^4)]$	0.537	0.506	0.462	0.394	0.279	0.056

We use the values of N_m in the four-loop approximation to obtain the estimates of t_5^M and t_6^M. Accurate extraction of N_m-values is extremely important upon studying their flavor dependence.

The results in Table are in good agreement with ones following from the now known expression for N_m:

$$N_m = \frac{2\beta_0}{\pi}\Gamma(1+b)\sum_{k\geq 0}\frac{S_k}{\Gamma(1+b+k)}$$

('01, Pineda; '14, Ayala et al.; '18, Hoang et al.)

N_m, $[\mathcal{O}(a_s^4)]$	0.526 $\pm$ 0.012	0.492 $\pm$ 0.016	0.446 $\pm$ 0.024	0.381 $\pm$ 0.038	0.271 $\pm$ 0.063	0.053 $\pm$ 0.097

The Naive-Nonabelianization procedure

The Naive-Nonabelianization procedure is to replace $n_l \to -6\beta_0 = n_l - 31/2$ in contributions proportional to n_l^{k-1} in terms t_k^M. They were calculated numerically *('95, Ball, Beneke, Braun)*.

$$t_2^M = -1.0414 n_l + 13.443,$$
$$t_2^{M,NNA} \approx -1.0414\left(n_l - \frac{31}{2}\right) \approx -1.0414 n_l + 16.142;$$

$$t_3^M = 0.6527 n_l^2 - 26.655 n_l + 190.60,$$
$$t_3^{M,NNA} \approx 0.6527\left(n_l - \frac{31}{2}\right)^2 \approx 0.6527 n_l^2 - 20.234 n_l + 156.81;$$

$$t_4^M = -0.6781 n_l^3 + 43.396 n_l^2 - 745.72 n_l + 3567.61,$$
$$t_4^{M,NNA} \approx -0.6781\left(n_l - \frac{31}{2}\right)^3 \approx -0.6781 n_l^3 + 31.532 n_l^2 - 488.74 n_l + 2525.2.$$

The application of the NNA in the 5 and 6-th orders gives:

$$t_5^{M,\,NNA,\,\overline{m}} \approx 0.9 n_l^4 - 59 n_l^3 + 1469 n_l^2 - 16156 n_l + 66641,$$

$$t_6^{M,\,NNA,\,\overline{m}} \approx -1.5 n_l^5 + 125 n_l^4 - 4127 n_l^3 + 68088 n_l^2 - 561727 n_l + 1853698.$$

The NNA procedure leads to the sign-alternating flavor structure of these contributions.

The effective charges approach

The "effective" spectral density $T(s) = \overline{m}_q(s)\left(1 + \sum_{k=1}^{\infty} t_k a_s^k(s)\right)$, defined in the Minkowskian domain, enters in the following model dispersion relation *('97, Chetyrkin, Kniehl, Sirlin)*:

$$\boxed{F(Q^2) - F(0) = Q^2 \int_0^\infty ds \frac{T(s)}{(s+Q^2)^2} = \overline{m}_q(Q^2)\left(1 + \sum_{k\geq 1} f_k^E a_s^k(Q^2)\right)},$$

$$M_q \approx \frac{1}{2\pi i} \int_{-\overline{m}_q(\overline{m}_q^2)-i\epsilon}^{-\overline{m}_q(\overline{m}_q^2)+i\epsilon} ds' \int_0^\infty \frac{\overline{m}_q(s)(1+\sum_{k\geq 1} t_k^M a_s^k(s))}{(s+s')^2} ds\,,$$

Consideration of the effects of the running of $\alpha_s(\mu^2)$ and $\overline{m}_q(\mu^2)$ gives:

$$Q^2 \int_0^\infty ds \frac{\{1; l; l^2; l^3; l^4; l^5; l^6\}}{(s+Q^2)^2} = \Big\{1;\ \mathfrak{L};\ \mathfrak{L}^2 + \frac{\pi^2}{3};\ \mathfrak{L}^3 + \pi^2\mathfrak{L};\ \mathfrak{L}^4 + 2\pi^2\mathfrak{L}^2 + \frac{7\pi^4}{15};$$

$$\mathfrak{L}^5 + \frac{10}{3}\pi^2\mathfrak{L}^3 + \frac{7}{3}\pi^4\mathfrak{L};\ \mathfrak{L}^6 + 5\pi^2\mathfrak{L}^4 + 7\pi^4\mathfrak{L}^2 + \frac{31}{21}\pi^6\Big\},$$

where $l = \log(\mu^2/s)$ and $\mathfrak{L} = \log(\mu^2/Q^2)$.

$$\boxed{f_k^E = t_k^M + \Delta_k},\ k = \overline{1,6}.$$

Here $\Delta_k = \Delta_k\left(\pi^2, t^M_{\{1,2,3,4\}}, \beta_{\{0,1,2,3,4\}}, \gamma_{\{0,1,2,3,4\}}\right)$ — contributions of the analytical continuation effects. Δ_6 contains five-loop coefficients β_4 *('17, Baikov, Chetyrkin et al.; Herzog et al.)* and γ_4 *('14, Baikov, Chetyrkin et al.; '17, Luthe et al.)*.

