Determination of the strong coupling constant from moments of quarkonium correlators revisited

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Determination of the strong coupling constant and heavy quark masses from the 2+1 flavor lattice QCD calculations of the moments of pseudo-scalar quarkonium correlators

PP, J. Weber, PRD 100 (2019) 034519
PP, J. Weber, arXiv:2012.06193

Extension of the previous work
Maezawa, PP, PRD PRD 94 (2016) 034504

QWG workshop, March 18, 2021

Moments of quarkonium correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G_n = \sum_t t^n G(t), \;\; G(t) = a^6 \sum_{\mathbf{x}} (am_{h0})^2 \langle j_5(\mathbf{x},t) j_5(0,0)\rangle \qquad j_5 = \bar{\psi}\gamma_5\psi$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_h)}{am_h^{n-4}(\mu_m)}$$

To cancel lattice effects consider the reduced moments

Note: $G_n \sim M_{2m+2}, \;\; m = 1, 2, \ldots$

$$R_n = \left(\frac{G_n}{G_n^0}\right)^{1/(n-4)}$$

Allison et al, PRD78 (2008) 054513

and similarly on the weak coupling side:

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_{h0}/m_h(\mu_m)) & (n \geq 6) \end{cases},$$

$$r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu/m_h) \left(\frac{\alpha_s(\mu)}{\pi}\right)^j$$

+ contribution from condensate

$$\sim \frac{1}{m_h^4} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$R_4, \;\; R_6/R_8, R_8/R_{10} \Rightarrow \alpha_s(\mu)$

$R_6, \;\; R_8 \;\; R_{10} \Rightarrow m_h(\mu_m)$

Some lattice details

Highly improved Staggered Quark (HISQ) action and tree-level improved gauge action

HotQCD gauge configurations : 2+1 flavor QCD
physical m_s , $m_l=m_s/20$: $m_K=504$ MeV, $m_\pi=161$ MeV
Bazavov et al, PRD 90 (2014) 094503

statistics for the $T=0$ runs:

24^3x32:	4-8K TU
32^4, 32^3x64:	7-40K TU
48^4:	8-16K TU
48^3x64:	8-9K TU
64^4 :	9K TU

in molecular dynamic time units (TU)

Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dE_0(r)}{dr}\right)_{r=r_1} = 1$$

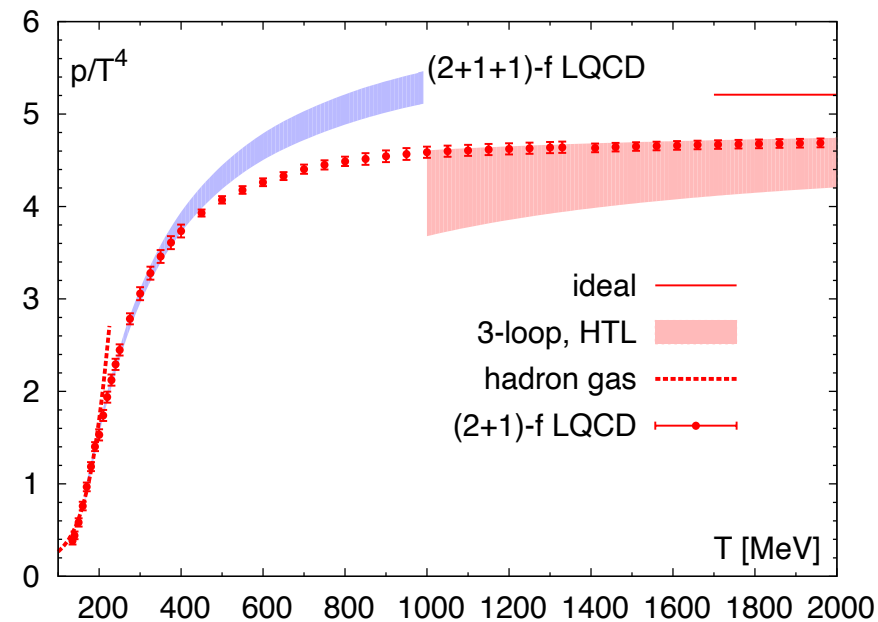
$r_1=0.3106(14)(8)(4)$ fm (pion decay constant)

Temperature is varied by the lattice spacing a

$T = (1/N_\tau a)$

Many lattice spacings available, $a_{min}=0.041$fm

Additional gauge configurations for $m_l=m_s/5$ on 64^4 lattices with $a=0.035$ fm, 0.029 fm and 0.025 fm to obtain the static quark potential at shorter distances
Bazavov, PP, Weber, PRD 97 (2018)



am_c^0 from spin averaged 1S charmonium mass or from $\eta_c(1S)$ mass

Lattice results on the moments of quarkonium correlators

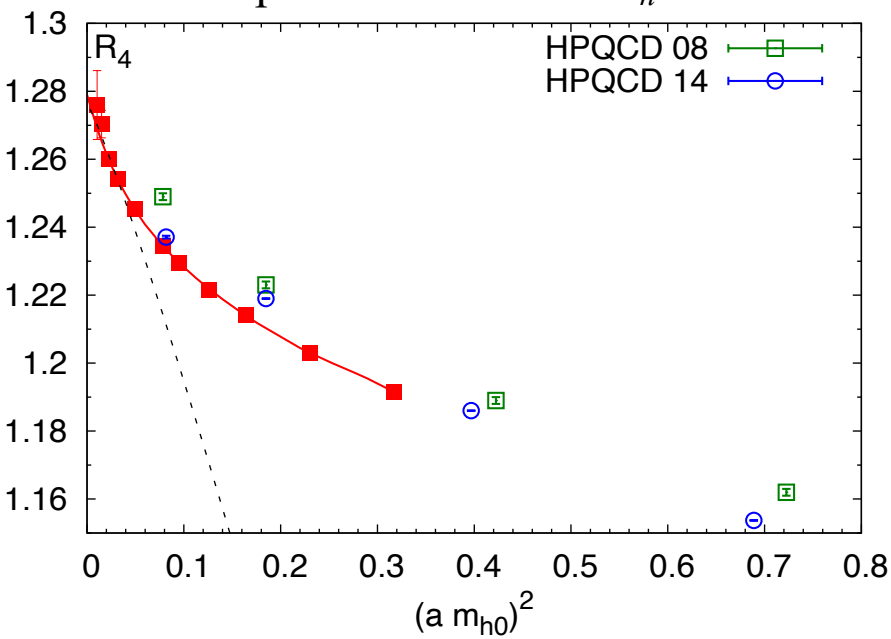
$$m_h = m_c,\ 1.5m_c,\ 2m_c,\ 3m_c,\ 4m_c,\ m_b$$

Random color wall sources => statistical errors are negligible; Dominant errors are the finite volume errors and errors due to mistuning of the heavy quark mass.

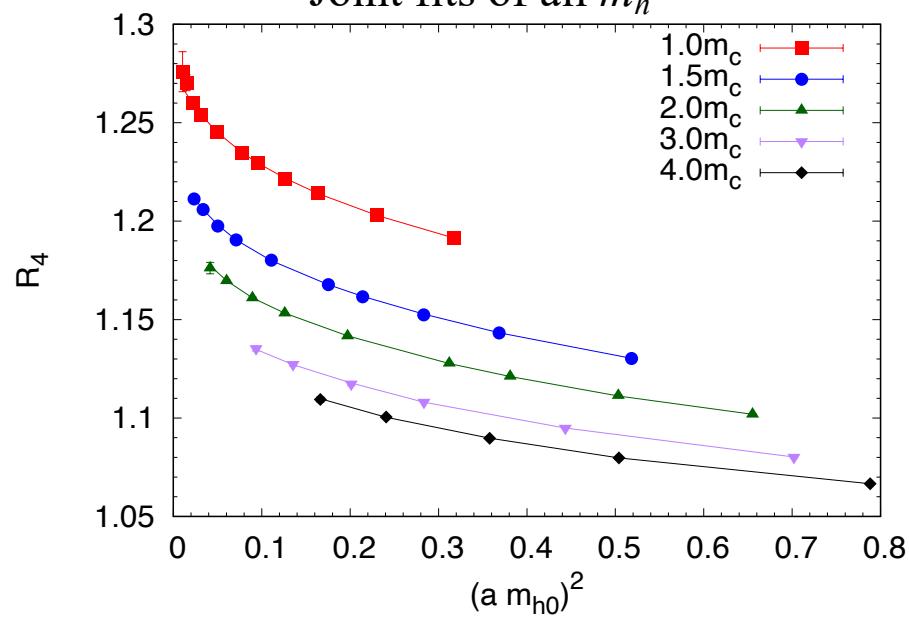
Volume errors are estimated from the free theory calculations (upper bound), are largest at small a

Sea quark mass effects are smaller than statistical errors $\Rightarrow$ combine $m_s/20$ and $m_s/5$ data