The numerical values of Δ_k are not negligible – they are comparable with values of coefficients t_k^M.

The effective charges approach

$a_s^{eff}(Q^2)$ – the effective charge *('84, Grunberg; '95, Kataev, Starshenko)*:

$$\frac{F(Q^2)-F(0)}{\overline{m}_q(Q^2)} = 1 + f_1^E a_s^{eff}(Q^2)\,,$$

$$a_s^{eff}(Q^2) = a_s(Q^2) + \sum_{k=2}^{\infty} \phi_k a_s^k(Q^2) \;\Rightarrow\; \beta^{eff}(a_s^{eff}) = -\sum_{i\geq 0} \beta_i^{eff}(a_s^{eff})^{i+2}$$

$$\beta_4^{eff} = \beta_4 - 3\phi_2\beta_3 + (4\phi_2^2-\phi_3)\beta_2 + (\phi_4 - 2\phi_2\phi_3)\beta_1 + (3\phi_5 - 12\phi_2\phi_4 - 5\phi_3^2 + 28\phi_2^2\phi_3 - 14\phi_2^4)\beta_0\,,$$

$$\phi_k = f_k^E / f_1^E .$$

Approximate equalities $\boxed{\beta_k^{eff} \approx \beta_k}$ allow to estimate values of the coefficients f_{k+1}^E.

$$\boxed{t_{k+1}^M = f_{k+1}^E - \Delta_{k+1}}$$ — ECH-estimates.

The method of effective charges could be applied directly to the quantity $T(s)/\overline{m}_q(s)$ in the Minkowskian region — ECH direct-estimates.

Numerical values

n_l	$t_5^{M,\, r-n}$	$t_5^{M,\, NNA,\, \overline{m}}$	$t_5^{M,\, NNA,\, M\to\overline{m}}$	$t_5^{M,\, ECH}$	$t_5^{M,\, ECH\ direct}$
3	33859	29864	20432	28435	26871
4	22602	21951	14924	17255	17499
5	13942	15725	10757	9122	10427
n_l	$t_6^{M,\, r-n}$	$t_6^{M,\, NNA,\, \overline{m}}$	$t_6^{M,\, NNA,\, M\to\overline{m}}$	$t_6^{M,\, ECH}$	$t_6^{M,\, ECH\ direct}$
3	825382	679654	522713	476522	437146
4	507235	462561	353810	238025	255692
5	285136	304866	233282	90739	133960

The presented estimates are obtained using the renormalon asymptotic formula ($r-n$), NNA procedure for two various initial normalizations of the n_l^4 and n_l^5 terms ($NNA,\ \overline{m}$ and $NNA,\ M \to \overline{m}$) and two variants of the effective charges method (ECH and $ECH\ \ direct$).

The obtained estimates are consistent with each other at the factor 1.5-2. For $t_5^M(n_l = 4)$ they are in agreement (in order of magnitude) with ones found by *('18, Mateu, Ortega)* using the scale-dependent mass defined in the MSR-scheme *('08, Hoang et al.)*.

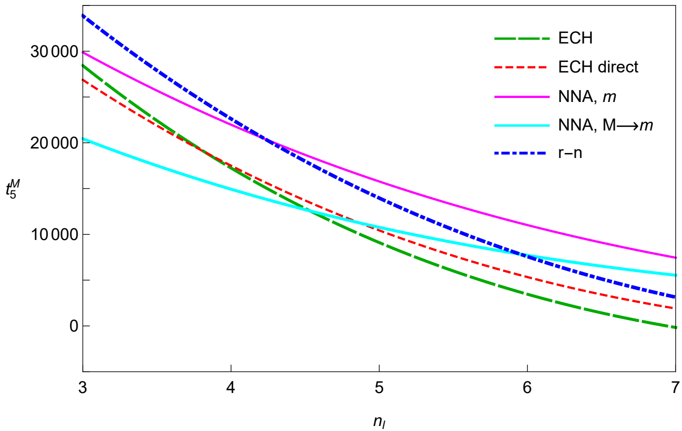
Flavor dependence

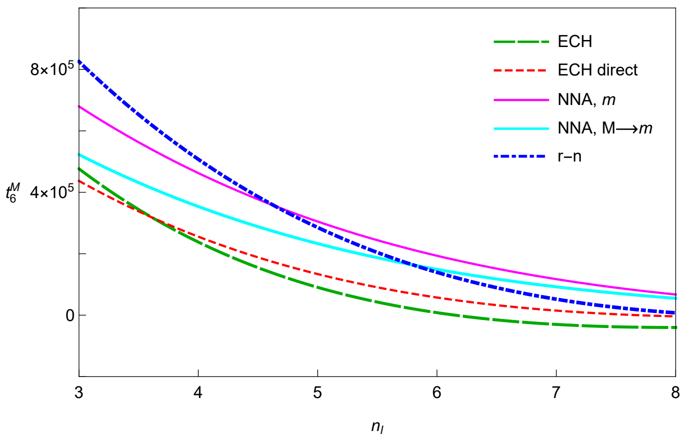
Using estimates presented above, one can find their flavor dependence:

$$
\begin{aligned}
t_5^{M,\,r-n} &\approx 1.6n_l^4 - 85n_l^3 + 2164n_l^2 - 23534n_l + 87157\,,\\
t_5^{M,\,ECH} &\approx 2.5n_l^4 - 136n_l^3 + 2912n_l^2 - 26976n_l + 86620,\\
t_5^{M,\,ECH\ direct} &\approx 1.2n_l^4 - 77n_l^3 + 1959n_l^2 - 20445n_l + 72557\,,\\
t_5^{M,\,NNA,\,\overline{m}} &\approx 0.9n_l^4 - 59n_l^3 + 1469n_l^2 - 16156n_l + 66641,\\
t_5^{M,\,NNA,\,M\to\overline{m}} &\approx 0.9n_l^4 - 56n_l^3 + 1256n_l^2 - 12383n_l + 47721\,.
\end{aligned}
$$

$$
\begin{aligned}
t_6^{M,\,r-n} &\approx -9.9n_l^5 + 372n_l^4 - 8052n_l^3 + 115164n_l^2 - 883651n_l + 2629567\,,\\
t_6^{M,\,ECH} &\approx -4.9n_l^5 + 352n_l^4 - 9708n_l^3 + 131176n_l^2 - 855342n_l + 2096737\,,\\
t_6^{M,\,ECH\ direct} &\approx -2.2n_l^5 + 148n_l^4 - 4561n_l^3 + 71653n_l^2 - 538498n_l + 1519440\,,\\
t_6^{M,\,NNA,\,\overline{m}} &\approx -1.5n_l^5 + 125n_l^4 - 4127n_l^3 + 68088n_l^2 - 561727n_l + 1853698\,,\\
t_6^{M,\,NNA,\,M\to\overline{m}} &\approx -1.5n_l^5 + 120n_l^4 - 3779n_l^3 + 58846n_l^2 - 460910n_l + 1468466\,.
\end{aligned}
$$

As in the case of known coefficients t_2^M, t_3^M, t_4^M, the approximate terms for t_5^M and t_6^M also satisfy the sign-alternating character of n_l-dependence, supported by the results of the large-β_0 expansion.





One can see that the obtained five- and six-loop estimates are consistent with each other for different values of n_l (physical $3 \leq n_l \leq 5$ and nonphysical $6 \leq n_l \leq 8$) at the factor 1.5 – 2.

Asymptotic behavior in QCD

For c, b and t-quarks $\overline{\text{MS}}$-on-shell relation contains significantly increasing and strictly sign-constant coefficients ($\overline{a}_s = \alpha_s(\overline{m}_q^2)/\pi$):

$$M_c^{ECH} \approx \overline{m}_c(\overline{m}_c^2)(1 + 1.3333\,\overline{a}_s + 10.318\,\overline{a}_s^2 + 116.49\,\overline{a}_s^3 + 1702.70\,\overline{a}_s^4 + \underline{28435\,\overline{a}_s^5 + 476522\,\overline{a}_s^6})\,,$$

$$M_b^{ECH} \approx \overline{m}_b(\overline{m}_b^2)(1 + 1.3333\,\overline{a}_s + 9.277\,\overline{a}_s^2 + 94.41\,\overline{a}_s^3 + 1235.66\,\overline{a}_s^4 + \underline{17255\,\overline{a}_s^5 + 238025\,\overline{a}_s^6})\,,$$