Separate fits at each m_h



Joint fits of all m_h



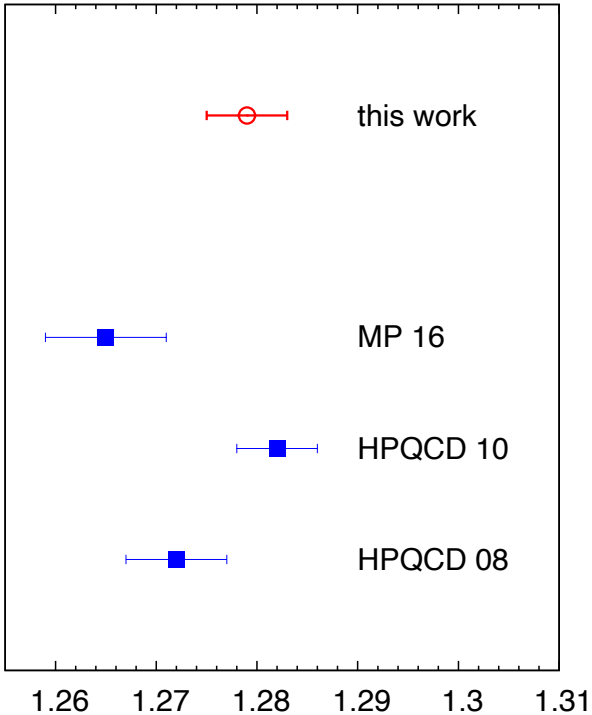
Continuum results are needed but there is a significant dependence on the lattice spacing

Lattice cutoff effects: $\sim \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_s^i (am_{h0})^{2j}$

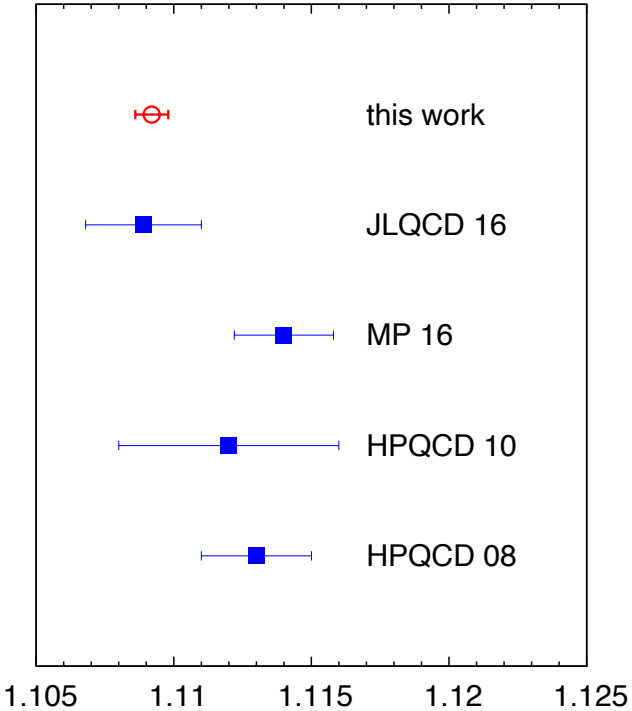
$\Rightarrow$ fits with $I = 2$ and various $J = 5$ $\alpha_s = \frac{g_0^2}{4\pi u_0^4}$

Continuum results on the moments @ m_c

$R_4,\ m_h = m_c$



$R_6/R_8,\ m_h = m_c$



$R_8/R_{10},\ m_h = m_c$

Finite volume effects are too large !

HPQCD: 2+1 flavor improved staggered (asqtad) sea + valence HISQ,
Allison et al, PRD 78 (2008) 054513; McNeile, PRD 82 (2010) 034512

JLQCD: 2+1 flavor Domain-Wall Fermions,
Nakayama, Fahy, Hashimoto, PRD 94 (2016) 054507

MP 16: 2+1 flavor HISQ (sea and valence sectors)
Maezawa, PP, PRD PRD 94 (2016) 034504

Discrepancies are understood to be due simple $a^2 + a^4$ extrapolations

Extracting the strong coupling constant in 3f QCD

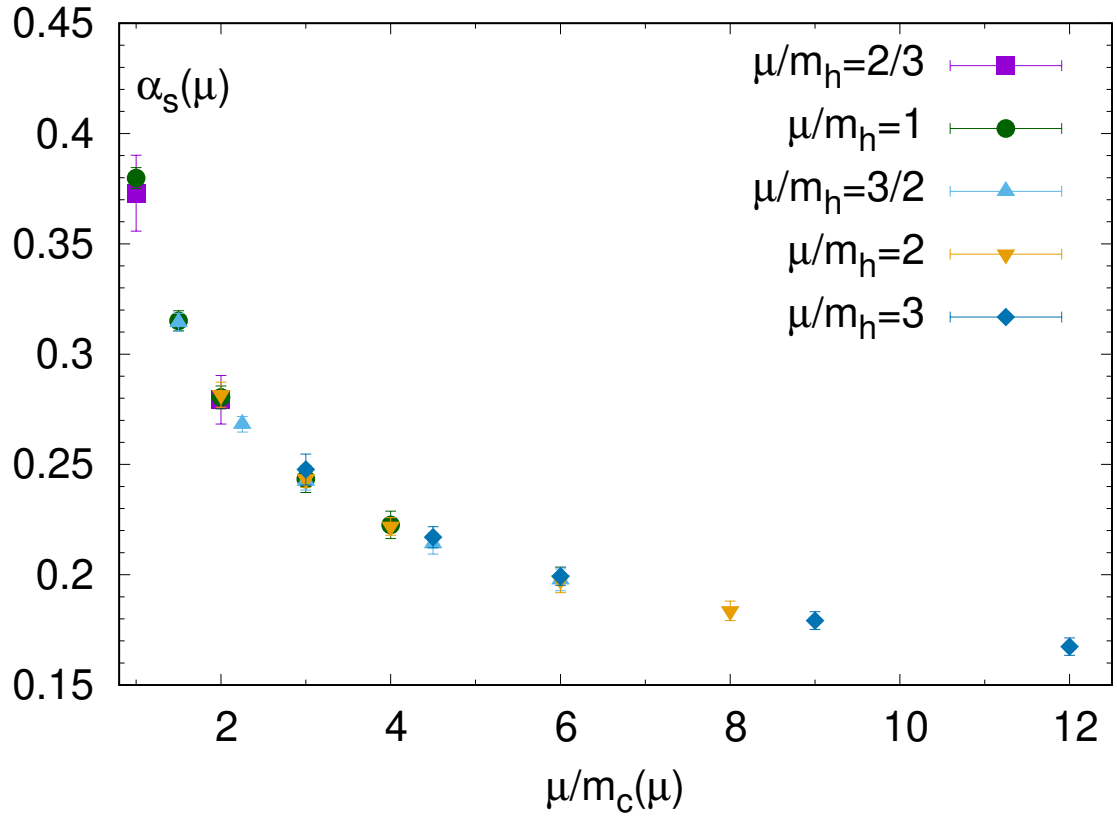
Natural choice: $\mu = \mu_m$

Use different renormalization scales: $\mu = 2/3m_h,\ m_h,\ 3/2m_h, 2m_h,\ 3m_h$

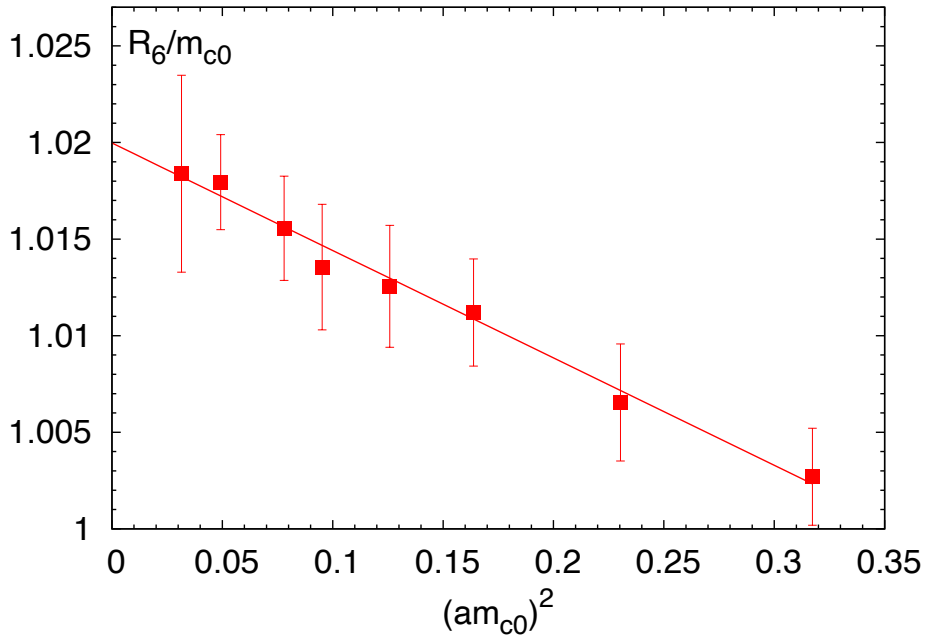
$R_4 \to \alpha_s(m_h)$

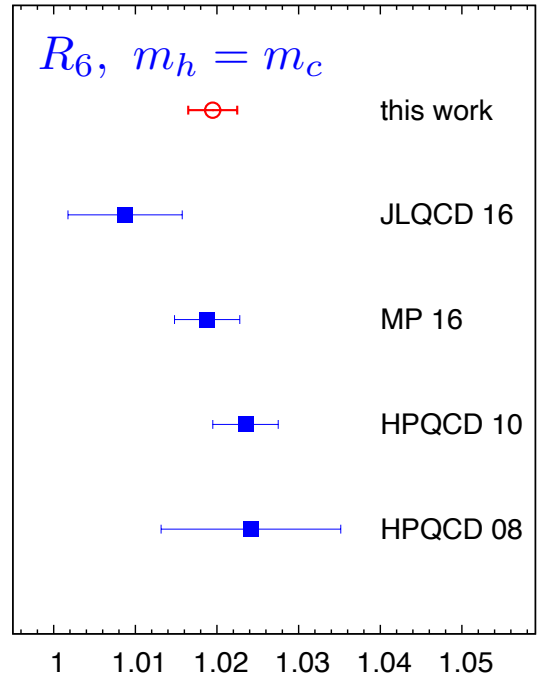
perturbative error: $\pm 5 \times r_{n3} \left(\frac{\alpha_s}{\pi}\right)^4$

condensate error: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.006 \pm 0.012)\ \mathrm{GeV}^4$



Determination of the quark masses

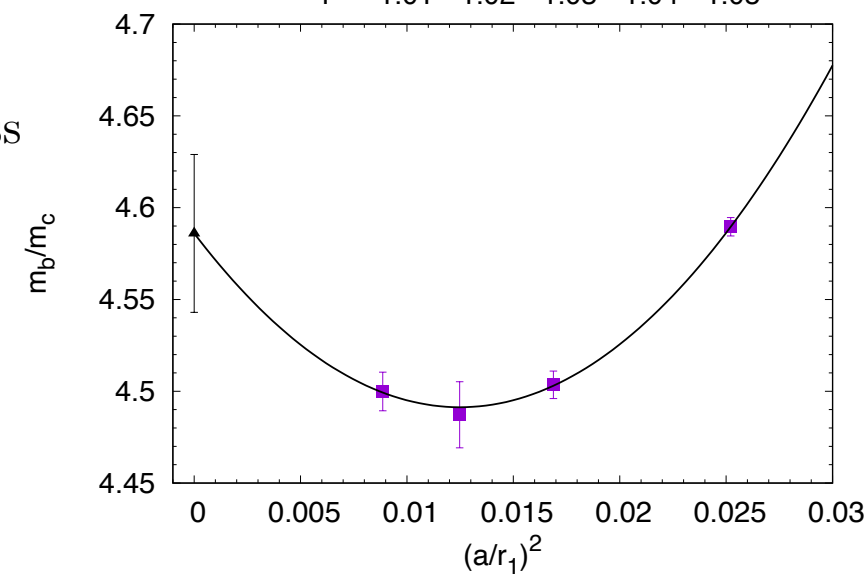




Determine am_b^0 by fixing the $\eta_b(1S)$ mass to its physical value from PDG

$$m_b/m_c = 4.586(43)$$

agrees with other lattice determinations



Determination of the quark masses

m_h	R_6	R_8	R_{10}	av.
$1.0m_c$	1.2740(25)(17)(11)(61)	1.2783(28)(23)(00)(43)	1.2700(72)(46)(13)(33)	1.2754(39)
$1.5m_c$	1.7147(83)(11)(03)(60)	1.7204(42)(14)(00)(40)	1.7192(35)(29)(04)(30)	1.7191(38)
$2.0m_c$	2.1412(134)(07)(01)(44)	2.1512(71)(10)(00)(29)	2.1531(74)(19)(02)(21)	2.1507(52)
$3.0m_c$	2.9788(175)(06)(00)(319)	2.9940(156)(08)(00)(201)	3.0016(170)(16)(00)(143)	2.9949(153)
$4.0m_c$	3.7770(284)(06)(00)(109)	3.7934(159)(08)(00)(68)	3.8025(152)(15)(00)(47)	3.7956(110)
m_b	4.1888(260)(05)(00)(111)	4.2045(280)(07)(00)(69)	4.2023(270)(14)(00)(47)	4.1985(163)