$$M_t^{ECH} \approx \overline{m}_t(\overline{m}_t^2)(1 + 1.3333\,\overline{a}_s + 8.236\,\overline{a}_s^2 + 73.63\,\overline{a}_s^3 + 839.14\,\overline{a}_s^4 + \underline{9122\,\overline{a}_s^5 + 90739\,\overline{a}_s^6})\,.$$

$$\frac{M_c}{1\text{ GeV}} \approx 1.270 + 0.212 + 0.205 + 0.289 + 0.529 + \Big\{\underbrace{1.316 + 4.011}_{\text{r-n}};\ \underbrace{1.105 + 2.316}_{\text{ECH}};\ \underbrace{1.044 + 2.124}_{\text{ECH direct}};\ \underbrace{1.160 + 3.303}_{\text{NNA},\ \overline{m}};\ \underbrace{0.794 + 2.540}_{\text{NNA},\ M\to\overline{m}}\Big\}\,,$$

$$\frac{M_b}{1\text{ GeV}} \approx 4.180 + 0.398 + 0.198 + 0.144 + 0.135 + \Big\{\underbrace{0.176 + 0.283}_{\text{r-n}};\ \underbrace{0.135 + 0.135}_{\text{ECH}};\ \underbrace{0.137 + 0.143}_{\text{ECH direct}};\ \underbrace{0.171 + 0.258}_{\text{NNA},\ \overline{m}};\ \underbrace{0.117 + 0.197}_{\text{NNA},\ M\to\overline{m}}\Big\}\,,$$

$$\frac{M_t}{1\text{ GeV}} \approx 164.300 + 7.552 + 1.608 + 0.496 + 0.195 + \Big\{\underbrace{0.112 + 0.079}_{\text{r-n}};\ \underbrace{0.073 + 0.025}_{\text{ECH}};\ \underbrace{0.083 + 0.037}_{\text{ECH direct}};\ \underbrace{0.126 + 0.084}_{\text{NNA},\ \overline{m}};\ \underbrace{0.086 + 0.064}_{\text{NNA},\ M\to\overline{m}}\Big\}\,.$$

Structure of the $M_l/\overline{m}_l(\overline{m}_l^2)$-relation in PT QED

Using results of the recent computations in QED ('20, Laporta), confirming previous calculations ('16, Marquard et al.), one can obtain the analogous expressions for e, μ and τ-leptons ($\overline{a} = \alpha(\overline{m}_l^2)/\pi$):

$$M_e \approx \overline{m}_e(\overline{m}_e^2)(1 + \overline{a} + 0.1659\,\overline{a}^2 - 2.1314\,\overline{a}^3 + 7.473\,\overline{a}^4)\,,$$

$$M_\mu \approx \overline{m}_\mu(\overline{m}_\mu^2)(1 + \overline{a} - 1.3961\,\overline{a}^2 - 0.6460\,\overline{a}^3 + (3.153 \pm 0.178)\,\overline{a}^4)\,,$$

$$M_\tau \approx \overline{m}_\tau(\overline{m}_\tau^2)(1 + \overline{a} - 2.9582\,\overline{a}^2 + 4.7556\,\overline{a}^3 + (-21.253 \pm 0.356)\,\overline{a}^4)\,.$$

There is no sign-constant or sign-alternating structure of the considered series in QED.

Pay attention!

For t-quark the EW corrections are extremely important.

In the difference $M_t - \overline{m}_t(\overline{m}_t^2)$ the numerical values of the LO PT QCD and EW corrections are close to each other and **have different signs** (see '95 Hempfling and Kniehl; '03 Jegerlehner, Kalmykov; '13 Jegerlehner et al.; '16 Martin).

The EW effects should be taken into account when the transition $\overline{m}_t \leftrightarrow M_t$ is considered both in theoretical and experimentally related studies!!

Conclusion

- Starting from the $\mathcal{O}(\alpha_s^3)$ level for the c-quark it is necessary to use not the pole but its running mass when processing experimental data.

- Starting from the $\mathcal{O}(\alpha_s^4)$ level for the b-quark it is more justified to use the concept of its running mass.

- In the case of the t-quark the asymptotic behavior in QCD is observed after the 7-th order (NNA argument). The concept of the pole mass of top-quark in PT QCD is well-defined.

- Reminding note: upon extraction of M_t the careful treatment of the EW effects is important as well.