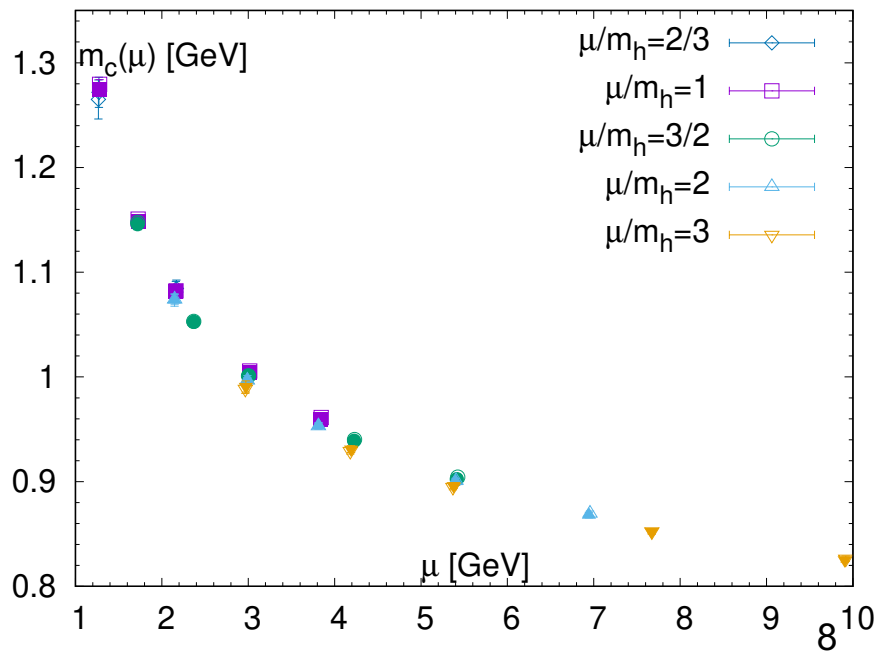
R_6, R_8 and R_{10} give consistent results for $m_h(m_h)$

$m_h(m_h)/h$, $h = m_h/m_c$ is consistent with expected $m_c(\mu)$

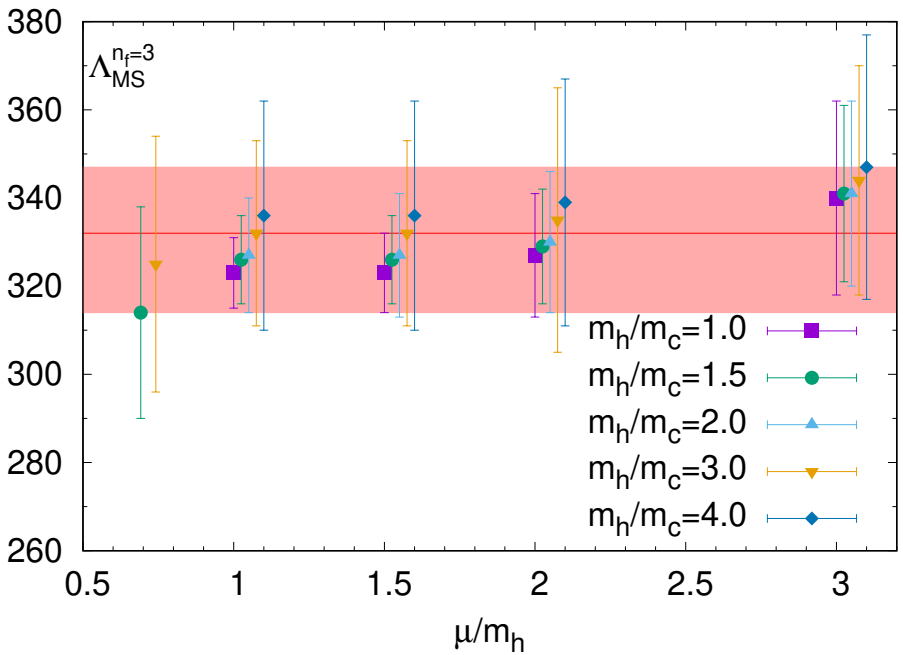
Final results after running with RunDeC

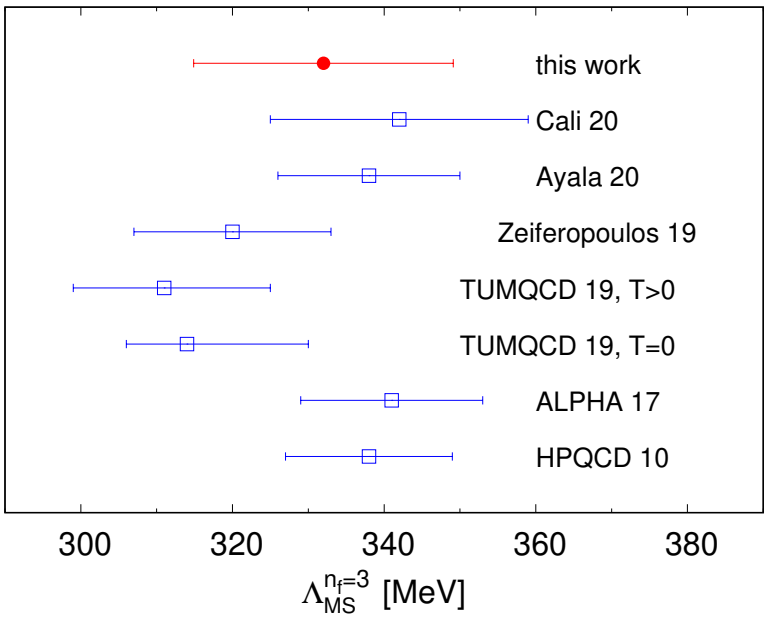
$$m_c(\mu = m_c, n_f = 4) = 1.265(10) \text{ GeV}$$

$$m_b(\mu = m_b, n_f = 5) = 4.188(37) \text{ GeV}$$



The Λ-parameter and strong coupling constant in 5f QCD





Average the results at different μ/m_h and m_h use the spread as an error estimate

$$\Lambda_{\overline{MS}}^{n_f=3} = 332 \pm 17 \pm 2\ (scale)\ \text{MeV}$$

$\Lambda_{\overline{MS}}^{n_f=3}$ is agreement with other lattice determinations

Matching to 4 and 5 flavor theory using RunDeC at $M_c = 1.5$ GeV and $M_b = 4.8$ GeV or at $m_c(\mu)$, $\mu = m_c - 2$ GeV and $m_b(m_b)$

Error from running and matching: 0.0003

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1177(12)$$

Summary

- Precise determination of the strong coupling constant from the moments of quarkonium correlators is challenging because:

1) Large lattice cutoff dependence of the lowest moment and the ratios of the moments
2) Uncertainties in the weak coupling expansion

- The most recent determination gives

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1177(12)$$

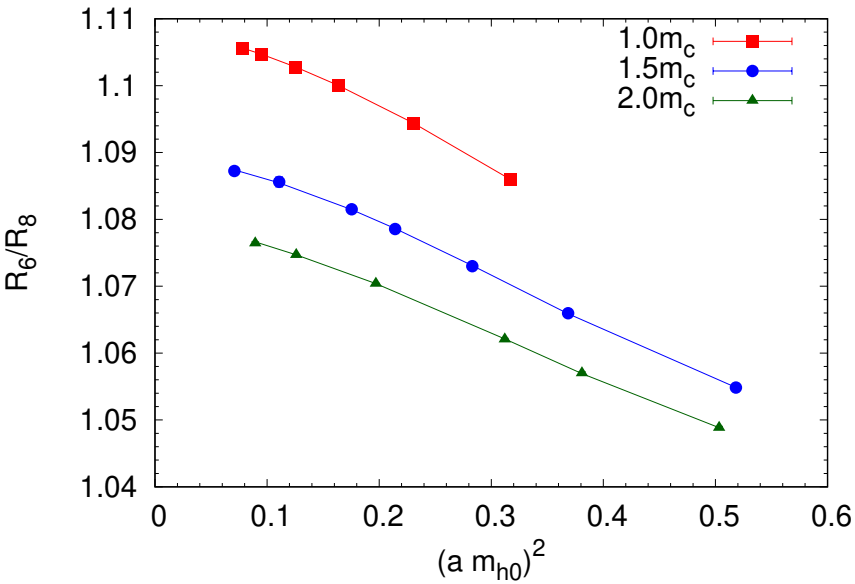
- The heavy quark masses can be well determined from the moments of quarkonium correlators

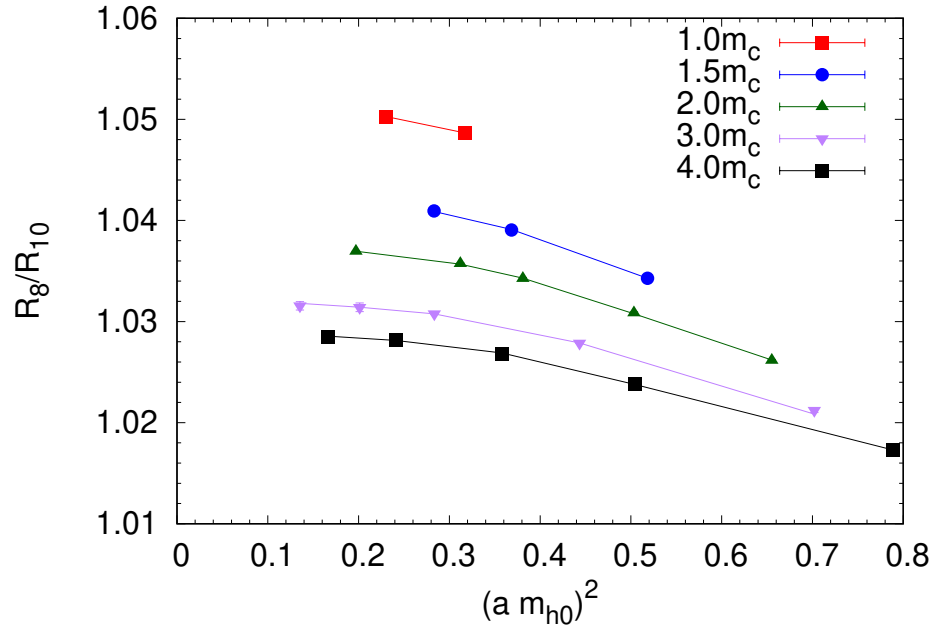
$$m_c(\mu = m_c, n_f = 4) = 1.265(10)\ \text{GeV} \qquad m_b(\mu = m_b, n_f = 5) = 4.188(37)\ \text{GeV}$$

- There are some tension in the continuum extrapolated values of the moments from different groups, but the main cause of discrepancy could be due to the perturbative error

➡ 5-loop calculations will certainly help.

Back-up





R_6/R_8			R_8/R_{10}		
m_h/m_c	continuum	$\alpha_s(m_h)$	m_h/m_c	continuum	$\alpha_s(m_h)$
1.0	1.10895(32)	0.3826(14)(178)(39)	1.0	-	-
1.5	1.09100(25)	0.3137(10)(76)(8)	1.5	1.04310(45)	0.3166(34)(82)(17)
2.0	-	-	2.0	1.03830(68)	0.2808(51)(50)(4)
3.0	-	-	3.0	1.03249(94)	0.2382(69)(24)(1)
4.0	-	-	4.0	1.02987(106)	0.2191(293)(17)(0)

Back-up

2020:

m_h	R_4	R_6/m_{c0}	R_8/m_{c0}	R_{10}/m_{c0}	$\alpha_s(m_h)$
$1.0m_c$	1.2778(20)	1.0200(16)	0.9166(17)	0.8719(21)	0.3798(28)(31)(22)
$1.5m_c$	1.2303(30)	1.0792(20)	0.9860(20)	0.9462(23)	0.3151(43)(14)(4)
$2.0m_c$	1.2051(37)	1.1182(23)	1.0317(23)	0.9944(26)	0.2804(51)(9)(1)
$3.0m_c$	1.1782(44)	1.1729(27)	1.0923(26)	1.0574(31)	0.2434(61)(5)(0)
$4.0m_c$	1.1631(45)	1.2098(31)	1.1321(30)	1.0985(31)	0.2226(62)(4)(0)

2019:

m_h	R_4	R_6/R_8	R_8/R_{10}	av.	$\Lambda_{\overline{MS}}^{n_f=3}$ MeV
$1.0m_c$	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3782(65)	314(10)
$1.5m_c$	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	310(10)
$2.0m_c$	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2648(29)	284(8)
$3.0m_c$	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

No proper description of cutoff effects
Because of the limited amount of data

$m_h = 2m_c$ result is an outlier; weighted average + spread : $\Lambda_{\overline{MS}}^{n_f=3} = 298 \pm 16$ MeV

m_h	R_4	R_6/R_8	R_8/R_{10}
$1.0m_c$	1.279(4)	1.1092(6)	1.0485(8)
$1.5m_c$	1.228(2)	1.0895(11)	1.0403(10)
$2.0m_c$	1.194(2)	1.0791(7)	1.0353(5)
$3.0m_c$	1.158(6)	1.0693(10)	1.0302(5)